Compartment Level Progressive Collapse Analysis of Lightweight Ship
Structures

Abstract

The continued development of large high speed ships, often constructed from aluminium alloy, has raised important issues regarding the response of lightweight hull girders under primary hull girder bending. In particular, the response of lightly framed panels in compression may be influenced by overall panel buckling over several frame spaces. Therefore, to provide improved ultimate strength prediction for lightweight vessels, an extended progressive collapse methodology is proposed. The method has capabilities to predict the strength of a lightweight aluminium midship section including compartment level buckling modes. Nonlinear finite element analysis is used to validate the extended progressive collapse methodology.

1. Introduction

Recent advances in the high speed ship industry have led to increases in the size and operability of large lightweight craft for commercial and naval applications. This is exemplified by the recently completed littoral combat ship USS Independence commissioned by the US Navy. Such vessels are not only large, but also have increased operability requirements including exposure to deep ocean environments. This has raised important issues regarding the response of critical structure under primary hull girder bending.

Previous studies have demonstrated that the simplified progressive collapse method [1], often known as the Smith method, is a capable procedure to predict the ultimate strength behaviour of a conventional hull girder. An early and successful validation of the method was by Rutherford and Caldwell, who investigated the collapse of the Energy Concentration accident in Rotterdam harbour [2]. An experimental programme detailed by Dow [3] has become a benchmark study, presenting both experimental and numerical results for a 1/3 scale frigate model in vertical bending. The approach has been validated in several further studies [4, 5]. The international ship structure committee on ultimate strength have further applied the Smith method to numerous ship cross sections and found reasonable correlation to equivalent finite element analyses [6].

The standard progressive collapse approach assumes interframe buckling of the longitudinal structure, which is generally adequate for steel hulls with stocky transverse framing. However, recent work [7, 8, 9] has shown that stiffened panels typical of a lightweight ship may not necessarily buckle interframe, and may be significantly influenced by buckling modes over several frame spaces. Although this mode of collapse would usually be avoided by ensuring frames are sufficiently stocky, the increased drive towards lightweight designs means that the overall collapse mode is not necessarily always accounted for adequately in design. Therefore, even though overall collapse is not a recommended collapse mode for structural design, a method appropriate for determining ultimate strength at both the interframe and compartment level is still of value during design of new structures and analysis of existing ships.

Therefore this paper proposes an extended progressive collapse method, which possesses the ability to account for interframe and overall buckling modes between bulkheads or other discontinuous
transverse structure. The extended method is demonstrated using example lightweight girders for which the ultimate strength is significantly reduced due to the influence of overall buckling modes. The method is shown to adequately predict the compartment level strength of these structures.

2. Background

The interframe progressive collapse method has been established for quite some time. The approach has early origins in the design of aircraft [10] and was first applied to ship structures by Caldwell [11]. The method was developed into a more rigorous approach for the analysis of ship structures by Smith [1]. A complete description of the “Smith” progressive collapse method can be found in several papers relating to the subject [1, 2, 3, 4]. The following discussion follows the general approach given by Dow [3], which is appropriate for dealing with biaxial bending and unsymmetrical sections, and focuses on aspects relevant to the extended methodology proposed in this paper.

2.1. Principles

The progressive collapse method evaluates the strength of longitudinally effective structure between adjacent frames when the hull girder is subjected to longitudinal bending moment. A conventional cargo ship typically has a parallel middle body and thus an evaluation of the midship section only is normally sufficient for an ultimate strength analysis. For a lightweight, high speed ship without a significant parallel middle body the strength of sections away from amidships may also need to be evaluated. This is normal practice for a warship design.

In a conventional progressive collapse analysis the hull girder cross section is divided into discrete elements. Failure of the hull girder in global bending occurs by interframe failure of these elements in tension or compression. The transverse frames are assumed to be sufficiently strong to act as boundary supports.

The fundamental assumptions of the progressive collapse method are that plane sections remain plane; elements act independently; and buckling and collapse of the section is interframe. Following these assumptions the method follows a relatively straightforward incremental procedure, which is summarised as follows and then discussed in more detail in subsequent sections of this paper.

1. A cross section of the girder is selected. Only longitudinally effective structure is included in the cross section;

2. The cross section is divided into small elements;

3. Each element is assigned a “load shortening” curve, describing the behaviour of the element under incremental compression/tension. The load shortening curve may implicitly include other load effects;

4. The initial position of the cross section neutral axis is calculated;
5. Incremental vertical curvature is applied about the instantaneous neutral axis. At each increment of curvature:

   a. The incremental strain of each element is calculated assuming the cross section remains plane;

   b. Element incremental stresses are derived from the slope of the load shortening curve;

   c. Stresses are integrated over the cross section to obtain bending moment increments;

   d. The position of the neutral axis is adjusted to account for the loss of stiffness over areas of the hull exhibiting high compressive strains.

6. Incremental moments and curvatures are summed to obtain total cumulative values.

2.2. Element Subdivision

For the purposes of progressive collapse analysis, a hull girder cross section is divided into discrete elements. The element size must be small enough to provide sufficient fidelity in the solution. A standard element consists of a single stiffener with attached plating up to the midpoint between adjacent stiffeners on each side, as shown in Figure 1. This is referred to in this paper as a plate stiffener combination (PSC). The properties of a PSC sized beam column are conventionally used to predict the load shortening properties of the element for the progressive collapse analysis.

However, subdivision is not limited to PSC sized elements. More refined subdivision of the scantlings can be used in the discretisation of the cross section, for example by splitting a panel into separate stiffener and plate elements, as shown in Figure 1. This is usually done so as to increase the accuracy of the progressive collapse solution.

![Figure 1 – Element subdivision in the progressive collapse method](image-url)
Smaller elements allow a more accurate calculation of the neutral axis shift (the computation of which is discussed further below). Conversely, a larger element consisting of several stiffeners can be defined so long as accuracy in the progressive collapse solution is not significantly affected. In both of these cases the properties of the element are still usually defined using an equivalent PSC representation.

The concept of element subdivision in the progressive collapse method thus has two purposes: firstly to provide sufficient accuracy in the progressive collapse solution, and secondly so as to provide a suitable beam column model for deriving a representative load shortening curve. It is important to clearly differentiate between these two functions, particularly in reference to the extended methodology proposed in this paper. Whilst an element is usually assigned properties using an equivalent PSC representation, the actual discretisation is not limited to PSC sized elements.

2.3. Load Shortening Curves

The properties of each element are defined using a nonlinear curve which describes the relationship between axial load and end displacement. This curve is commonly known as a load shortening curve. A fundamental assumption of the progressive collapse method is that plane sections remain-plane. Therefore, each element is assumed to bear in-plane tensile or compressive loads only. A load shortening curve is thus an adequate representation for the progressive collapse method.

Established progressive collapse codes have developed several methods to predict the load shortening behaviour of each PSC making up the hull girder cross section. The tensile and compressive properties of each element are usually derived separately.

The tensile properties of the element are usually idealised to follow the material stress-strain relationship. In some circumstances, such as for structure close to a corner, the element is assumed sufficiently stocky or constrained to prevent buckling. Thus the compressive behaviour is also assumed to follow the material stress strain relationship. Hard corners are typically assumed to extend from corner points over a span of 30 times plate thickness.

For the purposes of defining the compressive load shortening properties, a PSC can normally be assumed equivalent to a beam-column. Several established methods are available to predict the load shortening behaviour of a beam column in compression, including the finite element method and analytical procedures. For example the code developed by Dow [4] has options to utilise a parametric database of load shortening curves, which have been derived using a combination of physical test results and beam column finite element analyses.

Alternatively, element load shortening curves can be derived using empirical approaches [12, 13], analytical methods [14] or by the nonlinear finite element method (FEM) [15]. These methods vary in their complexity and can also include secondary load effects, such as the influence of hydrostatic pressure loads on the strength of the beam column.

2.4. Calculation Procedure

The contribution of each element to the overall bending strength of the girder is calculated using a simple incremental approach. Increments of bending moments \( \Delta M_H, \Delta M_V \) are related to
incremental curvature ($\Delta \phi_H, \Delta \phi_V$) using the instantaneous tangent rigidities of the cross section ($D_H, D_V, D_{HV}$). The method assumes that incremental bending moment / curvature occurs about an instantaneous neutral axis, which is a function of the instantaneous tangent stiffness and area of each element. The relationship between bending moment and curvature is split into vertical and horizontal components:

$$\begin{bmatrix} \Delta M_H \\ \Delta M_V \end{bmatrix} = \begin{bmatrix} D_H & D_{HV} \\ D_{HV} & D_V \end{bmatrix} \begin{bmatrix} \Delta \phi_H \\ \Delta \phi_V \end{bmatrix}$$  

The above formulation is suitable for calculating incremental bending moment for fixed increments of curvature. The method can be used directly to calculate bending moment well into the post collapse region. However, this approach is not always suitable for biaxial bending moment problems, as the proportion of horizontal and vertical bending moments are unconstrained and will not necessarily follow a prescribed ratio. Therefore the equations can be rearranged to calculate curvature over fixed increments of bending moment:

$$\begin{bmatrix} \Delta \phi_H \\ \Delta \phi_V \end{bmatrix} = \begin{bmatrix} D_H & D_{HV} \\ D_{HV} & D_V \end{bmatrix}^{-1} \begin{bmatrix} \Delta M_H \\ \Delta M_V \end{bmatrix}$$  

Expanding this equation yields:

$$\begin{bmatrix} \Delta \phi_H \\ \Delta \phi_V \end{bmatrix} = \begin{bmatrix} D_V & -D_{HV} \\ -D_{HV} & D_V \\ \frac{1}{|A|} & 0 \\ 0 & \frac{1}{|A|} \end{bmatrix} \begin{bmatrix} \Delta M_H \\ \Delta M_V \end{bmatrix}$$

where:

$$|A| = \begin{vmatrix} D_H & D_{HV} \\ D_{HV} & D_V \end{vmatrix} = D_H D_V - D_{HV}^2$$

This formulation breaks down when the maximum bending moment is reached, which is signalled when the determinant of $A$ becomes negative. Thus the procedure must switch to applying incremental curvature if the post collapse behaviour is also required.

The contribution of each element to the overall bending strength of the girder is encapsulated in the tangent rigidity formulations. $D_H, D_V$ and $D_{HV}$ describe the instantaneous inertia of the cross section including the effects of the tangent stiffness of each element:

$$D_H = \sum_i E_i A_i y_i^2$$

$$D_V = \sum_i E_i A_i z_i^2$$

$$D_{HV} = \sum_i E_i A_i y_i z_i$$
These equations assume that each element is relatively small and therefore its own inertia can be neglected. \( y_i \) and \( z_i \) are the component distances of the element centroid from the vertical and horizontal position of the instantaneous neutral axis. In the first increment the instantaneous neutral axis is equivalent to the elastic neutral axis because all elements are assumed to have elastic stiffness. However, the stiffness of the element changes as the end displacement increases. To derive the tangent modulus \((E_i, i)\) of each element, the strain in each element is first calculated incrementally as:

\[
\Delta \varepsilon_i = \Delta \phi_{yi} y_i + \Delta \phi_{zi} z_i
\]

\[
\varepsilon_i = \varepsilon_{i-1} + \Delta \varepsilon_i
\]

The instantaneous stress \((\sigma_i)\) and tangent modulus is then derived from the element load shortening curve using appropriate interpolation methods (Figure 2).

![Figure 2 - Example element load shortening curve.](image)

The tangent modulus is also used to determine the shift in the instantaneous neutral axis. In the first increment the elastic neutral axes of the cross section are used to define the initial curvature axes. In subsequent steps the method assumes that incremental bending moment/curvature occurs about instantaneous neutral axes, which are a function of the instantaneous tangent stiffness and area of each element. Thus as the stiffness of elements change, the position of the axes shift according to the following formulae:

\[
\Delta N_{Ai} = \sum_i E_i A_i z_i, \quad \Delta N_{Ai} = \sum_i E_i A_i y_i
\]

3. The Extended Progressive Collapse Method

3.1. Concept

Two fundamental assumptions of the conventional progressive collapse method are that each plate-stiffener buckles in an interframe manner when loaded in compression and that each plate-stiffener
element acts independently. If the element is deemed stocky enough to prevent buckling it follows the stress-strain relationship of the material. This assumption implies that the behaviour of a panel with multiple stiffeners can be adequately represented by a single plate-stiffener model. Thus the beam-column representation of each element is assumed adequate for the derivation of load shortening curves.

However, a lightly framed panel such as may be used in a lightweight ship structure does not necessarily fail interframe. Previous work by the authors [7] has shown that overall collapse modes can occur in wide, lightly stiffened panels with scantlings equivalent to typical fast ferry structures. These overall collapse modes are associated with a very different load shortening response when compared to equivalent beam column type analysis. The ultimate strength of the orthogonal multi stiffened panel can be much lower than the equivalent interframe representation.

These findings highlight a requirement for a simplified method to predict hull girder strength over several frame spaces, typically spanning an entire compartment, rather than just assuming interframe panel behaviour and thus only analysing structure between adjacent frames. Thus an extension to the progressive collapse methodology is proposed. This follows the same overall principles and calculation procedure as the established method but with a fundamental different approach to defining elements and their associated load shortening curves.

### 3.2. Methodology

In the extended progressive collapse approach, the hull girder cross section is subdivided into small elements using the same approach as the standard method. However, the load shortening curve for each element is derived using the properties of the entire panel of which it is a part. Furthermore the element length stretches over the entire compartment rather than interframe.

Elements are thus grouped into “panel sets” which define the overall extents of orthogonal stiffened panels (usually flat panels) within the structure. The choice of panel extents is usually dictated by the form of the overall girder geometry. The panel width must encompass all the structure which may fail in an overall manner. For example, a deck panel may run the entire width of the ship, or the panel may be intersected by deep longitudinal frames or longitudinal bulkheads.

The panel length is usually set equal to the compartment length between bulkheads. The panel load shortening curve is then defined to reflect both interframe and overall collapse behaviour. Therefore, if all the panels within a cross section are predicted to fail interframe, the extended progressive collapse methodology will calculate the same girder response as the conventional approach. However, if some or all panels are predicted to fail in an overall manner, with reduced ultimate strength capacity, the extended methodology directly accounts for the corresponding reduction in primary hull girder strength.

### 3.3. Panel Strength

There are several possible ways to develop load shortening curves which account for overall panel collapse. The approach adopted in this paper is the semi analytical method [8], which is a direct incremental approach to derive compartment level panel load shortening curves using orthotropic
A summary of the semi analytical method is presented here – a more complete description including the formulations, are given in [8] and [9]. An alternative approach, which is not discussed in detail here, is to derive the load shortening relationship using nonlinear FEM analysis of an equivalent multi-frame panel representing the critical part of the global structure.

The semi analytical method makes recourse to large deflection orthotropic plate theory, which is an established approach to calculate the capacity of an orthogonal stiffened panel. Formulations are concisely summarised by Paik et al. [16]. Orthotropic theory calculates the ultimate capacity of the panel when represented as a single entity, with the properties of the individual plate and stiffener elements combined (smeared) to make an equivalent panel plate. This creates a plate with orthotropic properties, for which the elastic buckling strength can be calculated using established formulations.

When used in isolation, the large deflection orthotropic plate method can be said to be closed form, in that it only calculates the ultimate strength of the panel. It does not directly calculate the complete load shortening response of the panel to progressively increasing compressive load. Furthermore, the buckling strength is calculated using the elastic orthotropic properties of the panel. Thus, elasto-plastic effects, together with the nonlinear buckling characteristics of the constituent components of the panel (unstiffened plates bounded by an orthogonal framework of stiffeners), are not accounted for within the method.

The semi analytical method was therefore developed to enable a complete derivation of the load shortening curve for a panel which may buckle in either an interframe or overall manner, also accounting for the nonlinearity of the panel components under compressive load.

The semi analytical method makes recourse to a pre-defined database of non-dimensional plate and stiffener load shortening curves to define the response of the components of the panel. The database has been developed from multiple nonlinear FEM analyses of unstiffened plates and stiffeners in uniaxial compression, which are detailed in several papers by the authors [7, 17] and include formulations covering imperfections and other nonlinear parameters. The database includes data for various stiffener types (such as angles, tee bars and flat bars), different plate sizes, three geometric imperfection levels (slight, average and severe) and data for different materials including steel and several marine grade aluminium alloys. The aluminium alloy data also includes a range of parameters to account for the heat affected softening zone which is created close to a weld joint by the high heat input, which locally alters the material properties of the alloy. The database includes curves for a range of slenderness parameters covering the extents typical for a ship type stiffened panel. Representative curves for a component with specific dimensions can therefore be derived through interpolation.

To form the “pre-collapse” portion of the panel load shortening curve, the representative curves for the plate and the stiffener components of the panel are combined proportional to their relative cross section area. This procedure is carried out for incremental steps of end displacement, producing a relationship describing the resistance of the panel to a given end shortening.

At each increment step, the plate and stiffener dataset curves are also used to define the “instantaneous” stiffness and rigidity of the components at the given end shortening. This recognises that, as the panel is incrementally loaded and approaches collapse, the stiffness and
The flexural rigidity of the component plates and stiffeners do not remain constant. The orthotropic plate strength of the panel is then re-calculated, using the instantaneous stiffness properties of the components in place of the elastic Young’s modulus. This checks whether the instantaneous buckling strength of the panel is higher than the combined resistance of the panel components.

The nonlinear behaviour of the elements within a stiffened panel are therefore included in the orthotropic plate method by using the instantaneous stiffness of the plates and stiffeners (derived from the component load shortening curves) to replace the elastic constants in the orthotropic calculations. This usually has the effect of reducing the overall strength of the multi frame panel as incremental load is increased.

If the panel strength predicted by orthotropic plate theory drops below the interframe strength the panel is assumed to fail in an overall manner. The load shortening curve peak and post collapse behaviour is thus defined by the orthotropic plate calculations. Alternatively, if the orthotropic plate strength remains higher than the interframe panel strength, the panel is assumed to fail interframe and the load shortening curve is defined solely using the combined plate and stiffener load shortening curves.

3.4. Program Development

The extended methodology is implemented in a progressive collapse program (ProColl) with a direct interface to the semi analytical orthotropic plate method. The program follows a two-step process. Firstly the load shortening curves for all the elements are calculated and secondly the progressive collapse calculations are invoked to calculate the incremental bending moment-curvature relationship. The program can complete either an interframe analysis (ProColl-I) or a compartment level analysis (ProColl-O). If ProColl-I is invoked, the semi analytical method is restricted to calculating the interframe strength of each panel set, which effectively restricts the elements to PSCs. ProColl-O uses the full capabilities of the semi-analytical method and therefore requires information on the compartment length and the transverse frame size.

ProColl writes an output file containing the bending moment and curvature results. In addition, the program has capabilities to produce basic 2D contour plots of the instantaneous element tangent stiffness at a given increment of curvature/moment. An example contour plot is shown in Figure 3. The plot shows the position of the instantaneous neutral axis and highlights elements which have failed (i.e. those where the element tangent stiffness is less than zero) in red. Hard corners are highlighted yellow when they reach the yield/proof stress. Elements which are close to their ultimate strength are highlighted green.
4. Results

This section provides a validation of the compartment level capabilities of the extended method. The interframe progressive collapse method (ProColl-I), extended progressive collapse method (ProColl-O) and nonlinear FEM analyses are first compared using a simple case study box girder with properties typical of a large aluminium vessel. Further validation of the methods is then presented using an actual compartment level ship structure which is typical of a large aluminium multihull. Validation is with equivalent nonlinear FEM analyses of full ship sections. All FEM solutions are completed using an arc length solver. Average levels of initial geometric imperfections, residual stresses and material nonlinearities including heat affected zones are all represented appropriately in the FEM models. Further details regarding the FEM modelling approach are in [18, 19].

4.1. Aluminium Box Girders

The dimensions of the box girders are similar to the types of arrangement used in other studies investigating ship type aluminium structures [20]. These panels were sized specifically to have typical slenderness properties of a large lightweight ship. They show different overall buckling characteristics depending on the size of the transverse frames and are therefore highly suitable for assessing their strength as part of a 3D box girder structure. The load shortening curves predicted by the semi analytical method, which are accessed directly by the ProColl-O analyses, are discussed in [19].

The overall box girder is square in cross section with a side length of 8.4m, stiffened by 20 longitudinals spaced 400mm apart on each side. Four cross sections are considered, which are sized and named as given in Table 1 and Table 2. The frames are also consistent sizes with the panel analyses. The compartment length is set at 7 frame spaces, which is sufficient to show compartment level buckling characteristics. All FEM models use a 1 compartment + 1 bay representation. The bulkheads are modelled with very large thickness to keep the compartment ends straight. Geometric
and material imperfections are applied over the entire box using methods and amplitudes consistent with the panels discussed in the previous section. A 50mm element length is used to mesh the geometry, which was found through a mesh refinement study of box M1 to give sufficient convergence in the results.

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The results are compared to the extended progressive collapse methodology. The results can be compared both in their prediction of the ultimate strength of the box and the prediction of the entire load shortening relationship. The ultimate strength results are summarised in Table 3 and show close agreement. The results reflect similar characteristics to the panel strength results presented previously. If the transverse stiffening is lightened or the longitudinal stiffening is made stockier, the influence of the overall collapse mode is increased. Box girder M1 shows the most dramatic decrease in strength when framed with the light T1 flat bar (180x10) with a drop in ultimate strength of 39%.

<table>
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Mean Bias = 0.96, C.O.V. = 0.03
The entire progressive collapse behaviour of the four girder models are shown in Figure 4 with comparison to equivalent FEM models. In some instances (M2-Frame T2 and M4-Frame T2) the FEM models had problems converging and therefore a complete comparison is not possible. This is an occasional problem also discussed in other studies [9, 21]. At present the authors have found no obvious pattern as to why convergence using an arc-length approach has problems, and requires further investigation as to whether it also occurs with other FEM solvers.

The plots demonstrate the applicability of the extended progressive collapse method in firstly predicting the onset of overall collapse in stiffened panels making up a longitudinally stiffened structure, and secondly predicting the subsequent effect on the collapse characteristics of the structure under primary bending. The curves show good correlation in the ultimate strength region, although the curve characteristics near to the peak do have some significant differences. In particular, the FEM analyses show a much more gradual transition from the interframe mode to overall, which is characterised by a reduced gradient in the bending moment curve. In comparison, the ProColl analyses maintain a fairly linear relationship up to near the ultimate strength, at which point the curve diverges more sharply into an overall mode. This is a similar characteristic as found in the panel analyses, and thus the difference can be attributed to the way the semi-analytical method predicts the influence of overall collapse at the panel level.
Plots of the FEM mesh at the ultimate strength point of the analysis for girders M1 are shown in Figure 5. These show that the box collapses with an overall mode of failure in the top and side panels. Overall collapse occurs with both transverse frame sizes. As would be expected, the more lightly stiffened girder (T1) shows an increased influence of gross panel buckling between bulkheads and thus suffers a more severe degradation in strength as compared to the interframe result.

Figure 5 – Deformed mesh plots of box girder M1 with frames T1 (left) and T2 (right), magnification x3. U is in mm.
Plots of the FEM mesh at the ultimate strength point of the analysis for girders M3 are shown in Figure 6. This girder uses thinner plate, thus the longitudinal cross section is “lighter” than girder M1. The results show a degradation of strength in the lighter framed girder (T1), although the reduction is less than the equivalent result for girder M1. With the 360x10mm frames (T2), the box collapses interframe. This demonstrates that the influence of overall collapse is dictated by the longitudinal structure as well as the absolute sizing of the transverses.

4.2. Aluminium Multihull

The methods developed in this paper are directed towards large lightweight marine structures. Therefore, the study of a large aluminium ship structure provides an appropriate case study to validate the proposed progressive collapse methodology. Therefore, this section investigates the strength characteristics for the midship scantlings of an aluminium catamaran under primary longitudinal bending. The scantlings have been simplified for the purposes of providing a clear comparison between methods.

4.2.1. Model Definition

Based on the limited information contained in ship structure report SSC-438 [22], a representative multihull cross section is developed with scantling details as shown in Figure 7. These scantlings are not the exact configuration of the Pacificat hull investigated in the SSC report, for which a detailed structural layout is not openly available. The hull shape is broadly similar and was developed by scaling from a small scale general arrangement drawing contained in the SSC report. Similarly, the stiffener sizes and spacing were estimated based on reported information, but are not necessarily an accurate reproduction of the actual vessel scantlings. Therefore no inference from the results in this paper should be transferred to the actual vessel.
It is recognised that longitudinal bending moment is not necessarily the critical load condition for a multihull of this size. The critical load in such a vessel is usually the prying moment between the two demi-hulls. However, for the purposes of comparing the methodologies developed in this paper, the application of a longitudinal bending test as a theoretical case study is considered valid, particularly because the scantlings are such that overall and interframe failures occur in different areas of the cross section.

Table 4 – Aluminium Multihull Panels

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The structural details are specified to enable a concise assessment of component level strength of plate and stiffener combinations. The plate thickness ranges from 8mm in the decks to 20mm at the keel. Four stiffener sections are used with webs 95mm – 140mm high and are spaced between 200mm-275mm. Stocky longitudinals with 300mm webs are positioned intermittently along the decks at about 3000mm intervals. The transverse frames are 400mmx10mm and are spaced at 1200mm intervals. In total the cross section contains 13 distinct panel sizes, which are summarised in Table 4 with the panel locations indicated in Figure 7. The material is 5083-H116 throughout.

4.2.2. Interframe Analyses

The hull girder is first assessed in vertical bending assuming interframe collapse. A ½+1+½ bay FEM model is analysed under vertical hog and sag. The FEM analysis includes average imperfections and residual stresses. The geometric imperfections are only introduced into the central bay.

The section is also analysed using ProColl-I. Hard corners are assumed to extend 30 times the plate thickness from the deck corners and knuckle points. For the purposes of an interframe progressive collapse analysis, several of the panels listed in Table 4 have identical properties (Panels P5-P7 and P8-P13). These panels only differ in the number of longitudinals making up the total panel, which is irrelevant if assuming each PSC acts independently. Therefore there are only 6 PSC combinations. PSC FEM and semi analytical method generated load shortening curves for these panels are compared in Figure 8.

The curves generally show close agreement in terms of the initial stiffness (pre-collapse), ultimate strength value and post collapse response. The semi analytical method is only able to model the post collapse response using a simple linear relationship, the steepness of which is calculated as a function of the immediate pre-collapse stiffness of the panel. In contrast, the FEM analysis continues to predict a nonlinear response. In the cases shown here this simplified linear relationship is probably adequate for representing the post collapse response, showing a similar gradient to the FEM results. However in some instances (for example panel P3), the FEM response is more obviously nonlinear, with a steeper initial unloading then flattening out under further increases of end displacement. The post collapse response of certain panels within the progressive collapse method will have some effect on the resulting bending moment prediction. In particular, the post collapse behaviour of those panels which fail prior to the girder as a whole have some effect on the ultimate strength prediction.

However, without recourse to equivalent physical testing, it is difficult to say which method is more representative of an actual panel. The post collapse response is a difficult phenomenon to predict and is subject to some uncertainty. The semi analytical method performs well in these instances in predicting the general steepness of the unloading curve and indicates that the resulting hull girder bending moment result should also be well correlated between the numerical methods.
Figure 8 – Aluminium multihull PSC curves
These PSC load shortening curves together with deformed mesh plots from the FEM analysis and 2D section plots from the ProColl-I program are compared in Figure 9. The results show remarkably good agreement with the close correlation of the load shortening curves matched by the same pattern of buckling in the FEM and ProColl-I section plots. There are three clear transition points highlighted on the load shortening curve.

The bending moment – curvature relationship is almost linear up to position A at a curvature of 0.6x10⁶ m⁻¹, at which point the top deck and the uppermost area of side shell begin to buckle. The ProColl plot shows that the side shell is the first to collapse and a similar pattern is observed in the FEM plot although the exact areas which have collapsed are less distinct. The failure in these regions does not trigger complete collapse of the hull girder, but instead causes the bending moment curve gradient to drop considerably. A further effect of the loss of tangent stiffness in the top deck area is to lower the instantaneous neutral axis, which causes additional loads in the upper portions of the cross section.

The ultimate strength is reached at a curvature of approximately 0.8x10⁶ m⁻¹ (position B) at which point the entire top deck and side shell between the top deck and middle deck has collapsed. The collapse is triggered by the decrease in tangent stiffness of the middle deck, although the ProColl-I analysis shows that this deck has not actually collapsed at the ultimate strength point but instead has lost effective stiffness below half the elastic modulus. This is validated by the FEM mesh plot, which shows that buckling has not nucleated into the middle deck at the ultimate strength point.

The ultimate strength point is characterised by a clear transition on the FEM load shortening curve whereas the ProColl-I result is much smoother. However, the general characteristics are still broadly similar. The collapse is not as gradual as found for the 1/3 scale frigate, which is perhaps due to the influence of buckling in several decks rather than just a single main deck. The post collapse curves predicted by ProColl-I and FEM are closely correlated. The mesh plots show that buckling spreads across the entire middle deck and continues propagating down the side shell. The ProColl-I plot demonstrates the same pattern.

In summary, the interframe analysis demonstrates two key factors that result in a close correlation between FEM and the simplified method. Firstly, the element load shortening curves are closely matched between FEM and the semi analytical method. The elements in the progressive collapse method are therefore closely replicating the behaviour of the compressed portion of the global FEM model. Secondly, the pattern of the collapse and the spread of buckling in the cross section are similar between the FEM and simplified analyses, showing that the methods are broadly representing the curvature about the instantaneous neutral axis in a similar manner.
Figure 9 – Interframe bending moment – curvature of the aluminium multihull. FEM plots are magnified x10.
4.2.3. Compartment Analyses

The hull girder is also assessed using a compartment level FEM model and is compared to the bending moment – curvature relationship predicted by the extended progressive collapse method (ProColl-O).

The semi analytical orthotropic plate method predicts that the two uppermost decks in the aluminium multihull (which are identical) fail with an overall collapse mode. This is confirmed by comparison to an equivalent FEM analysis of the deck, which produces a very similar relationship as shown in Figure 10. The FEM plot of the deck in Figure 11 clearly demonstrates an overall mode of failure. The remaining panels in the multihull structure are predicted to fail interframe by the semi analytical method. Thus the load shortening curves used in ProColl-O are identical to the interframe curves as previously presented in Figure 8.

![Figure 10](image1.png)

**Figure 10** – Load shortening curve for the top deck (P8-10) of the aluminium multihull predicted by FEM and the semi analytical method.

![Figure 11](image2.png)

**Figure 11** – Buckling mode shape for the top deck FEM model. Half model shown.

This means that the compartment level hull girder model also shows significantly reduced primary strength and is thus an excellent comparator of the capabilities of the extended progressive collapse method.
The FEM model has 1 compartment + 1 bay extent. A half model is used with a symmetry boundary condition along the centreline. The imperfection and residual stress magnitudes are the same as used for the interframe model. The imperfections are introduced across the entire compartment. The mesh size is varied from fine elements in the top decks and the upper portion of the side shell to a coarse discretisation in the bottom structure.

The FEM results are compared to the equivalent ProColl-O and ProColl-I bending moment curves in Figure 12, together with associated mesh plots at critical points. The compartment level analyses show very good correlation. Although the ultimate strength of the FEM analysis is slightly higher than the ProColl-O prediction, the general shape of the curve is similar, showing that the propagation of the buckling mode is alike in both models. As expected the hull girder shows a significant loss of ultimate strength (approximately 25% compared to the interframe result), which is due to overall buckling of the two uppermost decks.

The plots show two clear transition points where each of the upper decks buckles overall. The top deck collapses first (position A in Figure 12) which causes a drop in the bending moment curve stiffness. This is similar to the girder behaviour in the interframe analysis. The ultimate strength is reached when the middle deck also fails. Both decks collapse in a clear overall pattern. The side shell fails in an interframe mode, which is predicted by both the FEM and the extended progressive collapse method.

The post ultimate strength behaviour shows the propagation of the collapsed region continuing down the side shell. The neutral axis continues to drop. The ProColl-O analysis can continue for a long period after ultimate strength has been surpassed, although the continued validity of the predicted bending moment curve at high curvature is questionable because the calculations rely on the reserve strength of the buckled members, which is difficult to quantify accurately in the load shortening curves. In contrast, and in this instance, the FEM analysis terminates shortly after the ultimate strength is surpassed due to poor convergence within the arc length solver calculations.
Figure 12 – Overall bending moment – curvature of the aluminium multihull. FEM plots are magnified x10.
5. Conclusions

This paper has proposed an extended progressive collapse method which utilises the semi analytical method to re-evaluate the load shortening curves used in the progressive collapse calculations. The extended method has been shown to predict the overall buckling characteristics for a range of box and hull girders. The method has proved to give representative results and encapsulate the characteristics of the overall collapse mode in lightly stiffened compartments under primary bending moments.

The extended progressive method is found to predict compartment level buckling with good reliability. The results generally show close correlation to equivalent compartment FEM analyses. Overall buckling modes are shown to have a significant effect in reducing the bending capacity of a lightly stiffened hull structure.

The global FEM and simplified progressive collapse analyses have shown that both approaches can produce reasonable and valid solutions for the compartment level progressive collapse problem. This means that the decision of which methodology to employ in practical design and analysis situations has as much to do with the ease of analysis and the purpose of the analysis as it does with the accuracy of the solution.

In terms of providing a relatively quick model definition and fast solution, the simplified methodologies remain valid and easily applied, and in terms of simplicity of use are superior to FEM. However, the FEM approach, if completed in a rigorous manner, is an acceptable and realistic alternative. The FEM approach also continues to hold significant advantages because it is employed within a general purpose analysis package. This flexibility means that it has capabilities beyond the simplified methods developed in this paper, such as modelling unusual scantling arrangements, irregular spacing of stiffeners, or multiple load combinations. These example scenarios are not readily dealt with by the simplified methods as developed in this paper.

6. References


