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A Quaternion Weighted Fourier Linear Combiner for Modeling Physiological Tremor

Kabita Adhikari, Student Member, IEEE, Sivanagaraja Tatiniati, Member, IEEE, Wei Tech Ang, Member, IEEE, Kalyana C. Veluvolu, Senior Member, IEEE, and Kianoush Nazarpour*, Senior Member, IEEE

Abstract—Goal: The present work offers a new approach to model physiological tremor aiming at attenuating undesired vibrations of the tip of microsurgical instruments. Method: Several tremor modeling algorithms, such as the weighted Fourier linear combiner (wFLC), have proved effective. They however treat the 3-dimensional (3-D) tremor signal as three independent 1-D signals in the x, y, and z axes. In addition, the force f by which a surgeon holds the instrument has never been taken into account in modeling. Hence conventional algorithms are inherently blind to any potential multi-dimensional couplings. Results: We first show that there exists statistically-significant subject- and task-dependent coherence between data in the x, y, z, and f axes. We hypothesize that a filter that models the tremor in 4-D (x, y, z, and f) yields a more accurate model of tremor. We therefore developed a quaternion version of the wFLC algorithm and termed it QwFLC. We tested the proposed QwFLC algorithm with real physiological tremor data that was recorded from five novice subjects and four experienced microsurgeons. Although compared to wFLC, QwFLC requires 6 times larger computational resources, we showed that QwFLC can improve the modeling by up to 67% and that the improvement is proportional to the total coherence between the tremor in xyz and the force signal. Conclusions: Taking into account the interactions of 3-D tremor data and the force enhances the modeling performance significantly. Significance: With more accurate physiological tremor modeling, enhanced tremor cancellation performance in microsurgery may be achieved.

Index Terms—Physiological tremor, quaternion algebra, weighted Fourier linear combiner

I. INTRODUCTION

TREMOR is an unintentional oscillatory movement of the body parts presented mainly in the hands. The tremor is broadly classified into two categories: pathological and physiological [1], [2]. Pathological tremor is one of the most extreme forms of movement disorders. Physiological tremor however is not a movement disorder and it can be observed in every normal individual. The amplitude of the physiological tremor ranges typically from 50 – 100µm in each of the xyz axes (Figure 1) [3], [4]. The spectrum of the physiological tremor can display multiple dominant peaks spread between 6Hz and 14Hz [3], [5]–[7].

With the advancement in medical technologies, robotic assisted surgery evolved to achieve minimally invasive surgery and enhance surgical dexterity. Since the creation of first laparoscopic tool in 1985 [8], robotic arms have improved dramatically. Systems such as LARS [9], ZEUS [10] and Da Vinci [11] provide several movement degree of freedom with capability to mimic natural wrist and hand motions. The “Steady Hand Robot” system [12] was designed to be simultaneously held by both surgeon and an actively controlled robotic arm which controls the force exerted by the surgeon to provide smooth, tremor free precise manipulation. In addition, there has been a rise of smart surgical hand-held devices to perform sub-millimetre manipulations. For instance, the “Micron” system proposed by Riviere [13] offered more flexibility and 3-D micromanipulation. Recently Gonenc et. al. [14] have shown that Steady Hand Robot and Micron can provide similar improvements in terms of tremor compensation in microsurgeries.

Physiological tremor, although not so much of a problem in day-to-day life, can lead to substantial imprecisions in microsurgery in which a positioning accuracy of 10 – 20µm is typically required [1], [3], [15]. Natural upper-limb movement is often a combination of regular sub-movements between 1Hz and 4Hz [16], [17]. Therefore physiological tremor can be filtered out readily with linear low-pass filters. However, to achieve sharp cut-offs, linear filters need a large number of taps. Consequently they lead to considerable time delays [18]. Therefore, they are not used in microsurgery in which real-time filtering is required [3]. Moreover, factors such as anxiety and fatigue can modulate the magnitude and the spectrum

Fig. 1. A representative trace of the recorded position of the tip of an instrumented stylus in the x axis during a task (A); and the physiological tremor extracted by bandpass filtering between 5Hz and 20Hz in (B).

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A MATLAB implementation of the QwFLC algorithm supporting this publication is openly available under an “Open Data Commons Open Database License”. Additional metadata are available at: 10.17634/137930-1. Please provide similar improvements in terms of tremor compensation in microsurgeries.
of the physiological tremor significantly [2]. Such dynamics render fixed linear filters ineffective.

Riviere and Thakor [3] proposed adaptive filtering for tremor cancellation in microsurgery. In this scheme to attenuate the tremor at the tip of a hand-held instrument adaptively, a signal analysis unit models the tremor and actuates the tip of the instrument with an anti-phase signal with minimal delay. They used a least mean square (LMS)-based adaptive filtering algorithm for modeling. Others followed a similar process to model or predict non-stationary amplitude and frequency of tremor [5], [7], [15].

Tremor modeling using adaptive filters typically require a reference signal that is highly correlated with the tremor signal. The amplitude of typical physiological tremor is however approximately 50 times smaller than that of the voluntary motion. This mismatch can push the adaptation towards the dominant components of the voluntary movement rather than that of the tremor [19]. Hence, the tremor signal is extracted in real-time using digital filters prior to adaptive modeling [5], [7], [15]. Building upon the Fourier linear combiner (FLC) structure proposed in [20], Riviere and Thakor [3] modeled the tremor signal with a truncated Fourier series representation in which the dominant tremor frequency was tracked by maintaining a running sum of the all preceding frequency updates. The resultant sum modulated a bank of sine and cosine bases and their harmonics. These bases were scaled then by a second set of weights to track the amplitude of the tremor signal. This algorithm was called weighted Fourier linear combiner (wFLC) and it has been tested widely in different settings and extended to other applications modeling of gait [21] or analysis of atological tremor [22]. The proposed QwFLC filter will still need the reference tremor signal extracted offline from a mixture of voluntary and tremulous movement.

The wFLC algorithm and all its extensions treat the 3-D (x, y, and z) tremor signal as three independent 1-D signals. In addition, no previous studies have tested whether inclusion of the force f by which a surgeon holds an instrument improves the modeling accuracy. We therefore hypothesized that an adaptive system which incorporates any multi-dimensional couplings can improve the tremor modeling performance.

Quaternion algebra can preserve natural representation of 3-D or 4-D processes. Due to such advantages quaternion operation has been popular in many areas, e.g. image processing [23] and wind modeling [24]. A quaternion LMS (QLMS) was recently developed in [25]. QLMS incorporates both the covariance and the pseudo-covariance of the input 4-D signals to update the new filter parameters to improve signal modeling or prediction [25].

In Section II we extend the conventional FLC and WFLC algorithms to their quaternion versions, that is, QFLC and QwFLC. We then evaluate their performance with synthetic and real data in Section III. Section IV and V are our Discussions and Conclusions, respectively. Preliminary results of this work were reported in [26], [27].

II. METHODS

A. Experimental Setup

Participants: Five medical students with basic surgical skills (novice group) and four experienced microsurgeons (surgeon group) took part in this study. All subjects gave their informed written consent before participation. The study was approved by the local ethics committee at Nanyang Technological University [28].

Measurement: With the motivation to develop smart hand-held surgical instrument capable of tracking and cancelling tremor during microsurgery, the optical micro motion sensing system (M²S²) was developed in [29]. The system was designed to track the 3-D displacement of the tip of a microsurgical instrument in a laboratory suite, while the subjects carried out some realistic task similar to the micro-manipulation performed during real microsurgeries [29], [30].

M²S² has an embedded microsensing system with two orthogonally-placed position sensitive detectors to track the 3-D displacement of the tip of a microsurgical instrument in real-time. The workspace is a target platform of dimension 10 × 10 × 10mm³ where tasks are performed. An IR diode illuminates the workspace and the 3-D displacement of a small white ball, that is placed at the tip of the stylus, is calculated by the centroid position of the reflected infra-red rays onto the detectors. For visual feedback a 19” flat LCD TV monitor is used which was placed 70cm away from the subjects.

Subjects were asked to seat facing the monitor and to hold an instrumented stylus between their index and thumb fingers with the wrist relaxed on an armchair and elbow held at 90 degrees. The stylus has similar mass characteristics to those of typical surgical forceps.

A force sensor (FSG15N1A, Honeywell Sensing and Control, USA) is mounted on the stylus to measure the grip force (range: 0 – 15N). The position and force data are digitized at 250 samples per second with a data acquisition card (PD-MF-16-150, United Electronic Industries, USA) with a 16 bits resolution. For further details, the reader is referred to [28]–[30].

To generate an appropriate reference signal for our adaptive algorithms, we pre-filtered (offline) the recorded position and force data in each axis independently with a zero-lag 5th-order Butterworth digital filter with a passband of 5 – 20Hz.

Tasks: Subjects performed two tasks:

- **Pointing**: Two dots were displayed on the monitor screen. One was a fixed white dot and another an orange dot which moved according to the tool tip position held by the subjects. The subjects were asked to overlap the orange dot with the white dot and to maintain the same position for 30s [28];

- **Tracing**: Subjects were asked to trace the circumference of a white circle (diameter: 4mm) which was shown on the screen. A dot was used as cursor for the subjects to move along the circle clockwise as accurately as possible with the speed which was realistic to surgical manipulation as the subjects had surgical knowledge [28]. The tracing task was carried out on the x-y horizontal plane. The total tracing time for each subject was 30s.
Average tracing speeds were 1.59mm{s}^{-1} and 1.96mm{s}^{-1} for novice and surgeon subjects, respectively. This speed did not vary significantly among subjects.

Data were shortened to 28s to counter the transient effect of filtering. Some subjects performed the two tasks under three different visual magnification conditions: ×1, ×2 and ×10. Here, we used data from the ×1 magnification condition in which data from all subjects are available.

B. Time- and Frequency-Domain Correlation Analysis

To investigate force and tremor dependency, we first performed a simple time-domain correlation analysis. Time-domain correlation \( R \) between \( x \), \( y \), \( z \), and \( f \) were calculated, for instance \( R_{yx}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \) where \( l \) denotes the time-lag. Time-domain correlation analysis can reveal broadband and strong correlations readily. To identify the associations when signals have moderate but frequency-specific correlation, frequency-domain techniques, such as coherence analysis, may be more effective [31], [32]. Coherence offers a bounded measure and reflects linear interaction between two processes, with 0 and 1 occurring in case of no and perfect linear dependency, respectively [33]. Coherence \( (\text{Coh}) \) between all pairs of \( x, y, z \) and \( f \) were calculated, for example coherence between \( x \) and \( y \) (denoted by \( \text{Coh}_{xy} \)) is calculated as

\[
\text{Coh}_{xy}(f) = \frac{|\text{PSD}_{xy}(f)|^2}{\text{PSD}_{xx}(f)\text{PSD}_{yy}(f)} \quad (1)
\]

where \( f \) is the frequency, and \( \text{PSD}_{xx} \) and \( \text{PSD}_{yy} \) denote the power spectral density of \( x \) and the cross-spectral density between \( x \) and \( y \), respectively. In calculating the PSDs, a standard Hanning window (width, 1s) was used.

Following the approach adopted in [32]–[34], each data file was split into \( D = 28 \) non-overlapping 1s segments. The statistical significance threshold value for the estimated coherence spectra was calculated under the hypothesis of independence with \( s = 1 - 0.1^{(D-1)} \). Coherence values larger than \( s \) were considered significantly different from zero with \( p \)-values smaller than \( \alpha = 0.05 \) [33].

Coherence analysis could show statistically significant but artificial coherence when the PSD magnitude of one or both of the signals is very small. In order to avoid such a problem, we first analyzed the PSD of all signals in the 5–15Hz frequency range. We ensured that coherence values are only accepted if the signal power is within the 3dB range of the peak power in each axis. The PSD analysis was carried out for all subjects, all four axes and in both tasks. We chose first to estimate coherence between the data in the \( x \), \( y \), and \( z \) axes. We then repeated this analysis between the data in the \( x \), \( y \), and \( z \) axes and the \( f \) data.

C. Algorithm Development

Here, we briefly review the FLC and wFLC algorithms that are both based on the LMS algorithm. We then describe the proposed QFLC and QwFLC structures.

1) Fourier Linear Combiner: Any periodic or quasi-periodic signal \( s \) of a known main frequency \( \omega_0 \) can be estimated adaptively as \( \delta \) by combining sine and cosine bases [20]:

\[
n_{h,t} = \begin{cases} 
\sin(h \omega_0 t) & 1 \leq h \leq H \\
\cos((h-H) \omega_0 t) & H + 1 \leq h \leq 2H 
\end{cases} \\
\hat{s}_t = w^T_n \n_t 
\]

where \( w_t = [w_{1,t}, w_{2,t}, \ldots, w_{2H,t}]^T \) is a vector of the adaptively calculated Fourier series coefficients, \( n_t = [n_{1,t}, n_{2,t}, \ldots, n_{2H,t}]^T \), \( t \) denotes the time, \( H \) is the number of harmonics in the model representing the measured signal \( s \) and \([.]^T \) denotes the vector transpose operation.

The unknown amplitude of the signal is then tracked using the adaptation law \( w_{t+1} = w_t + \mu \varepsilon_t \n_t \) with \( \varepsilon_t \), being the error and calculated as \( \varepsilon_t = \hat{s}_t - s_t - \hat{w}_t^T \n_t \). The parameter \( \mu \) is a learning rate governing the speed of adaptation to the changing amplitude of the signal. Whilst this algorithm serves as the basis for the methods developed in the following, it is not directly applicable in tremor modeling because physiological tremor has multiple unknown peaks in the frequency range of 6–14Hz [35], [36].

2) Weighted Fourier Linear Combiner: The wFLC algorithm is a frequency weighted version of FLC [3]. It can track multiple tremor frequencies by modeling them as a running sum of an initial reference frequency and replacing the fixed \( \omega_0 \) with its time-varying version \( \omega_{0,t} \). The formula for frequency update in wFLC are [3]:

\[
m_{h,t} = \begin{cases} 
\sin(h \sum_{k=0}^{t} \omega_{0,k}) & 1 \leq h \leq H \\
\cos((h-H) \sum_{k=0}^{t} \omega_{0,k}) & H + 1 \leq h \leq 2H 
\end{cases} \\
\omega_{0,t+1} = \omega_{0,t} + \mu \varepsilon_t \sum_{h=1}^{H} \sum_{k=0}^{t} h(W_{h,k}m_{h,t} + w_{h,k}h_{t}) 
\]

where \( m_{h,t} \) is the \( h \)-th element of the vector \( m_t \). The tremor amplitude is then tracked with the adaptive weights \( w_{t+1} = w_t + \mu w \varepsilon_t \m_t \) with adaptation error \( e_t = \hat{s}_t - s_t - \hat{w}_t^T \m_t \). Learning rates \( \mu_w \) and \( \mu_w \) govern the step size of the frequency and the amplitude updates.

Conventionally, with wFLC, the tremor in the \( x \), \( y \), and \( z \) dimensions are modeled independently. To account for the couplings, we propose the use of quaternion algebra. A brief review of quaternion algebra is presented in Appendix A.

D. Quaternion Fourier Linear Combiner

We extended the FLC into its quaternion version (QFLC) by utilizing the quaternion LMS (QLMS) formalism proposed in [25]. Figure 2 depicts a block diagram of the proposed structure. In this structure \( S_t \) is the quaternion variable whose real component is the recorded force \( s_{j,t} \) and its imaginary components are the tremor recorded in each axis, \( s_{x,t}, s_{y,t}, \) and \( s_{z,t} \) at time \( t \), that is, \( S_t = s_{x,t} + j s_{y,t} + k s_{z,t} \).

With QFLC, we can model the amplitude of a 4-D process of a known fundamental frequency \( \Omega_0 \) with harmonics \( h = 1, \ldots, H \). Each harmonic forms a quaternion vector. Consequently, vectors \( w_t \) and \( n_t \) in FLC, are transformed to their quaternion versions \( W_t \) and \( N_t \), respectively. Therefore for each harmonic \( h \):

\[
W_{h,t} = w_{h,x,t}i + w_{h,y,t}j + w_{h,z,t}k. 
\]
Following the standard LMS formulation, the quaternion adaptation error is calculated with

\[ E_t = S_t - \hat{S}_t = S_t - \Omega_t^T \tilde{N}_t, \]  

where \( \hat{S}_t \) denotes an approximation of \( S_t \). Finally, the weight update equation follows the standard QLMS algorithm \[25\]:

\[ \Omega_{t+1} = \Omega_t + \mu (2E_t \tilde{N}_t^* - M_t^* E_t^*), \]  

(7)

where \( \mu \) is the learning rate which determines speed of convergence and \( \cdot^* \) denotes the quaternion conjugate transpose operation.

Note that in equation (7), \( \Omega_t \) includes the weights for all \( H \) harmonics. In addition, it is straightforward to expand QFLC to have axis-specific learning rates.

To measure the efficiency of the QFLC algorithm, we simulated a 3-D reaching task. The hand velocity was modeled with a half-sine wave with a maximum velocity \( 350 \text{mms}^{-1} \). The instantaneous hand position was then calculated by integrating the velocity profile. QFLC, much like FLC, can track only one frequency. Hence the same 10 Hz signal was added to the signals in all 3-D axes to represent the tremor. Note that, in a real-life case, the tremor spectrum in the three dimensions can be different. In addition, independently-generated Gaussian noise was added to data in \( xyz \). Simulation was performed with noise levels from -15dB to 15dB (interval 5dB) signal-to-noise (SNR) levels. At each SNR level we repeated the simulations 10 times. In the first iteration we initialised the filter weights \( \Omega_t \) randomly and independently across \( xyz \) axes. To achieve a fair comparison, the same initialization \( w \) and \( \Omega_t \) values were used in the following repetitions. Finally to compare adaptation speed of the QFLC and the FLC algorithms, we calculated the time-lag of the peak cross-correlation function between the original and the modelled 10 Hz signals in consecutive non-overlapping \( 1s \) windows.

Since there was only one frequency to track, we fixed the number of harmonics \( H \) to 1. This made the filter length \( L = 2H + 1 = 3 \). To have a fair comparison, the learning rate for both algorithms was \( \mu = 7 \times 10^{-4} \). This learning rate was chosen empirically to optimize the error and convergence rate. To provide enough time for settling, the simulation was run for 20s.

\[ \Omega_{t+1} = \Omega_t + \mu (2E_t \tilde{N}_t^* - M_t^* E_t^*), \]  

(8)

where \( \Omega_t \) is the quaternion version of \( \mathbf{m}_t \) defined earlier in the derivation of wFLC. The frequency update equation in the quaternion domain will then be

\[ \Omega_{t+1} = \Omega_t + \mu (2E_t \tilde{G}^* - G^* E_t^*), \]  

(9)
where $G$ is

$$
G = \sum_{h=1}^{H} h(W_{h,t}M_{h+H,t} - W'_{h+H,t}M_{h,t}).
$$

Finally, adaptation error is calculated as $\epsilon_{t} = S_{t} - \hat{S}_{t} = S_{t} - W'_{t}M_{t}$. Similar to QFLC, we kept the learning rates $\mu_{W}$ and $\mu_{H}$ fixed across all axes.

For QwFLC, two cases were considered. In case 1, the data recorded by the force sensor was not included in $S_{t}$ and hence the quaternion was purely imaginary comprising only the position information in the $x$, $y$, and $z$ dimensions. We denote this case with QwFLC-3D. In the second case, the force data was included in the quaternion $S_{t}$ as its real part in addition to position information in the $x$, $y$, and $z$ dimensions. The algorithm developed for this case is referred to as QwFLC-4D. In order to provide a fair comparison of performance between WFLC and the proposed methods, we chose the wFLC and QwFLC parameter values empirically to best fit both algorithms. To observe the effect of variability of the parameters we have varied the frequency learning rate $\mu_{f}$ between $7 \times 10^{-5}$ to $9 \times 10^{-5}$ (resolution $2 \times 10^{-6}$) and the amplitude learning rate $\mu_{W}$ between 0.07 to 0.08 (resolution 0.001). Increasing these learning rates further can shorten the transition time slightly however could result in instability. The standard values of $M = 1$, $w = 0$, and $W = 0$ were chosen and the initial frequency value was set to 0.1508 (6Hz). We fixed the number of harmonics $H$ to 1 and hence the filter length was $L = 2H + 1 = 3$.

F. Performance Indices

To quantify the modeling performance of all algorithms we used two performance indices:

1) Root-mean square error (RMSE): Where $X_{org,i}$ and $X_{mod,i}$ represent the $i$-th sample of the observed and modeled quaternion signals and $K$ denotes their length, the RMSE index is defined as

$$
RMSE = \sqrt{\frac{K}{\sum_{i=1}^{K} (X_{org,i} - X_{mod,i})^2}}.
$$

In the case of perfect reconstruction, RMSE will be zero. Therefore the lower the RMSE, the better the modeling.

2) Time delay: In an ideal adaptive modeling system, the time delay between the original and modeled signal is zero. As a second measure of performance, therefore, we calculated the cross-correlation between 100 randomly-selected 5s long segments of the $X_{org}$ and the $X_{mod}$ signals in each axis. This was repeated in all subjects, for both tasks and the three algorithms: wFLC, QwFLC-3D, and QwFLC-4D. For each data segment, we registered the time-lag index of the cross-correlation function peak (argument of maximum cross-correlation):

$$
\tau_{delay} = \arg \max[(X_{org} \star X_{mod})(t)]
$$

where $\star$ denotes the cross-correlation operation.

Finally, for comparison purposes, we normalised the counts of 0, 1, or 2 time-lags to the total number of cross-correlations function that we computed across both subject groups and both tasks for all algorithms.

G. Computational Complexity

To realise the suitability of the proposed algorithm in real-time applications, we have compared computational complexity of FLC, QFLC, wFLC, and QwFLC. The complexity is measured by calculating the total number of multiplications and additions required per input data while executing the algorithm. The running time complexity is also expressed in $O(.)$ notation to express rate of growth of computational time in terms of order of the filter length. A full description of the derivation of the computational complexity is provided in the Supplementary Material.

III. Results

A. Correlation Analysis Results

With a standard correlation analysis, we found that the tremor components in the $x$, $y$, and $z$ dimensions are not independent time series and there exists a subject-specific and task-specific coupling between axes. As an example, Figure 4 (A.1 and B.1) shows strong correlations in the $xyz$ axes for a surgeon carrying out the two tasks. However, in the time-domain correlation between $xyz$ and $f$ plots, it is very difficult to observe any clear correlation structure. We tested the hypothesis that moderate narrowband correlations between tremor in the $xyz$ axes and force $f$ signal cannot be detected if the 5 – 20Hz band is considered. We therefore used the coherence analysis to examine correlations in the frequency domain.

Before we proceed with the coherence analysis, with the PSD analysis, we found that the tremor power was distributed in the frequency range of 5–15Hz for most subjects. Analysing the PSDs in all subjects for both tasks, we identified the lowest peak power as $5 \times 10^{-12}$ $\mu m^2 / Hz$ for the tracing task, and took our reference power level as its half power point (-3dB: $3.5 \times 10^{-12}$ $\mu m^2 / Hz$). For the pointing task we found the lowest peak power of $4 \times 10^{-12}$ $\mu m^2 / Hz$, and took our reference power level (-3dB: $2.8 \times 10^{-12}$ $\mu m^2 / Hz$). Gray boxes in Figure 4 (A.2 and B.2) show the selected power spectral densities above the calculated reference levels for a surgeon subject in the $xyz$ tremor axes and for the force signal.

We further evaluated the calculated coherence values with a significant threshold at $s = 0.1$ as explained earlier. The top three plots in Figure 4 (A.3 and B.3) show coherence estimates for the same surgeon subject with the dashed line signifying the threshold coherence. Note that in Figure 4 (A.3 and B.3), there are peaks above the threshold where the signal power is minimal, for an example in Figure 4 A.3 Coh$_{xy}$ has coherence value above the threshold between 4 – 12Hz and there is another peak at 17Hz. However PSD$_{xy}$ and PSD$_{yz}$ are only substantial in the ranges of 4.5–15Hz and 5–14Hz which gives an overlapping frequency window of 5 – 14Hz for Coh$_{xy}$. Therefore the peak occurring at 17Hz was discarded.

We adopted the above process in all conditions to verify whether there is a statistically significant coherence between
the $xyz$ and $f$ data. We found moderate coherence between the tremor and force data which was subject- and task-dependent.

Lower plots in Figure 4 (A.3 and B.3) show coherence plots for the data from the same subject. This observation provided a strong evidence that the $xyz$ tremor data are not independent and QwFLC-3D can lead to more accurate models. Moreover, the observed coherence between the tremor in the $xyz$ axes and the grip force $f$ motivated the test of QwFLC-4D. Supplementary Figure 1 depicts the same analysis for a representative novice subject.

**B. QFLC Results - Synthetic Data**

Figure 5A shows the modeling performance of QFLC and FLC for a representative segment of the simulated tremor data. QFLC showed better modeling of the tremor signal yielding smaller error than FLC. Figure 5B shows median RMSEs obtained from 10 iterations at each SNR value. The Figure 5B provides evidence of improved performance achieved with QFLC. Figure 5C shows that the time delay between the original and QFLC-modeled 10Hz signal reduces considerably faster than that between the original and FLC-modeled 10Hz signal.

**C. QwFLC Results - Real Data**

Figure 6 depicts a representative modeling performance in the same surgeon subject as in Figure 4. A close examination
of the modeled tremor and modeling error in Figure 6A and C suggests that the QwFLC-4D outperforms both conventional wFLC and QwFLC-3D. Figure 6B and D show that QwFLC-4D can approximate the spectrum of the original tremor signal more accurately than wFLC and QwFLC-3D and the enhancement is across the whole tremor spectrum and not only in the frequencies in which the tremor in \(xyz\) and force signals were coherent, as was depicted in Figure 4.

Figure 7A, B show the RMSE yielded with wFLC, QwFLC-3D, and QwFLC-4D for each subject, averaged across \(xyz\), in both pointing and tracing tasks for a special case of \(\mu_W = 0.075\) and \(\mu_Q = 8 \times 10^{-5}\). For this special case, the average reduction in modeling error was 27% when using QwFLC-3D instead of 3 independent wFLCs. This average error reduced further by 40% when QwFLC-4D was used. Therefore, compared to wFLC, QwFLC-4D improved the modeling accuracy by 67% across all subjects, on average. Figure 7C,D show the average RMSE across all subjects and \(xyz\) for the different values of \(\mu_W\) and \(\mu_Q\). Both variates of QwFLC outperformed the wFLC algorithm.

We then quantified the improvement in terms of the RMSE index versus the total area under the statistically significant coherence. Gray boxes in Figure 4 (A.3 and B.3) show an example of the included areas. In the top and the bottom subfigures of Figure 8, the total coherence area is calculated with \(Coh_{xy} + Coh_y + Coh_{xz}\) and with \(Coh_{x} + Coh_{y} + Coh_{z}\), respectively. We considered two cases: using QwFLC-3D instead of wFLC and using QwFLC-4D instead of QwFLC-3D. Linear regression was used to fit the data set from all 9 subjects in each fit. The top subfigures of Figure 8 show strong correlations \((R^2 = 0.63, p < 0.01\) and \(R^2 = 0.78, p < 0.01\)) between the improvement in RMSE (1D to 3D) and sum...
Moreover, we compared the performance of different algorithms in terms of the time delay of the peak cross-correlation function between the original and modeled signals. Figure 9 summarizes the percentage of the segments for which the time delay was 0, 1, or 2 lags (1 time lag is 4 ms) for each algorithm. Between three algorithms, wFLC produced the slowest tracking performance with $\tau_{delay} = 2$ in 100% of the cases in both subject groups. This finding implies that the average time delay between the original and the wFLC-modeled tremor was on average 8 ms. However, the time lag between the original tremor and the modeled tremor with QwFLC-4D was 0 (that is, smaller than 4 ms) for above 75% and above 95% of cases in the pointing and tracing tasks, respectively. The delay between the original and the estimated signals depends on the level of non-stationarity of the tremor signal and the adaptation delay. For the dataset that we analyzed the QwFLC-4D showed the fastest adaptation.

### D. Computational Complexity

Real-time tremor compensation in robotics surgical instruments, requires the processes of tremor sensing and compensation to finish within a very short time, maximum 30 ms [37]. Whether this time-delay can be considered appropriate in modern robotic microsurgery is understudied. wFLC can readily be implemented in real-time to compensate for the physiological tremor [13]. Table I presents a comparative study of computational complexity of FLC and wFLC versus QFLC and QwFLC. We have multiplied the complexity of FLC and wFLC by 3 to simulate parallel application in $\text{xyz}$. Using QwFLC, instead of 3 wFLC, increases the complexity linearly by 6 times (82L vs. 27L/2). Detailed derivation of computational complexity can be found in the Supplementary Material.

### IV. Discussion

The use of robotic-assisted microsurgery is increasingly becoming wide-spread due to its robustness and high precision. Yet, removal of surgeon physiological tremor from the tip of the surgical device has remained a challenge. In this paper, we proposed a new class of algorithms based on quaternion algebra for modeling of tremor in 3-D and 4-D. Through time-domain correlation and coherence analysis we showed that the tremor data in different axes are not independent. We also demonstrated that there is significant coherence between the tremor data recorded in the $\text{xyz}$ axes and the grip force $f$ by which the surgeons hold or move the microsurgery tool. We showed that by taking into account such coupling information, the tremor modeling and the subsequent filtering performance can be improved significantly. We demonstrated that quaternion-based filtering is more accurate than the conventional one dimensional algorithms. We used an off-line zero-lag filter in our simulation. The linear filter used to extract the voluntary motion however would lead to up to 20 ms delay in real-time application. This delay is within the range suggested by [37] but could degrade accuracy. Multi-step prediction is a potential approach to circumvent this problem [19], [35], [38]. QwFLC can be readily reformulated to achieve...
tremor prediction. Preliminary results of combining multi-step prediction and QwFLC are presented in the Supplementary Material, Figures S2-4.

A. The source of oscillations in f

We showed that the coherence between the $xyz$ tremor and $f$ is statistically significant. However we did not draw any conclusions on the source of oscillations in $f$ in the tremor frequency bands. In fact as early as 1956, Halliday and Redfearn [39] showed that at excessive force levels (up to 50N) (isometric contractions) the physiological tremor amplitude increases. This phenomenon was corroborated by others [40], [41]. However, to the best of the authors’ knowledge, there is no evidence to support the oscillations in small forces (0-15N) [42]. However, from a practical point of view, including the mutual coupling information between the tremor and grip force by which the device is held or moved. In order to exploit the coupling information between $xyz$ tremor axes and force we proposed a new class of algorithms based on quaternion algebra for modeling of tremor in 3-D and 4-D. Taking into account the coupling between the data in the $xyz$ axes and the force data, the error in tremor modeling reduced by 67%. The QwFLC algorithm can be implemented in real-time because it is only 6 times more complex than 3 parallel wFLC blocks.

V. Conclusions

In current tremor modelling algorithm, data recorded in the $xyz$ axes are analyzed independently. We showed that the $xyz$ axes of tremor are not independent and the data in these axes feature significant subject- and task-dependent linear coupling. We also demonstrated that there is a considerable level of coherence between the data recorded in the $xyz$ axes and the grip force by which the device is held or moved. Therefore, to fully exploit correlation between axes, WL-QLMS could have been employed. However, to maintain consistency in performance comparison, we used QLMS in both 3D and 4D versions of QwFLC.

VI. Appendix

A. Quaternions: Basic Algebra

Quaternion dimensions are defined by three orthogonal standard basis vectors $i$, $j$ and $k$ and $i j k = i^2 = j^2 = k^2 = -1$. The unit vectors are non-commutative; for instance $i = j k = -k j$. A quaternion variable $Q(Q_a, Q_b, Q_c, Q_d)$ is an augmented form of a complex number and can be written as

$$Q = Q_a + Q_b i + Q_c j + Q_d k$$

where the scalar (real) component is denoted by $Q_a$ and the imaginary parts are $Q_b$, $Q_c$ and $Q_d$. Addition of two quaternions $Q_1$ and $Q_2$ is a quaternion with

$$Q_{a,c,d} = (Q_{1,a} + Q_{2,a})i + (Q_{1,c} + Q_{2,c})j + (Q_{1,d} + Q_{2,d})k.$$
Multiplication of two quaternions is non-commutative $(Q_1Q_2 = -Q_2Q_1)$ and is given by
\[
Q_1Q_2 = (Q_{1,a} + Q_{1,b,c,d})(Q_{2,a} + Q_{2,b,c,d}) = Q_{1,a}Q_{2,a} - Q_{1,b,c,d}Q_{2,b,c,d} + Q_{2,a}Q_{1,b,c,d} + Q_{1,b,c,d}Q_{2,b,c,d}.
\]

Finally, $Q^*$, the conjugate of $Q$, is $(Q_{a,-} - Q_{b,c,d})$.

References

Kabita Adhikari (S’14) received her B.E. in Electronics and Communication Engineering from Institute of Engineering, Tribhuvan University, Nepal in 2004. She received her MSc degree in Optoelectronics and Communication Systems in 2007 from Northumbria University, UK. From 2008 to 2010 she worked as a lecturer in South Tyneside College. She then joined Newcastle University in 2010 as a Teaching Fellow. Currently along with her academic role, she reads for a Ph.D. degree at Newcastle University, UK. Her Research interests include biomedical signal processing and adaptive filtering.

Sivanagaraja Tatinati (S’13) received the B.Tech. degree in electronics and communication engineering from Acharya Nagarjuna University, Guntur, India, in 2010, and the Ph.D. degree in school of electronics engineering at Kyungpook National University, South Korea in 2015. His current research interests include robotics assisted medical instruments, adaptive filtering, machine learning techniques based regression, application of estimation and prediction algorithms in biomedical engineering.

Wei Tech Ang (S’98-M’04) received the B.E. and M.E. degrees in mechanical and production engineering from Nanyang Technological University, Singapore, in 1997 and 1999, respectively, and the Ph.D. degree in robotics from Carnegie Mellon University, Pittsburgh, PA, USA, in 2004. Since 2004, he has been with the School of Mechanical and Aerospace Engineering, Nanyang Technological University, where he is currently an Associate Professor and holds the appointment of Head of Engineering Mechanics Division. His research focuses on robotics technology for Biomedical applications, which include surgery, rehabilitation and cell micromanipulation.

Kalyana C. Veluvolu (S’03-M’06-SM’13) received the B.Tech. degree in electrical and electronic engineering from Acharya Nagarjuna University, Guntur, India, in 2002, and the Ph.D. degree in electrical engineering from Nanyang Technological University, Singapore, in 2006. During 2006-2009, he was a Research Fellow with the Biorobotics Group, Robotics Research Center, Nanyang Technological University. Since 2009, he has been with the School of Electronics Engineering, Kyungpook National University, Daegu, Korea, where he is currently an Associate Professor. He has been a Principal Investigator or a Co-investigator on a number of research grants funded by the National Research Foundation of Korea, and other agencies. He has authored or co-authored over 100 journal articles and conference proceedings. His current research interests include nonlinear estimation and filtering, braincomputer interface, biomedical signal processing, and surgical robotics.