Sensitivity Analysis for Hydraulic Models

JimW. Hall[1], Shawn. A. Boyce[2], Yeuling Wang[3], Richard J. Dawson[4], Stefano Tarantola[5], Andrea Saltelli[6]

Abstract

Sensitivity analysis is well recognised as being an important aspect of the responsible use of hydraulic models. This paper reviews a range of methods for sensitivity analysis and presents two illustrative examples of differing complexity, which illustrate the deficiencies of standardized regression coefficients, derivatives and other local methods of sensitivity analysis. The use of global variance-based sensitivity analysis is shown to be more general in its applicability and in its capacity to reflect non-linear processes and the effects of interactions among variables.

Subject headings: Sensitivity analysis; Variance analysis; Regression analysis; Uncertainty principles.

1 Introduction

Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation (Saltelli et al., 2004). To avoid distinctions between model input variables, boundary conditions and parameters, all of the inputs to a

---

[1] *Professor, School of Civil Engineering and Geosciences, Cassie Building, Newcastle University, NE1 7RU, UK. jim.hall@ncl.ac.uk*

[2] *Hydrologist, Caribbean Institute for Meteorology and Hydrology, Husbands, Barbados, sboyce@cimh.edu.bb*

[3] *PhD student, School of Civil Engineering and Geosciences, Cassie Building, Newcastle University, NE1 7RU, UK. yueling.wang@newcastle.ac.uk*

[4] *Research Fellow, School of Civil Engineering and Geosciences, Cassie Building, Newcastle University, NE1 7RU, UK. richard.dawson@ncl.ac.uk*

[5] *European Commission Joint Research Centre, Via Fermi, 1, 21020 Ispra (VA), Italy. stefano.tarantola@jrc.it*

[6] *European Commission Joint Research Centre, Via Fermi, 1, 21020 Ispra (VA), Italy. andrea.saltelli@jrc.it*

model are collectively referred to as ‘input factors’. Amongst the reasons for using sensitivity analysis are:

- To identify the factors that have the most influence on model output.
- To identify factors that may need more research to improve confidence in model output.
- To identify factors that are insignificant to model output and can be eliminated from further analysis.
- To determine if a model reproduces known influences upon the processes it is simulating.
- To identify regions in the space of inputs where the variation in model output is maximum.
- To find the optimal regions within the parameter space for use in calibration studies.
- To identify which, if any, factors or groups of factors interact with each other.
- To establish whether model predictions are robust to plausible variations in input factors, on the other hand, strongly dependent on fragile assumptions.

Sensitivity analysis is widely accepted as a necessary part of good modelling practice. However, the increasing complexity of computer models used in hydraulic engineering has not, in general, been accompanied by a corresponding and necessary increase in the rigour and sophistication of sensitivity analysis. The increasing use of coupled models from different disciplines, for example the coupling of hydrodynamic, structural reliability and impacts models to provide estimates of risk (Dawson et al., 2005), provides additional motivation for improved sensitivity analysis. Since these models derive from different scientific communities, model developers and users cannot be expected to have reliable intuitions about the model behaviour and interactions without a systematic approach to exploring the model response to varying inputs.

Understanding and analysing uncertainties has concerned hydraulic engineers for many years (Johnson, 1996; Melching, 1995; Tang and Yen, 1972; Yen and Tung, 1993). Uncertainty analysis involves quantification of uncertainties in model inputs and propagating them through to model predictions. Sensitivity analysis can be thought of as addressing the inverse of this problem, which is to diagnose the influence that model input factors, individually and in combination, have on the variation in the model prediction. Typically uncertainty analysis is applied in situations where quantities in a system being analysed are not known precisely (e.g. channel roughness) or vary in nature (e.g. river discharge). As will be demonstrated herein, sensitivity analysis is more general in that it can also usefully be applied to *design variables*, *i.e.* quantities that will be decided upon by the engineer. Whilst it is not meaningful to apply uncertainty analysis to these variables (as they will in future be fixed, subject to some tolerance) it is useful to apply sensitivity analysis to identify which design variables are important in their influence on system performance and which are less important.

To achieve the aims outlined above a method of sensitivity analysis should have the following desirable properties:

- The method should be able to diagnose the effect of input factors acting individually or in combination, in the latter case in order to identify the effect of interactions between factors.
- The method should test the influence of a model input factor over its entire range of variation.
- The method should be applicable, within the range of input variation, to linear and non-linear models.
- The method should be model independent.
- The method should be computationally efficient.
- The method should be applicable to both discrete and continuously varying inputs.

In the remainder of this paper a variety of methods for sensitivity analysis are reviewed against the criteria set out above. Section 2 reviews the main approaches to sensitivity analysis and in Section 3 these approaches are applied to a text book example in hydraulic engineering and then a more complex numerical model. The choice of sensitivity analysis method is discussed in Section 4, before the concluding Section 5.

2 Approaches to sensitivity analysis

The main approaches to sensitivity analysis that are applicable in hydraulic engineering are introduced in the following. More theoretical discussion can be found in Saltelli et al. (2000), Cacuci (2003), Saltelli et al. (2005), Saltelli et al. (2008).

2.1 Derivatives based sensitivity indices

Model output derivatives, with respect to input factors, are intuitive sensitivity indices. In general consider a model $Y = f(X_1,\ldots, X_k)$ with k input factors. In the following capital notation (e.g. Y) is used to denote a random variable and lower case (e.g. y) to denote a fixed value of that variable. The partial derivative of Y with respect to an input factor X_i, $\frac{\partial Y}{\partial X_i}$, measures how sensitive the output is to a perturbation of the input. If factors are uncertain within a known or hypothesized range, then the measure

$$S_i^{\sigma} = \frac{\sigma_i}{\sigma_Y}\frac{\partial Y}{\partial X_i} \tag{1}$$

provides a standardized index, where σ_i, σ_Y are the the standard deviations of the inputs and output of uncertainty analysis, respectively. The sensitivity measures $\frac{\partial Y}{\partial X_i}$ can be efficiently computed by algorithmic differentiation, where the computer program that implements the model is modified so that the derivatives are computed with a modicum of extra execution time (Grievank, 2000). There is furthermore a variety of methods to compute these derivatives for large systems of differential equations, such as the Green functions method, the direct method, the decoupled direct method, the adjoint method and others (Turanyi and Rabitz, 2000). A less sophisticated but much more commonplace approach is to vary each input factor in turn by a positive and negative increment around its central value, whilst keeping all other input factors at their central value. This "one-at-a-time" sensitivity analysis can be thought of as providing informal sensitivity indices using arbitrary finite difference estimates of the partial derivatives (Rabitz, 1989).

It is clear that if X_i does not vary linearly with Y over the space of X_i, the calculated derivatives at a single point may misrepresent the sensitivity of the output to uncertainty in the input factors, so derivative-based methods sensitivity indices are referred to as *local* in the following. Notwithstanding these limitations, the majority of sensitivity analysis met in hydraulic engineering and indeed hydrology, where there has been more attention paid to the problems of uncertainty estimation, are local, derivative-based (Chen and Chen, 2003; Cornell, 1972; Horritt, 2006; Indelman et al., 2006; Kabala, 2001; Nash and Karney, 1999; Oliver and Smettem, 2005; Podsechin et al., 2006; Renault and Hemakumara, 1999; Rocha et al., 2006; Swaminathan et al., 1986).

2.2 Linear regression

For linear models, the linear regression coefficients between input and output provide natural sensitivity indices. In the case of numerical models this can be achieved by constructing a Monte Carlo sample of the model inputs and regressing the corresponding outputs, Y against the inputs X_i using multiple regression analysis model of the form:

$$Y = b_0 + \sum_{i=1}^{k} b_i X_i \tag{2}$$

where b_i are fixed regression coefficients. The linear regression coefficients will have dimensions but can be standardized so that:

$$\tilde{Y} = \beta_0 + \sum_{i=1}^{k} \beta_i \tilde{X}_i \tag{3}$$

where $\tilde{Y} = \dfrac{Y - \mu_Y}{\sigma_Y}$, $\tilde{X}_i = \dfrac{X_i - \mu_i}{\sigma_i}$, and $\beta_i = \dfrac{\sigma_Y}{\sigma_i} b_i$. $\tilde{Y}$ and $\tilde{X}_i$ are the standardized variables, μ_Y, σ_Y and μ_i, σ_i are the means and standard deviations of the output and input factors respectively and β_i are known as standardized regression coefficients (SRCs) (Helton et al., 2006). The values of β_i can be estimated by evaluating y at each point in a Monte Carlo sample of the input variables X_i and then applying regression analysis to the sample of points.

For linear models $\beta_i^2 = (S_i^{\sigma})^2$ and if the model is non-linear, SRCs are still a reflection of the contribution of the variance of each input factor to the overall output variance and are more attractive than local derivatives as they offer a measure of the effect of each given factor on Y, which is averaged over a sample of possible values, as opposed to being computed at the fixed point. SRCs are therefore a global sensitivity measure, their limitation being in their applicability to non-linear models.

The sum of the squares of the SRCs represents the proportion of the model output variance explained by the regression model and gives insight into model linearity. This sum can be formulated from a Monte Carlo sample of model simulation data and is expressed as the model coefficient of determination, R_Y^2:

$$R_Y^2 = 1 - \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{(y_i - \mu_Y)^2} \tag{4}$$

where n is the number of simulations, y_i are the simulation results for model realisation i and $\hat{y}_i$ are the values of y provided by the regression model for input vector x_i. When the coefficient of determination this number is high, e.g. 0.7 or higher, then the SRCs can be used for sensitivity analysis, albeit at the price of remaining ignorant about that fraction of the model variance not explained by the SRCs.

SRCs have been used with some success as sensitivity measures in hydraulic engineering. Even though many hydraulic models are in principle non-linear, they are often found in practice to be nearly linear over the range of variation of the inputs, so SRCs are a convenient sensitivity measure, with the added advantage that R_Y^2 provides a diagnostic of the appropriateness of the linear assumption. For example, the regression analysis of Yeh and Tung’s (1993) pit-migration model yielded an R_Y^2 of 0.981. Jaffe and Ferrara (1984) analysed water quality model sensitivity using ranked inputs and outputs, which can yield higher values of R_Y^2 for non-linear but still monotonic models. Melching (2001) and Manache and Melching (2004) reported R_Y^2 values in excess of 0.9 for features of the simulated dissolved oxygen concentration from water resource simulation models (Melching and Bauwens, 2001). Siebera and Uhlenbrook (2005) used a regression-based sensitivity analysis to verify the structure of a distributed catchment model. Pappenberger et al. (2006) used a regression-based technique to determine the influence of uncertainties in rating curves and boundary conditions on flood inundation predictions. SRCs therefore provide a convenient starting point for sensitivity analysis, recognising that if the models concerned are demonstrably non-linear over the range of input variation then the following more sophisticated methods will be required.

2.3 FAST: Fourier Amplitude Sensitivity Test

Whilst regression methods have seen quite wide application in hydraulic engineering, when the coefficient of determination of a regression model is low then it is necessary to adopt a sensitivity method that is applicable to non-linear models. The Fourier Amplitude Sensitivity Test (FAST) (Cukier et al., 1973; Cukier et al., 1978; Cukier et al., 1975; Schaibly and Shuler, 1973) works irrespective of the degree of linearity or additivity of the model.

In FAST the variance $V(Y)$ of Y is decomposed using spectral analysis, so that $V = V_1 + V_2 + ... + V_k + r$, where V_i is that part of the variance of Y that can be attributed to X_i alone and *r* is a residual. The ratio $S_i = \frac{V_i}{V}$ can be taken as a measure of the sensitivity of Y with respect to X_i. For linear models $\beta_i^2 = S_i = (S_i^\sigma)^2$. Saltelli et al. (2004) show how S_i^σ is an effective measure for linear models, β_i is an effective measure for moderately non-linear models, for which an effective linear regression model can be built, and S_i is the model free extension that works even for strongly non-linear models.

There is a natural and intuitive interpretation of these variance-based sensitivity indices in terms of the amount by which the output variance from a model is reduced by fixing an input at a given value. $V(Y)$ is the variance of Y and $V\left(Y \mid X_i = x_i^*\right)$ the variance that would be obtained if X_i were to be fixed to some value x_i^*, in other words the conditional variance of *Y* given $X_i = x_i^*$. x_i^* could be thought of as the true value of X_i determined with a measurement. However, it is not known *a priori* what x_i^* happens to be, but if the distribution of X_i is known it is natural to compute the average $E_{X_i}(V(Y \mid X_i))$ over all possible values of x_i^*. The quantity

$V(Y)-E_{X_i}\left(V(Y \mid X_i)\right)$ is the average amount by which the variance $V(Y)$ will be reduced if the uncertainty in X_i is removed.

A known algebraic result is that

$$V(Y)=E_{X_i}\left(V(Y \mid X_i)\right)+V\left(E_{\mathbf{X}_{\sim i}}(Y \mid X_i)\right) \tag{5}$$

where $V\left(E_{\mathbf{X}_{\sim i}}(Y \mid X_i)\right)$ is referred to as the 'variance of conditional expectation' of Y given X_i and the subscript $\mathbf{X}_{\sim i}$ denotes the vector of all factors other than X_i. The FAST based sensitivity index is simply

$$S_i=\frac{V\left(E_{\mathbf{X}_{\sim i}}(Y \mid X_i)\right)}{V(Y)} \tag{6}$$

In classic FAST only the main effect terms V_i are computed, and the success of a given analysis is empirically evaluated by the sum of these terms. If this is high, as rule of the thumb greater than 0.6 (Liepmann and Stephanopoulos, 1985), then the analysis is successful. The V_i describe the so called 'additive' part of a model and additive models are defined as those for which $\sum_i S_i =1$. Note that in this context the condition of 'additivity' is more general than linearity, to which SRCs are restricted. Extended FAST (Saltelli et al., 1999) allows the computation of higher order terms:

$$V(Y)=\sum_i V_i+\sum_{\substack{i,j\\ i<j}} V_{ij}+\sum_{\substack{ijl\\ i,j\\ j<l}} V_{ijk}+\ldots+V_{123..k} \tag{7}$$

where

$$V_i=V\left(E_{\mathbf{X}_{\sim i}}(Y \mid X_i)\right) \tag{8}$$

$$V_{ij}=V\left(E_{\mathbf{X}_{\sim ij}}(Y \mid X_i, X_j)\right)-V_i-V_j \tag{9}$$

$$V_{ijl}=V\left(E_{\mathbf{X}_{\sim ijl}}(Y \mid X_i, X_j, X_l)\right)-V_{ij}-V_{jl}-V_{il}-V_i-V_j-V_l \tag{10}$$

and so on, and their corresponding sensitivity indices, are obtained from normalisation by V. Equation 8 is the same variance decomposition as is employed in ANOVA (Analysis of Variance) (Box et al., 1978).

Computing all 2^k terms in Equation (8) is computationally prohibitive for all but functions with very small k. A more practical approach is to estimate the k total sensitivity indices, S_{Ti}, where (Homma and Saltelli, 1996)

$$S_{Ti} = 1 - \frac{V\left[E(Y \mid X_{\sim i} = x_{\sim i}^{*})\right]}{V(Y)} \tag{11}$$

The total sensitivity index therefore represents the average output variance that would remain as long as X_i stays unknown. The total sensitivity indices help finding interactions within the model. For example, factors with small first order indices but high total sensitivity indices affect the model output Y mainly through interactions. The presence of such factors is indicative of redundancy in the model parameterisation.

Unlike derivative and regression analysis, there has to date been rather limited published application of these more general variance based sensitivity indices in the hydraulic engineering literature. The only two reported applications (Hall et al., 2005; Pappenberger et al., 2008) have been in the field of flood inundation modelling. Ratto et al. (2007b) include the use of variance-based sensitivity indices in their discussion of rainfall-runoff modelling.

2.4 Other methods

Specialist application situations have led to development of specific sensitivity methods. For example, the problems of reliability analysis lead to a natural family of sensitivity measures related to the local derivatives at the point on the limit state function where the structure is most likely to fail (Cawlfield, 2000; Vrijling, 2001).

Another useful sensitivity measure, which is computationally cheaper than the variance based methods, is the measure of Morris (1991), which is useful when the number of uncertain factors is high and/or the model is expensive to compute. It belongs thus to the family of screening sensitivity analysis methods (Campolongo et al., 2000). Xu and Mynett (2006) applied the Morris method to identify the most influential parameters on a river basin management model’s output.

Hydrological model calibration exercises have yielded sensitivity indices as a by-product. Where the calibration is implemented as an optimisation problem the local derivatives from the optimisation can give insight into the sensitivity of model response .Tang et al. (2006) apply this approach to a non-linear parameter estimation tool. Alternatively, if a population of model parameter sets is partitioned between those that reproduce observations acceptably and those that do not, then statistical methods such as the Kolmogorov-Smirnov test can be used to analyse if a parameter significantly influences the calibration. This is a specific example of Monte Carlo Filtering, where the results of a sequence of Monte Carlo model experiments are 'filtered' to remove the instances that perform unacceptably (Hornberger and Spear, 1981).

Measures of entropy can be used to estimate the degree of association between model inputs and outputs (Helton et al., 2006), with the attraction that entropy is less reliant than variance-based methods on the second moment as a description of dispersion. Pappenberger et al. (2008) used Kullback–Leibler entropy as a sensitivity measure. The extension to the case of imprecise probabilities is explored by Hall (2006).

The field of Bayesian statistics provides a sound decision-theoretic justification for sensitivity indices in terms of the 'partial Expected Value of Perfect Information' (partial EVPI) (O'Hagan et al., 1999; Oakley and O'Hagan, 2004). This requires the existence of a utility function, but if such a function does exist or can be assumed (e.g. a quadratic loss function), it allows the quantification of the economic value of reducing the uncertainty in any given input factor. That value of uncertainty reduction is a natural sensitivity index, and indeed Oakley and O'Hagan demonstrate that the first order variance-based sensitivity measures are a special case of this Bayesian measure of sensitivity. A further generalisation was provided by Hall (2006), who extended both the variance based measures and the partial EVPI to the situation in which the input probability distributions are not precisely known, the consequence of which is that the various sensitivity indices are output as intervals rather than points. The use of sensitivity indices in the context of imprecise information is further explored by Ferson and Tucker (2006).

3 Demonstration

3.1 A didactic example: Force on a pipe bend

A simple problem that will be familiar to all hydraulic engineers concerns the force exerted on a pipe by water flowing steadily around a horizontal bend (Figure 1). This ‘model’ is so simple as to allow a characterisation of the system sensitivity by analytic methods, but it will be dealt with here as if it were a more complex computer model. The well known solution for F_x and F_y, the orthogonal force components on the bend, are as follows:

$$F_x = p_1 a_1 - p_2 a_2 \cos\theta + \rho q (v_1 - v_2 \cos\theta) \tag{12}$$

$$F_y = \rho q v_2 \sin\theta + p_2 a_2 \sin\theta \tag{13}$$

where q is the discharge in the pipeline (m^3/s), v_1 and v_2 are the velocities before and after the bend respectively (m/s), a_1 and a_2 are the areas of the pipe before and after the bend respectively (m^2), p_1 and p_2 are the pressures immediately before and after the bend (N/m^2), ρ is the density of water (kg/m^3) and θ is the angle of the bend. The magnitude of the resultant force is $F_R = \sqrt{F_u^2 + F_v^2}$ and $p_2 = p_1 + \frac{\rho}{2}(v_1{}^2 - v_2{}^2)$.

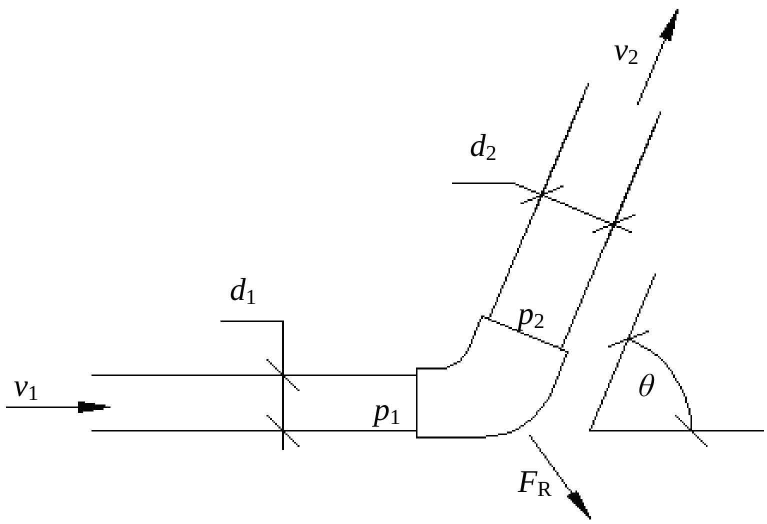

Figure 1 Diagram of notation in pipe bend calculation

In order to explore the sensitivity of response of this system to variation in the input factors, the input factors were assigned the distributions in Table 1. This range has deliberately been chosen to be quite wide in order to illustrate the potential for non-linear response even in this very simple example. Figure 2 illustrates the resultant force for a range of values of q and d_1. Even for this simple example it is clear how the behaviour of partial derivatives (Figure 3) varies over the range of the input factors, indicating that they are only locally informative as a sensitivity measure.

Table 1 Distributions of input factors for pipe bend example

Variables	**Distribution type**	**Parameters**
D_1	Uniform	D_1~U(0.05, 0.15)
D_2	Uniform	D_2~U(0.05, 0.15)
Q	Lognormal	ln$Q$~N(0.01,0.3)
P_1	Lognormal	lnP_1~N(11.5,0.5)
θ	Normal	θ~N(70°,15°)

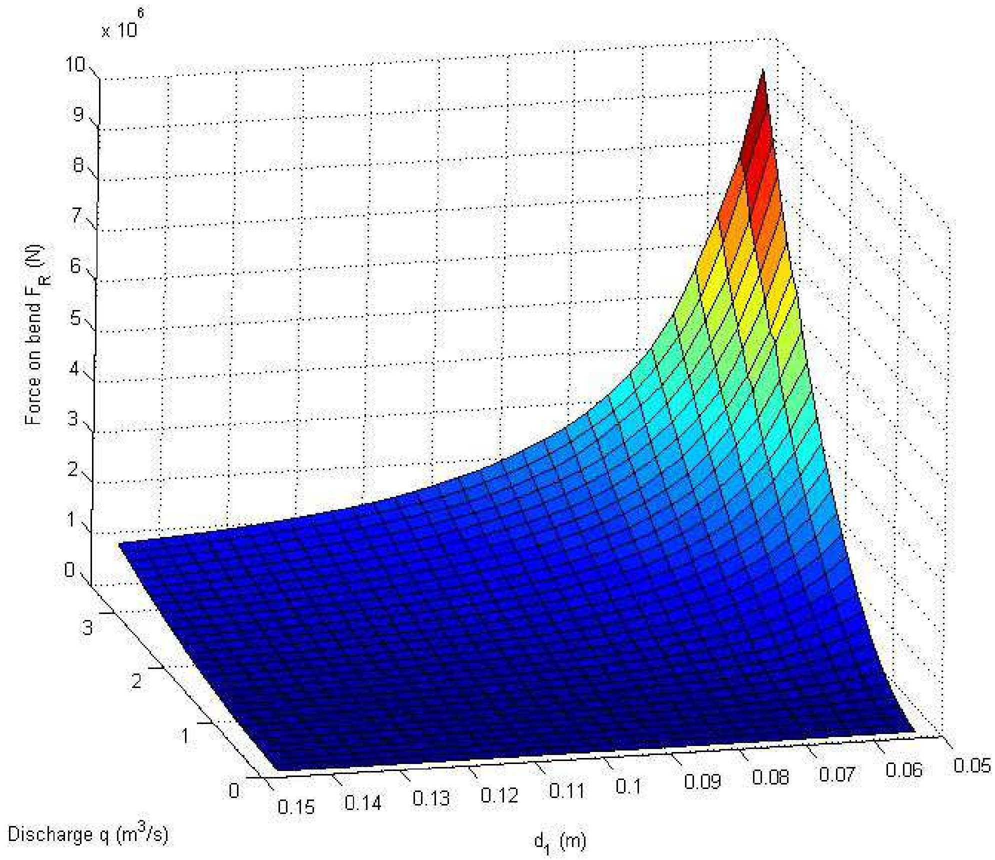

Figure 2 Resultant force on a pipe bend as a function of discharge q and diameter d_1 (d_2 = 0.1m, θ = 70°, P_1 = 1×10^{5}N/m^2)

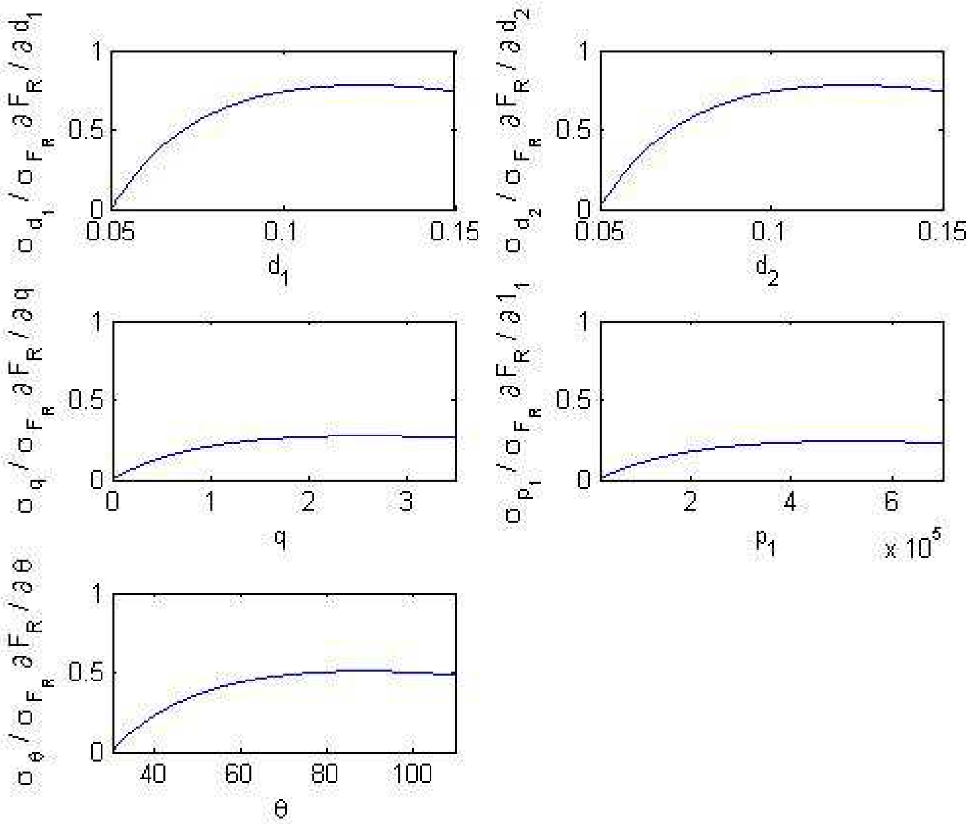

Figure 3 Standardized derivatives

Global sensitivity analysis was based on Monte Carlo sampling of the input factors and calculating the resultant force on the pipe bend. The system response can be visualised in scatter-plots (Figure 4) (Helton, 1993; Kleijnen and Helton, 1999), which plot the resultant force for each member of the Monte Carlo sample as a function of the five input factors. Scatter plots are the simplest form of analysis and can reveal nonlinear relationships, parameter thresholds and, if plotted in two dimensions, variable interactions, which can aid in the understanding of model behaviour (Saltelli et al., 2005). Figure 4 at a glance indicates noticeable sensitivity to Q and non-linear sensitivity to $D_{1.}$

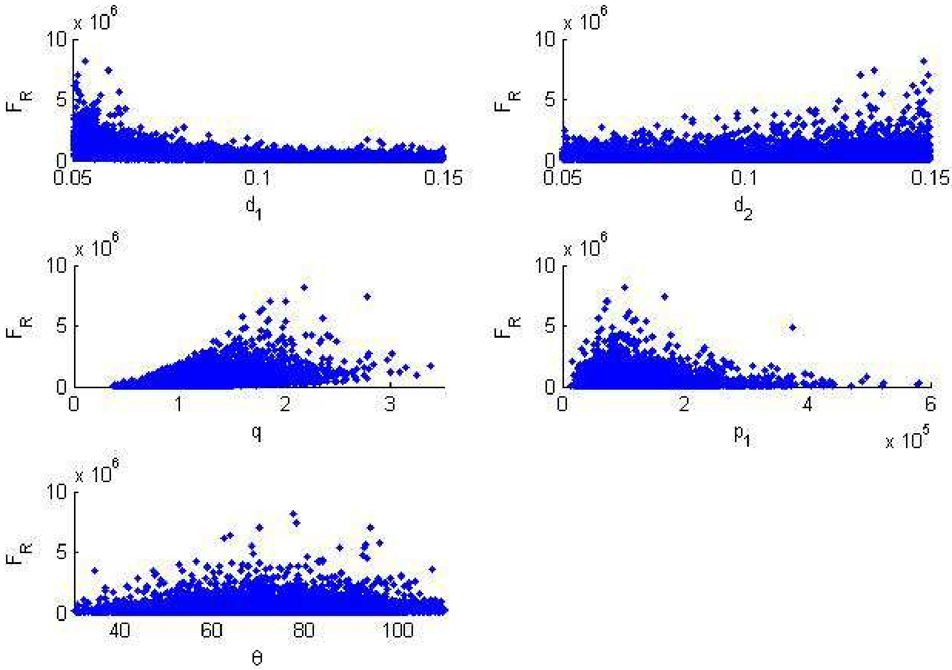

Figure 4 Scatter plots for the resultant force F_R (N) as a function of input factors

Table 2 Sensitivity measures for force on pipe bend

Variables	$\boldsymbol{\beta_i^2}$	$\boldsymbol{S_i}$	$\boldsymbol{S_{Ti}}$
D_1	0.263	0.381	0.765
D_2	0.002	0.014	0.188
Q	0.210	0.215	0.370
P_1	0.000	0.000	0.044
θ	0.015	0.010	0.060
Total	**0.49**	**0.62**	

Table 1 shows that the sensitivity based on the SRCs β_i only captures 49% of the variance of the model output. The coefficient of determination R_Y^2 = 0.53, incidating that the regression model is not appropriate for analysis of this problem. The first order FAST based sensitivity indices, S_i, explain 62% of the total variance. This means that the additive component of the model that is not linear accounts for 13% (ie, 62-49%) of the variability of the model output and implies that at least 38% of the variation is due to nonlinear effects as a result of higher order interactions taking place among the uncertain variables. The same shares can be seen in Table 1 for each input factor separately. Whilst the stanardised derivatives plotted in Figure 3

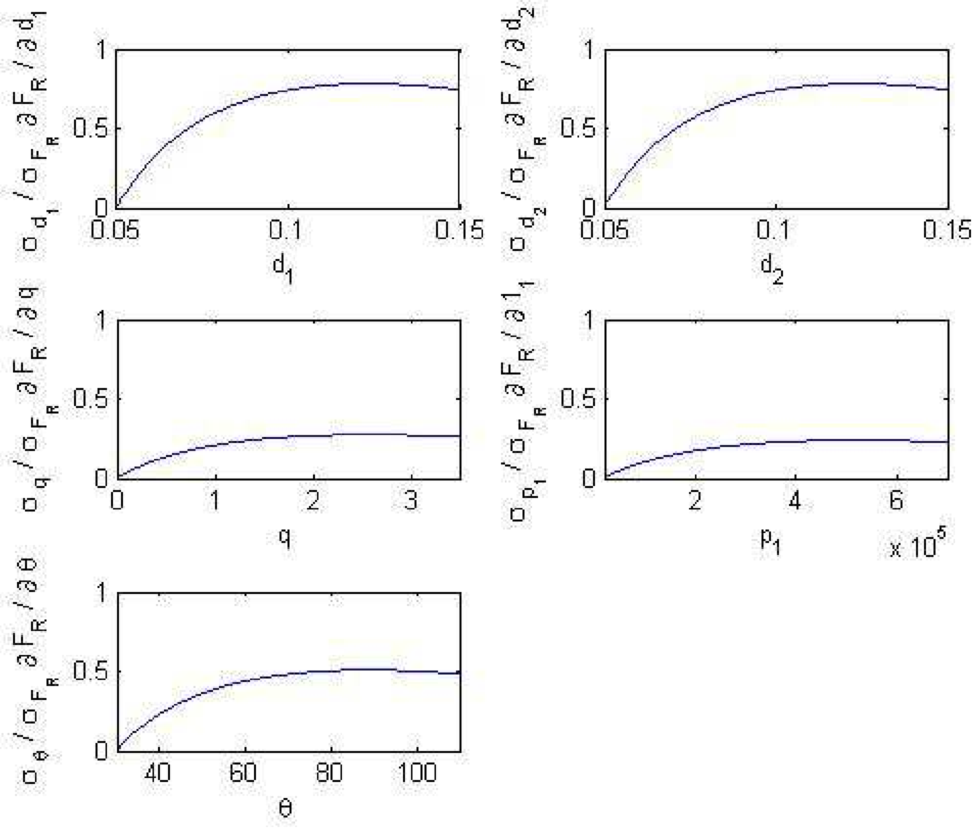

Figure 3indicate that d_2 has as much influence on F_R as d_1, this is not refected in the variance-based sensitivity indices. This reinforces the importance of global analysis and of properly constraining the range of variation of input factors.

Based on the first order analysis, it is evident that the factors that offer the best chance at minimizing the variation of the force on the pipe bend are D_1 and Q, which together account for just under half of the variations the indices capture. However, the size of the unknown interactions suggests that a much larger reduction can be achieved if the interacting factors can be identified. The total sensitivity indices, S_{Ti}, (Table 1) are all more than twice the corresponding first order indices, demonstrating the importance of interactions between the variables.

3.2 Sensitivity analysis of a hydrodynamic model of a dam break experiment

Advanced hydrodynamic simulation codes can exhibit non-linear behaviour and may be sensitive to boundary conditions. They are therefore candidates for more rigorous use of sensitivity analysis methods. Here the shallow water equation solver of Liang, Borthwick and colleagues (Liang et al., 2004; Liang et al., 2008; Liang et al., 2007) has been applied to the benchmark example of a sudden release of water in a flume including an adverse slope.

The simulation model setup corresponds to the flume experiment reported by Brufau (2002) and is illustrated in Figure 5, in which water is retained behind a gate ('dam') which is suddenly removed vertically, allowing the water to rush from left to rigt over the adverse slope. The whole channel is 38m long. The 15.5m long reservoir is connected with the bump by a straight channel of 10m length. The adverse-slope of the bump is 3m long and 0.4m high followed by a slope with 3m length. Initially, the water in the reservoir is still with free water depth z_0. In the dry bed case tested here, the water depth of the channel downstream of the bump is zero. The time taken to remove the gate (dam), is denoted t_r. The experiment lasted roughly 40s. The gauging points, 'G', are defined by the distance (in metres) between the gauging point and the dam. Water surface elevation and velocity were measured at these points.

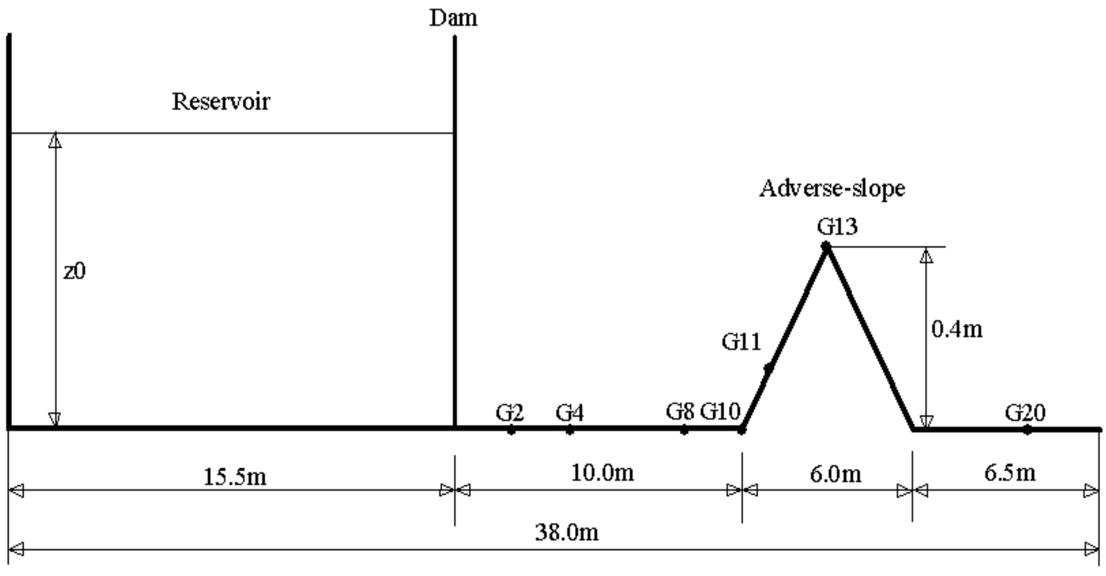

Figure 5 Hydrodynamic model setup

The 2-d shallow water equation solver was set up to reproduce these experimental conditions. The regular orthogonal grid was 100 cells by 10 cells. The slip boundary conditions are imposed to whole channel except the right hand end of the channel, which is set to be open outflow. Each simulation took on average about 1.5s of CPU time to run on a fast PC. Typical outputs of water surface elevation, z, (which can be compared with observations) and velocity, u, at the seven gauges are illustrated in Figure 6.

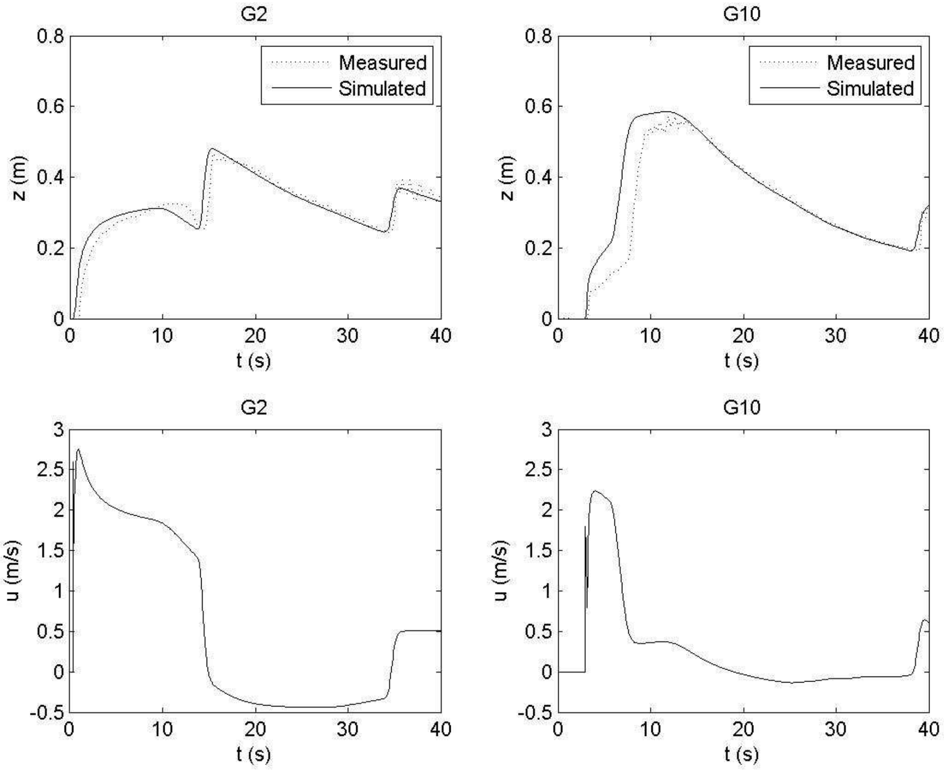

Figure 6 Typical outputs of the shallow water equation solver

The system is sensitive to the dam removal time (t_r), the free water surface elevation in the reservoir (z_0) and the Manning's friction coefficient (n) in the flume, so these were investigated using the distributions given in Table 3. Sensitity analysis was conducted using standardised regression coefficients and extended FAST. Sensitivity analysis results for two gauging points are presented in Table 4. At gauge G4 the SRCs β_i captures 65% of the variance in the prediction of water surface elevation and 81% of the variance in the velocity. The SRCs and FAST sensitivity indices give rather different indications of sensitivity, whith FAST ranking n as the most important variable for prediction of z and t_r as the most important variable for prediction of u. The situation at G10 is more problematic, with a very low coefficient of determination and small sum of the first order FAST indices indicating strong non-linearity and interaction between the variables. To illustrate the variation in sensitivity through the experiment, the FAST first order indices for z and u are plotted over the whole length of the flume (right of the dam) and duration of the experiment in Figure 7. Whilst the pattern of sensitivity illustrated in Figure 7 is complex, it is clear that to the right hand side of the adverse slope (right of G13) Manning's n is the most important variable for determining both z and u. To the left of G13 the interactions between variables are stonger and the sensitivity oscillates, though from t = 15s to roughly t = 30s t_r is the most significant variable in determining both z and u. To understand the complex patters of sensitivity in non-linear models like the one tested here requires computation of the full range of indices and careful scrutiny of their variation on space and time.

Table 3 Distributions of input factors for dam break example

Variables	**Distribution type**	**Parameters**
Dam removal (t_r)	Lognormal	ln(tr) ~ N(0.8, 0.5)
Free surface (z_0)	Normal	z_0 ~ N(0.75, 0.03)
Manning's (n)	Lognormal	ln(n) ~ N(-4.8, 0.5)

Table 4 Comparison of FAST and SRCs for selected gauge points (t=8s) (sensitivity of water surface elevation, z, and velocity, u)

Gauge	**G4: *z***			**G4: *u***			**G10: *z***			**G10: *u***		
Variables	$\boldsymbol{\beta_i^2}$	$\boldsymbol{S_i}$	$\boldsymbol{S_{Ti}}$	$\boldsymbol{\beta_i^2}$	$\boldsymbol{S_i}$	$\boldsymbol{S_{Ti}}$	$\boldsymbol{\beta_i^2}$	$\boldsymbol{S_i}$	$\boldsymbol{S_{Ti}}$	$\boldsymbol{\beta_i^2}$	$\boldsymbol{S_i}$	$\boldsymbol{S_{Ti}}$
t_r	0.37	0.19	0.27	0.15	0.36	0.52	0.01	0.05	0.70	0.01	0.06	0.68
z_0	0.07	0.11	0.16	0.12	0.09	0.17	0.03	0.03	0.59	0.02	0.03	0.53
n	0.21	0.52	0.62	0.55	0.31	0.45	0.26	0.16	0.89	0.25	0.19	0.90
Total	**0.65**	**0.83**		**0.81**	**0.77**		**0.29**	**0.24**		**0.27**	**0.28**	

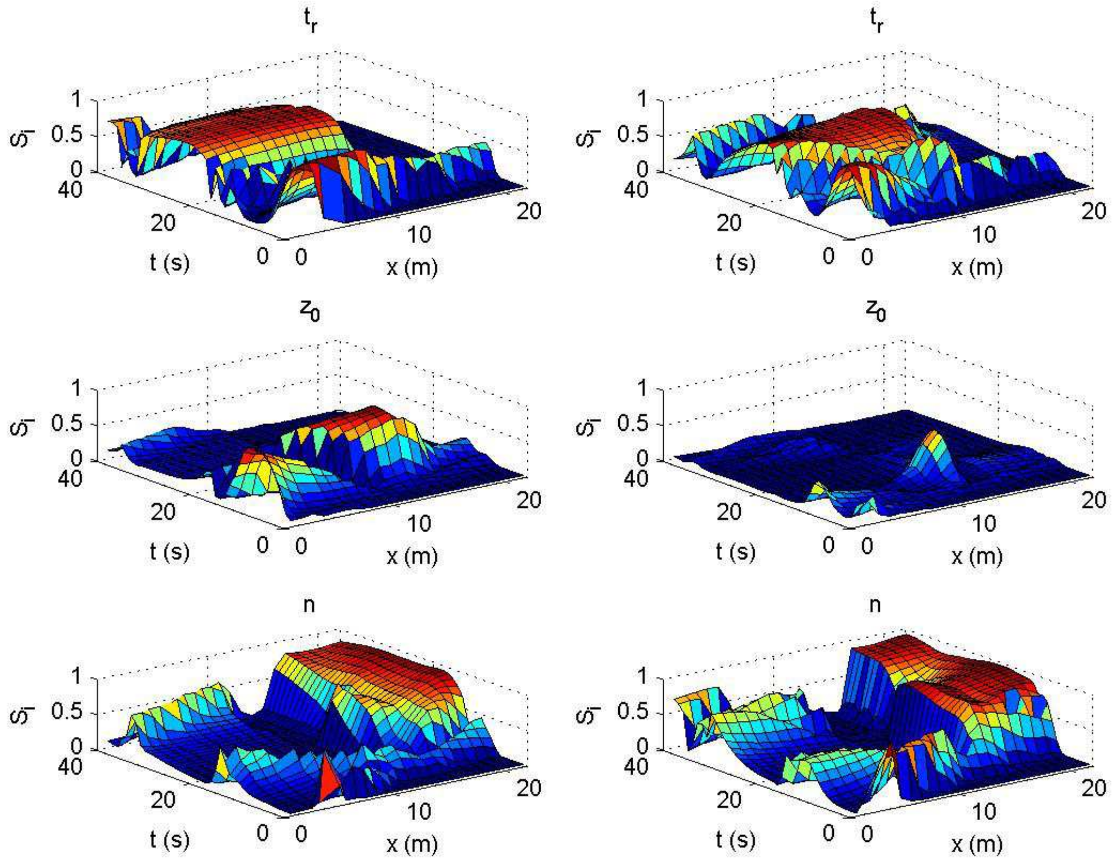

Figure 7 First order FAST sensitivity indices for plotted against distance down the flume (*x*) and time during the experiment (*t*) (left panels = surface elevation, *z*; right panels = velocity, *u*)

4 Choice of methods

The choice of the most appropriate sensitivity analysis technique depends upon:

- the computational cost of running the model,
- the number of input factors,
- the degree of non-linearity of the model over the range of input factors,
- the degree of complexity of the model coding,
- the amount of analyst's time available for sensitivity analysis, and,
- the setting for the analysis.

Exploratory analysis with scatter plots and computation of standardized regression coefficients (SRCs) can provide initial insights into the degree of non-linearity in model response and a computationally cheaper alternative to the variance-based methods. With a single batch of n sampled points the SRCs and their rank transformed version can be estimated for all the input factors, though, as already discussed, the SRC's are only effective for linear or quasi-linear models, i.e. for R^2 greater than about 0.7. Manache and Melching (2008) provide a thorough review of regression and correlation measures measures and their properties and provide guidance on selecting the most robust and reliable measures for practical use

If the coefficient of determination of a linear model is high then SRCs are a convenient sensitivity index to use. If this is not the case, then for models that require a modest amount CPU time (i.e. up to the order of 1 minute per run), and with a number of input factors which does not exceed, say, 20, the variance-based techniques yield the more accurate pattern of sensitivity (Tang et al., 2006). Both the method of Sobol' (Saltelli, 2002; Saltelli et al., 2000; Sobol', 1993) and the extended FAST (Saltelli et al., 1999) provide all the pairs of first order and total indices at a cost of $(k+2)n$ model runs for Sobol' and $\approx kn$ model runs for the extended FAST. Typically n is between 500 and 1000. To give an order of magnitude of the computational requirement, for a model with 10 factors and 0.5 minutes of CPU time per run, a good characterization of the system via S_i and S_{Ti} can be obtained at the cost of ~50 hours of CPU time.

With the method of Sobol', in addition to the first order and total indices computed with $(k+2)n$ model runs, all the interaction terms of order k-2 can be obtained at no extra cost. At the additional cost of kn model runs, *double* estimates of all the first, second, (k-2)-th order and total indices can be obtained. Finally, any other interaction term between the third and the (k-3)-th can be estimated at the further additional cost of n model runs each (Saltelli, 2002).

When the input factors are correlated, an *ad hoc* computational scheme must be adopted. An efficient and unbiased estimation procedure is available for first order indices and is based on *replicated* Latin hypercube sampling (Hamm et al., 2006; McKay et al., 1979), in which the cost to estimate all the first order indices is nr

model runs, where r is the number of replicates needed (usually around 100), and the cost is independent of the number of factors.

For higher order indices as well as total indices in the case of correlated input one has to apply a brute force approach whereby the operators V and E (Equation (5)) are to be written in explicit form i.e. as the variance of a mean, involving a double computing loop. The computational cost is thus nr model runs for each index.

When the CPU time increases (say, up to ten minutes per run), or the number of factors increases (say, up to one hundred), the method of Morris (1991) offers the best result. The number of sampled points required is $n_{Morris}=r(k+1)$, where r is generally set to between 4 and 8 and k is the number of input factors. To make an example, with 80 factors and 5 minutes CPU time per run, all the model outputs can be ready in 27 hours if $r = 4$ is taken.

When the number of input factors and/or the CPU time is so large as to even preclude the use of the method of Morris (1991), then supersaturated fractional factorial designs, where factors are iteratively perturbed in batches, can be used (Campolongo et al., 2000; Iman and Hora, 1990). However, these methods do not provide an effective exploration of the space of the inputs, as they mostly operate at very few factor levels and require strong assumptions on the model behaviour.

Automatic differentiation techniques can also be used when CPU time is very large. Derivatives are inherently local sensitivity indices. In addition, they require intervention of the analyst in the computer code that implements the model. However, for expensive models, these methods may provide some insight into the importance of input factors. If higher order derivatives are computed, these give information about multi-factor curvature effects e.g. second order term of the type $\frac{\partial^2 Y}{\partial X_i \partial X_j}$ gives information about a possible interaction effect between X_i and X_j.

An alternative to the direct use of sampling-based sensitivity methods is to construct a computationally efficient emulator of the numerical model in question and conduct sensitivity analysis using the emulator function. Emulator methods based on the use of Gaussian processes can yield variance-based sensitivity indices directly (Oakley and O'Hagan, 2004). Gaussian process emulators have proved to be very efficient, even for complex non-linear models (Hankin, 2005). However, they are based upon an assumption of smoothness over a continuous input space, so are not universally appropriate, and can be hard to identify in high-dimensional problems. In high dimensional problems a variety of kernel regression (Storlie and Helton, 2006), HDMR (Li et al., 2002) and filtering methods (Ratto et al., 2007a) have been developed for constructing emulators over the most influential factors.

Sensitivity analysis is also driven by the setting. When the purpose of the analysis is to prioritise factors, the first order sensitivity indices S_i have a strong motivation for use. If the objective is to fix non-influential factors, then the total sensitivity indices S_{Ti}, or the measure of Morris (1991), are more appropriate. If a particular region in the space of the output (e.g. above or below a given threshold) is of interest, then Monte Carlo filtering can complement the measures just mentioned. In all of these settings, the computation of derivatives, especially if achieved with a modicum of extra computing, is advisable for a general understanding of the model.

Sensitivity analysis can also be extended to address issues of model choice. For example there may be alternative representations of a particular process within a simulation model. These alternative process modules can be indixed by an integer “switching” variable, which is then included in the uncertainty analysis, usually with a discrete uniform distribution over the set of possible modules (Tarantola et al., 2002). The sensitivity index of the switching variable can be compared with the sensitivity to parameter uncertainties. This type of analysis is particularly attractive in the context of complex coupled systems of models, where there may be a number of permutations of model choice and the influence of those choices on predictions are not necessaryily intuitive.

5 Conclusions

Sensitivity analysis is an essential aspect of responsible model use, particularly at a time when models are becoming more complex and are being coupled in order to address multidisciplinary problems. Decision makers who make use of hydraulic model results can legitimately request thorough analysis of the sensitivity of the results to plausible variations in the model inputs. This information can be used to target data acquisition and engineering design decisions more effectively by identifying the parameters that exert the greatest influence on system performance.

This paper has sought to introduce members of the hydraulic engineering and research community to methods of sensitivity analysis with which they may not be familiar. It has been demonstrated how the incautious use of local derivatives or regression coefficients can lead to misleading conclusions which do not represent the full range of model behaviour for non-linear models. Scatter and surface plots provide a useful visual impression of sensitivity but for a limited number of input factors. Variance-based methods can deal with interacting input factors and provide an impression of sensitivity averaged over the range of input response. However, the use of variance-based methods requires probability distributions for the input factors. Some sense of the range of variability of the input factors is to be expected for model applications, but the meaning of input variance is less clear in the context of a numerical model being tested prior to application. In that case the use of uniform distributions over the range of applicability is perhaps the most sensible approach.

The use of first order and total variance sensitivity indices has been demonstrated to diagnose the effect of input factors acting individually or in combination, in the latter case in order to identify the effect of interactions between variables. A further attraction of the use of variance-based methods is that they are model independent and can be applied with no modification to the model code.

The computational limitations of sensitivity analysis have been discussed. In situations with computationally very expensive models or large numbers of inputs a sequential approach to sensitivity analysis, which proceeds through a process of screen and hierarchical decomposition of groups of variables is required. Where model response is reasonably smooth but very expensive to compute, the use of emulator functions can yield large computational savings.

6 Acknowledgements

The research described in this paper was funded in part by the UK Engineering and Physcial Sciences Research Council under grants GR/S76304/01 and EP/F020511/1.

Notation

b_i	regression coefficient
D_1, D_2	random variables for diameters
d_1, d_2	pipe diameters
E	expectation operator
E_{X_i}	expectation with respect to input factor X_i
$E_{\mathbf{X}_{\sim i}}$	expectation with respect to all input factors other than X_i
F_x, F_y	orthogonal pipe force components
F_R	magnitude of resultant force
g	acceleration due to gravity
i, j, l	counters
k	number of input factors
n	number of realisations in a statistical simulation
P_1, P_2	random variables for pressures
p_1, p_2	pressures
Q	random variable for discharge in pipe
q	discharge in pipe
R_Y^2	coefficient of determination
r	residual
S_i	first order variance-based sensitivity index for factor X_i
S_{Ti}	total order variance-based sensitivity index for factor X_i
S_i^r	non-dimensionalised derivatives based sensitivity index
S_i^σ	sigma standardized derivatives based sensitivity index
V	variance operator
V_i	variance component from factor i
X_i	model input factor, i (random variable)
$\tilde{X}_i$	standardized variable X_i
$\mathbf{X}_{\sim i}$	the vector of all factors other than X_i.

x_i	model input factor, i
x_i^*	known value of model input factor X_i
$\overline{x}_i$	is the nominal value of factor X_i
Y	model output (random variable)
$\tilde{Y}$	standardized variable Y
y	model output
$\overline{y}$	value taken by Y when all input factors are at their nominal value
$\hat{y}_i$	prediction of y_i from a regression model
β_i	standardized regression coefficient
θ	pipe bend angle
μ_i	mean of factor X_i
μ_Y	mean of Y
σ_i	standard deviation of factor X_i
σ_Y	standard deviation of Y

References

Box, G., Hunter, W. and Hunter, J., 1978. Statistics for Experimenters, An Introduction to Design, Analysis, and Model Building. John Wiley & Sons, New York.

Brufau, P., Vázquez-Cendón, M.E. and García-Navarro, P., 2002. A numerical model for the flooding and drying of irregular domains. International Journal for Numerical Methods in Fluids, 39: 247-275.

Cacuci, D.G., 2003. Sensitivity & Uncertainty Analysis, Volume 1: Theory. Chapman & Hall/CRC, New York.

Campolongo, F., Kleijnen, J. and Andres, T., 2000. Screening methods. In: A. Saltelli, K. Chan and M. Scott (Editors), Sensitivity Analysis. John Wiley & Sons, Chichester, pp. 65-80.

Cawlfield, J.D., 2000. Reliability Algorithms: FORM and SORM Methods. In: A. Saltelli, K. Chan and M. Scott (Editors), Sensitivity Analysis. John Wiley & Sons, Chichester, pp. 155-166.

Chen, X. and Chen, X., 2003. Sensitivity Analysis and Determination of Streambed Leakance and Aquifer Hydraulic Properties. Journal of Hydrology, 284: 270-284.

Cornell, C.A., 1972. First-Order Analysis of Model and Parameter Uncertainty, Proc. Int. Symp. on Uncertainties in Hydrology and Water Resources Systems, University of Arizona, Tucson, pp. 1245-1272.

Cukier, R., Fortuin, C., Shuler, K., Petschek, A. and Schaibly, J., 1973. Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients. I Theory. Journal of Chemical Physics, 59(8): 3873-3878.

Cukier, R., Levine, H. and Shuler, K., 1978. Nonlinear Sensitivity Analysis of Multiparameter Model Systems. Journal of Computational Physics, 26: 1-42.

Cukier, R., Schaibly, J. and Shuler, K., 1975. Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients. III. Analysis of the approximations. Journal of Chemical Physics, 63(3): 1140-1149.

Dawson, R.J., Hall, J.W., Sayers, P.B., Bates, P.D. and Rosu, C., 2005. Sampling-based flood risk analysis for fluvial dike systems. Stochastic Environmental Research and Risk Analysis, 19(6): 388-402.

Ferson, S. and Tucker, W.T., 2006. Probability Bounds Analysis Is a Global Sensitivity Analysis. Reliability Engineering and Systems Safety, 91(10-11): 1435-1442.

Grievank, A., 2000. Principles and Techniques of Algorithmic Differentiation: Evaluating Derivatives. SIAM Publisher, Philadelphia, PA.

Hall, J.W., 2006. Uncertainty-based sensitivity indices for imprecise probabilities. Reliability Engineering and Systems Safety, 91(10-11): 1443-1451.

Hall, J.W. et al., 2006. Attribution of flood risk in urban areas, Proc. 7th Int. Conf. on Hydroinformatics. IWA, Nice.

Hall, J.W., Tarantola, S., Bates, P.D. and M.S., H., 2005. Distributed sensitivity analysis of flood inundation model calibration. Journal of Hydraulic Engineering, 131(2): 117-126.

Hamm, N.A., Hall, J.W. and Anderson, M.G., 2006. Variance based sensitivity analysis of the probability of hydrologically induced slope instability. Computers and Geosciences, 32(6): 803-817.

Hankin, R., 2005. Introducing BACCO, an R Bundle for Bayesian Analysis of Computer Code Output. Journal of Statistical Software, 14(16).

Helton, J., 1993. Uncertainty and sensitivity analysis techniques for use in performance assessment for radioactive waste disposal. Reliability engineering & systems safety, 42(2-3): 327-367.
Helton, J.C., Johnson, J.D., Sallaberry, C.J. and Storlie, C.B., 2006. Survey of sampling-based methods for uncertainty and sensitivity analysis. SAND2006-2901, Sandia National Laboratory.
Homma, T. and Saltelli, A., 1996. Importance measures in global sensitivity analysis of nonlinear models. Reliability Engineering and System Safety, 52(1): 1-17.
Hornberger, G.M. and Spear, R.C., 1981. An approach to the preliminary analysis of environmental systems. Journal of Environmental Management, 12(1): 7-18.
Horritt, M.S., 2006. A linearized approach to flow resistance uncertainty in a 2-D finite volume model of flood flow. Journal of Hydrology, 316: 13-27.
Iman, R. and Hora, S., 1990. A robust measure of uncertainty importance for use in fault tree system analysis. Risk Analysis, 10(3): 401-406.
Indelman, P., Dagan, G., Cheng, A. and Ouazar, D., 2006. Sensitivity Analysis of Flow in Multilayered Leaky Aquifer Systems. Journal of Hydraulic Engineering, 122(1): 41-45.
Jaffe, P.R. and Ferrara, R.A., 1984. Modeling Sediment and Water Column Interactions for Hydrophobic Pollutants. Parameter Discrimination and Model Response to Input Uncertainty. Water Research, 18(9): 1169-1174.
Johnson, P.A., 1996. Uncertainty of hydraulic parameters. Journal of Hydraulic Engineering, 122(2): 112-114.
Kabala, Z.J., 2001. Sensitivity Analysis of a Pumping Test on a Well with Wellbore Storage and Skin. Advances in Water Resources, 24: 483-504.
Kleijnen, J.P. and Helton, J.C., 1999. Statistical Analyses of Scatterplots to Identify Important Factors. Reliability Engineering and System Safety, 65: 147-185.
Li, H., Wang, S.-W. and Rabiz, H., 2002. Practical approaches to construct RS-HDMR component functions. Journal of Physical Chemistry, 106: 8721-8733.
Liang, Q., Borthwick, A.G.L. and Stelling, G., 2004. Simulation of Dam and Dyke-break Hydrodynamics on Dynamically Adaptive Quadtree Grids. International Journal for Numerical Methods in Fluids, 46: 127-162.
Liang, Q., Du, G., Hall, J.W. and Borthwick, A.G.L., 2008. Flood inundation modelling with an adaptive quadtree grid shallow water equation solver. Journal of Hydraulic Engineering, ASCE, in press.
Liang, Q., Zang, J., Borthwick, A.G.L. and Taylor, P.H., 2007. Shallow Flow Simulation on Dynamically Adaptive Cut-cell Quadtree Grids. International Journal for Numerical Methods in Fluids, 53(12): 1777-1799.
Liepmann, D. and Stephanopoulos, G., 1985. Development and global sensitivity analysis of a closed ecosystem model. Ecological Modelling, 30(1): 13-47.
Manache, G. and Melching, C.S., 2004. Sensitivity Analysis of a Water-Quality Model Using Latin Hypercube Sampling. Journal of Water Resources Planning and Management, 130(3): 232-242.
Manache, G. and Melching, C.S., 2008. Identification of Reliable Regression- and Correlation-Based Sensitivity Measures for Importance Ranking of Water-Quality Model Parameters. Environmental Modelling and Software, 23(5): 549-562.
McKay, M., Conover, W. and Beckman, R., 1979. A comparison of three methods for selecting values of input variables in the analysis ofoutput froma computer code
Technometrics, 21: 239-245.

Melching, C.S., 1995. Reliability Estimation. In: V.P. Singh (Editor), Computer Models of Watershed Hydrology. Water Resources Publications, Littleton, CO, pp. 69-118.

Melching, C.S., 2001. Sensitivity Measures for Evaluating Key Sources of Modeling Uncertainty, Third International Symposium on Environmental Hydraulics. Mira Digital Publishing, Tempe, Arizona.

Melching, C.S. and Bauwens, W., 2001. Uncertainty in Coupled Nonpoint Source and Stream Water-Quality Models. Journal of Water Resources Planning and Management, ASCE, 127(6): 403-413.

Morris, M., 1991. Factorial Sampling Plans for Preliminary Computational Experiments. Technometrics, 33(2): 161-174.

Nash, G.A. and Karney, B.W., 1999. Efficient Inverse Transient Analysis in Series Pipe Systems. Journal of Hydraulic Engineering, 125(7): 761-764.

O'Hagan, A., Kennedy, M. and Oakley, J., 1999. Uncertainty analysis and other inference tools for complex computer codes. Bayesian Statistics, 6: 503-524.

Oakley, J.E. and O'Hagan, A., 2004. Probabilistic sensitivity analysis of complex models: a Bayesian approach. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 66 (3): 751-769.

Oliver, Y.M. and Smettem, K.R.J., 2005. Predicting water balance in a sandy soil: model sensitivity to the variability of measured saturated and near saturated hydraulic properties. Australian Journal of Soil Research, 43: 87-96.

Pappenberger, F. et al., 2006. Influence of uncertain boundary conditions and model structure on flood inundation predictions. Advances in Water Resources, 29(10): 1430-1449.

Pappenberger, F., Ratto, M., Beven, K.J. and Matgen, P., 2008. Multi-method global sensitivity analysis of flood inundation models. Advances in Water Resources, 31(1): 1-14.

Podsechin, V., Tejakusuma, I., Schernewski, G. and Pejrup, M., 2006. On parameters estimation in dynamic model of suspended sediments. Journal of Hydrology, 318: 17-23.

Rabitz, H., 1989. Systems Analysis at the Molecular Scale. Science, 246(4927): 221.

Ratto, M., Pagano, A. and Young, P., 2007a. State Dependent Parameter metamodelling and sensitivity analysis. Computer Physics Communications, 177(11): 863-876.

Ratto, M. et al., 2007b. Uncertainty, sensitivity analysis and the role of data based mechanistic modeling in hydrology. Hydrology and Earth Systems Science, 11: 1249-1266.

Renault, D. and Hemakumara, H.M., 1999. Irrigation Offtake Sensitivity. Journal of Irrigation and Drainage Engineering, 125(3): 131-136.

Rocha, D., Abbasi, F. and Feyen, J., 2006. Sensitivity Analysis of Soil Hydraulic Properties on Subsurface Water Flow in Furrows. Journal of Irrigation and Drainage Engineering, 132(4): 418-424.

Saltelli, A., 2002. Making best use of model valuations to compute sensitivity indices. Computer Physics Communications, 145: 280-297.

Saltelli, A., K Chan and Scott, M., 2000. Sensitivity Analysis. Wiley, New York.

Saltelli, A., Ratto, M., Andres, T. and Campolongo, F., 2008. Global Sensitivity Analysis: The Primer Wiley, New York.

Saltelli, A., Ratto, M., Tarantola, S. and Campolongo, F., 2005. Sensitivity Analysis of Chemical Models. Chemical Reviews, 105(7): 2811-2827.

Saltelli, A., Tarantola, S., Campolongo, F. and Ratto, M., 2004. Sensitivity Analysis in Practice: A Guide to Assessing Scientific Models. Halsted Press, New York.

Saltelli, A., Tarantola, S. and Chan, K., 1999. A Quantitative Model-Independent Method for Global Sensitivity Analysis of Model Output. Technometrics, 41(1): 39-56.

Schaibly, J. and Shuler, K., 1973. Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients. II Applications. Journal of Chemical Physics, 59(8): 3879-3888.

Siebera, A. and Uhlenbrook, S., 2005. Sensitivity analyses of a distributed catchment model to verify the model structure. Journal of Hydrology, 310: 216-235.

Sobol', I., 1993. Sensitivity analysis for non-linear mathematical models. Mathematical Modelling & Computational Experiment, 1: 407-414.

Storlie, C.B. and Helton, J.C., 2006. Multiple predictor smoothing methods for sensitivity analysis, Sandia National Laboratory.

Swaminathan, M., Rankin, G. and Sridhar, K., 1986. Evaluation of the basic systems of equations for turbulence measurements using the Monte Carlo technique. Journal of Fluid Mechanics, 170: 1-19.

Tang, W.H. and Yen, B.C., 1972. Hydrologic and Hydraulic Design Under Uncertainties, Proc. Int. Symp. on Uncertainties in Hydrology and Water Resources Systems, University of Arizona, Tucson, pp. 868-882.

Tang, Y., Reed, P., Wagener, T. and van Werkhoven, K., 2006. Comparing sensitivity analysis methods to advance lumped watershed model identification and evaluation. Hydrology and Earth System Sciences Discussions, 3: 3333-3395.

Tarantola, S., Giglioli, N., Jesinghaus, J. and Saltelli, A., 2002. Can Global Sensitivity Analysis steer the implementation of models for environmental assessments and decision-making? Stochastic Environmental Research and Risk Assessment, 16(1): 63-76.

Turanyi, T. and Rabitz, H., 2000. Local methods. In: A. Saltelli, K. Chan and E.M. Scott (Editors), Sensitivity Analysis. Wiley, New York, pp. 81–99.

Vrijling, J.K., 2001. Probabilistic design of water defense systems in The Netherlands. Reliability Engineering and System Safety, 74: 337-344.

Xu, Y. and Mynett, A.E., 2006. Application of uncertainty and sensitivity analysis in river basin management. Water Science and Technology, 53(1): 41-49.

Yeh, K.-C. and Tung, Y.-K., 1993. Uncertainty and Sensitivity Analyses of Pit-Migration Model. Journal of Hydraulic Engineering, ASCE, 119(2): 262-283.

Yen, B.C. and Tung, Y.-K., 1993. Reliability and Uncertainty Analyses in Hydraulic Design. ASCE, New York.