Removal of the Eye-Blink Artifacts From EEGs via STF-TS Modeling and Robust Minimum Variance Beamforming

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Abstract—A novel scheme for the removal of eye-blink (EB) artifacts from electroencephalogram (EEG) signals based on a novel space–time–frequency (STF) model of EEGs and robust minimum variance beamformer (RMVB) is proposed. In this method, in order to remove the artifact, the RMVB is provided with *a priori* information, namely, an estimation of the steering vector corresponding to the point source EB artifact. The artifact-removed EEGs are subsequently reconstructed by deflation. The *a priori* knowledge, the vector corresponding to the spatial distribution of the EB factor, is identified using the STF model of EEGs, provided by the parallel factor analysis (PARAFAC) method. In order to reduce the computational complexity present in the estimation of the STF model using the three-way PARAFAC, the time domain is subdivided into a number of segments, and a four-way array is then set to estimate the STF-time/segment (TS) model of the data using the four-way PARAFAC. The correct number of the factors of the STF model is effectively estimated by using a novel core consistency diagnostic- (CORCONDIA-) based measure. Subsequently, the STF-TS model is shown to closely approximate the classic STF model, with significantly lower computational cost. The results confirm that the proposed algorithm effectively identifies and removes the EB artifact from raw EEG measurements.

Index Terms—Eye-blink (EB) artifact removal, parallel factor analysis (PARAFAC), robust minimum variance beamformer (RMVB), space–time–frequency (STF)-time/segment (TS) modeling.

I. INTRODUCTION

THE ELECTROENCEPHALOGRAM (EEG) signal is the manifestation of brain activity recorded as changes in electrical potentials at multiple locations over the scalp. The electrooculogram (EOG) generated by eye movements or blinks is found to be the most significant and common artifact in

EEG [1]. The EOG is of the order of ten times larger in amplitude than the average cortical signals, and lasts for approximately 300 ms. Due to the reasonably high magnitude of the blinking artifacts and the high resistance of the skull and scalp tissues, the EOG may contaminate the majority of the electrode signals, even those in the occipital area. In recent years, various methods for EB artifact removal from EEGs have been proposed, which are mainly based on linear regression [2] and independent component analysis (ICA) [1]. Approaches such as trial rejection, eye fixation, EOG subtraction, principal component analysis (PCA) [3], blind source separation (BSS) using ICA [4]–[6], spatial [7], and H^∞ [8] adaptive filters have also been documented as having varying success. Despite no quantitative comparison for any reference dataset being available, it has been shown that the regression- and BSS-based methods are the most reliable ones [1], [2], [4]–[6].

Spatial filtering or simply “beamforming” that falls within array processing methods, has been widely used in communications and radar signal processing applications [9]. In recent years, beamforming methods have also been widely utilized in and customized for brain signal processing, e.g., multiple signal classification (MUSIC), recursively applied and projected MUSIC (RAP-MUSIC), and first principle vectors (FINE) that have been reviewed in [10]. Genuinely speaking, source (dipole) localization has been the main application of beamforming in EEG analysis [10]–[13], where one takes the advantage of high-dimensional EEG recordings and designs the beamformers so that they pass brain electrical activities originating from a specific location while attenuating other activities emanating from other locations. Note that, *preferably* these interfering sources should not be spatially or temporally correlated with the source of interest. Theoretically, this results in the equivalence of the variance (energy) of the filter output with the electrical signal coming from that location of interest. Beamforming has also been very recently utilized in extraction and localizing of the spatially confined sources of interest, i.e., event-related potentials that come from specific locations, from the EEG recordings by estimating a (sub)optimal transformation in order to suppress other interfering sources elicited from other locations [14], [15].¹ However, to the authors’ best knowledge, beamforming-based methods have not been specifically

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¹The so-called transformation can be either a linear [9] or nonlinear [16] combination of EEGs recorded from spatially distributed electrodes.

considered in the extraction and removal of the EB artifacts from EEG recordings.² This is understandable since these schemes suffer a significant performance degradation when the array response vector for the source of interest (EB in our case) is not exactly known [17]–[20]. The problem arises when methods used in [17]–[21] deal with the electromagnetic waves of known propagation pattern arriving at mostly linear uniform (rarely nonuniform or sparse) arrays of receivers. However, in EEG analysis, although the 10/20 electrode positioning standard is usually followed, the electrodes are positioned on the subject's scalp manually, which causes major uncertainties about the precise electrode locations. Therefore, we are always confronted with an *ad hoc* configuration of electrodes that affects the steering vectors of propagating brain sources.

In order to overcome these uncertainties, in [22], a method for solving the forward problem has been introduced, by which, in [11], the localization of the brain electrical sources has been firstly very well solved. In [11], the steering vector corresponding to each grid point within the brain toward the scalp electrodes is found, and then linearly constrained minimum variance beamformers are solved for all of the grid points within the brain to localize the brain electrical sources. This approach is promising, however it suffers from complex computations incurred while solving the forward problem [22].

In this regard, our contribution is the estimation of the steering vector corresponding to the EB artifact regardless of the conventional forward solutions to EEGs. Since the sparsely occurring EB is the dominant source in the ongoing EEGs, this estimation is trustworthy and could be utilized in the beamforming procedure to remove the EB effect from EEGs. Rationally, the beamforming approach can identify and extract the EB artifact due to its independence from the neural brain activities, i.e., EEGs.

Statistically, nonstationary EEGs yield spatio-temporal information about active parts of the brain. This knowledge has been efficiently exploited for localizing the sources of background EEG and the removal of various artifacts from EEG measurements using the PCA and the ICA [3]. However, in these conventional methods, other priors, such as spectral information, are not taken into account. A topographic time–frequency decomposition method is proposed in [23] and consolidated in [24], where the STF model of multichannel EEGs is introduced. For further details, see [6] and the references therein. More recently, we have utilized the STF model for the identification and removal of EB artifacts and brain computer interfacing [1, Ch. 7], [6], [25], [26]. Although STF modeling is effective, it suffers from high-computational complexity when applied to long-term data sequences recorded from a high number of electrodes [26].

In this paper, a novel technique for removing the EOG artifacts from multichannel EEGs is presented. Our method is based on the robust minimum variance beamformer (RMVB) [21], where the spatial *a priori* knowledge of the mixing process obtained by parallel factor analysis (PARAFAC) [27]³ is exploited

as an estimation of the steering vector corresponding to the EB source.

The major advantage of the proposed method is that unlike the respective regression- and BSS-based methods presented in [2] and [4], it needs neither the reference EOG channel recordings nor any objective criterion for distinguishing between EB and spurious peaks in the ongoing EEGs. Reducing the computational complexity in the estimation of the STF model using the PARAFAC is achievable by subdividing the time domain into a number of segments, and a four-way array is then set to estimate the STF-time/segment (TS) model of the data using the four-way PARAFAC. Subsequently, the STF-TS model results in the classic STF model is achievable with significantly lower computational cost. It is also interesting to notice that, in this approach, there is no need to separate the dataset into training and testing subsets to tune the parameters. As long as, by using any primitive method, we make sure that an EB artifact has happened, the presented method can be utilized to remove the artifact from EEGs.

There are two major differences between the approach we follow in this paper and what we have proposed in [6]. First, assuming that the estimation of the steering vector corresponding to the EB artifact is precise, in [6], this vector has been used in a semiblind source extraction framework. Moreover, in this paper, we do not estimate the steering vector corresponding to the EB source by using the ordinary STF model. In contrast, by introducing the STF-TS model, we significantly reduce the computational complexity occurred while estimating the STF model. Note that during the estimation of the STF-TS model, there is tradeoff between the computational requirements and the proper unbiased estimation of the aforementioned steering vector. The bias is compensated by using the RMVB.

This paper is organized as follows. In Section II, we briefly review the RMVB and introduce the spatial signature of the STF-TS model as an estimation of the array response vector corresponding to the EB artifact. Afterward, the proposed STF-TS based STF model estimation methodology is described. The results are subsequently reported in Section III, followed by concluding remarks in Section IV.

II. ALGORITHM DEVELOPMENT

Assume N zero-mean real and mutually uncorrelated *geometrically stationary* sources $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_N(t)]'$, where $[\cdot]'$ denotes the vector transpose and t denotes the discrete time index, are mixed by an $N \times N$ full column rank matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_N]$, where $\mathbf{a}_i$ is the i th column of $\mathbf{A}$. The vector of time mixture samples $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]'$ is given as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t) \quad (1)$$

where $\mathbf{v}(t) = [v_1(t), v_2(t), \dots, v_N(t)]'$ is the additive white Gaussian zero-mean noise, which is assumed to be spatially uncorrelated with the sensor data and temporally uncorrelated. The sources are presumed to be uncorrelated,⁴ therefore, the

²Although in [14], the ocular artifact removal is a by-product.

³The interested reader is referred to [28]–[30] for further mathematical details of the PARAFAC model, the uniqueness conditions, and its robust iterative fitting.

⁴Specifically, it is reasonable to assume that the EB artifact source is at least uncorrelated with EEGs [4]–[6].

time-lagged-symmetrized autocorrelation matrix $\mathbf{R}_{\mathbf{x}\mathbf{x}}^k$ can be calculated as

$$\mathbf{R}_{\mathbf{x}\mathbf{x}}^k = E[\mathbf{x}(t)\mathbf{x}'(t - \tau_k)] = \sum_{i=1}^N r_i(\tau_k) \mathbf{a}_i \mathbf{a}_i' \quad (2)$$

for $k = 1, 2, \dots, K$, where K is the maximum number of time lags, i.e., τ_K and $E[\cdot]$ denotes the statistical expectation operator. In (2), $r_i(\tau_k) = E[s_i(t)s_i(t - \tau_k)]$ is the time-lagged autocorrelation value of $s_i(t)$. The vector $\mathbf{x}(t)$ in (1) is a linear combination of the columns of the mixing matrix, i.e., the $\mathbf{a}_i$ s, weighted by the associated source and contaminated by the noise $\mathbf{v}(t)$.

A. Robust Minimum Variance Beamformer

The most straightforward way to extract the j th source is to project $\mathbf{x}(t)$ onto the space orthogonal to, denoted by $\perp$, all of the columns of $\mathbf{A}$ except $\mathbf{a}_j$, i.e., $\{\mathbf{a}_1, \dots, \mathbf{a}_{j-1}, \mathbf{a}_{j+1}, \dots, \mathbf{a}_N\}$. Since $\mathbf{a}_j$ performs as the steering vector of the j th source, by defining a vector as a *spatial filter* $\mathbf{w}_j$, we may write [6]

$$y(t) = \mathbf{w}_j' \mathbf{x}(t) \quad (3)$$

where $y(t)$ is an estimation of the source $s_j(t)$ corresponding to $\mathbf{a}_j$. The spatial filter can be determined by applying the unit-gain constraint $\mathbf{w}_j' \mathbf{a}_j = 1$, and by minimizing the variance of the filter output, i.e., $y(t)$ [11]. However, in practice, the steering vector $\mathbf{a}_j$ is not always known [17]–[20]. Hence, the approach based on the theoretically rigorous worst-case performance optimization, recently developed in [20], is used here in order to compensate for the deviation vector δ of $\hat{\mathbf{a}}_j$ from the actual steering vector $\mathbf{a}_j$, i.e., $\delta = \mathbf{a}_j - \hat{\mathbf{a}}_j$. Note that δ is l_2 , denoted by $\|\cdot\|$, -norm-bounded by some known positive constant ϵ . Denoting $\mathbf{R} = 1/K \sum_{k=1}^K \mathbf{R}_{\mathbf{x}\mathbf{x}}^k$, as outlined in [21, Ch. 2], the beamformer is obtained by minimizing

$$J_c = \mathbf{w}_j' \mathbf{R} \mathbf{w}_j \text{ s.t. } \min_{\|\delta\| \leq \epsilon} |\mathbf{w}_j' \hat{\mathbf{a}}_j + \mathbf{w}_j' \delta| = 1 \quad (4)$$

where $|\cdot|$ denotes the absolute value operator. Equivalently [31], we may rewrite J_c as

$$J_c = \mathbf{w}_j' \mathbf{R} \mathbf{w}_j \text{ s.t. } |\mathbf{w}_j' \hat{\mathbf{a}}_j - \epsilon \|\mathbf{w}_j\|| = 1. \quad (5)$$

Following the Lagrange multiplier method, we differentiate J_c with respect to $\mathbf{w}_j$ and set it to zero. Afterward, we have $\mathbf{R} \mathbf{w}_j + \lambda \epsilon \frac{\mathbf{w}_j}{\|\mathbf{w}_j\|} = \lambda \hat{\mathbf{a}}_j$. After dropping the unimportant constant [31] λ , the spatial filter can be computed using

$$\mathbf{w}_j = \left[\mathbf{R} + \frac{\epsilon}{\rho} \mathbf{I} \right]^{-1} \hat{\mathbf{a}}_j \quad (6)$$

where $\rho \triangleq \|\mathbf{w}_j\|$ and $\mathbf{I}$ denotes the identity matrix. In (6), the main concern in estimating $\mathbf{w}_j$ is to have an estimation of ρ , which may be determined by using the following procedure.

Eigenvalue decomposition of $\mathbf{R}$, i.e., $\mathbf{R} = \mathbf{U} \mathbf{\Xi} \mathbf{U}'$ results in the $N \times N$ unitary matrix $\mathbf{U}$, whose columns are the unit norm eigenvectors of $\mathbf{R}$, and $\mathbf{\Xi}$, the diagonal matrix of the real positive eigenvalues of $\mathbf{R}$, i.e., ξ_i , where $\xi_1 \geq \xi_2 \geq \dots \geq \xi_N > 0$. By defining

$$\mathbf{\Psi}(\rho) \triangleq \mathbf{\Xi} + \frac{\epsilon}{\rho} \mathbf{I} \quad (7)$$

and following the procedure suggested in [31], we may write

$$\|\mathbf{U} \mathbf{\Psi}^{-1}(\rho) \mathbf{U}' \hat{\mathbf{a}}_j\|^2 - \rho^2 = \|\mathbf{\Psi}^{-1}(\rho) \mathbf{g}'\|^2 - \rho^2 = 0 \quad (8)$$

where $\mathbf{g} = [g_1, g_2, \dots, g_N]' = \mathbf{U}' \hat{\mathbf{a}}_j$. Introducing

$$f(\rho) \triangleq \|\mathbf{\Psi}^{-1}(\rho) \mathbf{g}'\|^2 - \rho^2 = \sum_{i=1}^N \left[\frac{|g_i|}{\epsilon + \rho \xi_i} \right]^2 - 1 = 0 \quad (9)$$

in [21, Ch. 2], it is shown that the necessary and sufficient condition for (9) to have a unique real positive solution for ρ is that the norm of the mismatch vector is upper bounded by the norm of the estimated signal steering vector, i.e., $\|\delta\| = \epsilon < \|\hat{\mathbf{a}}_j\|$. Considering $\|\mathbf{g}\| = \|\hat{\mathbf{a}}_j\|$ and (9), the upper bound of $f(\rho)$ is achieved as

$$f(\rho) < \frac{\sum_{i=1}^N |g_i|^2}{(\epsilon + \rho \xi_N)^2} - 1 = \frac{\|\hat{\mathbf{a}}_j\|^2}{(\epsilon + \rho \xi_N)^2} - 1 \triangleq f_{\max}(\rho). \quad (10)$$

Note that $f(\rho)$ and $f_{\max}(\rho)$ are both decreasing functions of ρ , and the root of $f(\rho)$, say ρ_0 , is positive. Hence, we have [21, Ch. 2]

$$0 < \rho_0 < \rho_{\max} = \frac{\|\hat{\mathbf{a}}_j\| - \epsilon}{\xi_N}. \quad (11)$$

Therefore, the problem of estimating ρ , and consequently, the spatial filter $\mathbf{w}_j$, can be solved within an iterative scheme as in **Algorithm 1** in which $\nabla_{\rho} f(\rho_{i-1})$ is the derivative of $f(\rho)$ at $\rho = \rho_{i-1}$.

B. PARAFAC and STF Modeling

In this paper, by exploiting the PARAFAC, we extract the factor relevant to the EB artifact to be used within the beamforming procedure. The resulting spatial signature of the EB-related factor is exploited to formulate (6). Importantly, we have considered that the spatial signatures of this factor are directly related to the level of EB contamination for each electrode. This assumption is rational since the EB may be considered as a strong point source that is just attenuated while propagating from the frontal area to the central and occipital parts of the brain. Hence, the column of the mixing matrix $\mathbf{A}$, i.e., $\hat{\mathbf{a}}_j$, corresponding to the EB source, is estimated by the PARAFAC and used in (6). Hereafter, we introduce the novel approach for estimating the STF model of EEGs using the proposed STF-TS model.

The key idea behind this research is in considering the EEGs as the superposition of the electropotentials of the neurons measured by placing the electrodes on the scalp. EEGs may be represented by using the linear models that are defined in three domains, i.e., space, time, and frequency, in order to simultaneously investigate their spatial, temporal, and spectral dynamics [1], [6], [24]–[26]. Here, we have assumed that each distinct local EEG activity (on the scalp) is uncorrelated with the activities of the neighboring areas. EEGs can be modeled as the sum of the distinct components where each distinct component is formulated as the product of its basis in space, time, and frequency domains.

In order to decompose the EEGs into spatial, temporal, and spectral signatures, the three-way PARAFAC is applied to

Algorithm 1 Estimating $\mathbf{w}_j$ **Require:** $\rho_{\max}, f(\rho)$.**Ensure:** $i = 0$, using the binary search method, find $\rho_i \in (0, \rho_{\max})$
if $f(\rho_i) > 0$ & $f(\frac{13}{12}\rho_i) < 0$.**Ensure:** the stopping value κ , Maximum iterations I and $i = i + 1$.**while** $|f(\rho_i)| < \kappa$ & $i < I$ **do**

$$\rho_i = \rho_{i-1} - \frac{f(\rho_{i-1})}{\nabla_{\rho} f(\rho_{i-1})}$$

end while**Ensure:** Using (6) compute $\mathbf{w}_j$ where $\rho = \rho_0 = \rho_i$.

the three-way EEG data $\tilde{\mathbf{Y}}^{N \times F \times T} \triangleq \tilde{\mathbf{Y}}(1 : N, 1 : F, 1 : T)$,⁵ where N , F , and T are respectively the number of EEG channels, frequency bins, and time instants. Therefore, as in the sequel, $\tilde{\mathbf{A}}^{N \times M}$, $\tilde{\mathbf{C}}^{F \times M}$, and $\tilde{\mathbf{D}}^{T \times M}$ are, respectively, the spatial, spectral, and temporal signatures of $\tilde{\mathbf{Y}}^{N \times F \times T}$, where their elements are denoted by $\tilde{a}(t, m)$, $\tilde{c}(n, m)$, and $\tilde{d}(f, m)$. While retaining the consistency of formulation, we occasionally drop the superscripts to simplify the presentation.

The STF model is presented as

$$\tilde{\mathbf{Y}}^{N \times F \times T} = \hat{\mathbf{Y}} + \tilde{\mathbf{E}}^{N \times F \times T} \quad (12)$$

where $\hat{\mathbf{Y}} = \sum_{m=1}^M \tilde{a}(n, m)\tilde{c}(f, m)\tilde{d}(t, m)$ is an estimation, denoted by $(\cdot)$, of $\tilde{\mathbf{Y}}^{N \times F \times T}$, M stands for the maximum possible number of factors, and $\tilde{\mathbf{E}}^{N \times F \times T}$ is the three-way array of the residue of the model, which is mostly omitted for brevity. In order to find M , we utilize the known core consistency diagnostic (CORCONDIA) measure [32]. The signatures $\tilde{\mathbf{A}}$, $\tilde{\mathbf{C}}$, and $\tilde{\mathbf{D}}$ can be estimated by using the alternating least squares (ALS) algorithm, where the cost function is

$$[\hat{\mathbf{A}}, \hat{\mathbf{C}}, \hat{\mathbf{D}}] = \arg \min_{\tilde{a}, \tilde{c}, \tilde{d}} \|\tilde{\mathbf{Y}} - \hat{\mathbf{Y}}\|^2. \quad (13)$$

Intuitively, the spatial signature $\tilde{\mathbf{A}}$ obtained from the STF model represents the weighting parameters of the interchannel correlation among the time–frequency representations of each channel. However, in order to mitigate the high-computational cost occurring in using STF with three-way PARAFAC [27], in the sequel, we introduce a novel method for estimating the STF model. The strategy is based on the *divide and conquer* philosophy where, as will be detailed later on, instead of calculating the model signatures from the original data, we estimate these signatures by joining the weighted versions of their *local* temporal signatures.

C. STF-TS Modeling

For long-term EEG measurement, the calculations of both the time–frequency transform and STF-based PARAFAC are computationally intensive. Therefore, aiming at reducing this

computational complexity, we divide the time domain into a number of segments. After that, the time–frequency transform is applied [6] individually to each segment, forming a four-way array. We set up the four-way array $\underline{\mathbf{Y}}^{N \times S \times F_s \times T_s} \triangleq \underline{\mathbf{Y}}(1 : N, 1 : S, 1 : F_s, 1 : T_s)$, where N is the channel index and S is the maximum number of segments. The energies of the time–frequency transform for T_s time instants and F_s frequency bins are then computed. PARAFAC is then applied to the four-way array. This may be formulated in the same way as in [27], where $\mathcal{A}^{N \times M}$ is the spatial signature, $\mathcal{B}^{S \times M}$ is the temporal/segment signature, $\mathcal{C}^{F_s \times M}$ is the spectral signature, and $\mathcal{D}^{T_s \times M}$ is the temporal signature with matrix elements denoted, respectively, by $a(n, m)$, $b(s, m)$, $c(f_s, m)$, and $d(t_s, m)$. Hence,

$$\underline{\mathbf{Y}}^{N \times S \times F_s \times T_s} = \hat{\mathbf{Y}} + \underline{\mathbf{E}}^{N \times S \times F_s \times T_s} \quad (14)$$

where $\hat{\mathbf{Y}} = \sum_{m=1}^M a(n, m)b(s, m)c(f_s, m)d(t_s, m)$ and $\underline{\mathbf{E}}^{N \times S \times F_s \times T_s}$ are the negligible four-way residuals of the model array. In order to find the model, we have used the following cost function

$$[\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{D}}] = \arg \min_{a, b, c, d} \|\underline{\mathbf{Y}} - \hat{\mathbf{Y}}\|^2. \quad (15)$$

By decomposing the multichannel EEGs using the STF-TS model, the number of free parameters P_4 , i.e., the number of elements that has to be estimated by PARAFAC, is $M(N + S + F_s + T_s)$, while the number of free parameters of the STF model P_3 is as high as $M(N + F + T)$. Evidently, when T is large, $P_4 \ll P_3$. This means that less parameters need to be estimated, and therefore, the computational complexity of the PARAFAC algorithm is reduced. Here, we show how to estimate the signatures of the STF model using the signatures of the STF-TS model. In this paper, the tri-linear least squares (TALS) method [33] is used to compute the parameters of the STF model-trilinear model. Similarly, a customized quadlinear version of the TALS is used to compute the parameters of the STF-TS model, i.e., (15). By using the STF-TS model, the poor convergence of TALS can be avoided by selecting the appropriate size (number) of segments S .

According to (14), the temporal signatures of the long-term EEGs are estimated by cascading all S segments of the temporal signatures $\mathcal{D}$, which are weighted by their corresponding TS signatures $\mathcal{B}$. In order to effectively estimate the STF model from the STF-TS model, the suggested number of segments S and the number of components M should maximize the CORCONDIA value as

$$[S, M] = \arg \max_S \left\{ \arg \max_M [\text{CORCONDIA}(\underline{\mathbf{Y}}, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D})] \right\}. \quad (16)$$

The main concept behind (16) is that by decomposing $\underline{\mathbf{Y}}$ to as many as M possible factors for the STF model, we firstly guarantee that the correct number of factors for STF is achieved, and then, we progress to the process of temporal segmentation. In other words, since the ultimate goal of the STF-TS model is to approximate the STF model, M should be identified for the STF model using the conventional approach of [27] before adjusting S to maximize the CORCONDIA criterion for the STF-TS model.

⁵Note that the MATLAB matrix notation has been utilized.

When the residual is considered negligible, the STF model (12) can be written in a matrix form as

$$\tilde{\mathbf{Y}}_{N \times F \times T} = \tilde{\mathbf{D}} \Sigma_{\tilde{\mathbf{A}}_n} \tilde{\mathbf{C}}' \quad (17)$$

where $\Sigma_{\tilde{\mathbf{A}}_n}$ is the diagonal matrix with the n th row of $\tilde{\mathbf{A}}$ as its diagonal elements, $n = 1, 2, \dots, N$. Similarly, the STF-TS model (14) is written in the matrix form as

$$\mathbf{Y}_{N \times S \times F_s \times T_s} = \mathcal{D} \Sigma_{\mathcal{B}_s} \Sigma_{\mathcal{A}_n} \mathcal{C}' \quad (18)$$

where $\Sigma_{\mathcal{A}_n}$ is a diagonal matrix with the n th row of $\mathcal{A}$ as its diagonal elements, $n = 1, 2, \dots, N$. Similarly, $\Sigma_{\mathcal{B}_s}$ is a diagonal matrix with the s th row of $\mathcal{B}$ as its diagonal elements for $s = 1, 2, \dots, S$. According to (17) and (18), $\tilde{\mathbf{D}}$ for the STF model can be estimated by the scaled version of $\mathcal{D}$ from the STF-TS model as

$$\tilde{\mathbf{D}} \approx [\mathcal{D} \Sigma_{\mathcal{B}_1}, \dots, \mathcal{D} \Sigma_{\mathcal{B}_S}]'. \quad (19)$$

In addition, in order to simultaneously achieve acceptable estimates of the temporal and spectral signatures, the following condition should be addressed:

$$\frac{1}{f_0} \leq \frac{L}{S} \leq T_{\text{int}} \quad (20)$$

where L is the length of the EEG in seconds, S is the number of segments, and T_{int} is the time interval that allows the temporal signatures to have smooth envelopes. We have defined the fundamental frequency f_0 , as the frequency of the first peak in the frequency spectrum of filtered EEGs. Bearing in mind that as long as $1/f_0 \leq L/S$, the spectral signatures are reconstructed faithfully. We have empirically found that, for various EEG recordings in order to achieve smooth reconstructions for the temporal signatures, T_{int} should take values between 0.7–0.9 s. After simple mathematical manipulations, (20) can be easily written as

$$\frac{L}{T_{\text{int}}} \leq S \leq f_0 L \quad \text{and} \quad S, \frac{f_s L}{S} \in \mathbb{Z}^+ \quad (21)$$

where f_s is the sampling rate and $\mathbb{Z}^+$ represents the set of positive integers. In this paper, as explained in Section III, f_0 is set to 2 Hz since the EEG measurements have been bandpass filtered between 2 to 30 Hz. If the aforementioned conditions are taken into account, the spectral signature $\tilde{\mathbf{C}}$ is also well approximated by $\mathcal{C}$, while the spatial signature $\tilde{\mathbf{A}}$ is approximately equal to $\mathcal{A}$. We indicate that the acceptable values for S mainly depend on the different terms in (21), namely, the sampling rate, the length of the data under study, and also the selection of T_{int} that are totally subjective.

From the original available data set, we selected an eye-blink contaminated segment of the EEG of 9.2 s length, i.e., 1820 sample points (Fig. 1). The STF model of EEG recordings of Fig. 1 has been shown in Fig. 2, where according to the second row of Table I, two factors can be extracted if $S = 1$, i.e., $M = 2$. Evidently, the first components (Factor 1) of the STF model demonstrate the EB-relevant factor due to the following reasons.

- 1) It mainly occurs in the frequency band of around 5 Hz, while the other factor exists in the entire band and represents the ongoing activity of the brain or perhaps a broad-band white noise-like component, Fig. 2(a).

Fig. 1. Set of real EB contaminated EEG recordings.

Fig. 2. Extracted factors by using STF decomposition of the EEG recording in Fig. 1; (a) and (b) illustrate, respectively, the spectral and temporal signatures of the extracted factors; (c) and (d) represent the spatial distribution of the factors, respectively. Evidently, Factor 1 demonstrates the EB phenomenon since it occurs in a frequency band of around 5 Hz [Fig. 2(a)], it is indeed transient in the time domain [Fig. 2(b)] and it is confined to the frontal area. (a) Frequency (Hz). (b) Time (s). (c) Spatial signature of the Factor 1. (d) Spatial signature of the Factor 2.

- 2) The temporal signature of the first factor definitely shows a transient phenomenon such as the EB while that of Factor 2 consistently exists during the course of the EEG segment, Fig. 2(b).
- 3) Unlike Fig. 2(d), in Fig. 2(c), the spatial distribution of the extracted factor is confined to the frontal area, which clearly demonstrates the effect of the EB. The other factor shows the background activity of the brain as it spreads all over the scalp.

For STF-TS modeling, we consider $L = 0.9$ s, and $T_{\text{int}} = 0.9$ s, and $f_s = 200$ in (21). Therefore, the initial candidates for S are 13 and 14. Although in (21), the lower bound for S is $L/T_{\text{int}} = 10.22$, we have intentionally included $S = 10$ in

TABLE I
COMPUTED CORCONDIA PERCENTAGE VALUES FOR DIFFERENT S AND M
CORRESPONDING TO THE EEG SEGMENT IN FIG. 5

Model	$M = 1$	$M = 2$
STF	100.00	98.405
STF-TS ($S=10$)	100.00	54.180
STF-TS ($S=13$)	100.00	<u>33.508</u>
STF-TS ($S=14$)	100.00	-11.9950

The bold value **98.405** demonstrates that $M=2$ is the correct number of factors for the STF model. Computing the CORCONDIA value for $M=2$ and $S=10, 13$ and 14 and selecting the maximum and best fitness – according to (21); the bold underlined value **33.508**, shows the right selection of the S and M for the STF-TS model. The CORCONDIA value for $M=2$ and $S=10, 54.180$ does not satisfy (21), see Fig. 3.

our analysis in order to demonstrate the accuracy of (21). The CORCONDIA values for $M=2$ and $S=10, 13$, and 14 have been calculated and shown in Table I. Here, as in (16), the maximum CORCONDIA value for maximum M and S should be selected. Apparently, disregarding (21), the best CORCONDIA candidate in Table I is 54.180 for $M=2$ and $S=10$. As plotted in Fig. 3, an acceptable decomposition is not achieved for $S=10$, although it presents the maximum CORCONDIA. Evidently, none of the six signatures, i.e., two spectral, two temporal, and two spatial, have been estimated correctly. Note that due to the leakage from the dominant EB factor to the brain activity factor during decomposition, there is a considerable similarity in their spectral and spatial signatures. The temporal signatures are also misidentified. Therefore, not only the CORCONDIA value is important, S should fulfil the inequalities and conditions of (21). In practice, this mismodeling can be avoided by carefully testing the marginal value of S , i.e., ten in this experiment, or proper selection of the T_{int} . Therefore, we select the next candidate, i.e., $S=13$ for which the CORCONDIA value is 33.5080. The results of the EEG STF-TS modeling for $M=2$ and $S=13$ have been plotted in Fig. 4 where it is illustrated how well the STF model is approximated by the STF-TS model. Factor 1 stands for the EB factor while Factor 2 again shows the brain background activity. In the sequel, the spatial signature of Factor 1 is used in the beamforming stage.

We would like to highlight that the acceptable values for S mainly depend on various terms of (21). For instance, depending on a specific application, if one selects the length of the data to be 4.2 s and the sample rate to be 1000 Hz, with $f_0 = 2$ and $T_{\text{int}} = 0.9$ s, then (16) should be solved for $S=5, 6, 7$, and 8 . However, we again suggest that it is likely that, for the smallest value of S , i.e., 5, an acceptable decomposition would not be obtained. Therefore, in order to avoid such cases, the solution is setting $T_{\text{int}} = 0.8$ s and computing (21) for $S=6, 7$, and 8 .

In summary, our method consists of the following steps. Given an artifact contaminated EEG data, we

- 1) bandpass filter the EEGs between 2 Hz and 30 Hz;
- 2) set up the four-way array, i.e., $\underline{\mathbf{Y}}_{N \times S \times F_s \times T_s}$, as stated in Section II-C;

Fig. 3. Extracted factors by using the STF-TS decomposition of the EEG recording in Fig. 1 when $M=2$ and $S=10$. Regarding Fig. 2, none of the six signatures, i.e., two spectral, two temporal, and two spatial, have been estimated correctly. Note that due to the leakage from the dominant EB factor to the brain activity factor while decomposition, there is a considerable similarity in their spectral and spatial signatures. The temporal signatures are also misidentified. (a) Frequency (Hz). (b) Time (s). (c) Spatial signature of the Factor 1. (d) Spatial signature of the Factor 2.

Fig. 4. Extracted factors by using the STF-TS modeling by $M=2$ and $S=13$. Interestingly, as expected, the spectral and spatial signatures of the extracted components are very similar to those of Fig. 2, and the temporal signatures effectively identify the transient EBs. (a) Frequency (Hz). (b) Time (s). (c) Spatial signature of the Factor 1. (d) Spatial signature of the Factor 2.

- 3) execute the four-way PARAFAC and select the EB artifact relevant factor, as will be described in Section III;
- 4) exploit the spatial signature of the EB artifact factor as $\hat{\mathbf{a}}_j$ and execute the beamforming procedure;
- 5) reconstruct the artifact removed EEGs by deflation [34, p. 192].

III. SIMULATION RESULTS

We applied our algorithm to real EEG measurements. The database was provided by the School of Psychology, Cardiff

Fig. 5. Results of the proposed EB artifact removal method for a set of real EEGs. The left subplot depicts highly EB contaminated EEGs before artifact removal, while in the right subplot the segment of EEGs after being corrected for EB artifact is illustrated.

University, U.K. It represents a wide range of EBs, and therefore, gives a proper evaluation of our method. The scalp EEG was obtained by using 25 silver/silver-chloride electrodes placed at locations defined by the conventional 10–20 system [1]. The data was sampled at 200 Hz, and the bandpass filtered with cutoff frequencies of 2 and 30 Hz. Twenty real highly EB contaminated EEG recordings, each 9 s long, have been artifact removed by using our method. The performance of the algorithm can be observed by comparing the EEGs obtained at the electrodes in Fig. 5(a), and the same segment of data after being processed by the proposed algorithm in Fig. 5(b).

In what follows, we provide a detailed comparison between the results of STF modeling using the two mentioned approaches in Section II, i.e., direct three-way PARAFAC (Section II-B), see Fig. 6, and the STF modeling by using the STF-TS model of EEGs (Section II-C), see Fig. 7. Averaged CORCONDIA values for three independent runs with different initialization as detailed in [27] have been computed for methods of STF and STF-TS modeling. In Fig. 6, the number of components M is selected as $M = 2$ according to the computed CORCONDIA value, i.e., 98.425%, whereas the CORCONDIA for the proposed STF-TS model was 17.117% when the number of segments was $S = 18$ [(16) and Table II].

Fig. 7(a)–(d) illustrate, respectively, the estimated spectral, temporal, and spatial signatures of the under study EEGs. The results of the STF-TS model in comparison to that of the STF model, i.e., Fig. 6, demonstrate the reliability of the STF-TS modeling, since both methods result in approximately the same signatures, and as expected, the STF-TS method is a faster algorithm. Small deviations in spectral and temporal signatures of the STF model using the STF-TS are negligible, since they are merely utilized to identify the EB relevant factor. Moreover, experimentally we have found that, due to the fact that the EB factor is the dominant factor, it is always effectively identified, if the conditions in (21) are met; any probable de-

Fig. 6. Extracted factors by using the STF modeling; (a) and (b) illustrate respectively the spectral and temporal signatures of the extracted factors; (c) and (d) represent the spatial distribution of the factors, respectively. Factor 1 demonstrates the EB phenomenon. (a) Frequency (Hz). (b) Time (s). (c) Spatial signature of the Factor 1. (d) Spatial signature of the Factor 2.

TABLE II
COMPUTED CORCONDIA PERCENTAGE VALUES FOR DIFFERENT S AND M CORRESPONDING TO THE EEG SEGMENT IN FIG. 5

Model	$M = 1$	$M = 2$	$M = 3$
STF	100.00	98.979	46.248
STF-TS ($S=10$)	100.00	-13.445	-
STF-TS ($S=12$)	100.00	-5.0723	-
STF-TS ($S=15$)	100.00	-8.663	-
STF-TS ($S=18$)	100.00	<u>17.117</u>	-

The bold value **98.979** demonstrates that $M = 2$ is the correct number of factors for the STF model. Estimating the CORCONDIA for $M = 2$ and $S = 10, 12, 15$, and 18 and selecting the maximum fitness, the bold underlined value **17.117**, shows the right selection of the S and M for the STF-TS model. “-” denotes that CORCONDIA does not converge for that specific S and M .

TABLE III
NUMBER OF ESTIMATED FREE PARAMETERS FOR THE STF AND STF-TS MODELS AND THEIR RESPECTIVE COMPLEXITY

Models	STF $M = 2$	STF-TS $M = 2, S = 18$
Free parameters	4,010	646
Complexity	1	0.161

viation only perturbs the signatures of the background EEG activities.

By using the STF model, we have to calculate the parallel factors of the three-way array of size $N \times F \times T$. This process takes a longer period of time due to the calculations of more free parameters P_3 as compared the P_4 values with the STF-TS model. The first row of Table III shows that the number of free parameters is greatly reduced by using the STF-TS

Fig. 7. Extracted factors by using the STF-TS modeling; (a) and (b) illustrate respectively the spectral and temporal signatures of the extracted factors; (c) and (d) represents the spatial distributions of those extracted factors. Interestingly, as expected, the spectral and spatial signatures of the extracted components are very similar to those of Fig. 6 and the temporal signatures effectively track the transient EBs of the ongoing EEGs. (a) Frequency (Hz). (b) Time (s). (c) Spatial signature of the Factor 1. (d) Spatial signature of the Factor 2.

model, where the size of the three-way $\tilde{\mathbf{Y}}_{N \times F \times T}$ for the STF model is $25 \times 1800 \times 180$, i.e., 4010 parameters to be estimated, and the size of the four-way $\mathbf{Y}_{N \times S \times F_s \times T_s}$ for the STF-TS model is $25 \times 18 \times 180 \times 100$, i.e., 646 parameters to be estimated. Consequently, the second row of Table III illustrates the relative calculation time of the STF and STF-TS models. For the EEGs used in this experiment, the relative calculation time of the STF-TS model, presuming that the calculation time of the STF model compared to the method proposed in [6] is 1, is 0.161.

At this stage, we are only interested in the spatial signature of the EB artifact relevant factor to be used in the RMVB algorithm as an approximation to $\mathbf{a}_j$, i.e., $\hat{\mathbf{a}}_j$. Again, with the similar reasoning to what has been done for Fig. 2, the first components (Factor 1) of both STF models resulted from the two approaches demonstrate the EB-relevant factor since it mainly occurs in the frequency band of around 5 Hz, and the temporal signature shows a transient phenomenon. Moreover, unlike Fig. 6 and Fig. 7(d), in Fig. 6 and Fig. 7(c), the spatial distribution of the extracted factor, to be used as $\hat{\mathbf{a}}_j$, is confined to the frontal area, which clearly demonstrates the effect of the EB. The other factor shows the background activity of the brain as it spreads all over the scalp.

Using $\hat{\mathbf{a}}_j$ in (6), we find the beamformer $\mathbf{w}_j$ and extract the EB source. The artifact removed EEGs are then reconstructed by using the batch deflation method [34, p. 192]. The interested reader is referred to APPENDIX I for further details.

In order to provide a quantitative measure of performance for the proposed artifact removal method, the correlation coefficient (CC) between the extracted EB artifact source and the original EEGs and the artifact removed EEGs are computed, see Fig. 8. The CC of two discrete random variables x and y over a fixed

Fig. 8. Averaged CC values (and their corresponding standard deviations) between the extracted EB and the restored EEGs, before and after artifact removal of different channels in (a) and (b), respectively. The experiments have been performed for 20 different EB contaminated EEG recordings. Note that the scales are different by 10^3 .

interval is mathematically defined as

$$CC = \frac{|\sum_{i=1}^w x(i)y(i)|}{\sqrt{\sum_{j=1}^w x^2(j)} \sqrt{\sum_{j=1}^w y^2(j)}} \quad (22)$$

where w is the number of time samples.

The values reported in Fig. 8 have been computed as follows. For each of the 20 different EB artifact contaminated EEGs, we executed our proposed method. The aforementioned CC s for each run were then computed between the extracted EB and the EEGs, before and after the artifact removal. These values have been subsequently averaged and shown in Fig. 8. Furthermore, their corresponding standard deviations have also been reported. As expected, the CC values have been significantly decreased by using the proposed method. Simulations for 20 EEG measurements demonstrate that the proposed method can efficiently identify and remove the EB artifact from the raw EEG measurements.

As a second criterion for measuring the performance of the overall system, we selected a segment of EEG, called x_{seg} and the reconstructed EEG $\hat{x}_{\text{seg}}$, which does not contain any artifact, and measured the waveform similarity by

$$\eta_{dB} = 10 \log \left[\frac{1}{M} \sum_{i=1}^M \left(1 - E\{x_{\text{seg}}(i) - \hat{x}_{\text{seg}}(i)\} \right) \right]. \quad (23)$$

When the value of η_{dB} is zero, the original and reconstructed waveforms are identical. From the 20 sets of EEGs, the average waveform similarity was as low as $\eta_{dB} = 0.008$ dB (standard deviation 10^{-3} dB). These results suggest that the observations have been faithfully reconstructed.

IV. CONCLUDING REMARKS

We have presented a robust method for removing EBs from EEGs by employing the robust minimum variance beamforming

method to allow for the deviation of the estimate of the steering vector corresponding to the EB source from the actual steering vector. The vector of spatial distribution of the EB factor has been identified using the proposed four-way PARAFAC, which enjoys much less computational complexity in comparison with the conventional STF modeling using the three-way PARAFAC [6]. For the first time in this paper, we have utilized the spatial signature of the EB factor as an estimation of the steering vector that introduces the EB source to the EEGs. This assumption is rational since the EB can be considered as a strong point source that is just attenuated while propagating from the frontal area to the central and occipital parts of the brain.

Our approach can also be implemented in the conventional paradigms by adaptive learning a global steering vector from the training data set and use it for removing the artifacts from the test data. However, we would like to stress on the following three issues that prevented us from following the adaptive learning procedure.

- 1) EB artifacts can be very different in terms of the amplitude and how they contaminate other channel signals. They may just contaminate the EEGs recordings from the frontal electrodes or nearly all the recordings, even those recorded from the electrodes in the occipital area. Due to these diverse artifact strengths, we are faced with different steering vectors. This diversity makes the learning of the optimum steering vector from the training set rather difficult, and the training procedure may suffer from the poor generalization while implementing on the test data.
- 2) Although we have presented our method just for the EB artifact removal, its potential for the removal of the eye movements (vertical/lateral movements) and saccade artifacts is currently investigated. Even if it was possible to identify a steering vector for EB artifacts, for the eye movement artifacts especially the lateral ones the corresponding steering vectors show considerable intersession and intra-subject variability. Therefore, we may not find a single steering vector for removal of the lateral eye movement artifacts. In removing the saccade artifacts, the situation can be worse depending on the angular speed of the eye that may reach up to 1000 degree per second, and also the temporal pattern of saccade that lasts upto approximately 200 ms. Thus, we have developed our method for the EB artifact removal on a trial by trial basis. We would also like to cite the research by Parra *et al.* [35], where the problem of the EB and eye-movement artifact removal has been very well solved. However, their approach without using the reference ocular electrodes would not be effectively applicable.
- 3) The online implementation of our method is possible. As shown in Table III, the estimation of the STF-TS model is fairly straightforward. If the algorithm is expected to work in the recording session, i.e., in the clinical examinations and mainly for fast reviewing purposes, the STF-TS modeling can be just estimated for the first few segments, and then it introduces, for instance, the average of steering vector to the robust beamformer. The beamformer will relatively compensate the deviations of averaged steering

vector of the recent EBs from that of the new EBs and extracts the artifact. This approach can also be regarded as a learning paradigm, where the learning process is simply an averaging operation. However, in the offline analysis, we follow all the steps and identify a steering vector for each set of the EEG recordings to be restored from artifacts.

The results show that the proposed method extracts and removes the effect of eye-blinking artifacts from EEGs. The EEGs are processed using the RMVB algorithm, and the artifact is autonomously extracted; then, the EEGs are reconstructed in a deflation stage. Based on our experiments, the proposed framework consistently removes the EB artifacts from the EEG signals.

APPENDIX I

DEFLATION METHOD

In order to achieve EB-free EEG recordings, $\mathbf{x}_{\text{flt}}(t)$, after the extraction of the EB source $y(t)$ using (3), we apply the deflation procedure that eliminates the previously extracted signal $y(t)$, from the recording mixtures, i.e., $\mathbf{x}(t)$

$$\mathbf{x}_{\text{flt}}(t) = \mathbf{x}(t) - \tilde{\mathbf{w}}_j y(t) \quad (24)$$

where, as in [34, Sec. 5.2.5], $\tilde{\mathbf{w}}_j$ can be estimated either adaptively or simply after the minimization of the mean square cost function J_j with respect to $\tilde{\mathbf{w}}_j$

$$\begin{aligned} J_j(\tilde{\mathbf{w}}_j) &= E\{\mathbf{x}_{\text{flt}}(t)' \mathbf{x}_{\text{flt}}(t)\} \\ &= E\{\mathbf{x}_j(t)' \mathbf{x}_j(t)\} - 2\tilde{\mathbf{w}}_j' E\{\mathbf{x}_j(t)y(t)\} \\ &\quad + \tilde{\mathbf{w}}_j' \tilde{\mathbf{w}}_j E\{y(t)^2\}. \end{aligned} \quad (25)$$

This results in the following efficient batch one-step formula to estimate $\tilde{\mathbf{w}}_j$ as

$$\tilde{\mathbf{w}}_j = \frac{E\{\mathbf{x}(t)y'(t)\}}{E\{y(t)^2\}} = \frac{E\{\mathbf{x}(t)\mathbf{x}'(t)\mathbf{w}_j\}}{E\{y(t)^2\}} \quad (26)$$

where $\mathbf{w}_j$ is obtained by (6). In fact, $\tilde{\mathbf{w}}_j$ is an estimation of $\mathbf{a}_j$, the j th column of $\mathbf{A}$, neglecting arbitrary scaling and permutations of columns ambiguities.

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