Stock Return Predictability Despite Low Autocorrelation

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Abstract

This paper shows that short horizon stock returns can be predicted to a much greater degree by past price movements than would be anticipated given their low autocorrelation. This raises doubts over the reliability of the autocorrelation statistic as a measure of stock market predictability.

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JEL classification: G1, G14
1. Introduction:

From the early work of Fama (1970) low autocorrelations in short term stock returns (price changes) have been used to infer a lack of predictability. However, returns are only truly unpredictable when they are statistically independent (i.e. the value of a return in one time period has no bearing on the value of subsequent returns) and the absence of autocorrelation does not imply the independence of returns; independence requires that all nonlinear functions of returns also have no autocorrelation (Cont, 2001).

The purpose of this paper is to show that short horizon stock returns can be predicted to a much greater degree by past returns than would be anticipated given their low autocorrelation. In informal terms, if an overall series of returns has low autocorrelation but a subset of it, for example observations after large price movements, is predictable one might expect to see some offsetting predictability amongst observations not in that subset.

In this study we formalize the above proposition by reference to the mathematical properties of the autocorrelation measure and then provide empirical evidence for the proposition.

The rest of the paper is organized as follows. Section 2 provides a review of the existing literature. Section 3 presents our theoretical analysis. The data are described in section 4. Section 5 presents the empirical results and section 6 provides conclusions.

2. Existing Literature – Large Price Changes

To show that predictability of stock prices can be achieved even with very low levels of autocorrelation, we extend the literature on large price changes. A substantial body of work has dealt with the short term reaction of stock market securities to large preceding price movements (for example, Brown et al., 1988, 1993, Atkins and Dyl, 1990, Bremer
and Sweeney, 1991, Cox and Peterson, 1994, Park, 1995, Lasfer et al., 2003, Mazouz et al., 2009). This literature has found strong elements of short term predictability amongst securities following large price changes, with price reversals often being observed, although the implications of this for the general predictability of security price returns after price changes of all sizes remain unexplored.

3. Theoretical Analysis

In this section we outline the relevant properties of autocorrelation and how these relate to previous empirical work on large price changes. We then show how previous approaches can be extended and generalized to examine responses to the full range of price changes.

3.1 Properties of Autocorrelation

The autocorrelation of a time series with constant expected return is given by

\[
\frac{\text{E}[\text{R}_t, \text{R}_{t-1}]}{\text{E}[\text{R}_t] \text{E}[\text{R}_{t-1}]} = \frac{\text{E}[\text{R}_t, \text{R}_{t-1}]}{\mu^2} - (1)
\]

Where \( \mu \) is a constant equal to the expected return.

Now, if (1) is small this could either imply that \( \text{R}_t \) and \( \text{R}_{t-1} \) are independent or that there could be one of a set of non-linear relationships between them.

Discrete or continuous empirical investigations can be used to investigate the nature of the relationship between \( \text{R}_t \) and \( \text{R}_{t-1} \) as discussed below.

3.2 Discrete Empirical Analysis

\( \text{R}_{t-1} \) can be divided into a number of groups (bands) by size

Say we use bands \( B_i, i = 1, 2, \ldots, n \)
Where $R_{t-1}$ falls in:

$B_1$ if $R_{t-1} < -c_1$; $B_2$ if $-c_1 \leq R_{t-1} < -c_2$; ....; $B_n$ if $c_n \leq R_{t-1}$

Where $c_1, c_2, \ldots, c_n$ are constant such that $c_i \leq c_{i+1}$

Applying Bayes Theorem

$$E[R_t, R_{t-1}] = \sum_{i=1}^{n} E[R_t, R_{t-1} | R_{t-1} \in B_i] \Pr[R_{t-1} \in B]$$ (2)

3.2.1 Implications of Previous Findings

Previous empirical investigations have focused on expected returns after large price movements. It is easily shown that if returns are predictable after large price changes this implies predictability after price changes of all sizes. For example, we may assume, given the findings of many prior studies, that reversals occur after large price movements i.e.

$$E[R_t | R_{t-1} < -c] > 0$$
$$E[R_t | R_{t-1} > c] < 0$$

Where $c$ is positive and relatively large.

From equation (2)

$$E[R_t, R_{t-1}] = E[R_t, R_{t-1} | R_{t-1} < -c] \Pr[R_{t-1} < -c]$$
$$+ E[R_t, R_{t-1} | -c \leq R_{t-1} \leq c] \Pr[-c \leq R_{t-1} \leq c]$$
$$+ E[R_t, R_{t-1} | R_{t-1} > c] \Pr[R_{t-1} > c]$$

Now if autocorrelation is small say $\delta$

$$E[R_t, R_{t-1}] = E[R_t, R_{t-1} | R_{t-1} < -c] \Pr[R_{t-1} < -c]$$
$$+ E[R_t, R_{t-1} | -c \leq R_{t-1} \leq c] \Pr[-c \leq R_{t-1} \leq c]$$
$$+ E[R_t, R_{t-1} | R_{t-1} > c] \Pr[R_{t-1} > c] = \delta$$

But

$$E[R_t, R_{t-1} | R_{t-1} < -c] < 0 \ \text{as} \ R_{t-1} < 0 \ \text{by definition and} \ E[R_t | R_{t-1} < -c] > 0$$
$$E[R_t, R_{t-1} | R_{t-1} > c] < 0 \ \text{as} \ R_{t-1} > 0 \ \text{by definition and} \ E[R_t | R_{t-1} > c] < 0$$

Therefore $E[R_t, R_{t-1} | -c \leq R_{t-1} \leq c] \geq \delta$
Thus if price reversals are observed after large absolute price movements in a series with low autocorrelation price trends must be more likely to continue after small price movements.

3.3 Extensions to Previous Findings

3.3.1 Discrete Analysis using Multiple Bands.

Previous studies have generally investigated returns after large, arbitrarily chosen, price movements. One problem with this approach is that different large return thresholds could give different conclusions about the properties of the same time series. Another problem is that this type of analysis provides no information about the shape of the main part of the distribution of returns after prior returns which are not considered large in magnitude. Both these problems can be mitigated by using a number of bands that cover the entire space of prior returns.

3.3.2 Continuous Analysis

A drawback of discrete bands is that the bands are arbitrary and will introduce discontinuities into what is presumably a continuous underlying distribution.

We can assume that there is a continuous function $G$ such that

$$R_i = G(R_{i-1}) + \epsilon_i$$

As there are an infinite number of possible functions $G$ we consider polynomial series of the form:

$$R_i = a_0 + a_1 R_{i-1} + a_2 R^2_{i-1} + \ldots + \epsilon_i$$

These can be fitted parsimoniously using information on the shape of the curve from the discrete case.

4. Data

The daily behaviour of 30 stocks traded on the London Stock Exchange is examined for the period from August 1987 through to August 2007. To ensure a range of company sizes, ten stocks have been chosen randomly from amongst those that have traded throughout the period as part of each of the FTSE 100, FTSE 250 and FTSE SmallCap...
indices. Table 1 shows descriptive statistics for the data. The typical characteristics of such series are observed with a low daily mean return, relatively high standard deviation with the presence of some large absolute returns and low autocorrelation.

5. Empirical Results

5.1 Discrete Analysis
Returns are classified into one of a number of bands covering the entire range of stock price movements as shown in Table 2 which also reports the average returns and standard deviation of returns within each band.

Different tests have been performed to test the null hypothesis that the average return on the day after a return in each band is equal to the average daily return of the stocks over the whole period of investigation. Initially a t-test is calculated using the standard deviation of the daily returns of all the stocks over the whole period of investigation. Secondly a t-test is calculated using the standard deviation of returns (on the day after an event) for events falling into particular band under consideration. This approach has been taken by a number of previous studies (see Brown et al., 1988, and Cox and Peterson, 1994) and allows for the fact that returns are unlikely to be generated from a distribution with constant variance. Finally a nonparametric binomial test which assumes neither constant variance nor that returns are normally distributed is calculated (see Bremer and Sweeney, 1991, and Cox and Peterson, 1994).

The three tests generally indicate that the null hypothesis can be rejected with a high level of significance. For positive returns between 0% and 10% and negative returns between 0% and -10% there is strong evidence that the average return on the next day is greater/smaller than the average daily return of the stocks over the whole period, respectively. Indeed the average daily return is negative after returns between 0% and -10%. For returns less than -10% the average next day return is positive and there is strong evidence it is greater than the average daily return of the stocks over the whole period.
For positive returns over 10%, the average return on the next day is negative and there is some evidence that it is significantly smaller than the average daily return of the stocks over the whole period. In general, the results presented in the table indicate that after very large price changes the market tends to reverse, while after smaller price changes there is a tendency for price trends to continue.

5.2 Continuous Analysis
Polynomial regression analysis is used to investigate the data in a continuous context as discussed in section 3.3.2. Given the observed reversals after large price changes and price continuation after small price changes there are evidently two turning points in the function indicating that a cubic polynomial is the most parsimonious that can capture the principle features of the data, thus we fit:

\[ R_t = a_0 + a_1 R_{t-1} + a_2 R^3_{t-1} + \epsilon_t \]

A panel data technique using a fixed effect model was chosen. The results are reported in Table 3 and show that the model has a very high level of significance.

6. Conclusions
In this study we document strong evidence for short term predictability of individual stock returns in the London Stock Exchange. We find strong empirical evidence for price reversals after large price changes and trend continuation after small price changes. We show that our findings are consistent with both previous research on large price changes in stock markets and the low measured autocorrelations of the return series involved. This raises doubts over the reliability of the autocorrelation statistic as a measure of the predictability of stock returns.
References:


