AN INVESTIGATION OF THE EFFECT OF WAVE PROFILE ON THE MANOEUVRING DERIVATIVES OF A FISHING VESSEL

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Ship behaviour in following seas is a complex phenomenon; with influence from many contributing variables. The effect of waves on the manoeuvring derivatives can be investigated through a comprehensive model testing program. However, this can be too expensive for the designers of smaller vessels; and is certainly impractical at the preliminary design stage. To address the problem, this paper presents an approximate method that may be used to evaluate changes in the derivative with respect to wave height. To achieve this, a method is proposed that evaluates steering behaviour in waves using only data that should be readily available at the preliminary design stage. The evaluation assumes waves of the same length as the vessel and that are travelling at approximately the same velocity. Validation of some of the main assumptions are made by comparison with dedicated model tests; using the oceanographic research vessel R.V. Bernicia as case-study. For completeness, methods are proposed to aid decision making when assessing the manoeuvring performance of any given design.

1. INTRODUCTION

Over the past few decades, a significant body of work has been compiled regarding ship motion in extreme following seas. Probably the most up-to-date and comprehensive review of such work is provided by Ayaz [2] – where he considers both theoretical and experimental research and efforts to develop numerical tools for simulation. Many experimental studies have been undertaken including both captive and free-running analysis. For example: Renilson & Driscoll [14] investigated the effects of a steady wave using a flow channel with a standing wave; while Chou et al [3] conducted extensive free-running model tests to investigate the mechanisms leading to capsize. After some early attempts to develop limited steady-state theoretical approaches [16], focus moved to the development of mathematical models for use in time-domain simulation [8]. Ayaz [2] goes on to provide a contemporary methodology and evaluation of ship motions in extreme following seas.

Much of the previous work aims to understand broaching events; where the yawing moment from the wave exceeds the restoring moment from the rudder. However, long before such extreme events occur, steering performance can degrade because the effective underwater shape of the ship is changed by the presents of waves. This can be especially important for fishing vessels, for two main reasons. Firstly, smaller vessels can be particularly susceptible to the effect of following seas, as they may likely encounter steeper waves of the same length as the vessel. Secondly, while the masters of many ships can choose to change heading to avoid unfavourable conditions, the mission profile of these ships may be more restrictive. If for example a fishing vessel is making a tow along a sub-sea trench then the ships heading cannot be so readily altered.

2. AIMS AND OBJECTIVES

The aim of the study is to investigate the effect of changes in the underwater form, due to waves, on the manoeuvring derivatives of a fishing vessel. This will be used to better understand which design changes may best achieve an improved solution. It is intended to use only data that can be readily available at the preliminary design stage; and thus economically accessible to the designers of small vessels.

The main objectives of this study are:

- to estimate the manoeuvring derivatives while taking into account the underwater shape when on a wave;
- to validate the main assumptions by comparison with suitable model tests;
- to investigate the steering behaviour of a case-study vessel as a function of wave-height;
- to provide a method of parametric evaluation that can be used to assess the effects of design changes on steering behaviour.

The study is limited to the consideration of waves with the same length as the ship and which are travelling at approximately the same velocity; and does not consider extreme events such as broaching. The assessment will be limited to ensuring that the vessel can remain at least controllable in a suitably wide range of wave conditions.
3. DESCRIPTION OF APPROACH

The work uses as case-study the R.V. Bernicia – which is an oceanographic research vessel that has been converted from a fishing vessel. It is considered relevant as, in addition to having the typical characteristics of a fishing vessel, the mission profile bears many of the same restrictions. A profile view of the vessel’s hull-form is included in Fig. 1 and the general particulars can be found in Table 1. The method assumes a model wherein the force coefficients are obtained in terms of the sectional added-masses and the bilge-vortex effect; described by Clarke [4]. For this analysis the model takes into consideration the actual sectional draught associated with the trim and wave-profile. The sectional areas are obtained by direct integration of the off-set data and other sectional particulars are obtained directly from the same data. It is assumed that the mass distribution of the ship remains unchanged and that consequently so must the longitudinal centre of buoyancy (LCB).

Table 1 General particulars; R.V. Bernicia

<table>
<thead>
<tr>
<th></th>
<th>Ship</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaling Factor, ( \lambda ) (-)</td>
<td>1/12</td>
<td>1/12</td>
</tr>
<tr>
<td>Length, ( L ) (m)</td>
<td>13.870</td>
<td>1.156</td>
</tr>
<tr>
<td>Beam, ( B ) (m)</td>
<td>3.528</td>
<td>0.294</td>
</tr>
<tr>
<td>Draught, ( T ) (m)</td>
<td>2.000</td>
<td>0.167</td>
</tr>
<tr>
<td>Centres, ( LCB/G ) (m)</td>
<td>-0.461</td>
<td>-0.038</td>
</tr>
<tr>
<td>Block Coefficient, ( C_B ) (-)</td>
<td>0.498</td>
<td>0.498</td>
</tr>
</tbody>
</table>

To this end, it is assumed that the ship must be free to trim and heave so that it may achieve constant displacement and LCB. For any given wave-height the displacement and LCB are obtained by direct integration of the sectional areas. Then, the error between the current values and the equilibrium values are minimised in terms of draught and trim; using a penalty-function method [15]. Figure 1 shows an example of the wave profile, for a wave-height of 1.2 meters, and includes the mean trim shown as a broken line. In all cases herein the worst case is assumed to be when the wave-length is equal to the length of the ship. On the relevant plots the wave-height is examined up to a value of 1.6 meters. Negative wave-amplitudes correspond to the case where the wave-trough is amidships and positive wave-amplitudes correspond to the case where the wave-crest is amidships. By way of example, Fig. 2 shows the change in trim and mean-draught as a function of wave-amplitude.

4. NUMERICAL METHODOLOGY

To examine the manoeuvring behaviour it is first necessary to estimate the manoeuvring derivatives. For this analysis the derivatives are assumed to take the form described in [4]. The nomenclature within this text assumes the suffix-notation for partial derivatives; commonly used for manoeuvring analysis. \( Y \) and \( N \) represent the sway force and yaw moment respectively. The suffixes \( v \), \( r \) and \( \delta \) represent the sway velocity, yawing rate and rudder angle respectively; with the dot-notation indicating the first derivative (acceleration). The suffixes \( h \) and \( f \), used outside parenthesis, indicate that the derivative contribution is from either the hull-form or fin-effect respectively. Also, the prime-notation indicates that terms are non-dimensional; herein with respect to a power of length and where necessary the ship design speed. The hull-form acceleration derivatives are given in Eqs. 1–3 and the velocity derivatives are given in Eqs. 4–7. In each of the derivative equations, \( L \) and \( T \) are the ship length and mean draught respectively. The term \( T_S \) represents the local sectional draught; \( C_H \) and \( I_H \) represent the sectional added-mass and vortex-effect respectively and \( x' \) is the non-dimensional sectional lever from amidships.
(Y')_{h} = -\pi \left( \frac{T}{L} \right)^2 \int_{STERN}^{BOW} \left( \frac{T_x}{T} \right)^2 \left( C_H + I_H \right) \ dx' 

(2)

\begin{align*}
(Y')_{h} &= (N')_{h} = -\pi \left( \frac{T}{L} \right)^2 \int_{STERN}^{BOW} \left( \frac{T_x}{T} \right)^2 \left( C_H + I_H \right) \ x^2 \ dx' 

\end{align*}

(3)

\begin{align*}
(Y')_{h} &= -\pi \left( \frac{T}{L} \right)^2 \left[ \left( \frac{T_s}{T} \right)^2 \left( C_H + I_H \right) \right]_{STERN} 

&+ \int_{STERN}^{BOW} \left( \frac{T_x}{T} \right)^2 \left( C_H + I_H \right) \ dx' 

\end{align*}

(4)

\begin{align*}
(Y')_{h} &= -\pi \left( \frac{T}{L} \right)^2 \left[ \left( \frac{T_s}{T} \right)^2 \left( C_H + I_H \right) \right]_{STERN} 

&+ \int_{STERN}^{BOW} \left( \frac{T_x}{T} \right)^2 \left( C_H + I_H \right) \ x' \ dx' 

\end{align*}

(5)

To estimate the sectional added-mass $C_{ih}$, two sectional forms are chosen from those proposed by Lewis [11]. The first of these is considered appropriate for sections in the middle of the ship and at the forward end; shown in Fig. 3. Here, the sectional added-mass is obtained in terms of the sectional area $S$, the breadth $B$, and the draught $T_s$. To mechanise the process of obtaining the added-mass values, an equation is fitted to the data given by Lewis. For sections typical of those given in Fig. 3 the added-mass is assumed to take the form given in Eq. 11 where the coefficient terms are obtained from the polynomials given in Eqs. 12–14. The results are shown in Fig. 4 where the solid lines represent the fitted equation and the crosses represent the original data.

\begin{align*}
C_H &= 1 + \frac{k_1}{k_2} \frac{B}{T} - k_1 

\end{align*}

(8)

\begin{align*}
k_1 &= -6.637 \left( \frac{S}{BT_s} \right)^3 + 9.779 \left( \frac{S}{BT_s} \right)^2 

&- 5.010 \left( \frac{S}{BT_s} \right) + 1.133 

\end{align*}

(9)

\begin{align*}
k_2 &= 13.644 \left( \frac{S}{BT_s} \right)^2 

&- 23.332 \left( \frac{S}{BT_s} \right) + 11.505 

\end{align*}

(10)

Fig. 3 Curve fitted to Lewis data for mid-sections

Fig. 4 Curve fitted to Lewis data for aft-sections
The rudder yaw derivative is in and given in Fig. 5.

In addition to the added-mass it is assumed that, as the bilge-vortices move aft, they influence the pressure around the aft-body. Clarke [4] shows that this can manifest in a significant increase in the measured forces at the stern; compared to the case where only added-mass is considered. To estimate the magnitude of the vortex-effect the characteristics of Eq. 4 are exploited. It can be seen from the equation that this derivative is dependent only on the contribution from the deepest stern section (in this case the back of the deadwood: \( x'_{\text{STERN}} \approx -0.4 \) in Fig. 1). Then, the difference between the expected derivative and that calculated using only the added-mass, gives the magnitude of the vortex-effect contribution at the stern; shown in Eq. 15. In the first instant the vortex-effect \( I_{H} \), can be estimated from the semi-empirical approach give in Eq. 16; derived from [4]. Later it may be possible to update this value using a simple model test that can be conducted in the most basic towing tank (expanded further in the validation section of this paper).

Next, the vortex-effect is assumed to have a parabolic distribution; starting with the maximum value at the stern and reducing to zero amidships; described by Eq. 17. By way of example the added-mass and the vortex-effect are calculated for the still-water case and given in Fig. 5.

\[
(I_{H})_{\text{STERN}} = \frac{Y'_H}{T^2} - (C_H)_{\text{STERN}}^{1/2} \tag{15}
\]

\[
(I_{H})_{\text{STERN}} = \left(1 + 0.4C_B \frac{B}{T}\right) - (C_H)_{\text{STERN}} \tag{16}
\]

\[
I_{H} = (I_{H})_{\text{STERN}} \left(\frac{x'}{x'_{\text{STERN}}}\right)^2 \tag{17}
\]

The rudder sway derivatives are estimated using Eq. 18 in terms of the rudder area \( A_R \) and the rudder aspect-ratio \( \sigma \). The rudder yaw derivative is obtained as a function of the sway derivative; as given in Eq. 19. The stabilising effect (or fin effect) of the rudder is obtained in terms of the rudder sway derivative, given in Eq. 20, where the term in parentheses accounts for the flow straitening effect of the hull-form in terms of the block coefficient \( C_B \), as proposed in [1].

\[
Y'_x \approx 1.44 A_R \left(\frac{2\pi + \sigma}{\sigma + 2}\right) \tag{18}
\]

\[
N'_x = Y'_x x'_{\text{RUDDER}} \tag{19}
\]

\[
(Y'_x)_{f} = -Y'_x \left(\frac{1}{1 + C_B}\right) \tag{20}
\]

Once each of the above described contributions are calculated the total velocity derivatives can be obtained according to Eqs. 21–24 [6].

\[
Y' = (Y'_x)_{h} + (Y'_x)_{f} \tag{21}
\]

\[
Y' = (Y'_x)_{h} + (Y'_x)_{f} x'_{\text{RUDDER}} \tag{22}
\]

\[
N' = (N'_x)_{h} + (Y'_x)_{f} x'_{\text{RUDDER}} \tag{23}
\]

\[
N' = (N'_x)_{h} + (Y'_x)_{f} x'^2_{\text{RUDDER}} \tag{24}
\]

In a similar manner the acceleration dependent stabilising effects are obtained according to Eq. 25 and the total acceleration derivatives are obtained as given in Eqs. 26–29 [6]; now also including a term for the rudder span, \( s_R \); and the flow straightening effect (as in Eq. 20).

\[
(Y'_x)_{f} = -\frac{2\pi A_R s_R}{L^3 \sqrt{\sigma^2 + 1}} \left(\frac{1}{1 + C_B}\right) \tag{25}
\]

Again, once each of the described contributions are calculated the total acceleration derivatives can be obtained according to Eqs. 26–29.
5. PREDICTION OF THE CHANGE IN DERIVATIVES IN WAVES

Collecting together all the terms derived so far, it is now possible to examine the behaviour of the manoeuvring derivative with respect to wave-height. Figure 6 presents the change in velocity derivatives as a function of wave-amplitude. As before, negative amplitudes correspond to a trough amidships and positive amplitudes correspond to a crest. As can be seen, all derivatives demonstrate an approximately linear change with respect to wave-amplitude.

Also, in most cases, derivative magnitude can be seen to approximately double over the wave-amplitudes examined. It is interesting to note that \( N'_r \) is almost tripled over the wave-range as this derivative can be shown to be the most influential regarding manoeuvring performance [7].

\[
Y'_v = (Y'_v)_b + (Y'_v)_f \tag{26}
\]
\[
Y' = (Y'_v)_b + (Y'_v)_f x'_RUDDER \tag{27}
\]
\[
N'_v = (N'_v)_b + (Y'_v)_f x'_RUDDER \tag{28}
\]
\[
N'_r = (N'_r)_b + (Y'_v)_f x'^2_{RUDDER} \tag{29}
\]

6. VALIDATION THROUGH COMPARISON WITH CAPTIVE MODEL TEST

To validate the estimation of the derivative used to calibrate the vortex-effect, limited captive model tests were conducted. Also, the above described method makes the assumption that the vortex-effect is not modified by the presence of the wave. To validate this assumption, dedicated captive model tests were conducted in waves. The tests were performed in a standard towing tank making it possible to measure only the sway velocity derivatives. Nevertheless, a novel approach was adopted when conducting the tests; yielding valuable results. A 12\textsuperscript{th} scale model was chosen as its scaled design speed is the same as the celerity of a wave of the same length as the model. In this manner it was possible to tow the model down the tank (away from the wave-maker) so that it remained approximately stationary with respect to the wave-form. In fact, a slight precession of the wave was allowed so that the maximum and minimum values could be obtained; and attributed to either crest or trough accordingly. The wave-profile used was of sinusoidal form, the same length as the model and with a wave-height equivalent to 1.2 meters in the full-scale. For reference, the model particulars are given in Table 1.

The tests yielded an approximately sinusoidal result for the side-forces measured at each drift angle. To find the forces corresponding to wave-trough or -crest, the significant third-highest and -lowest values are obtained according to Eqs. 30 and 31 respectively. Here the significant values are obtained in terms of the \( \bar{F} \) force measurement \( F' \); according to Eqs. 32 and 33. The data samples include values for only the middle-third of the test run; to remove error associated with accelerating the model (at the start of the run) or wave reflections (at the end of the run).

\[
(F'_v)_{\text{hi}} = \bar{F} + 4.0 \sqrt{m_0} \tag{30}
\]
\[
(F'_v)_{\text{lo}} = \bar{F} - 4.0 \sqrt{m_0} \tag{31}
\]
\[
\bar{F} = \frac{1}{n} \sum_{i=1}^{n} F_i \tag{32}
\]
\[
m_0 = \frac{1}{n} \sum_{i=1}^{n} (F_i - \bar{F})^2 \tag{33}
\]
The results are presented in Fig. 8 including the values obtained for a still-water run and the estimated crest and trough values. Then, regression is used to find the gradient of each data set (assuming a third-order odd-function); and thus the derivatives. The results are compared to the predicted values in Table 2; together with linear attained values for comparison.

To investigate assumptions made about the wave position, the numerical model is used to examine the effect on derivative magnitude. The influence of the position of the wave-trough was examined and is given in Fig. 9. From the results it is clear that the maximum and minimum values do not occur when the wave is centred amidships, as initially assumed, but when the wave is slightly further astern. In fact, the results indicate that the maximum and minimum values occur when the wave is centred closer to the LCB; which in retrospect seems more logical. Then, the values predicted with the wave centred at the LCB are used for comparison with those obtained from the model tests.

Considering the results given in Table 2, the prediction would appear to be promising. The regression coefficients are above 95% in all cases; some small improvement can be observed (in the linear derivative) when using the non-linear fit. It should be noted that the measured still-water derivative was used to update the calibration of the vortex-effect; thus demonstrating no difference between the measured and predicted values. As for the wave cases, reasonable prediction of the derivative magnitude can be observed for both cases; with the ‘trough-case’ presenting the best results. The results would indicate that the initial assumption (that the vortex-effect is not significantly modified by the presence of the wave) can be used for this simplified analysis. Nevertheless, this conclusion is drawn from only one example and is only applicable to the hull-form tested. Clearly, caution should be exercised if extrapolating for other applications.

### 7. PREDICTION OF THE CHANGE IN MANOEUVRING PERFORMANCE IN WAVES

Optimisation of the manoeuvring performance of a ship is a difficult subject at any stage of the design. Not least, because any optimal solution is more qualitative than quantitative. At the one extreme we could aim for a design that is difficult to turn but that maintains a course very well. At the other extreme we can opt for a design that can turn readily but that is more difficult to keep on course. It goes without saying that all designs must be a compromise between the two. For practical implementation at the preliminary design stage, it is here considered sufficient that the design be neither too difficult to turn or too difficult to keep on course, over a suitable range of wave-heights. Clearly, the decision as to exactly what that wave-height should be is subjective. However, at a minimum, a top-down approach could be adopted to serve as a guideline to aid in decision making; to stop operations or to changing heading.

To analyse the manoeuvring performance, the derived terms are transposed into the time- and gain-constant format [12]. These transpositions are given in Eqs. 34–37 and the remaining necessary identities are given in Eqs. 38–40.

\[
T'_x T'_y = \frac{(Y'_x - m') (N'_x - I'_x) - (Y'_x - m'x'_y) (N'_x - m'x'_y)}{Y'_x (N'_x - m'x'_y) - N'_x (Y'_x - m')}
\]

\[
T'_x + T'_y = \frac{\left[ (Y'_x - m') (N'_x - m'x'_y) + (N'_x - I'_x) Y'_x \right]}{Y'_x (N'_x - m'x'_y) - N'_x (Y'_x - m')}
\]

(Attained correlation coefficients given in parentheses)
From the results of Eqs. 34 and 35, the first two time-contents are obtained from the quadratic given in Eq. 41. In addition, the time response of the steering system $T_E'$, is approximated according to Eq. 42 [5].

$$T_1', T_2' = \frac{(T_1' + T_2') \pm \sqrt{(T_1' + T_2')^2 - 4T_1'T_2'}}{2}$$

$$T_E' \approx 2.5 \frac{L}{u}$$

Collecting the above terms together it is possible to investigate the course-keeping behaviour of the vessel from the result of Eq. 43. The Phase and Gain of the system is obtained, in terms of frequency $\omega'$, using Eqs. 44 and 45. The term $\phi_{\omega}$, used in Eq. 44, is dependent on the roots of Eq. 43; taken as -270 deg. for a stable system and -90 deg. for an unstable system. To assess the course-keeping ability, the phase-margin is calculated from Eq. 44, in terms of the frequency corresponding to zero Gain in Eq. 45. The phase-margin is taken as the phase magnitude smaller than -180 deg. for a stable system and greater than -180 deg. for an unstable system.

The phase-margin for the case-study vessel is calculated and presented in Fig. 10. It is not strictly necessary that the open-loop system be stable [4]; as a pilot could introduce as much as 20 deg. phase into the closed-loop system [13] [10]. Nevertheless, the existing vessel appears to demonstrate positive stability over the entire wave-range examined. This indicates that the vessel should remain at least controllable even in the more extreme wave conditions.

Once it is established that the vessel remains controllable, the next step is to ensure that the ship has sufficient turning ability. To examine this, the initial-turning ability is investigated, as defined by IMO [9]. The distance travelled in ship-lengths $t'$, corresponding to a 10 deg. heading change induced by a 10 deg. applied helm angle is calculated. This can be obtained from an iterative solution to Eq. 46.

$$t' = \frac{1}{-K'} + T' - T_e \left( \frac{T_e'}{T'} \right)^{\frac{1}{2}}$$

where $T' = T_1' + T_2' - T_E'$. The initial-turning performance for the case-study vessel is calculated and presented in Fig. 11. Though not necessarily required for vessels of less than 100 meters in length, the IMO recommends that this value should not exceed 2.5 ship-lengths. Ship-length aside, this limit can be very useful when
benchmarking respective designs. From the results it is clear that the vessel has good initial-turning performance over most of the wave-range tested. In fact, the value is only exceeded for higher waves when the trough is at the LCB; and then not by very much.

8. PARAMETRIC EVALUATION AT THE PRELIMINARY DESIGN STAGE

On analysis, the case-study vessel is close to optimal. However, showing the effects of design changes on manoeuvring performance still serves to demonstrate how the optimisation process can be achieved.

At the preliminary design stage the naval architect generally has two possibilities available for modifying the manoeuvring performance. Firstly, changes to the hull-form derivatives can be brought about through modifications to the hull-form. Second, changes to both the hull-form derivatives and the rudder derivatives can be brought about through modifications to the rudder. To investigate these effects, changes are made to both independently and the effects on phase-margin and initial-turning performance are considered. In the first instant, changes are made to the hull-form alone; equivalent to a 50% reduction in the fin-effect. In the second instant, changes are brought about by a 50% reduction in rudder size; affecting both the rudder derivatives and the fin-effect. For both of these cases the phase-margin and initial-turning performance are calculated for a range of wave-heights.

Figure 12 makes comparison of the phase-margin for the two modified designs (as marked) and includes the performance of the basis-design (as a broken line). Both of the modified designs present an increase in the range of phase-margin. In addition, both of the modified designs present an extreme phase-margin in the order of -20 deg.; making the vessel difficult, if not impossible, to keep on course. On examination, increasing the rudder size would be the most effective way of improving on a poor initial design.

9. DISCUSSION

The aim of this study was to investigate the effect of changes in the underwater form, due to waves, on the manoeuvring derivatives of a fishing vessel. This aim was set with an objective (amongst others) of providing a method of parametric evaluation that can be used to assess the effects of design changes on steering behaviour. To this end, a numerical tool is derived, that requires only general particulars (hull and rudder) and some hull-form definition (such as off-set data). A methodology is proposed for assessing the manoeuvring performance over a chosen range of wave-heights. In addition, a method is proposed for identifying the most appropriate action for improving an unsatisfactory initial design. Thus, the primary aims and objectives of this study are met.

A method of model testing is demonstrated that can be used to validate the assumptions; and that can be performed with relatively basic towing tank facilities. It should however be noted that the model testing method may well exhibit significant uncertainties. Beyond the usual errors, the generated waves must overtake the model before the run is started; thus modifying the wave-form in the process. In addition, only one derivative has been validated by this method; only inferring satisfactory validation of the others. Obviously, the limitations of this method should be fully recognized if applying it to other designs. It should also be remembered that this study does not consider broaching events. Satisfactory adherence to this linear analysis does therefore not preclude the possible occurrence of more extreme non-linear events.
Nevertheless, the proposed method is not limited to smaller vessels nor wave-form effects. In fact, the method may be used at the preliminary design stage for assessing manoeuvring performance par-say and (pending validation) may also be useful for assessing the effects of trim.

10. CONCLUSION

The proposed method is shown to approximate well the change in manoeuvring derivatives with wave-height, for the specific case investigated. Also, for the specific case investigated, the vortex-effect would appear to be not significantly modified by the presents of the wave. Finally, the parametric investigation finds that increasing the rudder derivative gives the most effective way of improving an initially unsatisfactory design.

REFERENCES


**NOMENCLATURE**

- $a$: Form-dependent half-breadth (m)
- $A_r$: Rudder area ($m^2$)
- $b$: Form-dependent body-section draught (m)
- $B$: Ship hull beam (m)
- $C_B$: Block coefficient (-)
- $C_H$: Sectional added-mass coefficient (-)
- $f$: Suffices indicating term associated with fin-effect (-)
- $F_i$: $i^{th}$ force measurement (N)
- $(F)_{1/2}$: Significant third-highest value of sample (N)
- $(F)_{1/-2}$: Significant third-lowest value of sample (N)
- $G$: Result of Laplace transform (-)
- $h$: Suffices indicating term associated with hull-form (-)
- $H$: Form-dependent section deadwood (m)
- $I_H$: Vortex-effect coefficient (-)
- $I_Z$: Moment of inertia about z-axis (Nm)
- $k_n$: Result of $n^{th}$ polynomial (-)
- $K$: Nomoto gain index (1/s)
- $L$: Ship length at water line (-)
- $m$: Mass displacement of ship (kg)
- $m_o$: Moment of sample (-)
- $N$: Yaw moment about body z-axis (Nm)
- $N_r$: Partial derivative of yaw moment with respect to yaw rate (-)
- $N_y$: Partial derivative of yaw moment with respect to yaw acceleration (-)
- $N_{y\alpha}$: Partial derivative of yaw moment with respect to sway velocity (-)
- $N_{y\alpha}$: Partial derivative of yaw moment with respect to sway acceleration (-)
- $N_{y\delta}$: Partial derivative of yaw moment with respect to the rudder angle (-)
- $s$: Laplace operator (-)
- $s_{rB}$: Rudder span (m)
- $t$: time [non-dimensional form taken in ship-lengths] (-)
- $T$: Draught (m)
- $T_E$: Time constant for steering engine (s)
- $T_n$: Nomoto equation $n^{th}$ time constant (s)
- $T_S$: Local section draught (m)
- $X$: Force [Surge] in direction of body x-axis (N)
- $x$: Ship body axis in surge coordinate (m)
- $x_g$: Distance from amidships to centre of mass in body x-axis (m)
- $Y$: Force [Sway] in direction of body y-axis (N)
- $Y_r$: Partial derivative of sway force with respect to yaw rate (-)
- $Y_{\alpha}$: Partial derivative of sway force with respect to yaw acceleration (-)
- $Y_{\alpha}$: Partial derivative of sway force with respect to sway velocity (-)
- $Y_{\alpha}$: Partial derivative of sway force with respect to sway acceleration (-)
- $Y_{\delta}$: Partial derivative of sway force with respect to the rudder angle (-)
- $\omega$: Frequency (1/s)
- $\omega_h$: Initial frequency [dependant on root] (1/s)
- $\sigma$: Rudder aspect-ratio (-)

Note: Where used in the text, the prime notation indicates that the values are non-dimensional.