Size and Power of Tests based on Permanent-Transitory Component Models

Fabrizio Casalin*

Abstract

The literature has recently proposed a new type of tests for the Efficient Market Hypothesis based on Permanent-Transitory Component Models. We compare the power of these statistics with conventional tests based on linear regressions. Simulation results suggest that the former dominate the latter for a wide range of data generating processes. We propose an application to spot and forward interest rates. Empirical results show that the two types of tests can yield conflicting results which can be explained by the size distortions and reduced power which affect the statistics based on linear regressions.

JEL classification: C32, F47, G17

Keywords: Efficient Markets Hypothesis; Permanent-Transitory Component Models; MC Simulations

*Newcastle University Business School, Newcastle upon Tyne, NE1 4SE, UK. E-mail: fabrizio.casalin@newcastle.ac.uk, Tel.:+44-(0)191-208-1595, Fax: +44-(0)191-208-1738. I am grateful to Enzo Dia, Giorgio Fazio and two anonymous referees for their useful comments.

1 Introduction

A large number of studies builds on the idea that the price of financial securities is driven by a common stochastic trend - which can be thought of as the fundamental value - and transient disequilibrium terms. For instance, Campbell and Shiller (1987) emphasize how there is only one non stationary common driving force which can be interpreted as something exogenous to the system of the term structure.[1] The presence of common stochastic trends in financial securities has been traditionally modelled through the concept of co-integration.[2] This last has become a convenient frame within which tests for the Efficient Market Hypothesis (EMH) can be carried out. For instance, considering forward and spot rates, the EMH is evaluated by estimating linear regressions between levels of the two rates (levels regressions), or between excess forward returns and forward premia (forward-spot regressions), and by testing that the parameter attached to the regressor is equal, respectively, to one and zero (see, e.g., Cuthberson 1996; Fama and Bliss 1987). However, a well-known limitation of these approaches is that - being the EMH a joint hypothesis of rational expectation and constant term premia - they cannot decompose the relative contribution of the two factors to the invalidation of the hypothesis. Moreover, the presence of serial correlation in the disturbance term of levels regressions may induce estimation bias and invalidate asymptotic inference (see Li and Maddala 1997).

Recently, an important strand of research has documented the presence of different forms of non-linearities in interest rate movements, and shown that these last can affect the finite sample performances of tests for the EMH.[3] For instance, Clarida et al. (2006), and Bansal and Zhou (2002) use different tests for EMH and document how their outcome depends on the ability of empirical models to properly detect regime shifts in interest rates series, thus suggesting that the presence of these last might be an important source of misspecification. Popular modelling strategies were to enable regime shifts in specific parameters featuring univariate or multivariate models with error correction, as well as in the parameters governing the term premia component or the conditional volatility. Regime shifts, in turn, have been traditionally modelled through Markov-switching processes - therefore enabling for multiple switchings - or single structural breaks

[1] Similarly, in Mussa's (1982) sticky-price model exchange rates are represented as a combination of fundamental and transient disequilibrium terms.

[2] See Granger (1986) for a comprehensive coverage of the topic.

[3] Non-linear dynamics can be induced by factors such as business cycle expansions and contractions, asymmetric transaction costs or infrequent trading.

(see, e.g. Gray (1996) and Brooks and Rew (2002)).

In the last decade, a number of scholars has modelled co-integration by following an alternative approach based on Permanent-Transitory Component Models (PTCMs). For instance, Iyer (2000) applies PTCMs to spot and forward interest rates whereas Hai et al. (1997) use the same modelling strategy to study the forward discount bias in foreign exchange markets. Recently, Casalin (2013) has proposed a PTCMs representation of spot and forward rates which makes it possible the identification of specific restrictions for the EMH and rational expectations which can be tested by means of standard Likelihood Ratio (LR) statistics. By making use of the moving average representation of spot and forward rates, the author shows that the above statistics are linked to conventional tests based on levels and forward-spot regressions.

This paper aims to advance the understanding of the finite sample properties for the LR statistics based on PTCMs by comparing their power with that of conventional tests based on linear regressions. We carry out the empirical analysis under a large spectrum of data-generating processes featuring normal disturbance terms, volatility clustering, misspecification of term premia, multiple regime shifts, and integrated versus near-integrated series for forward and spot rates. Empirical results suggest that LR tests for the null of EMH present approximately correct size and stronger power than their counterparts based on forward-spot and levels regressions. Moreover, conventional tests based on levels regressions are affected by size distortions which lead to over rejections of the null. All in all, our simulation exercises suggest that LR statistics based on PTCMs perform better than conventional tests based on linear regressions over a wide range of DGPs. The power of the above statistics tend to weaken for DGPs that depart from the benchmark case of normal disturbance terms. More specifically, the presence of near-integrated series as well as misspecified term premia are the two elements with the strongest power reducing effect, whereas both volatility clustering and regime shifts present negligible impacts. We propose an application of the above tests to series for three-month Eurodollar and Sterling Libor spot and forward interest rates. When applied to the two datasets, the tests for EMH agree in rejecting the null at standard significance levels. Similarly, tests for rational expectations consistently reject the null when applied to Sterling series. However, the same tests deliver inconsistent results when applied to Eurodollar series. More specifically, conventional tests based on linear regressions soundly reject the null of rational expectations, whereas statistics based on PTCMs fail to reject the same null at the 10% level. The conflict between the two competing tests can be resolved by recurring to our simulation results which show that, conditional on the data-generating process which characterizes spot and forward series, the former is affected by significant size distortions whereas the latter presents approximately correct size and stronger power.

The rest of the paper is organized as follows. Section 2 sets out the baseline relationship on which tests based on PTCMs are built, and it highlights the link between these last and conventional tests based on linear regressions. Section 3 illustrates the design of the simulation experiments. Section 4 compares size and power of tests based on PTCMs with their counterparts based on linear regressions. Section 5 checks the robustness of the above results when spot and forward rates evolve as stationary highly persistent processes. Section 6 proposes an application of the above tests to actual data. Section 7 concludes the paper.

2 Tests based on linear regressions and PTCMs

Defining $S_m(t)$ the m-period spot rate and $F_i^{i+m}(t)$ the m-period futures rate, i.e. the rate at trade date t prevailing between periods $(t+i)$ and $(t+i+m)$, we can specify the baseline relationships for spot and forward rates as follows:

$$F_i^{i+m}(t) = E_t[S_m(t+i)] + \varpi + \gamma(t) \tag{1}$$

$$S_m(t+i) = E_t[S_m(t+i)] + e_{S_m}(t+i) \tag{2}$$

where $E_t[S_m(t+i)]$ denotes the expected spot rate at time t, ϖ and $\gamma(t)$ denote the constant and time-varying component of the term premium and $e_{S_m}(t+i)$ is a random forecast error orthogonal to the information set available at time t.[4] Conventional tests based on linear regressions can be constructed by estimating the following two relationships:

$$F_i^{i+m}(t) - S_m(t+i) = \alpha_0 + \beta_0[F_i^{i+m}(t) - S_m(t)] + e(t+i) \tag{3}$$

$$F_i^{i+m}(t) = \alpha_1 + \beta_1 S_m(t+i) + \xi(t+i) \tag{4}$$

[4]For series of spot and forward rates in stock, foreign exchange and commodity markets m is set equal to 1, whereas for series in bond markets $m \geq 1$.

where the validity of the EMH implies $\beta_0 = 0$ and $\beta_1 = 1$. The above restrictions are tested through the statistics $t_0 = \widehat{\beta}_0/se(\widehat{\beta}_0)$ and $t_1 = (\widehat{\beta}_1 - 1)/se(\widehat{\beta}_1)$.

Tests based on PTCMs exploit Stock and Watson's (1993) observation that co-integrated variables can be expressed as a linear combination of I(1) common stochastic trends and I(0) components. By applying this result to spot and forward rates, it becomes possible to write:

$$F_i^{i+m}(t) = \mu_{F_i^{i+m}}(t) + x_{F_i^{i+m}}(t), \quad \mu_{F_i^{i+m}}(t) = \mu_{F_i^{i+m}}(t-1) + \epsilon_{F_i^{i+m}}(t) \tag{5}$$

$$E_t[S_m(t+i)] = \mu_{S_m}(t), \quad \mu_{S_m}(t) = \mu_{S_m}(t-1) + \epsilon_{S_m}(t) \tag{6}$$

where $\mu_{F_i^{i+m}}(t)$ and $\mu_{S_m}(t)$ are random walk processes, $\epsilon_{F_i^{i+m}}(t)$ and $\epsilon_{S_m}(t)$ are independently distributed white noise disturbances and $x_{F_i^{i+m}}(t)$ is a transient deviation from the stochastic trend.[5] By using eqs.(2) and (6) to specify the observable spot rate at time (t+i) and assuming co-integration, spot and forward rates can be specified as follows:

$$F_i^{i+m}(t) = k_{2,1} \cdot \mu^*(t) + x_{F_i^{i+m}}(t) \tag{7}$$

$$S_m(t+i) = \mu^*(t) + e_{S_m}(t+i) \tag{8}$$

$$\mu^*(t) = \mu^*(t-1) + \epsilon(t) \tag{9}$$

where $k_{2,1}$ is a constant parameter, $\epsilon(t) \sim iid\ N(0, \sigma_\mu^2)$ and $e_{S_m}(t+i) \sim iid\ N(0, \sigma_S^2)$. Eqs.(7)-(9) is the PTCMs representation of spot and forward rates. It shows that the two rates are driven by the same stochastic trend $\mu^*(t)$, stationary "*omnibus*" terms modelled by $x_{F_i^{i+m}}(t)$, and a forecast error $e_{S_m}(t+i)$ which encompasses all the residual forces which affect the two rates.

The rational expectations leg of the EMH is modeled through the parameter $k_{2,1}$. More specifically, when $k_{2,1}$ equals 1, then expectations are formed "correctly", i.e. the forward rate at time t will match, in conditional expectations, the future spot rate. In this case, any difference between the two rates is driven by a term premium plus a random noise which are modelled through the term $x_{F_i^{i+m}}(t) + e_{S_m}(t+i)$. The

[5] In line with Stock and Watson (1993), no restrictions are imposed on the stochastic properties of $x_{F_i^{i+m}}(t)$ beyond being ARMA stationary.

hypothesis of constant term premium is then encompassed in the condition that the transient component $x_{F_i^{i+m}}(t)$ is constant.

By assuming that the process $x_{F_i^{i+m}}(t)$ evolves as an ARMA(1,1) as follows:

$$(1-\phi L)x_{F_i^{i+m}}(t) = \varpi + (1-\theta L)\varepsilon_{F_i^{i+m}}(t) \tag{10}$$

with $\varepsilon_{F_i^{i+m}} \sim \ iid \ N(0,\sigma_F^2)$ and exploiting the moving average representation of Eqs.(7)-(9), it can be shown that the population value of β_0 is as follows:

$$\beta_0 = \frac{(1-k_{2,1})\sigma_\mu^2[(1-k_{2,1})(T-m)+m)] + \frac{1+\theta^2-2\theta\phi}{1-\phi^2}\sigma_F^2}{\sigma_\mu^2[(1-k_{2,1})^2(T-m)+m] + \frac{1+\theta^2-2\theta\phi}{1-\phi^2}\sigma_F^2 + \sigma_S^2} \tag{11}$$

whereas the population value of β_1 converges to the following expression:

$$\lim_{T\to\infty} \frac{k_{2,1}T\sigma_\mu^2}{k_{2,1}^2 T\sigma_\mu^2 + \frac{1+\theta^2-2\phi\theta}{1-\phi^2}\sigma_F^2} = \frac{1}{k_{2,1}} \tag{12}$$

where T is the number of observations used to estimate eqs.(3) and (4) (see Casalin (2013)). Eqs.(11) and (12) shed light on the link between tests based on PTCMs and their counterparts based on linear regressions. On the one hand, the baseline case of EMH, which is tested through the null $H_0 : \beta_0 = 0$ in forward-spot regressions, is equivalent to the null $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$ when tested through PTCMs. Departures from the EMH can be modelled through the following data-generating processes:

i - *Departures from RE and time varying TP:* $k_{2,1} \neq 1 \cap \sigma_F > 0$

ii - *Departures from RE and constant TP:* $k_{2,1} \neq 1 \cap \sigma_F = 0$

iii - *RE and time varying TP:* $k_{2,1} = 1 \cap \sigma_F > 0$

In the first case the population value of the parameter β_0 takes the general specification of eq.(11), whereas in the last two cases it assumes restricted specifications which depend on whether $k_{2,1} = 1$ or $\sigma_F = 0$. On the other hand, when spot and forward rates are co-integrated, the parameter β_1 detects only departures from rational expectations, since the variability induced by time varying term premia vanishes as the number of observations increases. Thus, the baseline case of rational expectations, which is tested through the null

$H_0 : \beta_1 = 1$, is equivalent to the null $H_0 : k_{2,1} = 1$ when tested with PTCMs.

The PTCMs representation of spot and forward rates of eqs.(7)-(9) can be estimated by means of Kalman Filter and Maximum Likelihood (ML). The null hypotheses of EMH and rational expectations can be specified as $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$ and $H_0 : k_{2,1} = 1$, and tested by means of Likelihood Ratio (LR) tests. Throughout the paper the two statistics will be denoted, respectively, by LR_0 and LR_1. Given that the null of EMH implies that one parameter value (i.e. σ_F^2) is placed on the boundary of the parameter space, the asymptotic distribution of the statistic LR_0 can be approximated by a mixture of central chi-square distributions known as chi-bar square, and defined as $\overline{\chi}^2_{(2)} = 0.5\chi^2_{(1)} + 0.5\chi^2_{(2)}$ (see Shapiro (1985), Self and Liang (1987)).[6] However, as pointed out by Stoel et al. (2006), the chi-bar square specification holds exactly as long as the empirical distribution of the parameters under constraint is symmetrical. When the condition of symmetry does not hold then the combination of weights departs from 0.5-0.5 and the above specification becomes only an approximation. Conventional tests for the null of EMH and rational expectations are carried out by means of the statistics t_0 and t_1. Since the latter is based on co-integrating regressions, inference is made by applying Fully Modified OLS (FM-OLS) (see Phillips and Hansen 1990).

3 Simulation design

Simulated series for future spot and forward rates are obtained through the following DGP:

$$F_i^{i+m,j}(t) = k_{2,1} \cdot \mu^{*j}(t) + x^j_{F_i^{i+m}}(t) \tag{13}$$

$$S_m^j(t+i) = \mu^{*j}(t) + e^j_{S_m}(t+i) \tag{14}$$

$$\mu^{*j}(t) = \rho\mu^{*j}(t-1) + \epsilon^j(t) \tag{15}$$

$$\epsilon^j(t) = \sigma_\mu^j(t) z^j(t) \tag{16}$$

$$\sigma_\mu^{2,j}(t) = \alpha + \alpha' SW_t^j + \beta\epsilon^{2,j}(t-1) + \gamma\sigma_\mu^{2,j}(t-1) \tag{17}$$

[6] Thus, the conventional $\chi^2_{(2)}$ distribution has too heavy a tail, leading to too-conservative hypothesis tests.

$$\phi(L)x^{j}_{F_i^{i+m}}(t) = \omega + \omega^{'} SW^{j}_{t} + \theta(L)\varepsilon^{j}_{F_i^{i+m}}(t) \quad (18)$$

where j=1,..,1,999, and SW_t is a first-order Markov switching state variable that takes on values 0 or 1, such that $P(SW_t = 0|SW_{t-1} = 0) = p_{00}$ and $P(SW_t = 1|SW_{t-1} = 1) = p_{11}$. Thus, the DGP of eqs. (13)-(18) accounts for integrated and near-integrated forward and spot series, volatility clustering, as well as multiple regime shifts in the levels of the transitory component $x_{F_i^{i+m}}(t)$ and conditional variance of the common trend $\mu^*(t)$. The modelling of volatility and regime shifts is in line with the strand of literature which documents the regime switching behavior of nominal interest rates. More specifically, we follow Bansal and Zhou (2002), and Gray (1996) by setting two alternative regimes, the former characterized by low volatility, small term premia and transition probability p_{00} equal to 0.975, and the latter by high volatility, larger term premia and transition probability p_{11} equal to 0.90.[7] We also set the switching parameters $\omega^{'}$ and $\alpha^{'}$ to 0.1 and 0.25 respectively.

Simulations are carried out by drawing the scalar sequence $[z^j(t), \varepsilon^j_{F_i^{i+m}}(t),\ e^j_{S_m}(t+i)]^T_{t=1}$ from normal distributions with mean 0 and variances equal respectively to unity, σ^2_F and σ^2_S. The sequences of observations $[(\mu^{*j}(t), x^j_{F_i^{i+m}}(t), e^j_{S_m}(t+i))]^T_{t=1}$ are then generated and combined according to eqs.(13)-(18) to construct series for the spot and forward rates with $i = 1$ and $m = 1$. To fully specify the empirical model of eqs.(7)-(10) we assume that the process $x_{F_i^{i+m}}(t)$ evolves as an AR(2), whereas simulated series for the same process are generated through ARMA(2,2) with parameters ϕ_1, ϕ_2, θ_1 and θ_2. Values for the LR statistics are then computed by fitting the PTCM of eqs.(7)-(9) to the computer-generated series with and without the restrictions implied by the null of EMH and rational expectations.[8] Similarly, values for the statistics t_0 and t_1 are obtained by fitting eqs.(3) and (4) to simulated spot and forward rates. The above simulations are carried out for different sets of parameters ρ, $k_{2,1}$, σ^2_F, ϕ_1, ϕ_2, θ_1, θ_2, α, β, γ, p_{00} and p_{11}. The only parameters kept constant across the different simulation exercises are the switching parameters, as well as the constant term ω which is set to zero to ease the computational burden.[9]

In the first simulation exercise we set $\rho = 1$, $\alpha = 1$, $\beta = \gamma = \theta_1 = \theta_2 = 0$, $p_{00} = 1$ and $p_{11} = 0$ in order to measure the size and power of the above statistics under the assumption that forward and spot series are

[7] With such transition probabilities the average number of shifts over 1,999 simulations is 2.6 for T=100, and 10.5 for T=400.

[8] Empirical estimates of PTCMs are worked out by using the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm in Gauss.

[9] The initial values of $\mu^{*j}(t)$ and S^j_t are set equal to 0 in all j repetitions.

driven by a common I(1) stochastic trend with homoscedastic disturbance terms, a term premium component which evolves as an AR(2), and no regime shifts. We then evaluate the resilience of the four statistics to departures from the above DGPs by incorporating fat tails, misspecification in the term premia, as well as regime shifts as previously set out.[10] We do so by carrying out a second simulation exercise where we set $\rho = 1$, $\alpha = 0.25$, $\beta = 0.3$ and $\gamma = 0.45$, $\theta_1 = \theta_2 = 0.3$, $p_{00} = 1$ and $p_{11} = 0$ to measure the finite sample performances of the above statistics in presence of volatility clustering and misspecified term premia.[11]. We then conduct a third empirical exercise by setting $\rho = 1$, $\alpha = 0.25$, $\beta = 0.3$ and $\gamma = 0.45$, $\theta_1 = \theta_2 = 0.0$, $p_{00} = 0.975$, $p_{11} = 0.9$ to evaluate the performances under volatility clustering and regime shifts. Finally, we carry out a last exercise where we amend the previous set of parameters by setting $\theta_1 = \theta_2 = 0.3$ to gauge the impact of misspecified term premia, on top of the volatility clustering and regime shifts effects already considered. We then repeat the above analysis by setting $\rho = 0.975$, i.e. under the assumption that spot and forward rates are driven by a stationary - yet highly persistent - common stochastic trend. Empirical results for the two types of analysis are set out in the next two sections.

4 Power with I(1) spot and forward rates

We begin our analysis by investigating whether the statistics LR_0 and LR_1 under the null $\text{H}_0 : k_{2,1} = 1 \cap \sigma_F = 0$ and $\text{H}_0 : k_{2,1} = 1$ can be approximated, respectively, by the mixture of chi-square distributions $\overline{\chi}^2_{(2)}$, and by a standard $\chi^2_{(1)}$ distribution, as the asymptotic theory suggests. We then carry out a similar exercise to ascertain whether the statistics t_0 and t_1 under the respective null are well approximated by Student-$t_{(T-2)}$ and N(0,1) distributions. Simulated series for spot and forward rates are generated under the null of EMH by following the procedure set out in the previous section and by setting T=400.[12] Empirical values of the statistics LR_0, LR_1, t_0 and t_1 under the respective null are obtained by simulating independently 100 times series for forward and future spot rates. By denoting with q_i the $100i$-th quantile of the null distribution of

[10] In this sense, our simulation strategy follows closely other studies that carried out power comparisons among existing and newly proposed statistical tests (see, e.g., Kim (1996), and Wright (2000)).

[11] Such set of parameters specifies GARCH processes with unconditional variance and kurtosis equal to 1 and 5.

[12] Since this last hypothesis encompasses the null of rational expectations, the same simulated series are used to construct the empirical pdfs of the four statistics under scrutiny. The series are generated through the DGPs of eqs.(13)-(18) under the four cases of normal disturbance terms, volatility clustering with misspecified term premia, with regime shifts, and with both misspecified term premia and regime shifts, and by setting the remaining parameters as follows: $\phi_1 = \phi_2$=0 and σ^2_R=0.5.

the above statistics, an estimate of this last is provided by the $100i$-th ordered statistic in the sequence of 100 replications. Such sequence is then simulated 20 times for a total of 2,000 simulations. The mean and standard deviation of q_i are then used to assess the true, null distribution of the statistics for i=(0.05, 0.25, 0.5, 0.75, 0.90, 0.95, 0.975). The top panel of Table 1 reports sample means (standard deviations) of the quantiles q_i for the empirical distributions of the four statistics under the respective null for the benchmark case of disturbance terms drawn from normal distributions. The mid-top, mid-lower and lower panels report the same figures for the three alternative DGPs characterized by volatility clustering with misspecified term premia, with regime shifts, and with both misspecified term premia and regime shifts. Confidence intervals for each quantile q_i can be constructed by using the respective sample means and standard deviations. Empirical results suggest that such confidence intervals encompass the theoretical cumulated pdf of $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$ and N(0,1) for a wide range of cumulated values, and for the four types of DGPs under scrutiny.

TABLE 1 HERE

We then proceed by comparing the power in finite samples of the above statistics as a function of two parameters T and $\lambda = \sigma^2_\mu/\sigma^2_F$ where the assumed null are $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$ or $H_0 : k_{2,1} = 1$. The parameter λ captures the variability induced by the common trend relative to the time varying term premium. Simulations are carried out for T equal to 100 and 400 observations and for $\lambda = (0.1, 0.5, 1, 2, 10, \infty)$.[13] In the first four columns of Table 2 we investigate the power of the statistics LR_0 and t_0 by simulating series for spot and forward rates under rational expectations and time varying term premia. This is equivalent to generate the above series by imposing on eqs.(13)-(18) the restrictions $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0\}$. The special case when $\lambda \to \infty$ implies DGPs with constant term premia, and it enables a further evaluation of the size of both the statistics under the null of EMH.

The four panels of the same table report the empirical results when the simulated forward and spot series are generated under the benchmark case of normal distributions with zero mean and variances σ^2_μ, σ^2_F and σ^2_R (top panel panel), as well as under the three alternative DGPs characterized by volatility clustering with misspecified term premia (mid-top), with regime shifts (mid-lower), and with both misspecified term premia and regime shifts (lower). Empirical results suggest that for a given T, both the statistics present

[13] The values of λ are computed by setting $\sigma^2_\mu = 1$ and $\sigma^2_F = (10, 5, 1, 0.5, 0.1, 0.0)$.

strong size-adjusted power for values of $\lambda \leq 2$. For values greater than 2 the statistic LR_0 loses power only marginally, whereas the loss of the statistic t_0 is far more severe, with a drop as large as 85% for T=100. Thus, the stronger the co-integrating relationship between forward and spot rates, the lower the power of the statistic t_0. Moreover, for fixed values of λ, power increases with T which is presumably a reflection of the consistency of the tests. Such increases are stronger for the statistic LR_0. Thus, for values of $\lambda > 2$ tests based on PTCMs become preferable to conventional tests based on forward-spot regressions. By setting $\lambda \to \infty$ it becomes possible the evaluate the size of the two statistics under the null of EMH. Empirical results suggest that the statistic LR_0 is affected by small size distortions, whereas t_0 presents approximately correct size. The empirical levels of the statistic LR_0, however, do not differ significantly from the nominal size. As pointed out by Stoel et al. (2006), such differences can be explained by the departures of the empirical distributions of the parameter σ_F^2 under constraint from the condition of symmetry. Such departures, in turn, imply that the chi-bar square specification becomes only an approximation for the distribution of the statistic under the null.

In the subsequent four columns we evaluate the power of the same statistics under the alternative of departures from rational expectations and time varying term premia, i.e. when the DGPs are specified as $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0\}$. These DGPs are equivalent to the case of deviations from rational expectations with either white noise or ARMA term premia. For a given T, both the statistics present strong size-adjusted power for $\lambda \leq 2$. However, also in this case the statistic t_0 tends to lose power for $\lambda > 2$, whereas the power of LR_0 remains close to unity. For instance, the case $\lambda \to \infty$ specifies DGPs where the EMH is rejected solely because of departures from rational expectations. In this special case the power of t_0 decreases by more than 90% for T=100. For fixed values of λ the power improves with T, yet remaining substantially lower than the power of LR_0. All in all, the above empirical results suggest that for values of $\lambda > 2$ the LR_0 tests become preferable to conventional tests based on forward-spot regressions, whereas for $\lambda \leq 2$ the two statistics are equivalent.

We then proceed by evaluating the power of the statistics LR_1 and t_1 when the DGPs are $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0.3\}$. The ninth and tenth columns of Table 2 report the performances of both the statistics when the DGPs are equivalent to the case of rational expectations and time varying term premia. This enables the investigation of the size of the two statistics. Moreover, the special case of $\lambda \to \infty$ enables to gauge the size under the null of EMH. Empirical results suggest that the statistic LR_1 has approximately correct size for the entire range of parameters λ and T, whereas the results for the statistic t_1 deserve a more careful analysis. In fact, the presence of time varying term premium can induce serial correlation in the error terms driving the co-integrating relationships. In this case, conventional large sample theory might provide a poor approximation for the distribution of the statistic in small samples (see Li and Maddala 1997). Empirical results suggest that the statistic t_1 is actually oversized for values of λ which range from 0.1 to 10, with distortions that tend to reduce for larger values of λ. The statistic achieves approximately correct size in the special case of $\lambda \to \infty$ with no regime shifts, i.e. when the variability of the term premia becomes negligible.[14]

The last two columns report the performances of the two statistics when the DGPs are equivalent to the case of departures from rational expectations and time varying term premia. For a given T, the statistic LR_1 presents low power for values of λ equal to 0.1. This last, however, quickly increases towards the unity when $\lambda \geq 0.5$, i.e. when the variability induced by the common stochastic trend dominates the variability of term premia. The special case of $\lambda \to \infty$ specifies DGPs characterized by departures from rational expectations and term premia either constant or affected by shifts in levels. Empirical results suggest that also in this case the power of LR_1 remains close to unity. Moreover, for a given λ, the above statistic gains substantial power when T increases. A similar pattern occurs when we evaluate the size-adjusted power of the statistic t_1. However, the statistic LR_1 retains consistently stronger power over the entire range of values taken by λ and T. The above pattern of results holds across the four broad classes of DGPs considered, with the size-adjusted power of the four statistics which tends to decrease when departures from the benchmark case of normal disturbance terms occur. The presence of misspecified term premia is the element which deploys the strongest impact, whereas volatility clustering and regime shifts show less severe power reducing effects.

TABLE 2 HERE

[14] We supplement the above analysis by evaluating the power of the Dynamic OLS (D-OLS) estimator of the statistics t_1 (see Saikkonen 1991; Stock and Watson 1993). Empirical results suggest that such estimator is affected by even stronger size distortions. Such results are not reported to save space but are available from the author upon request.

5 Power with highly persistent spot and forward rates

The main premise of the PTCM previously set out is that spot and forward rates are I(1) stochastic processes. In finance, there is large consensus that foreign exchange rates as well as share prices evolve as I(1) processes. It follows that the above statistics can be readably applied to test for EMH and rational expectations on these markets. However, when the above hypotheses are tested on bond markets, the evidence of I(1) processes is less clear. In fact, many existing studies have modelled interest rates as mean-reverting highly persistent processes. It is, in fact, difficult to imagine explosive interest rates even with the evidence of hyper inflation in the data. In this section we investigate the size and power of LR and t-statistics when spot and forward rates are stationary processes which are treated as I(1). More specifically, we replicate the analysis conducted in Section 4 where simulations of spot and forward rates are carried out by replacing the I(1) common stochastic trend with a stationary - yet highly persistent - process. In Table 3 we evaluate the empirical pdfs of the statistics LR_0, LR_1, t_0 and t_1 under the null $H_0 : k_{2,1} = 1 \cap \sigma_F^2 = 0$ and $H_0 : k_{2,1} = 1$. In Table 4 we compare the power of the same statistics as a function of the parameters T and λ under the same DGPs previously considered. The above simulation exercises are carried out by setting $\rho = 0.975$ in eq.(15).

We begin our analysis by investigating whether the four statistics under the respective null can be approximated by the $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$, Student-$t_{(T-2)}$ and N(0,1) distributions. Also in this case, the series for spot and forward rates are generated by following the procedure reported in Section 3, and their empirical distributions are assessed by computing the mean (standard deviation) of the $100i$-th quantile. In line with the evidence previously reported, the quantile confidence intervals for LR_0 and LR_1 encompass the theoretical cumulated pdfs of $\overline{\chi}^2_{(i,2)}$ and $\chi^2_{(i,1)}$ for large sets of values of the index i. Similar evidence is obtained for both the statistics t_0 and t_1. The above pattern of results holds when simulations are carried out under the four broad classes of DGPs featuring normal disturbance terms, volatility clustering, misspecified term premia and regime shifts. Thus, also in this case departures of various nature from the benchmark case of normality do not have any sizeable impact on the empirical distribution of the four statistics.

TABLE 3 HERE

We then proceed by comparing the power in finite samples of the four statistics where the assumed null

are $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$ or $H_0 : k_{2,1} = 1$. The first four columns of Table 4 report the power of the LR_0 and t_0 statistics when the DGPs are $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0\}$. For a given T, both the statistics present strong power for values of $\lambda \leq 2$, whereas they tend to lose power for values of λ greater than 2. The loss of power, however, is much more severe for the statistic t_0. Thus, also in the case of highly persistent spot and forward rates, the statistic t_0 tends to lose power when the variability induced by the common trend dominates the variability of term premia. It follows that for values of $\lambda > 2$ tests based on PTCMs become preferable to conventional tests based on forward-spot regressions. Moreover, for fixed values of λ, power increases with T. The size of the two statistics can be evaluated when $\lambda \to \infty$. Empirical results suggest that the statistic LR_0 is affected by small size distortions, whereas t_0 presents approximately correct size.[15]
The subsequent four columns report the power of the two statistics when the DGPs are $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0\}$. For a given T, the statistic LR_0 presents power close to unity for the entire range of the parameter λ. On the contrary, the statistic t_0 drastically loses power for $\lambda > 2$. In the limit case of $\lambda \to \infty$, i.e. when the EMH is rejected solely because of departures from rational expectations, the power collapses to levels as low as 4.5%. This last result suggests that when spot and forward rates are stationary persistent processes mainly driven by fluctuations in the common trend, then tests based on forward-spot regressions become unable to reject the null of EMH.

We then proceed by evaluating the power of the statistics LR_1 and t_1 when the DGPs are $\{k_{2,1} = 1 \cap \phi_1 = \phi_2 = 0.3\}$ and $\{k_{2,1} = 0.95 \cap \phi_1 = \phi_2 = 0.3\}$. Empirical results reported in the ninth and tenth columns suggest that the LR_1 statistic presents approximately correct size for the entire range of parameters λ and T. Similarly to the results of Section 4, the statistic t_1 presents size distortions which gradually vanish as λ increases. The statistic achieves approximately correct size only in the special case of $\lambda \to \infty$ with absence of regime shifts.[16] The last two columns of the table report the performance of the two statistics when the DGPs are equivalent to the case of departures from rational expectations and time varying term premia. For a given T, the statistic LR_1 loses power when compared to the figures reported in Table 2. For instance, when

[15] Also in this case, such small size distortions can be explained by Stoel et al.'s (2006) argument that when the empirical distributions of the parameter σ_F^2 under constraint depart from the condition of symmetry, then the $\overline{\chi^2}_{(2)}$ specification becomes only an approximation for the distribution of the statistic under the null.

[16] Also in this case, we investigate the size of the D-OLS estimator for the statistic t_1, finding even more severe distortions.

$\lambda \leq 0.1$ the power drops by 50% or more. This last, however, quickly increases towards the unity for values of $\lambda \geq 1$. Moreover, the statistic gains substantial power when T increases from 100 to 400 observations. A similar pattern occurs when we evaluate the size-adjusted power of the statistic t_1. However, the statistic LR_1 retains stronger than t_1 for the full spectrum of parameter λ and T. The above pattern of results holds for the benchmark case of normal disturbance terms, as well as for the alternative DGPs encompassing volatility clustering, misspecified term premia and regime shifts. Thus, the better performance of LR statistics survives also in the case of stationary - yet highly persistent - forward and spot series.

TABLE 4 HERE

6 Testing the EMH on Eurodollar and Sterling interest rates

We illustrate an application of the above statistics to monthly series of 3-month spot and forward interest rates for Eurodollar and Sterling Libor contracts over the period January 1987 - August 2013. The forward rates are the implicit rates extracted from the yield curve by using the three- and six-month rates.[17] Figure 1 depicts the four series under analysis.

In Table 5 we carry out a preliminary analysis to assess the non stationarity of spot and forward rates by using ADF-GLS, Modified Phillips-Perron, Sargan-Barghava, Optimal and Modified Optimal Point unit root tests as well as tests for bounded series with lower bound at zero (see Ng and Perron 2001, Elliott et al 1996, Cavaliere and Xu 2011).[18] The eight statistics consistently fail to reject the null of unit root at the 5% for the Eurodollar, and 1% level for Sterling spot and forward series.[19] We then test for co-integration by using standard Trace and Max Eigenvalues statistics, a version of the Trace statistic consistent with structural breaks, as well as residuals-based tests for the null of no co-integration (see Johansen et al 2000). These statistics are reported in the lower panel of the same table. They consistently suggest the presence of one co-integrating relationship between spot and forward rates. All in all, the above results provide convincing evidence that spot and forward Eurodollar and Sterling rates are non stationary and co-integrated processes.

[17] The dataset is obtained from the British Bankers' Association.

[18] We thank Cavaliere and Xu for kindly providing their Gauss code for bounded unit root tests.

[19] Specifications of the above unit root tests include both constant and trend.

We thus proceed by carrying out empirical estimates for the PTCMs of eqs.(7)-(9) as well as OLS (with Newey-West covariance matrix) and FM-OLS estimates of eqs.(3) and (4) so that LR tests as well as conventional tests based on linear regressions for the null of EMH and rational expectations can be computed. The top panel of Table 6 sets out the empirical estimates for the Eurodollar market of the unrestricted PTCM as well as of the same model with restrictions for rational expectations and EMH.[20] The asymptotic standard errors are generally small relative to the point estimates, suggesting that the parameters are precisely estimated.[21] The statistics LR_1 and LR_0 are used to test, respectively, the null of rational expectations and EMH. Marginal significance levels (p-values) indicate that the former cannot be rejected even at the 10% significance level, whereas the latter is soundly rejected at standard levels. These results suggest that the only cause of rejection of the EMH would be the presence of time varying term premium. However, conventional tests based on linear regressions depict a quite different scenario. In fact, the OLS point estimate is 0.354 with the statistic t_0 equal to 2.27, whereas the FM-OLS point estimate is 1.037 with the statistic t_1 equal to 3.36. Thus, according to the conventional tests based on linear regressions, both the null of EMH and rational expectations should be rejected at standard significance levels. While the LR_0 and t_0 statistics are concordant in rejecting the null of EMH, the LR_1 and t_1 statistics yield conflicting results. However, we can say something more on the statistical reliability of the above tests by recalling the simulation results of Table 2 which applies to the case of I(1) series. On the one hand, given the value of the parameter λ equal to 1.55, the statistics LR_0, t_0 and LR_1 present good size and power. On the other hand, the statistic t_1 is oversized with a tendency of having reduced power in comparison to LR_1. These results hold for four broad classes of DGPs featuring normal disturbance terms, volatility clustering, misspecified term premia and regime shifts. Thus, in the choice between the competing tests LR_1 and t_1 the researcher should opt for the former, as the latter is affected by significant size distortions.

The bottom panel of Table 6 displays the empirical estimates of the unrestricted and restricted PTCMs for the Sterling spot and forward rates. Empirical estimates are similar to the previous case where the Eurodollar market was considered. However, this time the interpretation of the results seems more straight

[20]We specify the term premia as AR(2) processes. To ensure stationarity we have imposed appropriate restrictions on the autoregressive parameters ϕ_1 and ϕ_2.

[21]We carry out a sensitivity analysis to check the robustness of the empirical results by feeding the BFGS algorithm with different starting values as well as with the final estimates in order to check that the algorithm delivers estimates consistent with those obtained in previous stages.

forward, as the two types of tests are concordant in rejecting both the null of EMH and rational expectations at standard significance levels.

To check the adequacy of the PTCM specifications used, Ljung-Box portmanteau tests are applied to the vector of residuals of the ARMA model, as proposed in Lütkepohl (1993, p. 300). Both $Q(12)$ and $Q(24)$ suggest the presence of moderate serial correlation in the residuals. Table 6 displays also the implied values as well as the OLS and FM-OLS empirical estimates of the parameters β_0 and β_1. The former are calculated by using the parameter estimates of the unrestricted PTCMs and eqs.(11) and (12). Together with the above figures we also report in square brackets an interval for the implied values as well as the 95% confidence interval for the empirical estimates. The interval of implied values is calculated by substituting in the same equations the combinations of the 95% confidence interval upper and lower bounds which yield maximum width. Under the "eyeball" metric, the PTCMs do quite a fair job of matching these two types of interval. In fact, the overlapping between the two intervals spans from 5% (Eurodollar) to 24% (Sterling) for the parameter β_0, and from 41% (Sterling) to 100% (Eurodollar) for the parameter β_1. Overall, these results suggest that the above PTCMs are reasonably well specified.

FIGURE 1 HERE

TABLES 5 AND 6 HERE

7 Conclusions

We evaluate the small sample performances of a new type of statistics based on Permanent-Transitory Components Models (PTCMs) used to test for the Efficient Market Hypothesis (EMH) and rational expectations in financial markets. A comparison between these last and conventional tests based on linear regressions is carried out under a wide range of different data-generating processes featuring integrated and near-integrated spot and forward rates, volatility clustering, misspecified term premia, as well as multiple regime shifts. Empirical results suggest that tests based on PTCMs dominate over the full spectrum of data-generating processes considered, as they present either stronger power or better size. We illustrate an application using Eurodollar and Sterling Libor spot and forward interest rates. Empirical results for Sterling Libor rates suggest that both types of tests are concordant in rejecting the null of EMH and rational expectations. However, when applied to Eurodollar rates the two types of tests yield results of more difficult interpretation. On the one hand, tests based on linear regressions soundly reject both the null. On the other hand, when tests based on PTCMs are applied to the same data, the null of EMH is still rejected whereas the null of rational expectations cannot be rejected at the 10% significance level. We resolve this conflicting result by recalling the findings of our simulation exercises which show that, for integrated spot and forward series, conventional tests based on linear regressions are significantly over sized unlike tests based on PTCMs which present approximately correct size and stronger power.

Some aspects of this study would benefit from further investigation. Firstly, the simulation exercises developed in the paper can be expanded to explore how PTCM-based tests perform in comparison to other tests available in the literature, such as the VAR-based tests proposed by Campbell and Shiller (1991). Secondly, it would be interesting to evaluate the finite sample performances of PTCMs-based statistics under DGPs featuring breaks in the common stochastic trend. We keep the above as possible avenues for future research.

References

Bansal, R. and H. Zhou (2002). Term structure of interest rates with regime shifts. *Journal of Finance*, 1997–2043.

Brooks, C. and A. Rew (2002). Testing for non-stationarity and cointegration allowing for the possibility of a structural break: An application to eurosterling interest rates. *Economic Modelling 19*, 65 – 90.

Campbell, J. Y. and R. J. Shiller (1987). Cointegration and tests of present values models. *Journal of Political Economy 95*, 1063– 1088.

Campbell, J. Y. and R. J. Shiller (1991). Yield spread and interest rates movements: A bird's eye view. *The Review of Economic Studies 58*, 495– 514.

Casalin, F. (2013). Testing the expectations hypothesis of the term structure with permanent-transitory component models. *Journal of Banking and Finance 37*, 3192–3203.

Cavaliere, G. and F. Xu (2011). Testing for unit roots in bounded time series. *Journal of Econometrics 178*, 259 – 272.

Clarida, R., L. Sarno, M. Taylor, and G. Valente (2006). The role of asymmetries and regime switching in the term structure of interest rates. *Journal of Business 79*, 1193 – 1224.

Cuthberson, K. (1996). The expectations hypothesis of the term structure: The UK interbank market. *The Economic Journal 106*, 578–592.

Elliott, G., T. Rothenberg, and J. H. Stock (1996). Efficient tests for an autoregressive unit root. *Econometrica 64*, 813–836.

Fama, E. and R. Bliss (1987). The information in long maturity forward rates. *American Economic Review 77*, 680–692.

Giles, D. E. and R. T. Godwin (2012). Testing for multivariate cointegration in the presence of structural breaks: P-values and critical values. *Applied Economics Letters 19*, 1561 1565.

Granger, C. W. (1986). Developments in the study of cointegrated economic variables. *Oxford Bulletin of Economics and Statistics 48*, 213–230.

Gray, S. (1996). Modeling the conditional distribution of interest rates as a regime switching process. *Journal of Financial Economics 42*, 27–62.

Hai, W., N. C. Mark, and Y. Wu (1997). Understanding spot and forward exchange rate regressions. *Journal of Applied Econometrics 12*, 715–734.

Iyer, S. (2000). The relationship between short-term and forward interest rates: a structural time-series analysis. *Applied Financial Economics 10*(2), 143–153.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control 12*, 231–254.

Johansen, S., R. Mosconi, and B. Nielsen (2000). Cointegration analysis in the presence of structural breaks in the deterministic trends. *Econometrics Journal 3*, 216–249.

Kim, J. (1996). Wild bootstrapping variance-ratio tests. *Economics Letters 92*, 38–43.

Lee, J. and M. C. Strazicich (2003). Minimum lagrange multiplier unit root test with two structural breaks. *The Review of Economics and Statistics 85*, 1082–1089.

Li, H. and G. Maddala (1997). Bootstrapping cointegrating regressions. *Journal of Econometrics 80*, 297–318.

Lütkepohl, H. (1993). *Introduction to Multiple Time Series Analysis* (First ed.). Springer-Verlag.

Mussa, M. (1982). A model of exchange rate dynamics. *Journal of Political Economy 90*, 74–104.

Ng, S. and P. Perron (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica 69*, 1519–1554.

Phillips, P. C. and B. E. Hansen (1990). Statistical inference in instrumental variables regression with I(1) processes. *Review of Economic Studies 57*, 99 – 125.

Saikkonen, P. (1991). Asymptotically efficient estimation of cointegration regressions. *Econometric Theory 7*, 1–21.

Self, S. G. and K.-Y. Liang (1987). Asymptotic properties of maximum likelihood estimators and likelihood ratio tests under nonstandard conditions. *Journal of the American Statistical Association 82*, 605–610.

Shapiro, A. (1985). Asymptotic distribution of test statistics in the analysis of moment structures under inequality constraints. *Biometrika 72*, 133–144.

Stock, J. H. and M. W. Watson (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica 4*, 783–820.

Stoel, R., F. Garre, C. Dolan, and G. van den Wittenboer (2006). On the likelihood ratio test in structural equation modeling when parameters are subject to boundary constraints. *Psycological Methods 11*, 439–455.

Wright, J. (2000). Alternative variance-ratio tests using ranks and sings. *Journal of Business and Economic Statistics 18*, 1–9.

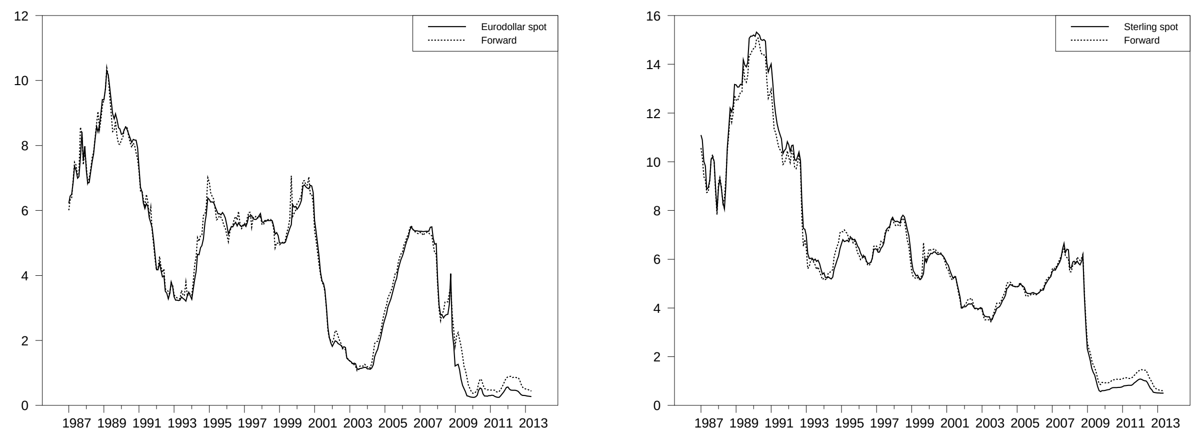

Figure 1: Left panel: Three-month spot (solid line) and forward (dotted line) Eurodollar interest rates for the period 1987:01 - 2013:08. Right panel: Three-month spot (solid line) and forward (dotted line) Sterling Libor interest rates for the period 1987:01 - 2013:08.

Table 1: Simulated quantiles of LR_0, LR_1, t_0 and t_1 probability distribution functions and theoretical $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$ and N(0,1) counterparts when spot and forward rates are integrated processes.

	$q_{0.05}$	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$q_{0.90}$	$q_{0.95}$	$q_{0.975}$
Normal disturbance terms							
LR_0^a	0.004	0.114	0.473	1.299	2.672	3.654	4.381
	(0.004)	(0.060)	(0.164)	(0.269)	(0.708)	(0.864)	(1.124)
LR_1^b	0.005	0.111	0.452	1.250	2.470	3.443	4.537
	(0.007)	(0.059)	(0.164)	(0.379)	(0.709)	(0.933)	(1.766)
t_0^d	-1.645	-0.59	0.045	0.702	1.281	1.630	1.854
	(0.241)	(0.133)	(0.123)	(0.131)	(0.152)	(0.213)	(0.236)
t_1^d	-1.862	-0.756	-0.037	0.590	1.280	1.532	1.884
	(0.242)	(0.149)	(0.128)	(0.146)	(0.236)	(0.238)	(0.310)
Volatility clustering & Misspecified term premia							
LR_0^a	0.006	0.115	0.464	1.375	2.673	3.757	4.499
	(0.009)	(0.061)	(0.214)	(0.502)	(0.549)	(1.005)	(1.240)
LR_1^b	0.006	0.105	0.460	1.361	2.628	3.525	4.627
	(0.007)	(0.068)	(0.173)	(0.392)	(0.659)	(0.884)	(1.710)
t_0^d	-1.707	-0.682	0.045	0.619	1.203	1.525	1.751
	(0.246)	(0.121)	(0.099)	(0.139)	(0.157)	(0.180)	(0.234)
t_1^d	-1.835	-0.783	-0.074	0.664	1.211	1.568	1.888
	(0.213)	(0.176)	(0.082)	(0.129)	(0.219)	(0.197)	(0.221)
Volatility clustering & Regime shifts							
LR_0^a	0.006	0.238	0.923	2.250	4.052	5.673	6.732
	(0.005)	(0.249)	(0.735)	(1.389)	(2.157)	(2.886)	(4.128)
LR_1^b	0.009	0.145	0.526	1.587	3.282	4.344	5.763
	(0.010)	(0.069)	(0.206)	(0.497)	(1.041)	(1.289)	(2.089)
t_0^d	-1.581	-0.519	0.112	0.819	1.386	1.752	1.991
	(0.231)	(0.114)	(0.126)	(0.139)	(0.175)	(0.207)	(0.273)
t_1^d	-2.217	-0.931	-0.120	0.801	1.119	1.980	2.554
	(0.324)	(0.200)	(0.179)	(0.138)	(0.228)	(0.253)	(0.343)
Volatility clustering & Misspecified term premia & Regime shifts							
LR_0^a	0.005	0.126	0.495	1.526	3.093	4.284	5.698
	(0.005)	(0.072)	(0.145)	(0.406)	(0.914)	(1.084)	(1.794)
LR_1^b	0.006	0.135	0.580	1.559	3.246	4.400	5.841
	(0.006)	(0.052)	(0.143)	(0.361)	(0.965)	(1.266)	(1.874)
t_0^d	-1.578	-0.579	0.107	0.789	1.397	1.708	1.969
	(0.216)	(0.112)	(0.140)	(0.146)	(0.185)	(0.180)	(0.213)
t_1^d	-2.170	-0.943	-0.108	0.844	1.159	2.012	2.383
	(0.267)	(0.170)	(0.183)	(0.201)	(0.191)	(0.192)	(0.292)
Theoretical quantiles							
$\overline{\chi}^2_{(2)}$	0.015	0.250	0.870	2.090	3.810	5.140	6.845
$\chi^2_{(1)}$	0.004	0.102	0.454	1.323	2.710	3.841	5.023
$N(0,1)$	-1.646	-0.675	0.000	0.675	1.282	1.646	1.962

Notes: Simulations of four statistics carried out by generating series for forward $F_1^2(t)$ and spot $S_1(t+1)$ under the null of EMH for T=400 through the DGPs of eqs.(13)-(18) with parameters $\rho = 1$, $k_{2,1} = 1$, $\phi_1 = \phi_2 = 0$, ω=0, $\omega' = 0.1$, $\alpha' = 0.25$ and $\sigma_R^2 = 0.25$. The remaining parameters are set as follows: $\alpha = 1$, $\beta = \gamma = \theta_1 = \theta_2 = 0$ and $q_{11} = 1$ (top panel), $\alpha = 0.25$, $\beta = 0.3$, $\gamma = 0.45$, $\theta_1 = \theta_2 = 0.3$ and $q_{11} = 1$ (mid top panel), $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.0$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (mid lower panel), and $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.3$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (lower panel). Number of replications equal to 2,000. q_i are 100i-th quantiles of simulated distributions with standard deviation in parentheses. Theoretical quantiles for $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$ and N(0,1) reported in the bottom panel.

[a] LR statistic for $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$.

[b] LR statistic for $H_0 : k_{2,1} = 1$.

[c] t-statistic for $H_0 : \beta_0 = 0$.

[d] t-statistic for $H_0 : \beta_1 = 0$.

Table 2: Size and power of the statistics LR_0, LR_1, t_0 and t_1.

T	λ	DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.0$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.0$		DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.3$	
		Normal disturbance terms											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.079	0.235	0.121	0.068
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.068	0.223	0.299	0.053
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.065	0.219	0.425	0.145
	2	0.999	0.998	0.999	0.941	0.999	0.998	0.999	0.997	0.066	0.208	0.547	0.206
	10	0.763	0.409	0.627	0.157	0.999	0.428	0.999	0.346	0.085	0.169	0.701	0.503
	∞	0.023	0.047	0.028	0.048	0.999	0.066	0.999	0.074	0.088	0.083	0.941	0.913
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.061	0.202	0.459	0.252
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.065	0.208	0.826	0.409
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.059	0.199	0.921	0.556
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.052	0.188	0.972	0.690
	10	0.995	0.710	0.990	0.523	0.999	0.821	0.999	0.976	0.058	0.133	0.999	0.941
	∞	0.023	0.061	0.029	0.049	0.999	0.619	0.999	0.634	0.058	0.070	0.999	0.996
		Volatility clustering & Misspecified term premia											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.054	0.201	0.056	0.058
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.088	0.189	0.134	0.049
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.057	0.214	0.211	0.06
	2	0.999	0.999	0.999	0.997	0.999	0.999	0.999	0.995	0.07	0.208	0.323	0.094
	10	0.952	0.728	0.646	0.365	0.968	0.726	0.968	0.405	0.079	0.204	0.593	0.241
	∞	0.031	0.051	0.026	0.051	0.957	0.077	0.957	0.089	0.077	0.095	0.962	0.860
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.047	0.145	0.451	0.146
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.057	0.170	0.827	0.224
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.045	0.160	0.924	0.281
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.062	0.183	0.978	0.409
	10	0.994	0.951	0.999	0.620	0.999	0.971	0.999	0.792	0.048	0.171	0.999	0.771
	∞	0.031	0.058	0.026	0.045	0.999	0.234	0.999	0.240	0.051	0.053	0.999	0.999
		Volatility clustering & Regime shifts											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.076	0.239	0.093	0.073
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.081	0.232	0.205	0.081
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.075	0.209	0.282	0.162
	2	0.999	0.997	0.999	0.989	0.999	0.997	0.999	0.990	0.087	0.223	0.363	0.178
	10	0.772	0.402	0.583	0.268	0.950	0.448	0.902	0.336	0.086	0.171	0.600	0.534
	∞	0.056	0.050	0.024	0.051	0.817	0.091	0.822	0.081	0.060	0.127	0.840	0.804
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.055	0.189	0.333	0.169
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.056	0.191	0.671	0.522
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.054	0.189	0.876	0.706
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.049	0.188	0.932	0.856
	10	0.999	0.937	0.986	0.809	0.999	0.992	0.999	0.972	0.047	0.169	0.999	0.993
	∞	0.085	0.053	0.023	0.058	0.999	0.687	0.999	0.663	0.082	0.122	0.999	0.999
		Volatility clustering & Misspecified term premia & Regime shifts											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.064	0.199	0.067	0.059
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.042	0.213	0.123	0.050
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.084	0.202	0.178	0.074
	2	0.999	0.999	0.999	0.995	0.999	0.999	0.999	0.995	0.050	0.208	0.231	0.064
	10	0.953	0.711	0.586	0.340	0.981	0.723	0.886	0.383	0.049	0.198	0.454	0.251
	∞	0.055	0.052	0.051	0.054	0.804	0.089	0.836	0.085	0.056	0.110	0.862	0.860
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.031	0.109	0.145	0.089
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.027	0.122	0.405	0.216
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.025	0.140	0.601	0.360
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.031	0.160	0.929	0.535
	10	0.999	0.999	0.999	0.885	0.999	0.999	0.999	0.986	0.049	0.188	0.999	0.928
	∞	0.045	0.055	0.025	0.058	0.999	0.683	0.999	0.626	0.083	0.124	0.999	0.999

Notes: Simulations of four statistics carried out by generating series for forward $F_1^2(t)$ and spot $S_1(t+1)$ through the DGPs of eqs. (13)-(18) with parameters $k_{2,1}$, ϕ_1, ϕ_2, λ set to values reported in the header of columns from 3 to 14 and column 2, and $\alpha=1$, $\beta=\gamma=\theta_1=\theta_2=0$ and $q_{11}=1$ (top panel), $\alpha=0.25$, $\beta=0.3$, $\gamma=0.45$, $\theta_1=\theta_2=0.3$ and $q_{11}=1$ (mid top panel), $\alpha=0.25$, $\alpha=0.3$, $\alpha=0.45$, $\theta_1=\theta_2=0.0$, $q_{11}=0.975$ and $q_{22}=0.9$ (mid lower panel), and $\alpha=0.25$, $\alpha=0.3$, $\alpha=0.45$, $\theta_1=\theta_2=0.3$, $q_{11}=0.975$ and $q_{22}=0.9$ (lower panel). The remaining parameters are set as follows: $\rho=1$, ω=0, $\omega'=0.1$, $\alpha'=0.25$ and $\sigma_R^2=0.25$. Number of replications equal to 1,999. Size-adjusted power at 5% level.

[a] LR statistic for the null $H_0: k_{2,1}=1 \cap \sigma_F=0$.

[b] LR statistic for the null $H_0: k_{2,1}=1$.

[c] t-statistic for the null $H_0: \beta_0=0$.

[d] t-statistic for the the null $H_0: \beta_1=1$.

Table 3: Simulated quantiles of LR_0, LR_1, t_0 and t_1 probability distribution functions and theoretical $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$, Student-$t_{(198)}$ and N(0,1) counterparts when spot and forward rates are stationary highly persistent processes.

	$q_{0.05}$	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$q_{0.90}$	$q_{0.95}$	$q_{0.975}$
			Normal disturbance terms				
LR_0^a	0.005	0.095	0.428	1.343	2.672	3.564	4.454
	(0.007)	(0.052)	(0.155)	(0.385)	(0.747)	(0.969)	(1.476)
LR_1^b	0.006	0.105	0.480	1.304	2.730	3.737	4.628
	(0.007)	(0.051)	(0.153)	(0.402)	(0.784)	(1.135)	(1.555)
t_0^d	-1.640	-0.621	0.048	0.705	1.267	1.573	1.781
	(0.253)	(0.136)	(0.142)	(0.146)	(0.186)	(0.169)	(1.888)
t_1^d	-1.925	-0.890	-0.189	0.497	1.258	1.494	1.844
	(0.232)	(0.157)	(0.104)	(0.116)	(0.126)	(0.157)	(0.226)
			Volatility clustering & Misspecified term premia				
LR_0^a	0.006	0.102	0.465	1.412	2.604	3.567	4.466
	(0.013)	(0.055)	(0.182)	(0.399)	(0.691)	(1.015)	(1.364)
LR_1^b	0.007	0.117	0.480	1.406	2.791	3.828	4.827
	(0.01)	(0.079)	(0.147)	(0.394)	(0.816)	(1.125)	(1.542)
t_0^d	-1.624	-0.606	0.005	0.675	1.279	1.623	1.812
	(0.189)	(0.132)	(0.123)	(0.145)	(0.161)	(0.206)	(0.23)
t_1^d	-1.947	-0.897	-0.215	0.541	1.263	1.456	1.785
	(0.204)	(0.128)	(0.113)	(0.117)	(0.177)	(0.221)	(0.265)
			Volatility clustering & Regime shifts				
LR_0^a	0.003	0.106	0.389	1.362	2.683	3.444	4.379
	(0.004)	(0.056)	(0.163)	(0.446)	(0.843)	(1.151)	(1.507)
LR_1^b	0.008	0.166	0.617	1.759	3.496	4.782	5.942
	(0.009)	(0.078)	(0.211)	(0.421)	(1.012)	(1.215)	(1.649)
t_0^d	-1.538	-0.475	0.16	0.83	1.428	1.754	1.953
	(0.211)	(0.138)	(0.115)	(0.166)	(0.172)	(0.198)	(0.228)
t_1^d	-2.138	-0.995	-0.197	0.656	1.111	1.783	2.243
	(0.225)	(0.141)	(0.167)	(0.173)	(0.268)	(0.300)	(0.323)
			Volatility clustering & Misspecified term premia & Regime shifts				
LR_0^a	0.005	0.118	0.468	1.398	2.482	3.643	4.614
	(0.008)	(0.058)	(0.178)	(0.413)	(0.582)	(1.000)	(1.541)
LR_1^b	0.011	0.167	0.642	1.798	3.553	5.095	6.171
	(0.014)	(0.097)	(0.254)	(0.469)	(1.062)	(1.675)	(2.037)
t_0^d	-1.557	-0.562	0.09	0.78	1.414	1.765	1.973
	(0.170)	(0.134)	(0.131)	(0.128)	(0.132)	(0.196)	(0.249)
t_1^d	-2.171	-0.904	-0.057	0.718	1.135	1.789	2.251
	(0.219)	(0.184)	(0.141)	(0.160)	(0.210)	(0.198)	(0.329)

Notes: Simulations of four statistics carried out by generating series for forward $F_1^2(t)$ and spot $S_1(t+1)$ under the null of EMH for T=400 through the DGPs of eqs.(13)-(18) with parameters $\rho = 0.975$, $k_{2,1} = 1$, $\phi_1 = \phi_2 = 0$, ω=0, $\omega^{'} = 0.1$, $\alpha^{'} = 0.25$ and $\sigma_R^2 = 0.25$. The remaining parameters are set as follows: $\alpha = 1$, $\beta = \gamma = \theta_1 = \theta_2 = 0$ and $q_{11} = 1$ (top panel), $\alpha = 0.25$, $\beta = 0.3$, $\gamma = 0.45$, $\theta_1 = \theta_2 = 0.3$ and $q_{11} = 1$ (mid top panel), $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.0$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (mid lower panel), and $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.3$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (lower panel). Number of replications equal to 2,000. q_i are 100i-th quantiles of simulated distributions with standard deviation in parentheses. Theoretical quantiles for $\overline{\chi}^2_{(2)}$, $\chi^2_{(1)}$ and N(0,1) reported in the bottom panel.

[a] LR statistic for $\mathrm{H}_0 : k_{2,1} = 1 \cap \sigma_F = 0$.

[b] LR statistic for $H_0 : k_{2,1} = 1$.

[c] t-statistic for $H_0 : \beta_0 = 0$.

[d] t-statistic for $H_0 : \beta_1 = 0$.

Table 4: Size and power of the statistics LR_0, LR_1, t_0 and t_1 when spot and forward rates are stationary highly persistent processes.

T	λ	DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.0$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.0$		DGP: $k_{2,1}=1$ $\phi_1=\phi_2=0.3$		DGP: $k_{2,1}=0.95$ $\phi_1=\phi_2=0.3$	
		Normal disturbance terms											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.069	0.236	0.073	0.052
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.066	0.219	0.102	0.092
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.062	0.224	0.116	0.094
	2	0.999	0.997	0.999	0.993	0.999	0.998	0.999	0.995	0.073	0.202	0.196	0.142
	10	0.741	0.415	0.627	0.282	0.888	0.389	0.893	0.260	0.084	0.153	0.437	0.420
	∞	0.023	0.050	0.033	0.059	0.845	0.055	0.829	0.053	0.091	0.084	0.950	0.856
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.043	0.203	0.212	0.074
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.055	0.198	0.758	0.180
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.046	0.200	0.906	0.286
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.062	0.196	0.991	0.455
	10	0.999	0.946	0.991	0.831	0.999	0.696	0.999	0.523	0.060	0.131	0.998	0.955
	∞	0.030	0.047	0.021	0.047	0.999	0.052	0.999	0.063	0.059	0.058	0.999	0.999
		Volatility clustering & Misspecified term premia											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.066	0.188	0.067	0.055
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.061	0.210	0.085	0.052
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.072	0.213	0.101	0.049
	2	0.999	0.999	0.999	0.996	0.999	0.999	0.999	0.993	0.070	0.212	0.133	0.047
	10	0.963	0.727	0.628	0.345	0.973	0.707	0.876	0.339	0.093	0.202	0.285	0.180
	∞	0.024	0.047	0.024	0.056	0.779	0.048	0.785	0.045	0.080	0.100	0.919	0.854
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.039	0.118	0.103	0.035
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.054	0.142	0.295	0.042
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.051	0.157	0.468	0.064
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.046	0.179	0.695	0.124
	10	0.999	0.959	0.995	0.641	0.999	0.942	0.999	0.664	0.060	0.185	0.977	0.588
	∞	0.027	0.054	0.031	0.053	0.999	0.069	0.998	0.064	0.063	0.064	0.999	0.999
		Volatility clustering & Regime shifts											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.08	0.243	0.083	0.054
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.078	0.238	0.118	0.067
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.031	0.222	0.13	0.096
	2	0.999	0.995	0.999	0.990	0.999	0.995	0.999	0.983	0.074	0.209	0.188	0.155
	10	0.796	0.406	0.596	0.271	0.853	0.373	0.85	0.252	0.088	0.182	0.393	0.416
	∞	0.069	0.046	0.028	0.049	0.682	0.048	0.704	0.045	0.07	0.117	0.751	0.736
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.05	0.196	0.089	0.073
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.052	0.189	0.252	0.161
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.044	0.215	0.383	0.292
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.064	0.189	0.62	0.509
	10	0.999	0.934	0.999	0.794	0.999	0.954	0.999	0.854	0.077	0.164	0.953	0.946
	∞	0.021	0.055	0.025	0.043	0.999	0.133	0.999	0.118	0.098	0.119	0.999	0.999
		Volatility clustering & Misspecified term premia & Regime shifts											
		LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_0^a	t_0^c	LR_1^b	t_1^d	LR_1^b	t_1^d
100	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.079	0.220	0.086	0.054
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.076	0.213	0.093	0.046
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.073	0.222	0.94	0.043
	2	0.999	0.999	0.999	0.996	0.999	0.999	0.999	0.993	0.077	0.219	0.131	0.064
	10	0.963	0.730	0.634	0.330	0.947	0.673	0.802	0.329	0.058	0.207	0.279	0.207
	∞	0.075	0.05	0.058	0.047	0.683	0.051	0.729	0.05	0.059	0.124	0.763	0.782
400	0.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.045	0.124	0.066	0.035
	0.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.062	0.134	0.151	0.063
	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.053	0.166	0.282	0.063
	2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.050	0.179	0.411	0.109
	10	0.999	0.998	0.999	0.884	0.999	0.999	0.999	0.911	0.054	0.187	0.97	0.673
	∞	0.059	0.054	0.036	0.053	0.999	0.131	0.999	0.120	0.090	0.101	0.999	0.999

Notes: Simulations of four statistics carried out by generating series for forward $F_1^2(t)$ and spot $S_1(t+1)$ through the DGPs of eqs.(13)-(18) with parameters $k_{2,1}$, ϕ_1, ϕ_2, λ set to values reported in the header of columns from 3 to 14 and column 2, and $\alpha = 1$, $\beta = \gamma = \theta_1 = \theta_2 = 0$ and $q_{11} = 1$ (top panel), $\alpha = 0.25$, $\beta = 0.3$, $\gamma = 0.45$, $\theta_1 = \theta_2 = 0.3$ and $q_{11} = 1$ (mid top panel), $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.0$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (mid lower panel), and $\alpha = 0.25$, $\alpha = 0.3$, $\alpha = 0.45$, $\theta_1 = \theta_2 = 0.3$, $q_{11} = 0.975$ and $q_{22} = 0.9$ (lower panel). The remaining parameters are set as follows: $\rho = 0.975$, ω=0, $\omega^{'} = 0.1$, $\alpha^{'} = 0.25$ and $\sigma_R^2 = 0.25$. Number of replications equal to 1,999. Size-adjusted power at 5% level.

[a] LR statistic for the null $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$.

[b] LR statistic for the null $H_0 : k_{2,1} = 1$.

[c] t-statistic for the null $H_0 : \beta_0 = 0$.

[d] t-statistic for the the null $H_0 : \beta_1 = 1$.

Table 5: Unit root and co-integration tests for three-month spot and forward interest rates on Eurodollar and Sterling markets.

	EURODOLLAR		STERLING	
	Forward	Spot	Forward	Spot
DF-GLS[a]	-2.083	-1.805	-2.939*	-2.746
MZ_t^b	-2.089	-1.811	-2.943*	-2.788
MSB[c]	0.235	0.272	0.169	-0.179
MPT_α^d	10.32	13.69	5.275*	-5.867
ERS_α^e	10.69	13.97	5.296*	-5.909
MLN_t^f	-3.591	-3.555	-3.864*	-3.964*
DF-OLS[g]	-2.440	-1.911	-2.993	-2.733
MZ_t^g	-2.497	-1.953	-3.393	-2.919
Trace[h]	31.99**		23.58**	
Eigen[h]	28.59**		22.17**	
Trace[i]	40.86**		28.89**	
E-G - Z_t^l	-4.071**		-8.089**	
P-O - Z_t^l	-5.523**		-5.845**	
G-H - Z_t^m	-6.702**		-6.584**	

Notes: Sample periods span from 1987:01 to 2013:08 for Eurodollar and Sterling Libor interest rates. * (**) statistically significant at 5% (1%) level.

[a] Dickey-Fuller GLS de-trended test with critical values at 5 (1%) level equal to -2.890 (-3.480).

[b] Ng and Perrons (2001) Modified Phillips-Perron statistic with critical values at 5 (1%) level equal to -2.910 (-3.420).

[c] Modified Sargan-Barghava test with critical values at 5 (1%) level equal to 0.168 (0.143).

[d] Modified Optimal Point statistic with critical values at 5 (1%) level equal to 5.480 (4.030).

[e] Elliott et al (1996) Optimal Point test with critical values at 5 (1%) equal to 5.636 (3.996). Statistics computed using spectral GLS de-trended AR kernel based on SIC.

[f] Lee and Strazicich's 2003 Minimum LM test with critical values at 5% (1%) equal to -3.842 (-4.545) for two endogeneous structural breaks in the level.

[g] Dickey-Fuller OLS de-trended and Modified Phillips-Perron unit root tests for bounded series with lower bound at zero (see Cavaliere and Xu (2011)).

[h] Johansen's (1988) Trace and Eigenvalue statistic with critical values at 5% (1%) level equal to 15.41 (20.04) and 14.07 (18.63) for the null of no co-integrating relationship.

[i] Johansen et al (2000) Trace statistic with simulated critical values at 5% (1%) level equal to 12.3 (16.9) for the null of zero co-integrating relationships with v_1=0.8 for the 2 datasets (see Table 1 in Giles and Godwin (2012)).

[l] Engle-Granger and Phillips-Ouliaris residual-based τ statistics for the null of no co-integration with critical values at 5% (1%) equal to -3.338 (-3.900).

[m] Gregory-Hansen residual-based z_t^* statistic for the null of no co-integration with critical values at 5% (1%) equal to -4.610 (-4.340).

Table 6: Maximum likelihood estimates of the PTCM of eqs.(7)-(9) for Eurodollar and Sterling Libor three-month spot and forward interest rates.

PTCM							IMPLIED		EMPIRICAL	
$k_{2,1}$	σ_μ	ϖ	ϕ_1	ϕ_2	σ_F	σ_R	β_0	β_1	β_0	β_1
EURODOLLAR LIBOR										
log Lik = 261.11										
0.975	0.221	0.080	1.243	-0.386	0.227	0.077	0.682	1.027	0.354	1.037
(0.022)	(0.010)	(0.000)	(0.059)	(0.037)	(0.013)	(0.013)	[0.633;1.274]	[0.981;1.074]	[0.022;0.666]	[1.015;1.059]
Q(12)= 66.00 (0.018)				Q(24)= 97.39 (0.330)						
Restriction: $k_{2,1} = 1$										
log Lik = 260.57										
-	0.221	0.089	1.218	-0.371	0.226	0.078				
	(0.010)	(0.000)	(0.053)	(0.032)	(0.013)	(0.013)				
$\text{LR}_1^a = 1.08$ (0.298)										
Restrictions: $k_{2,1} = 1 \cap \sigma_F = 0$										
log Lik = 43.25										
-	0.254	0.267	1.904	-0.907	-	0.497				
	(0.010)	(0.033)	(0.038)	(0.036)		(0.020)				
$\text{LR}_0^b = 435.7$ (0.000)										
STERLING LIBOR										
log Lik = 130.55										
0.962	0.279	0.080	1.091	-0.297	0.332	0.032	0.668	1.039	0.466	1.075
(0.014)	(0.012)	(0.000)	(0.054)	(0.029)	(0.018)	(0.037)	[0.636;0.882]	[1.010;1.071]	[0.146;0.786]	[1.053;1.097]
Q(12)= 73.88 (0.003)				Q(24)= 94.13 (0.418)						
Restriction: $k_{2,1} = 1$										
log Lik = 126.78										
-	0.277	0.133	1.098	-0.301	0.333	0.039				
	(0.012)	(0.095)	(0.052)	(0.029)	(0.018)	(0.031)				
$\text{LR}_1^a = 7.540$ (0.006)										
Restrictions: $k_{2,1} = 1 \cap \sigma_F = 0$										
log Lik = 118.01										
-	0.323	0.106	0.335	-0.005	-	0.674				
	(0.013)	(0.039)	(3.117)	(0.018)		(0.027)				
$\text{LR}_0^b = 25.08$ (0.000)										

Notes: Dataset consists of future spot and three-month forward interest rates for the period 1987:01 - 2013:08. Asymptotic standard errors in parentheses.

[a] LR test for the null $H_0 : k_{2,1} = 1$. LR statistic distributed as $\chi^2_{(1)}$. P-values in parentheses.

[b] LR test for the null $H_0 : k_{2,1} = 1 \cap \sigma_F = 0$. LR statistic distributed as $\overline{\chi}^2_{(2)}$. P-values in parentheses.

Q(p) are p-th order Ljung-Box statistics for serial correlation. Q(12)$\sim \chi^2_{(44)}$ and Q(24)$\sim \chi^2_{(92)}$. P-values in parentheses.

Implied values for β_0 and β_1 computed by substituting in eqs.(11)-(12) the combination of point estimates of the unrestricted PTCMs reported in first and fourth panels. Intervals reported in squared brackets constructed by using combinations of upper and lower 95% confidence interval bounds that maximize width.

Empirical estimates of β_0 and β_1 carried out by means of OLS (Newey-West standard deviations) and FM-OLS estimates of eqs.(3)-(4). 95% confidence intervals reported in squared brackets.