Calculation Of Iron Loss In Electrical Generators Using Finite Element Analysis

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1 Abstract

The accurate calculation of iron loss from finite element analysis in electrical machines is essential if optimal machines are to be designed. This paper conducts a holistic review of the extensive literature field before examining, in detail, several methods in order to recommend an optimum engineering solution. Both frequency domain and time domain methods are discussed including the use of different orthogonal components as well as the relative merits of using all, or some, of the Eddy Current, Anomalous and Hysteresis loss components. A theoretical cubic meter of iron is simulated to quickly demonstrate the inaccuracies of Cartesian coordinate methods before calculation on several manufactured machines are undertaken showing the superior accuracies of major/minor loop calculation. Calculation undertaken using the radial tangential orthogonal plane is shown to have less than 1% average difference to the major/minor loop yet is over 6 times quicker. The peak percentage error in an individual element is shown to be less than 5%. Discussions are also made regarding the method of curve fitting to gain loss constants and any possible sources of inaccuracy particularly during manufacture.

2 Introduction

In order to calculate the final nameplate rating of a machine it is very important for the designer to be able to calculate any losses incurred. A fast accurate and reliable method for the calculation of losses is essential in order to avoid any design concessions and potential financial penalties in any completed products. The losses within machines can be attributed to several causes such as resistive heating, friction and windage and iron loss. This paper specifically looks at the calculation of iron loss from flux density within the iron circuit of the machine which is calculated using Finite Element Analysis (FE). Any loss calculation must be capable of being integrated into an existing FE design system which is currently used to analyse synchronous generators in the multi-megawatt range and be suitable to be used within a manufacturing design office. To this end the method must be consistent, quick, accurate and integrated to allow rapid design iteration leading to optimised designs. The calculation method must also take into account any variances created by machine manufacturing or material handling processes.

Several different models for iron loss have been presented by various authors and the field of literature is extensive. This paper is proposing a comparative study of the various methods weighing the merits and limitations of each. Bertotti et al [1][2] developed a frequency domain model which divides the loss down into 3 components - Eddy Current loss, Hysteresis loss and Anomalous (sometimes called Excess) loss. The losses originate from the dynamic losses of the Weiss domains under variable magnetic fields. The discontinuous movements within the block walls create fast Barkhausen jumps and then eddy currents [3]. The hysteresis loss is physically caused by localised irreversible changes during the magnetisation process making it only dependant upon peak induction [4]. Bertotti’s equations are expressed in equations (1)-(4) and are typically summed for each harmonic of any magnetic fields hence necessitating a Fourier transform.

$$P = P_e + P_h + P_a \quad (1)$$

Eddy Current Loss [W/kg] = $P_e = Ke \hat{B}^2 f^2 \quad (2)$

Anomalous Loss [W/kg] = $P_a = Ka \hat{B}^{1.5} f^{1.5} \quad (3)$

Hysteresis Loss [W/kg] = $P_h = Kh \hat{B}^\alpha f \quad (4)$

where $Kh$, $Ka$, $Ke$ and $\alpha$ are material dependant constants

2.1 Continuous time and Discrete time.

Frequency domain equations are especially useful when the data regarding the magnetisation is presented in frequency terms and many authors have used these equations with...
success [4][5][6]. This often is not the case when time stepping FE analysis has been undertaken and relies upon a Fourier transform upon the time domain data to create the correct peak sinusoidal data for equations (2)-(4). The use of a Fourier transform adds an extra process and as data is analysed on an element by element basis within the FE it can create a slower technique. To counter this the equations can be transformed into a the time domain leading to equations (5)-(7) below.

\[ \text{Eddy Current Loss [W/kg]} = Ke \cdot \frac{1}{2\pi^2} \int_0^T \frac{dB_x}{dt} \frac{dB_y}{dt} \, dt \quad \text{(5)} \]

\[ \text{Anomalous Loss [W/kg]} = Ka \cdot \frac{1}{8.76} \int_0^T \left( \frac{dB_x}{dt} \right)^2 + \left( \frac{dB_y}{dt} \right)^2 \, dt \quad \text{(6)} \]

\[ \text{Hysteresis Loss [W/kg]} = Kh \cdot \frac{1}{T} \hat{B}^a \quad \text{(7)} \]

The eddy current and anomalous loss is caused by the rate of cycling of the flux through the atomic structure and hence the equations within the time domain have derivatives. The hysteresis loss is highly dependant upon the magnitude of the peak flux and thus the saturation of the material. As a consequence equation (7) does not undergo a full averaging over a period when transformed only the peak magnitude of the flux is desired [4]. Equations (5)-(7) are within the continuous time domain transcribing them into the discrete time domain yields (8)-(10) which are the most useful set loss equations when dealing with finite element data.

\[ \text{Eddy Current Loss [W/kg]} = Ke \cdot \frac{1}{2\pi^2} \sum_{n=1}^{N} \frac{B_{x,n} - B_{x,n-1}}{t_{n} - t_{n-1}} \quad \text{(8)} \]

\[ \text{Anomalous Loss [W/kg]} = Ka \cdot \frac{1}{8.76} \sum_{n=1}^{N} \frac{B_{x,n} - B_{x,n-1}}{t_{n} - t_{n-1}} \quad \text{(9)} \]

\[ \text{Hysteresis Loss [W/kg]} = Kh \cdot \frac{1}{T} \hat{B}^a \quad \text{(10)} \]

Equations (8)-(10) have been used by multiple authors to yield successful results[7][8][9][10].

### 2.2 Orthogonal Components

Equations (8)-(10) assume that all flux is in a single direction and consequently can be described as one dimensional (1D). The minority of electrical machine have constant axial shape and to a good approximation can be analysed in a Two Dimensional (2D) plane. This 2D nature allows rotation of flux vectors creating oscillation in both orthogonal components, as shown in Figure 2. Most FE packages calculate magnetic flux density into orthogonal X and Y components. These can be processed in several ways. Commonly calculations are processed in the B_x and B_y components and loss components calculated accordingly [14][17].

\[ \begin{bmatrix} B_a \\ B_b \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ \sin(\phi) & -\cos(\phi) \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix} \quad \text{(13)} \]

where \( \phi \) is the angle of an element relative to the axis.

It should be noted that many authors such as [13] do not mention orthogonal components. The reason for this can only be guessed.

Additionally authors have chosen whether to use one of three different axis planes. As discussed the XY plane is often chosen because finite element software natively outputs data within this Cartesian plane. Alternatively Cartesian components are transformed using equation (13) into radial and tangential components and loss components calculated accordingly [14][17].

\[ B = B_x e_x + B_y e_y \quad \text{(11)} \]

where \( e_x \) and \( e_y \) are unit vectors on the Cartesian plane

\[ |B| = \sqrt{B_x^2 + B_y^2} \quad \text{(12)} \]

The axis selection can be made using the calculated vector information. For this, authors must on an elemental basis extract a full loop of data, inspect to find the angle at which major axis rests relative to the axis and transform the element's data using equation (13). This process can be time consuming due to the extra computational cycles. Figure 3. shows a diagramatic representation of potential calculation axes.
2.3 Alternative Calculations
Several authors have used variations on equations (1)-(10) for their loss calculations. Ma et al [14], Nam et al [6][15] and Smith et al [16] have used the frequency based approach in (1)-(4) but have not included the anomalous loss term, \( P_a \). Similarly Seo et al [17] use the same model within the time domain [equations (8) and (10)] but use functional fifth order polynomial coefficients for \( K_e \) and \( K_h \) and make \( \alpha = 2 \) in order to include the excess (anomalous) loss within the eddy current loss term. Additionally some authors include the effect of minor hysteresis loop magnetisation. This has not been investigated as test plots at several locations have not shown the magnitude of the loops to be significant and the required finite element step size to investigate this phenomenon would render any method too slow for use within an industrial design environment.

2.4 Summary of Methods

<table>
<thead>
<tr>
<th>Orthogonal components</th>
<th>Eddy + Hysteresis + Anomalous</th>
<th>Eddy + Hysteresis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Time</td>
<td>Frequency</td>
</tr>
<tr>
<td>Sum Losses</td>
<td>[4]</td>
<td>[7][8][9][13]</td>
</tr>
<tr>
<td>Use [8]</td>
<td>[12][19]</td>
<td></td>
</tr>
<tr>
<td>Unknown [1][2][20]</td>
<td>[10][21]</td>
<td>[3][5][6][15]</td>
</tr>
</tbody>
</table>

2.5 Use of Pseudo Rotating Superposition (PRS)

The periodic symmetry of a machine around each slot has been used to allow rapid calculation of total harmonic distortion [22]. The PRS method can be used to create element by element flux waveforms for complete cycles using simulations where rotation is over only a single slot provided the mesh maintains rotational symmetry as shown in Figure 4. The elements in the first slot of a machine will have the same properties as the elements in the second slot when the rotor has rotated 1 slot. Consequently from a single static simulation of two poles a whole cycle of flux for any element can be created containing as many points as there are slots. By using a rotating machine solver within the FE many steps can be made over a single slot and hence the resolution of the waveform increased. This allows detailed waveforms to be created in relatively short periods of time. The PRS method could be used to find full waveforms and calculate Fourier series via equations (2)-(4). However this is slow and a more efficient method is to include equations (8)-(10) within the FE code and calculate total losses as a continual sum. As average losses are being considered in equations (8)-(10) the PRS method can be further simplified by solving over 1 slot and rather than creating full waveforms for each element, summing all slots in a pole pair to yield to the average for a single slot for a whole period. This negates having to manage data for all elements in all slots, manipulate the data and perform calculations on each elemental trace. It therefore ultimately makes the whole process easier and quicker whilst maintaining the same level of accuracy.

2.6 Determination of Constants

Loss curves for materials at various frequencies are often supplied by manufacturers [23]. These curves showing the losses at various frequencies and levels of magnetisation are used via a curve fitting method to find the various constants for the loss equations. Equations (1)-(4) have been fit using a genetic algorithm (GA) with a fitness function based upon average coefficient of determination \( R^2 \) to gain the most accurate fit. The constant \( K_e \) can be approximated from the material properties as defined in equation (13). This allows the curve fit to have either 3 or 4 variables as shown in Figure 5 and Figure 6. Methods using alternate equations must have the loss curves fit using their own characteristic equations in order to be valid and likewise can be fit using GA and coefficient of determination.

\[
K_e = \frac{\gamma \pi^2 t^2}{6 \delta} = 1.414 \times 10^{-4} \quad \text{...(13)}
\]

where:
- \( \gamma \) = conductivity = \( \frac{1}{3.8 \times 10^{-7}} \) S/m
- \( t \) = lamination thickness \( = 0.5 \) mm
- \( \delta \) = density \( = 7650 \text{ kg/m}^3 \)

Figure 5 - Four Variable Curve fit for M400-50 grade material
3 The meter cube

Loss curves supplied by manufacturers [23] are tested to a standard method which is used to define the grade of a material. The material grades as described in DIN EN 10106 describe the loss in a cubic meter of material when a sinusoids field is applied which creates a flux density of peak value of 1.5T at 50Hz. An electrical steel described as M400-50A will be 0.5mm thick and have 4W/kg loss under the test field conditions. These defined conditions allow a very quick and simple condition to be simulated and hence methods examined under a controlled situation. As a result the iron loss within an ideal 1m cube of iron has been calculated when all the elements within the block were experiencing the same rotating field as depicted in Figure 11. The block is then rotated through 360 degrees around the z axis keeping the field constant with respect to the block. The loss calculated using different orthogonal components at various position throughout rotation is plotted in Figure 10.

The Loss is found not to be constant when calculated using Cartesian components. This is a result of the anomalous loss term in each axis being raised to the power of 1.5 before being summed. With Eddy current losses the power raised is 2 and hence it is immune to the rotation discrepancy. As Hysteresis losses are calculated off the peak induction [4] the maximum value which is non axis dependant and does not suffer any rotational problems is used. It is suggested that this discrepancy is the reason many people calculate loss without the anomalous component. Using major and minor loops the loss is constant and hence is a far more robust methodology to use in machine which are non linear. The calculated loss in using the major minor loop axes is 3.78W/kg which is below the maximum 4W/kg stated in the standard showing the methods and curve fitting to be correct.

4 The machine

Loss within a 17.5MVA synchronous generator has been calculated using several methods. The Major/Minor Loop and Radial/Tangential orthogonal axis systems have both been considered as well as methods with and without the anomalous loss components. From Figure 9 showing the loss calculated in the machine via different methods it can be seen that methods do not vary widely, there is less than 1% average difference between methods with peak elemental differences at just over 4%. This similarity can be easily attributed to the fact that Major/Minor axis often lie in line
with the Radial/Tangential ones for example within a tooth almost all flux is radial where as on the core back the significant direction is tangential. The major discrepancies occur at the back of the teeth where the flux is transitioning between the radial and tangential positions hence placing the angle of the major axis some where in between the two. The error in this area can clearly be seen in Figure 12. Calculating the Loss using the finite element package can be divided into 3 procedures: pre-processing, solving and post-processing. The First two procedures are common to both methods and took in total 20 minutes. The time for the final test varies depending upon the method selected. Calculating the major/minor axis is computationally intense as it involves inspecting every element's complete vector locus to find the axis angle before applying the rotation transform. The radial tangential method can have any transforms applied directly as the axis angle is found using the coordinates of the axis. For the examined machine the radial/tangential method took 6 minutes to calculate whereas the major/minor loop calculation took 40 minutes. The excessive post processing time within the major/minor loop method which yields an answer which is less than 1% different to the quicker radial/tangential method leads to the conclusion that within an integrated design package the use of radial/tangential method is the better compromise.

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It is accepted practise for large synchronous machines to have a design factor associated with them. The design factor exists as a method of accounting for the effect of a myriad of design and manufacturing processes each of which adds unaccounted loss. Magnetic steel lamination manufacturers conduct Epstein tests upon ideal steel before it has been through any manufacturing processes and consequently each process and material handling alters the laminations crystalline structure increasing losses. The major sources of this process loss include the lamination stamping where shear stress is elevated at edges and braising or welding which often occurs along the machine corebacks to maintain structural integrity (as can be seen in Figure 1). The design process for speed uses a 2D analysis and assumes the machine is consistent along its axial length where as in reality machines are not constant for example they have radial cooling ducts and end plates which will again alter the loss.

The difference between individual machines is clearly highlighted in a series of three identical machines which were constructed in the same manor, to the same standard yet the iron loss test results show a variance of 21% with individual results of 47.2kW, 44.0kW and 38.9kW. This variance although large at a macro level in comparison to the 15MVA full load rating of the machine it is acceptable. The size of the frame has little effect with the 3 frames having averages of 1.78, 1.73 and 1.64. The smaller machines - Frame A have the largest design factor and the larger machines of Frame C have lower design factors. As the stamping of laminations alters properties of the material close to the edges, the ratio of the edge area steel to non edge area steel will affect the design factor. Naturally this ratio will be higher in smaller machines confirming the hypothesis that the more manufacturing the less actuate any predictions will be. This study only compares 3 frame sizes and shouldn't be extrapolated to machines of vastly different types without more study. However the

\[
\text{Design Factor} = \frac{\text{Test Result}}{\text{Design Result}} \quad ... (14)
\]

Figure 12 - Diagram showing the individual elemental percentage differences between the Radial/Tangential and the Major/Minor loop methods.

5 Comparison To Test Results

The radial/tangential method has been implemented within a design system to allow systematic simulation and comparison to test results of machines. Forty six synchronous generators with rating of 5 to 20 MVA have been compared to test results acquired during factory tests to IEEE STD 115-1995. The test setup involves measuring field and rotor voltages and currents one the test machine as well as the driving motor and running them under a variety of open and closed circuit conditions allowing the iron loss to be found exclusive of any windage, friction or resistive losses. The machines are all four pole wound field synchronous generators which are one of three different frame sizes but are of various axial lengths. They are of a range of different ratings and of either 50Hz or 60Hz synchronous frequency. For each machine a design factor is calculated which is defined in equation (14). The results for the machines yield an average design factor of 1.71 and the series can be seen in Figure 13.
6 Conclusions

This paper compares several different methods of calculating iron loss from FE flux density plots. The paper investigates both time and frequency methods and looks at the affects of alternate orthogonal component calculation methodology including: Cartesian, Radial/Tangential and Major/Minor Loop. By examining both an ideal 1m² block, an actual synchronous generator and a series of forty six synchronous generators the main conclusions of the paper are as follows:

- Loss Calculation in the Cartesian plane should not be used as it does not give geometry independent consistent solutions
- The loss in accuracy in using a Radial/Tangential method compared the Major/Minor Loop method is typically less than 4% yet the post processing calculation time was found to be around 6 times quicker and as such is a good engineering compromise which can be easily incorporated into a design package.
- The capabilities of curve fitting routines, loss curves accuracies and material handing during manufacture can add larger inaccuracies than the method of loss calculation.

7 References and Acknowledgements

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