Fundamenta Informaticae XXI (2001) 1001–1019

DOI 10.3233/FI-2016-0000

IOS Press

Reversing Transitions in Bounded Petri Nets

Kamila Barylska*
Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University
Toruń, Chopina 12/18, Poland
kamila.barylska@mat.umk.pl

Evgeny Erofeev†
Parallel Systems, Department of Computing Science
Carl von Ossietzky Universität
D-26111 Oldenburg, Germany
evgeny.erofeev@informatik.uni-oldenburg.de

Maciej Koutny
School of Computing Science
Newcastle University
Newcastle upon Tyne, NE1 7RU, United Kingdom
maciej.koutny@newcastle.ac.uk

Łukasz Mikulski*
Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University
Toruń, Chopina 12/18, Poland
lukasz.mikulski@mat.umk.pl

Marcin Piątkowski*
Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University
Toruń, Chopina 12/18, Poland
marcin.piatkowski@mat.umk.pl

Abstract. Reversible computation deals with mechanisms for undoing the effects of actions executed by a dynamic system. This paper is concerned with reversibility in the context of Petri nets which are a general formal model of concurrent systems. A key construction we investigate amounts to adding 'reverse' versions of selected net transitions. Such a static modification can severely impact on the behaviour of the system, e.g., the problem of establishing whether the modified net has the same states as the original one is undecidable. We therefore concentrate on nets with finite state spaces and show, in particular, that every transition in such nets can be reversed using a suitable finite set of new transitions.

Keywords: Petri net, reversibility, reversible computation

*Partially supported by the Polish NCN grant No.2013/09/D/ST6/03928

†Partially supported by DFG (German Research Foundation) through grant Be 1267/14-1 CAVER (Design and Analysis Methods for Real-Time Systems) and Graduiertenkolleg GRK-1765 SCARE (System Correctness under Adverse Conditions).

Address for correspondence: Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, ul. Chopina 12/18, 87-100 Toruń, Poland

1. Introduction

Reversible computation deals with (typically local) mechanisms for undoing the effects of actions executed by a dynamic system. Such an approach has been applied, in particular, to various kinds of process calculi and event structures (see, e.g., [5, 6, 7, 8, 11, 14, 15, 13]), and in a category theory based setting [9]. Reversibility is a highly desirable feature in, e.g., hardware devices resetting themselves after a period of idleness, self-stabilising systems recovering from failures, or modern debuggers which have to be capable to perform step-back executions (which are, in fact, reverses). Application areas for which reversibility is crucial also include embedded systems and flexible manufacturing systems. Moreover, unlike the classical executions, in quantum computations any action must be reversible since, according to the laws of physics, information can never be completely lost.

This paper is concerned with reversibility in the context of Petri nets which are a general formal model of concurrent systems. In its basic formulation, reversibility amounts to the possibility of returning to the initial marking (a global state) from any reachable marking (and thus ensuring that the behaviour of the net is cyclic). However, it is not required that any specific transitions or markings are used in order to bring the net back to the initial marking. A key construction we investigate here amounts to adding 'reverse' versions of selected net transitions, e.g., a 'straightforward' reverse simply changes the directions of arcs adjacent to a transition being reversed. As shown in [3], such a static modification can severely impact on the behaviour of the system, e.g., the problem of establishing whether the modified net has the same states as the original one is undecidable.

We therefore concentrate in this paper on Petri nets with finite state spaces, more precisely bounded Place/Transition-nets (PT-nets). The state space of a PT-net can be represented by finite labelled transition system (FLTS) which is a convenient tool for specifying different variants of reversibility. One can therefore aim at constructing a PT-net with 'reversed' behaviour given by an FLTS.

In this paper we show that it is, in general, impossible to reverse a transition using its straightforward reverse. What is more, the situation does not change if we relax the notion of a reverse by only requiring that the effect of its execution is opposite to that of the original transition. We therefore relax the requirement further, by allowing several reverses for a single transition. This leads to our main result that every transition in a bounded PT-net can be reversed using a suitable finite set of new transitions. This is an extended version of the paper [4] presented at CS&P 2016 conference.

2. Preliminaries

The set of all integers is denoted by $\mathbb{Z}$, and the set of all non-negative integers by $\mathbb{N}$. The cardinality of a set X is denoted by $|X|$, and the set of all nonempty subsets of X by $\mathcal{P}(X)$. $\mathbb{N}^X$ denotes all mappings from X to $\mathbb{N}$, and $\mathbb{Z}^X$ denotes all mappings from X to $\mathbb{Z}$. If $X = \{x_1, \ldots, x_n\}$ is finite and its elements are implicitly ordered $x_1, \ldots, x_n$, then a mapping f in $\mathbb{N}^X$ or $\mathbb{Z}^X$ can be represented as the vector $[f(x_1), \ldots, f(x_n)]$. Any unary and binary operation on $\mathbb{Z}$ can be applied component-wise to the mappings in $\mathbb{Z}^X$. Also, the minimum ($\min$) and maximum ($\max$) operations can be applied to nonempty finite subsets of $\mathbb{Z}^X$ in the component-wise manner. If $Y, Z \in \mathbb{Z}^X$ then $Y \leq Z$ provided that $Y(x) \leq Z(x)$, for every $x \in X$. Moreover, $Y < Z$ if $Y \leq Z$ and $Y \neq Z$. Whenever we consider the minimal or maximal elements of a subset of $\mathbb{Z}^X$ w.r.t. $\leq$, we indicate this using the $\leq$ notation.

Transition systems. A *finite labelled transition system* (or FLTS) is a tuple $TS = (S, T, \rightarrow, s_0)$, where S is a finite set of *states*, T is a finite set of *labels*, $\rightarrow \subseteq (S \times T \times S)$ is a set of *arcs*, and $s_0 \in S$ is an *initial state*. TS may be seen and depicted as a finite state automaton without accepting states.

A state s is *reachable* if there is a directed path from s_0 to s.

The set of all t-labelled arcs is denoted by $\rightarrow_t$, and the set of all arcs by $\rightarrow_{TS}$. We assume that each $\rightarrow_t$ is nonempty.

TS is *isomorphic* to an FLTS $TS' = (S', T', \rightarrow', s'_0)$ if $T = T'$ and there is a bijection $\zeta \colon S \rightarrow S'$ such that $s'_0 = \zeta(s_0)$ and $\rightarrow'_t = \{(\zeta(s), t, \zeta(s')) \mid (s, t, s') \in \rightarrow\}$, for every $t \in T$. We denote this by $TS \approx_\zeta TS'$ or $TS \approx TS'$.

Petri nets. A *place/transition net* (or PT-net) is a tuple $N = (P, T, F, M_0)$, where P is a nonempty finite set of *places*, T is a disjoint finite set of *transitions*, F is a *flow function* in $\mathbb{N}^{(P \times T) \cup (T \times P)}$, and M_0 is an *initial marking* belonging to the set $\mathbb{N}^P$ of *markings*.

In the graphical representation, nodes represent places and transitions, places are indicated by circles, and transitions by boxes. Weighted arcs represent the flow function. Arcs with the weight zero are omitted, and arcs with the weight one are drawn without inscriptions. Markings are depicted by tokens drawn inside the circles.

The *entries* and *exits* of a transition $t \in T$ are mappings in $\mathbb{N}^P$ such that respectively $en_t(p) = F(p, t)$ and $ex_t(p) = F(t, p)$, for every $p \in P$. The *effect* of t is the mapping $\mathit{eff}_t = ex_t - en_t \in \mathbb{Z}^P$.

We assume that the sets of places and transitions are implicitly ordered, and so markings, effects, entries, and exits can be represented by vectors.

A transition $t \in T$ is *enabled* at a marking M if $en_t \leq M$, and the *firing* of such a transition leads to the marking $M' = M + \mathit{eff}_t$. We denote this by $M[t\rangle M'$. Transition enabledness is *monotonic*, which means that if a transition t is enabled at a marking M and $M \leq M'$, then t is also enabled at M'.

The set of markings *reachable* from a marking M is the smallest set $[M\rangle$ containing M such that $M' \in [M\rangle \wedge M'[t\rangle M''$ implies $M'' \in [M\rangle$. We assume that each transition is enabled at a marking reachable from M_0.

Bounded and complementary nets. A PT-net $N = (P, T, F, M_0)$ is *bounded* if $[M_0\rangle$ is finite, and its *reachability graph* is then defined as the following FLTS:

$$RG(N) = \Big([M_0\rangle, T, \{(M, t, M') \mid M[t\rangle M'\}, M_0\Big).$$

If an FLTS TS is isomorphic to the reachability graph of N, then N is a *solution* for (or *solves*) TS, and TS is *solvable*. If no PT-net solves TS, TS is *unsolvable*.

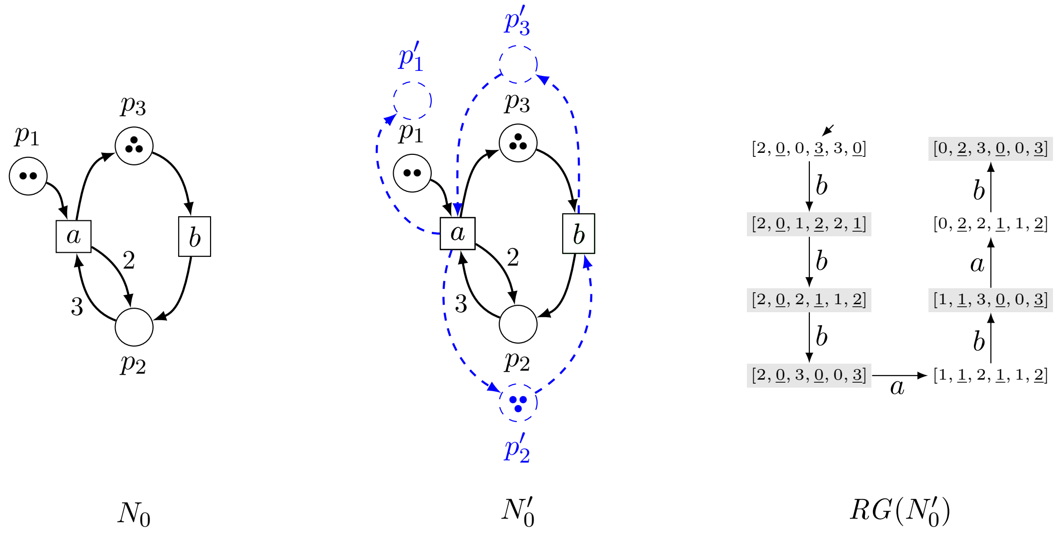


Figure 1. A bounded net N_0, its complementary net N'_0, and the reachability graph $RG(N'_0)$ of N'_0. New places and arcs are indicated by dashed lines. Vector coordinates corresponding to the new places are underlined.

A *complementary net*[1] of N is a PT-net $N' = (P \uplus P', T, F', M'_0)$ obtained by:

- adding complement places $P' = \{p' \mid p \in P\}$;
- extending the flow function so that $F'|_{(P\times T)\cup(T\times P)} = F$ and, for all $p' \in P'$ and $t \in T$:

$$F'(p',t) = \begin{cases} \mathit{eff}_t(p) & \text{if } \mathit{eff}_t(p) > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad F'(t,p') = \begin{cases} -\mathit{eff}_t(p) & \text{if } \mathit{eff}_t(p) < 0 \\ 0 & \text{otherwise;} \end{cases}$$

- setting $M'_0 = \widehat{M_0} \in \mathbb{N}^{P\cup P'}$, where for all $M \in [M_0\rangle$ and $p \in P$, $\widehat{M}(p) = M(p)$ and

$$\widehat{M}(p') = \max\{M''(p) \mid M'' \in [M_0\rangle\} - M(p).$$

Figure 1 depicts a PT-net N_0 together with its complementary net N'_0. It also shows the reachability graph of N'_0.

Adding complementary places does not change the behaviour of the original PT-net. Hence, the following holds (see [12] for details).

Fact 2.1. If N solves an FLTS TS, then N is bounded and its complementary net also solves TS. $\diamond$

[1] This is an extension of a similar notion from [12] defined for pure PT-nets.

Transition reverses. In this paper, reversibility of a net transition will be carried out by adding new (or *fresh*) transitions with the effect opposite to that of the original transition.

A *strict reverse* of a transition t is a fresh transition $\underline{t}$ such that $en_{\underline{t}} = ex_t$ and $ex_{\underline{t}} = en_t$. An *effect-reverse* is $\overline{t}$ such that $\mathit{eff}_{\overline{t}} = -\mathit{eff}_t$. A *minimal effect-reverse* is $\overline{t}$ such that $en_{\overline{t}} = ex_t - \min\{en_t, ex_t\}$ and $ex_{\overline{t}} = en_t - \min\{en_t, ex_t\}$.

All strict reverses and minimal effect-reverses are effect-reverses. However, as shown in Figure 2, not all effect-reverses are strict reverses.

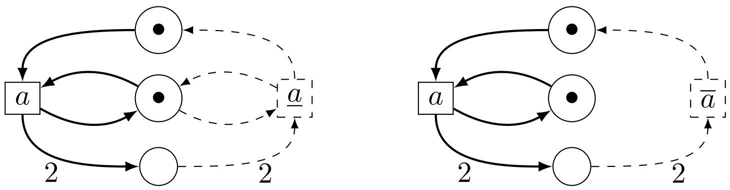


Figure 2. A transition a and its (strict) reverse $\underline{a}$ (lhs), and a minimal effect-reverse $\overline{a}$ which is not a strict reverse of a (rhs). Effect-reverses and adjacent arcs are indicated by non-solid lines.

Solvable and unsolvable words. A *word* is a finite sequence $w = t_1 t_2 \dots t_n$ of symbols, the *mirror image* of w is $w^{rev} = t_n t_{n-1} \dots t_1$, a *factor* of w is any subsequence $t_i t_{i+1} \dots t_j$ $(1 \le i \le j \le n)$. Moreover, w corresponds to an FLTS $TS_w = (\{0, \dots, n\}, \{t_1, \dots, t_n\}, \{(i-1, t_i, i) \mid 0 < i \le n\}, 0)$.

A PT-net N *solves* w if it solves TS_w, and w is then *solvable*. If no PT-net solves TS_w, w is *unsolvable*. If w is solvable, then so are all its factors. Thus, the unsolvability of any proper factor of w entails the unsolvability of w. For this reason, the notion of a *minimal unsolvable word*, defined as an unsolvable word with all proper factors being solvable, is well-defined (see [1] for details).

3. Solvability of FLTS with reverses

We now discuss the impact of adding transition reverses on FLTS solvability.

Definition 3.1. (FLTS reduction and extension)
Let $TS = (S, T, \rightarrow, s_0)$ be an FLTS, and $t \in T$ be a label.

The *reduction* of TS by deleting t is an FLTS $TS^{[-t]} = (S', T', \rightarrow', s_0)$ such that:

- S' comprises all states reachable in TS after deleting the arcs in $\rightarrow_t$;
- T' comprises all labels $u \in T \setminus \{t\}$ labelling arcs outgoing from the states in S';
- $\rightarrow'_u = \{(s, u, r) \in \rightarrow_u \mid s \in S'\}$, for every $u \in T'$.

The *extension* of TS by reversing t is an FLTS $TS^{[+\overline{t}]} = (S, T \uplus \{\overline{t}\}, \rightarrow', s_0)$ such that:

- $\rightarrow'_{\overline{t}} = \{(s', \overline{t}, s) \mid (s, t, s') \in \rightarrow_t\}$;
- $\rightarrow'_u = \rightarrow_u$, for every $u \in T$.

The notions of reduction and extension introduced above can be readily extended to a finite set of labels $\{t_1, \dots, t_n\} \subseteq T$. The resulting FLTSs will be denoted by $TS^{[-t_1,\dots,t_n]}$ and $TS^{[+\overline{t_1,\dots,t_n}]}$. ◇

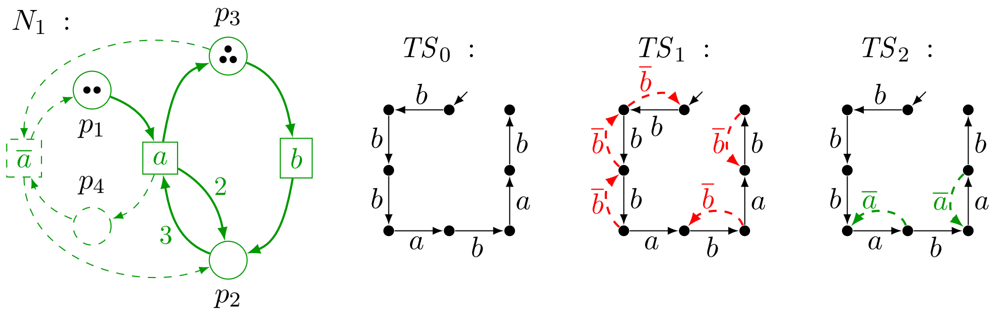


Figure 3. N_1 without the dashed part solves TS_0, and N_1 with the dashed part solves $TS_2 = TS_0^{[+\overline{a}]}$. Moreover, $TS_1 = TS_0^{[+\overline{b}]}$ is unsolvable.

The word $w = bbbabab$ corresponds to TS_0 shown in Figure 3, which is solved by N_1 without the dashed part[2]. After adding a reverse of a, we obtain TS_2 which is solved by N_1 with the dashed part. We will later show that adding a reverse of b yields an unsolvable TS_1. The transition $\overline{a}$ of N_1 in Figure 3 is an effect-reverse but not a strict reverse of a.

It turns out that if $TS^{[+\overline{t}]}$ is solvable, then there exists a solution with a strict reverse of t.

Proposition 3.2. Let t be a label of a solvable TS. If $TS^{[+\overline{t}]}$ is solvable, then it has a solution such that $\overline{t}$ is a strict reverse of t.

Proof:
Let $N = (P, T \uplus \{t, \overline{t}\}, F, M_0)$ be a solution for $TS^{[+\overline{t}]}$. We will show that $N' = (P, T \uplus \{t, \overline{t}\}, F', M_0)$ is also a solution for $TS^{[+\overline{t}]}$, where F' is defined as follows: (i) $en'_t = ex'_{\overline{t}} = \max\{en_t, ex_{\overline{t}}\}$; (ii) $ex'_t = en'_{\overline{t}} = \max\{ex_t, en_{\overline{t}}\}$; and (iii) $F'|_{(P\times T)\cup(T\times P)} = F|_{(P\times T)\cup(T\times P)}$. Clearly, we have: (iv) $\mathit{eff}_t = -\mathit{eff}_{\overline{t}} = \mathit{eff}'_t = -\mathit{eff}'_{\overline{t}}$; and (v) $en_t \le en'_t$ and $en_{\overline{t}} \le en'_{\overline{t}}$.

By (i, ii) and the fact that $\overline{t}$ is an effect-reverse of t in N, $\overline{t}$ is a strict reverse of t in N'. To demonstrate that N' is a solution for $TS^{[+\overline{t}]}$, we will show that $RG(N) = RG(N')$. The latter will follow from the fact that M_0 is the initial state in both $RG(N)$ and $RG(N')$, as well as $(M, u, M') \in \rightarrow_{RG(N)} \iff (M, u, M') \in \rightarrow_{RG(N')}$, for every marking M reachable in both N and N'.

[2] N_1 without the dashed part is N_0 of Figure 1.

The ($\Longleftarrow$) implication follows from (iii, iv, v). To show ($\Longrightarrow$), suppose that $(M, u, M') \in \rightarrow_{RG(N)}$. If $u \in T$ then, by (iii), $(M, u, M') \in \rightarrow_{RG(N')}$.
If $u = t$ then, by the definition of $TS^{[+\overline{t}]}$ and the fact that N is a solution for $TS^{[+\overline{t}]}$, we have $(M', \overline{t}, M) \in \rightarrow_{RG(N)}$. Hence $en_t \leq M$ and $ex_{\overline{t}} \leq M$, and so $en'_t = \max\{en_t, ex_{\overline{t}}\} \leq M$. Thus, by (iv), $(M, t, M') \in \rightarrow_{RG(N')}$.
If $u = \overline{t}$, then we proceed in a similar way as above. $\square$

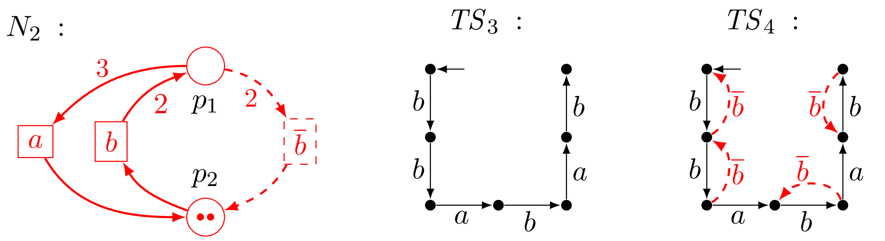


Figure 4. N_2 with the dashed part solves $TS_4 = TS_3^{[+\overline{b}]}$.

Consider N_2 of Figure 4 without the dashed part. It solves the word $bbabab$, and so its reachability graph is isomorphic to TS_3. Moreover, TS_4 obtained from TS_3 by adding a reverse for b is solvable by N_2 with the dashed part, where $\overline{b}$ is a strict reverse of b. Similarly, we may reverse a in TS_3, obtaining TS_5 of Figure 5. This FLTS is solvable by N_3 with the dashed part.

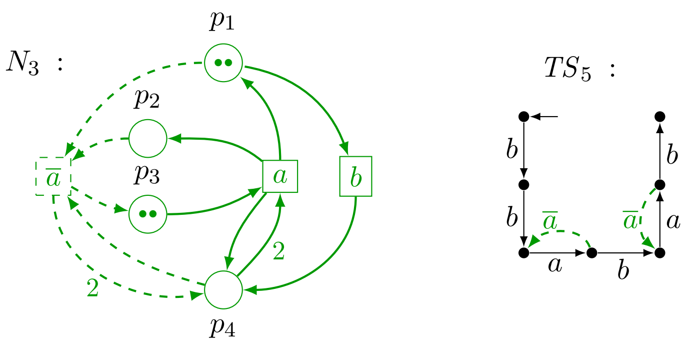


Figure 5. N_3 with the dashed part solves $TS_5 = TS_3^{[+\overline{a}]}$.

If adding reverses for two labels yields a solvable FLTSs, then the FLTS containing both reverses is also solvable.

Proposition 3.3. Let $TS = (S, T, \rightarrow, s_0)$ be solvable and $t \neq u \in T$. If both $TS^{[+\overline{t}]}$ and $TS^{[+\overline{u}]}$ are solvable, then so is $TS^{[+\overline{t,u}]}$.

Proof:
For $x \in \{t,u\}$, let $N^x = (P^x, T \uplus \{\overline{x}\}, F^x, M_0^x)$ be such that $TS^{[+\overline{x}]} \approx_{\zeta_x} RG(N^x)$ (i). Moreover, let $\widehat{N}^x = (P^x, T, \widehat{F}^x, M_0^x)$ be N^x after deleting $\overline{x}$. From the definition of $TS^{[+\overline{x}]}$ and (i), we have $TS \approx_{\zeta_x} RG(\widehat{N}^x)$ (ii). Without loss of generality, we assume that P^t and P^u are disjoint (iii).

Let $\widehat{N} = (P^t \uplus P^u, T, \widehat{F}, M_0^t \uplus M_0^u)$, where $\widehat{F}$ is given by $\widehat{F}^t$ and $\widehat{F}^u$. Then, by (ii, iii), we have $TS \approx_{\zeta} RG(\widehat{N})$, where $\zeta(s) = \zeta_t(s) \uplus \zeta_u(s)$, for every $s \in S$ (iv).

Let $N = (P^t \uplus P^u, T \uplus \{\overline{t}, \overline{u}\}, F, M_0^t \uplus M_0^u)$, where F is given by F^t and F^u as well as for $\{x,z\} = \{t,u\}$: $F(\overline{x}, p) = F^z(p,x)$ and $F(p,\overline{x}) = F^z(x,p)$, for every $p \in P^z$.[3] Hence, by Definition 3.1 and (iv), $RG(\widehat{N})$ is $RG(N)$ after deleting $\overline{t}$ and $\overline{u}$ (v).

It is then straightforward to show that $RG(N) \approx_{\zeta} TS^{[+\overline{t,u}]}$, using Definition 3.1 and (i, iv, v). $\square$

It follows from the proof of the last result that a solution for $TS^{[+\overline{t,u}]}$ can be obtained by synchronising any solutions for $TS^{[+\overline{t}]}$ and $TS^{[+\overline{u}]}$ with disjoint sets of places on the (common) transitions in T, and then making $\overline{t}$ a strict reverse w.r.t. the solution for $TS^{[+\overline{u}]}$, and making $\overline{u}$ a strict reverse w.r.t. the solution for $TS^{[+\overline{t}]}$.

Using Proposition 3.3 and starting from two solutions, N_2 for $TS_4 = TS_3^{[+\overline{a}]}$ and N_3 for $TS_5 = TS_3^{[+\overline{b}]}$, we can construct a solution N_4 for $TS_6 = TS_3^{[+\overline{a,b}]}$ (see Figure 6).

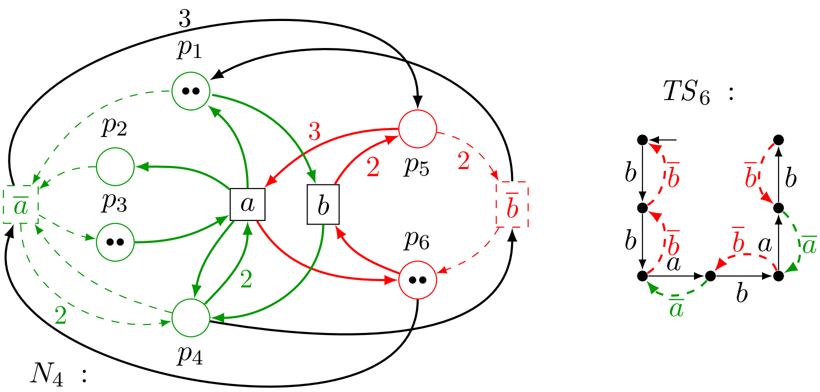


Figure 6. N_4 derived by synchronising N_2 and N_3 with the dashed parts solves $TS_6 = TS_3^{[+\overline{a,b}]}$.

We end this section looking at the solvability of words over a two-letter alphabet.

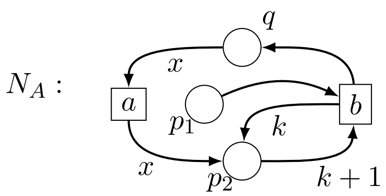
Proposition 3.4. If $w \in \{a,b\}^*$ is a minimal unsolvable word, then:

[3] $\overline{t}$ can be seen as the strict reverse of t w.r.t. P^u, and $\overline{u}$ as the strict reverse of u w.r.t. P^t.

- $TS_{w^{rev}}$ is solvable;
- $TS_{w^{rev}}^{[+\overline{a}]}$ or $TS_{w^{rev}}^{[+\overline{b}]}$ is unsolvable.

Proof:
(The proof uses the results and techniques of [2].) Let w be a minimal unsolvable word. According to [2], we can consider the following three cases (other cases can be obtained by swapping a and b).

Case 1: $w = ab^x ab^{x-k} a$ and $w^{rev} = ab^{x-k} ab^x a$ and $3 \leq k \leq x$.
N_A in Figure 7 is a solution for w^{rev}. Indeed, the initial marking M_0 allows only firing of a.
If $k = x$, this single firing of a does not enable b, and q contains enough tokens for one more firing of a. The second firing of a consumes all the tokens from q, disabling a, and enables b^x. Then a remains disabled until b^x has fired, and can be fired only once after that, finishing the firing of N_A.
If $x > k$, q allows a to be fired only once at the beginning. This first firing a enables b^{x-k} due to p_2. After the firing of b^{x-k}, a becomes enabled to fire once again. This firing puts x tokens into p_2 and, together with the tokens that have already been there, allows the firing of b^x. Then, due to p_1, only single a can fire, finishing the firing of N_A.

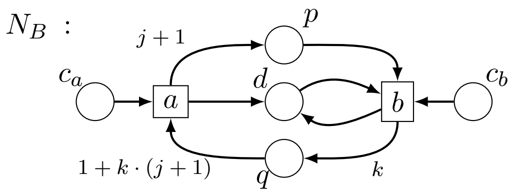


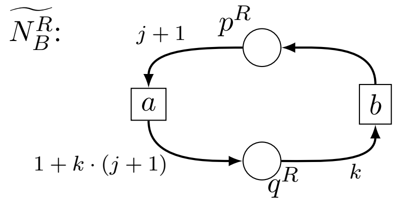
$$M_0 \begin{pmatrix} p_1 \\ p_2 \\ q \end{pmatrix} = \begin{pmatrix} 2x-k \\ 0 \\ x+k \end{pmatrix}$$

Figure 7. N_A solves $ab^{x-k}ab^x a$.

Case 2: $w = abb^j(bab^j)^k a$ with $j, k \geq 1$, or derived from it using the extension mechanism described in [2].

Consider the case $w = abb^j(bab^j)^k a$ (for its extensions the proof is analogous). Due to the minimality, the maximal proper prefix $w_1 = abb^j(bab^j)^k$ of w is solvable, and N_B in Figure 8 is a solution. Let us consider the *inversed* "core part" $\widetilde{N_B^R}$ of this PT-net, derived by reversing the arcs, and with the final marking of N_B as the initial marking. In [2], it has been shown that p and q completely define the order of firing of a and b inside w_1. a is not enabled initially in $\widetilde{N_B^R}$. Since the markings of p^R and q^R in this PT-net now repeat all reachable markings of N_B (restricted to places p and q) in a reverse order when firing w_1^{rev}, this net allows the firing of w_1^{rev}. We can add counter places c_a^R and c_b^R, for a and b, with initial markings $k+1$ and $(j+1)(k+1)$, respectively. These places will prevent firings after the firing of w_1^{rev}. We now show that the resulting net $\overline{N_B}$ (see Figure 9) does not have any additional behaviour except for w_1^{rev}. To the contrary, suppose that a marking $\overline{M}$ reachable in $\overline{N_B}$ enables a and b. Let M be the marking of N_B corresponding to $\overline{M}$, and consider the case when M is reached in N_B by firing transition a from some other marking M'. Due to c_b^R, $\overline{M}$ is not the marking before the very last a in w_1^{rev}. Hence, as a does not fire more than once in a row, there is marking M'' such

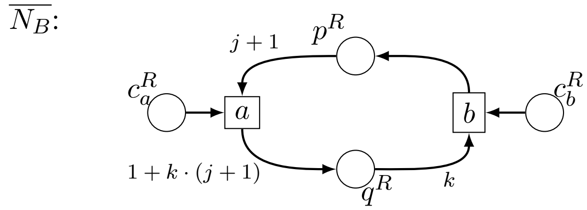




$$M_0\begin{pmatrix} p \\ q \\ d \\ c_a \\ c_b \end{pmatrix} = \begin{pmatrix} 1 \\ 1+k\cdot(j+1) \\ 0 \\ k+1 \\ (j+1)(k+1) \end{pmatrix} \qquad M_0\begin{pmatrix} p^R \\ q^R \end{pmatrix} = \begin{pmatrix} 1 \\ k\cdot j \end{pmatrix}$$

Figure 8. N_B solves the prefix w_1. $\widetilde{N_B^R}$ defines the internal firing of w_1^{rev}.

that $M''[b\rangle M'[a\rangle M$ in N_B. Since b is enabled at $\overline{M}$, we have $M(q) = \overline{M}(q^R) \geq k$. This implies $M'(q) \geq k+1+k\cdot(j+1)$. Since $M''[b\rangle M'$, $M''(q) \geq 1+k\cdot(j+1)$. Therefore, a is enabled at M'', a contradiction with b being enabled at M''. The case when M is reached from M' through firing of b is similar.



$$M_0\begin{pmatrix} p^R \\ q^R \\ c_a^R \\ c_b^R \end{pmatrix} = \begin{pmatrix} 1 \\ k\cdot j \\ k+1 \\ (k+1)\cdot(j+1) \end{pmatrix}$$

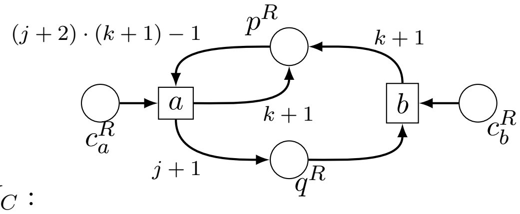
Figure 9. $\overline{N_B}$ solves the reversed prefix w_1^{rev}.

Thus, w_1^{rev} is solvable. We now can "unfire" one a in $\overline{N_B}$. In order to do this, we increase the arc weights between q^R and b by $1+k\cdot(j+1)$, and change the initial markings of c_a^R and p^R to $k+2$ and $j+2$, respectively. The PT-net obtained through such a transformation fires a initially and has the same behaviour as $\overline{N_B}$ afterwards. Hence, it solves w^{rev}.

Case 3: $w = bab^j(abb^j)^k b$ with $j \geq 0$, $k \geq 1$, or one of its extension-derivatives.
This case is similar to Case 2, and N_C in Figure 10 solves w^{rev}.

To show the second part, suppose that $TS_{w^{rev}}^{[+\overline{a}]}$ and $TS_{w^{rev}}^{[+\overline{b}]}$ are both solvable. Then, by the first part as well as Proposition 3.3, there is N which solves $TS_{w^{rev}}^{[+\overline{a,b}]}$. Let N' be obtained from N by deleting

N_C :



$$M_0 \begin{pmatrix} p^R \\ q^R \\ c_a^R \\ c_b^R \end{pmatrix} = \begin{pmatrix} 0 \\ j+2 \\ k+1 \\ (k+1)(j+1)+1 \end{pmatrix}$$

Figure 10. N_C solves $(bab^j(abb^j)^k b)^{rev}$.

a and b, renaming $\overline{a}$ and $\overline{b}$ as a and b, respectively, and then setting the initial marking to be the only marking reachable in N enabling neither a nor b. Then one can see that N' solves w, yielding a contradiction. □

We can now explain why $TS_1 = TS_0^{[+\overline{b}]}$ of Figure 3 is unsolvable. All we need to observe is that $z^{rev} = bbbabab$ is the mirror image of a minimal unsolvable word $z = bababbb$ (see [1]), and then recall that $TS_0 = TS_{z^{rev}}$, $TS_1 = TS_0^{[+\overline{b}]}$, and $TS_2 = TS_0^{[+\overline{a}]}$. Since TS_2 is solvable, by the second part of Proposition 3.4, TS_1 is unsolvable.

4. Splitting reverses

In this section, we discuss the possibility of 'splitting' reverses. More specifically, we investigate FLTSs in which a single label may have multiple reverses.

Consider N_5 of Figure 11 and its reachability graph TS_7, both with the non-solid parts. We have $\mathit{eff}_{\overline{b}_1} = \mathit{eff}_{\overline{b}_2} = -\mathit{eff}_b$, and so $\overline{b}_1$ and $\overline{b}_2$ are both effect-reverses for b. Moreover, we have already seen that TS_7 with $\overline{b}_1 = \overline{b}_2 = \overline{b}$ (i.e., TS_1 of Figure 3) is unsolvable. However, in terms of behaviour, N_5 is fully satisfactory provided that one allows more than one reverse for b. In what follows, we will show that by allowing multiple effect-reverses of transitions in bounded nets, one can successfully treat all possible reversibility scenarios we are concerned with in this paper.

Definition 4.1. (splitting reverses)
Let $TS = (S, T, \to, s_0)$ be an FLTS, $t \in T$ be a label, and $\overline{T}$ be a nonempty set of fresh labels. Moreover, let ϕ be a mapping from $\to_t$ to $\mathcal{P}(\overline{T})$ specifying all possible ways in which each of t-labelled arcs can be reversed.

Then the *extension* of TS by reverses of t w.r.t. ϕ is the FLTS $TS^{[+t\phi]} = (S, T \uplus \overline{T}, \to', s_0)$ such that:

- $\to'_u = \to_u$, for every $u \in T$;
- $\to'_{\overline{t}} = \{(s', \overline{t}, s) \mid (s, t, s') \in \to_t \ \wedge \overline{t} \in \phi((s,t,s'))\}$, for every $\overline{t} \in \overline{T}$.

The above notion is readily extended to a set of mappings ϕ_i from $\to_{t_i}$ to $\mathcal{P}(\overline{T}_i)$ $(1 \le i \le n)$ such

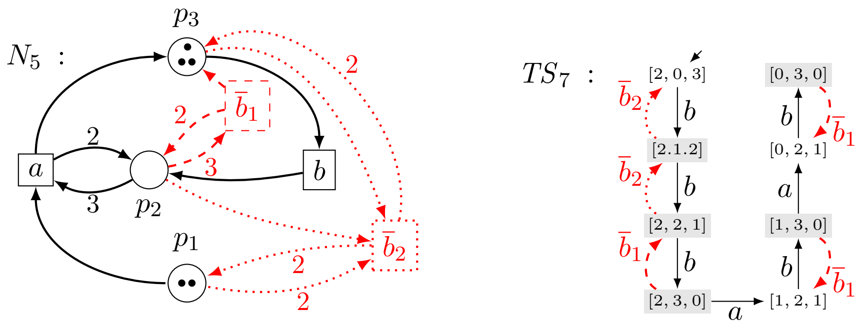


Figure 11. Splitting reverses in TS_7 results in solvability.

that $t_1, \ldots, t_n$ are distinct labels and $T, \overline{T}_1, \ldots, \overline{T}_n$ are mutually disjoint sets. The resulting FLTS will be denoted by $TS^{[+t_1\phi_1,\ldots,t_n\phi_n]}$. $\diamond$

Given a solvable FLTS, one can always add, without losing solvability, a new arc (s, t, s') provided that s corresponds to a $\leq$-maximal reachable marking, and t is a fresh label.

Lemma 4.2. Let $N = (P, T, F, M_0)$ be a bounded net, $RG(N) = ([M_0\rangle, T, \to, M_0)$ be its reachability graph, t be a fresh label, M be a $\leq$-maximal marking in $[M_0\rangle$, and $M' \in [M_0\rangle$.

Then $TS = ([M_0\rangle, T \uplus \{t\}, \to \uplus \{(M, t, M')\}, M_0)$ is a solvable FLTS.

Proof:

Let $N' = (P, T \uplus \{t\}, F', M_0)$, where $F'|_{(P\times T)\cup(T\times P)} = F$ and, for all $p \in P$, $F'(p, t) = M(p)$ and $F'(t, p) = M'(p)$. We then observe that $M[t\rangle M'$ and t is not enabled at any marking $M'' \neq M$ reachable in N. (Indeed, suppose that there exists such a marking M''. Then, by the definition of enabledness, $M(p) = F'(p, t) \leq M''(p)$, for every $p \in P$. Hence $M \leq M''$, contradicting the $\leq$-maximality of M.) Thus, the reachable markings of N and N' are the same, and $RG(N') = TS$. $\square$

Theorem 4.3. Let $TS = (S, T, \to, s_0)$ be a solvable FLTS, and $t \in T$. Then there exists a finite nonempty set $\overline{T}$ of fresh labels and a mapping ϕ from $\to_t$ to $\mathcal{P}(\overline{T})$ such that $TS^{[+t\phi]}$ is solvable.

Proof:

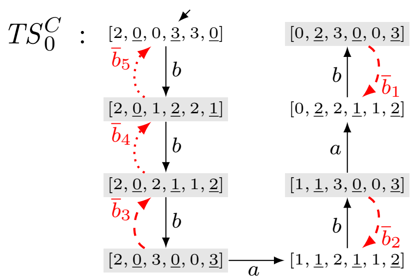
Let $N = (P, T, F, M_0)$ be a solution for TS. We first construct for N a complementary net $N' = (P \uplus P', T, F', M_0')$. By Fact 2.1, $RG(N')$ is isomorphic to TS.

For each pair of markings $M \neq M' \in [M_0'\rangle$, there is $p \in P$ such that $M(p) \neq M'(p)$. Moreover, by the definition of a complement place, either $M(p) > M'(p) \wedge M(p') < M'(p')$ or $M'(p) > M(p) \wedge M'(p') < M(p')$ holds. Hence, all distinct markings reachable in N' are $\leq$-incomparable, and so also $\leq$-maximal in $[M_0'\rangle$.

We then take a set of fresh labels $\overline{T} = \{t_{qp} \mid (p, t, q) \in \to_t\}$, and construct a mapping ϕ from $\to_t$ to $\mathcal{P}(\overline{T})$ so that $\phi((p, t, q)) = \{t_{qp}\}$, for every $(p, t, q) \in \to_t$.

By starting from N' and applying $|\rightarrow_t|$ times Lemma 4.2, we obtain that $TS^{[+t\phi]}$ is solvable. $\square$

Applying the construction from the last proof to TS_7 in Figure 11 leads to a PT-net with six places and five different effect-reverses for b, each such effect-reverse being enabled just at one marking. Of course, each such effect-reverse has exactly the same effect (opposite to the effect of b), but their entries are different. More precisely, the entry of $\overline{b}_i$ ($1 \leq i \leq 5$) is the same as the only marking at which $\overline{b}_i$ is enabled (see Figure 12 for details).



t	en_t	ex_t
$\overline{b}_1$	$[0,2,3,0,0,3]$	$[0,2,2,1,1,2]$
$\overline{b}_2$	$[1,1,3,0,0,3]$	$[1,1,2,1,1,2]$
$\overline{b}_3$	$[2,0,3,0,0,3]$	$[2,0,2,1,1,2]$
$\overline{b}_4$	$[2,0,2,1,1,2]$	$[2,0,1,2,2,1]$
$\overline{b}_5$	$[2,0,1,2,2,1]$	$[2,0,0,3,3,0]$

Figure 12. The reachability graph of the PT-net obtained from N_0 of Figure 1 by applying the construction from the proof of Theorem 4.3.

The construction described in the proof of Theorem 4.3 may lead to a substantial enlargement of the initial PT-net, as the number of places is doubled, and the number of newly created transitions can be as large as the number of arcs in the reachability graph of the initial net. However, as illustrated by the example depicted in Figure 11, there may also exist (multiple) solutions that are significantly smaller.

We will now present a possible optimisation of the last construction. The idea is to merge two or more effect-reverses obtaining another effect-reverse which is enabled at more than one reachable marking.

Definition 4.4. (merging reverses)
Let $N = (P, T, F, M_0)$ be a PT-net, $t \in T$ be a transition, and $\overline{T}$ be a finite nonempty set of effect-reverses of t. The *meet* of $\overline{T}$ is a fresh transition $\triangle\overline{T}$ such that $en_{\triangle\overline{T}} = \min_{\overline{t}\in\overline{T}} en_{\overline{t}}$ and $ex_{\triangle\overline{T}} = \min_{\overline{t}\in\overline{T}} ex_{\overline{t}}$. The *join* of $\overline{T}$ is a fresh transition $\triangledown\overline{T}$ such that $en_{\triangledown\overline{T}} = \max_{\overline{t}\in\overline{T}} en_{\overline{t}}$ and $ex_{\triangledown\overline{T}} = \max_{\overline{t}\in\overline{T}} ex_{\overline{t}}$. $\diamond$

We can apply merging to create new effect-reverses from already defined ones.

Proposition 4.5. Let $N = (P, T, F, M_0)$ be a PT-net, $t \in T$ be a transition, and $\overline{T}$ be a finite nonempty set of effect-reverses of t. Then $\triangle\overline{T}$ and $\triangledown\overline{T}$ are effect-reverses of t. Moreover, $\triangle\overline{T}$ is enabled at every marking at which at least one member of $\overline{T}$ is enabled, and $\triangledown\overline{T}$ is enabled at every marking at which all members of $\overline{T}$ are enabled.

Proof:
Clearly, $\mathit{eff}_{\triangle\overline{T}} = \mathit{eff}_{\triangledown\overline{T}} = -\mathit{eff}_t$. Hence $\triangle\overline{T}$ and $\triangledown\overline{T}$ are effect-reverses of t.

Suppose that $\bar{t} \in \overline{T}$ is enabled at a marking M. Then $en_{\bar{t}} \leq M$. Since, by Definition 4.4, $en_{\triangle\overline{T}} = \min_{\bar{t}\in\overline{T}} en_{\bar{t}}$, we have $en_{\triangle\overline{T}} \leq M$, and so $\triangle\overline{T}$ is enabled at M.

Suppose now that every $\bar{t} \in \overline{T}$ is enabled at a marking M. Then $en_{\bar{t}} \leq M$, for every $\bar{t} \in \overline{T}$. Since, by Definition 4.4, $en_{\nabla\overline{T}} = \max_{\bar{t}\in\overline{T}} en_{\bar{t}}$, we have $en_{\nabla\overline{T}} \leq M$, and so $\nabla\overline{T}$ is enabled at M. □

Consider again the example FLTS of Figure 12. Then $\mathit{eff}_{\triangle\{\bar{b}_1,\bar{b}_4\}} = \mathit{eff}_{\bar{b}_1} = \mathit{eff}_{\bar{b}_4} = [0,0,-1,1,1,-1]$ and $en_{\triangle\{\bar{b}_1,\bar{b}_4\}} = [0,0,2,0,0,2]$. Unfortunately, it means that $\triangle\{\bar{b}_1,\bar{b}_4\}$ is enabled not only at markings enabling $\bar{b}_1$ or $\bar{b}_4$ (i.e., $[0,2,3,0,0,3]$ and $[2,0,2,1,1,2]$, respectively), but also at $[2,0,3,0,0,3]$, $[1,1,2,1,1,2]$, $[1,1,3,0,0,3]$ and $[0,2,2,1,1,2]$. At markings $[2,0,3,0,0,3]$ and $[1,1,3,0,0,3]$ we would like some reverse of b to be enabled, hence no harm is done. On the other hand, at $[1,1,2,1,1,2]$ and $[0,2,2,1,1,2]$ no reverse of b should be enabled. We will address this unwanted situation by preferring those effect-reverses which are enabled in fewer markings outside a set of markings where they are supposed to reverse an original transition.

Definition 4.6. (transition restriction)
A transition t *restricts* transition u if $\mathit{eff}_t = \mathit{eff}_u$ and $en_u \leq en_t$. ⋄

Using the notion of restriction we can formulate

Proposition 4.7. Let t be a transition of a PT-net N, A be a nonempty set of markings M satisfying $ex_t \leq M$, and $R_{t,A}$ be the set of all effect-reverses $\bar{t}$ of t such that $en_{\bar{t}} \leq M$, for every $M \in A$.

Then there exists a unique effect-reverse $\bar{t}_A \in R_{t,A}$ that is a restriction of all members of $R_{t,A}$.

Proof:
By Dickson's Lemma [10], there exists a finite nonempty $A' \subseteq A$ comprising all $\leq$-minimal elements of A. Hence, $R_{t,A}$ is not only finite, but also effectively computable. Moreover, $R_{t,A}$ is nonempty as it contains the strict reverse of t. By Proposition 4.5, $\bar{t}_A = \nabla R_{t,A}$ is an effect-reverse of t. Moreover, $en_{\bar{t}} \leq en_{\bar{t}_A}$, for every $\bar{t} \in R_{t,A}$. Clearly, no other member of $R_{t,A}$ satisfies this. □

Intuitively, $\bar{t}_A$ is the best option for choosing a single reversing transition for t which is enabled at every marking of A as $\bar{t}_A$ is disabled at all markings where the other members of $R_{t,A}$ are disabled.

Definition 4.8. (maximally restrictive effect-reverse)
The unique effect-reverse $\bar{t}_A$ in Proposition 4.7 is the *maximally restrictive effect-reverse* of t for A. ⋄

Proposition 4.9. Let t be a transition of a PT-net N, and A, B be nonempty sets of markings M satisfying $ex_t \leq M$. Then $\bar{t}_{A\cup B} = \triangle\{\bar{t}_A, \bar{t}_B\}$.

Proof:
Clearly, $\bar{t}_{A\cup B} \in R_{t,A} \cap R_{t,B}$. Moreover, by Proposition 4.5, $\triangle\{\bar{t}_A, \bar{t}_B\} \in R_{A\cup B}$. Thus:

$$en_{\bar{t}_{A\cup B}} \leq \min\{en_{\bar{t}_A,\bar{t}_B}\} = en_{\triangle\{\bar{t}_A,\bar{t}_B\}} \leq en_{\bar{t}_{A\cup B}}$$

by Definitions 4.4 and 4.8 as well as Proposition 4.7. Hence the result holds. □

Let us now recall that in order to enable reversing of a label t of an FLTS TS solvable by a net $N = (P, T, F, M_0)$, we need to specify a set $\overline{T}$ of effect-reverses of t and a mapping ϕ in such a way that FLTS $TS^{[+t\phi]}$ is solvable (see Definition 4.1). To address this issue we propose to take an arbitrary net N solving T and employ its complementary net N'. Using the notion of maximally restrictive effect-reverse, we then improve the solution presented in the proof of Theorem 4.3, and determine one of the smallest possible sets $\overline{T}$ which is related to N and a net $\overline{N}$ solving $TS^{[+t\phi]}$.

Note that a suitable set $\overline{T}$ needs to satisfy two conditions:

- For every arc (M, t, M') in $RG(N)$, there is $\bar{t} \in \overline{T}$ such that $(M', \bar{t}, M)$ is an arc in $RG(\overline{N})$.
- For every $\bar{t} \in \overline{T}$, if $(M, \bar{t}, M')$ is an arc in $RG(\overline{N})$, then (M', t, M) is an arc in $RG(N)$.

Following this observation, we present a sketch of the construction of a minimal set of reverses for a given transition t of PT-net N. The idea of the construction is outlined below:

1. Start with N'' obtained by applying the construction from the proof of Theorem 4.3.
2. Compute the set $\mathcal{E}$ of all markings in $RG(N'')$ with an incoming t-labelled arc, and $\mathcal{A} = \mathcal{P}(\mathcal{E})$.
3. Remove from $\mathcal{A}$ any set A such that there is a reachable marking outside $\mathcal{E}$ enabling $\bar{t}_A$.
4. Compute a minimal cover $\mathcal{C} \subseteq \mathcal{A}$ of $\mathcal{E}$.[4]
5. Compute $\overline{T} = \{\bar{t}_A \mid A \in \mathcal{C}\}$.

Consider again TS_0 of Figure 3 and N_0 of Figure 1 solving it. In order to apply the above construction, we derive a complementary net $N'_0 = (\{p_1, p'_1, p_2, p'_2, p_3, p'_3\}, \{a, b\}, F', M'_0)$ (see Figure 1). On the basis of N'_0 we create N''_0 which contains five different reverses of b. These reverses are listed in Figure 12 using their entries and exits. The figure presents also the reachability graph TS_0^C of N''_0. Note that we obtain five markings in the set $\mathcal{E}$ (indicated by grey boxes in Figure 1 or Figure 12) and three markings in $[M'_0\rangle \setminus \mathcal{E}$. All markings from these two sets are also listed in Figure 13.

Out of the total number of 31 nonempty sets in $\mathcal{A}$ initially, only 13 remain in $\mathcal{A}$ after the third step. We can use them to determine three different covers of size 2 (see Figure 13). Note that, after removing all the complementary places, the second cover corresponds to the solution in Figure 11.

5. Infeasibility for reversing

To draw attention to an important issue, which becomes relevant during the analysis of an FLTS from the viewpoint of reversibility of transitions, let us consider the following example.

Suppose that one attempted to introduce a reverse for a in TS_8 of Figure 14, which can be solved by N_6. Although there exists a (strict) reverse $\overline{a}$ in N_6, depicted in Figure 14, the meaning of $\overline{a}$ may be confusing. We cannot regard it as an undoing of the firing of action a, since N_6 can fire $bc\overline{a}$, where a does not fire at all. What is more, we can keep repeating $bc\overline{a}$ indefinitely, without firing a even once.

[4]By Theorem 4.3, all singleton subsets of $\mathcal{E}$ belong to $\mathcal{A}$, and so such a cover can always be found.

$\mathcal{E}$	$[M'_0\rangle \setminus \mathcal{E}$
$[2,0,1,2,2,1]$	
$[2,0,2,1,1,2]$	$[2,0,0,3,3,0]$
$[2,0,3,0,0,3]$	$[1,1,2,1,1,2]$
$[1,1,3,0,0,3]$	$[0,2,2,1,1,2]$
$[0,2,3,0,0,3]$	

A	$en_{\overline{b}_A}$	$ex_{\overline{b}_A}$
$A_1 = \{M_1\}$	$[0,2,3,0,0,3]$	$[0,2,2,1,1,2]$
$A_2 = \{M_2\}$	$[1,1,3,0,0,3]$	$[1,1,2,1,1,2]$
$A_3 = \{M_3\}$	$[2,0,3,0,0,3]$	$[2,0,2,1,1,2]$
$A_4 = \{M_4\}$	$[2,0,2,1,1,2]$	$[2,0,1,2,2,1]$
$A_5 = \{M_5\}$	$[2,0,1,2,2,1]$	$[2,0,0,3,3,0]$
$A_6 = \{M_1, M_2\}$	$[0,1,3,0,0,3]$	$[0,1,2,1,1,2]$
$A_7 = \{M_1, M_3\}$	$[0,0,3,0,0,3]$	$[0,0,2,1,1,2]$
$A_8 = \{M_2, M_3\}$	$[1,0,3,0,0,3]$	$[1,0,2,1,1,2]$
$A_9 = \{M_3, M_4\}$	$[2,0,2,0,0,2]$	$[2,0,1,1,1,1]$
$A_{10} = \{M_3, M_5\}$	$[2,0,1,0,0,1]$	$[2,0,0,1,1,0]$
$A_{11} = \{M_4, M_5\}$	$[2,0,1,1,1,1]$	$[2,0,0,2,2,0]$
$A_{12} = \{M_1, M_2, M_3\}$	$[0,0,3,0,0,3]$	$[0,0,2,1,1,2]$
$A_{13} = \{M_3, M_4, M_5\}$	$[2,0,1,0,0,1]$	$[2,0,0,1,1,0]$

cover	$en_{\overline{b}_1}$	$ex_{\overline{b}_1}$	$en_{\overline{b}_2}$	$ex_{\overline{b}_2}$
(A_6, A_{13})	$[0,1,3,0,0,3]$	$[0,1,2,1,1,2]$	$[2,0,1,0,0,1]$	$[2,0,0,1,1,0]$
(A_{11}, A_{12})	$[2,0,1,1,1,1]$	$[2,0,0,2,2,0]$	$[0,0,3,0,0,3]$	$[0,0,2,1,1,2]$
(A_{12}, A_{13})	$[0,0,3,0,0,3]$	$[0,0,2,1,1,2]$	$[2,0,1,0,0,1]$	$[2,0,0,1,1,0]$

Figure 13. Markings of the sets $\mathcal{E}$ and $[M'_0\rangle \setminus \mathcal{E}$ for the net created by the construction from the proof of Theorem 4.3 for N_0 of Figure 1 (upper table). Thirteen sets of $\mathcal{A}$ after the third step (middle table), and three possible sets of two effect-reverses related to minimal covers of $\mathcal{E}$ (lower table).

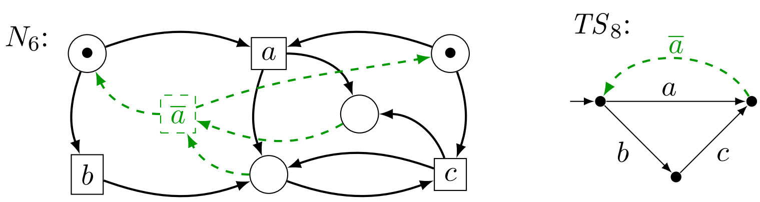


Figure 14. $TS^{[+\overline{a}]}$ can fire $\overline{a}$ without firing a.

Definition 5.1. (infeasibility for reversing)
Let $TS = (S, T, \rightarrow, s_0)$ be an FLTS. Then $t \in T$ is *infeasible (for reversing)*, if $TS^{[+\overline{t}]}$ has a directed path from s_0 with more occurrences of $\overline{t}$ than t. Otherwise, t is *feasible (for reversing)*. ⋄

There is an effective way of establish whether a label is (in)feasible for reversing.

Proposition 5.2. The following problem is decidable.

> **Feasibility for Reversing Problem**
> *Instance:* An FLTS $TS = (S, T, \rightarrow, s_0)$ and $t \in T$.
> *Question:* Is t feasible for reversing in TS?

Proof:
We provide a sketch of an algorithm which reduces the above problem to the problem of finding shortest directed paths in a weighted directed graph.

Input: An FLTS $TS = (S, T, \rightarrow, s_0)$ and $t \in T$.
Output: YES if t is feasible for reversing in TS; otherwise NO.
Procedure: Let $TS^{[+\overline{t}]} = (S, T \uplus \{\overline{t}\}, \rightarrow', s_0)$.

1. Construct a weighted directed graph $G = (S, E)$ where:

$$
\begin{array}{rcll}
E & = & \{(s,1,s') & | \quad (s,t,s') \in \rightarrow'\} \ \cup \\
 & & \{(s,-1,s') & | \quad (s,\overline{t},s') \in \rightarrow'\} \ \cup \\
 & & \{(s,0,s') & | \quad \exists u \in T \setminus \{t\} : (s,u,s') \in \rightarrow'\}.
\end{array}
$$

2. Search for a state s_{wit} such that the distance from s_0 to s_{wit} is negative.
3. If s_{wit} exists, return NO and otherwise YES.

For a transition system consisting of n states, the preprocessing phase (step 1) can be done in $O(n^2)$ time. The computation of step 2 can be performed in $O(n^3)$ time (using Bellman-Ford algorithm). Therefore the overall complexity of the algorithm is $O(n^3)$. □

6. Concluding remarks and future work

In this paper, we have investigated reversibility of transitions in bounded nets. In particular, we have shown that each transition in such nets can be reversed using a suitable finite set of new transitions, but not necessarily a single reverse transition. We have also discussed a procedure of the minimization of the number of effect-reverses for a particular solution of a given FLTS. The challenging problem which for the moment remains unsolved is to determine the best possible solution for an extension with reverses of a given FLTS.

Acknowledgement

We are grateful to the reviewers for their useful comments and suggestions for improvement.

References

[1] Kamila Barylska, Eike Best, Evgeny Erofeev, Łukasz Mikulski, and Marcin Piątkowski. On binary words being Petri net solvable. *Proceedings of the International Workshop on Algorithms & Theories for the Analysis of Event Data, ATAED 2015*, 1371:1–15, 2015.

[2] Kamila Barylska, Evgeny Erofeev, Łukasz Mikulski, and Marcin Piątkowski. Generating all minimal Petri net unsolvable binary words. In *Proceedings of Prague Stringology Conference 2015 (to appear)*, 2016.

[3] Kamila Barylska, Maciej Koutny, Łukasz Mikulski, and Marcin Piątkowski. Reversible computation vs. reversibility in Petri nets. *Reversible Computation - 8th International Conference, RC 2016, Proceedings*, 9720:105–118, 2016.

[4] Kamila Barylska, Łukasz Mikulski, Marcin Piątkowski, Maciej Koutny, and Evgeny Erofeev. Reversing transitions in bounded Petri nets. In *Proceedings of the 25th International Workshop on Concurrency, Specification and Programming, Rostock, Germany, September 28-30, 2016*, pages 74–85, 2016.

[5] Gérard Berry and Gérard Boudol. The chemical abstract machine. *Theoretical Computer Science*, 96(1):217–248, 1992.

[6] Luca Cardelli and Cosimo Laneve. Reversible structures. In François Fages, editor, *Proceedings of 9th International Computational Methods in Systems Biology (CMSB'11)*, pages 131–140. ACM, 2011.

[7] Vincent Danos and Jean Krivine. Reversible communicating systems. In *Proceedings of 15th International Conference on Concurrency Theory (CONCUR'04)*, volume 3170 of *Lecture Notes in Computer Science (LNCS)*, pages 292–307. Springer, 2004.

[8] Vincent Danos and Jean Krivine. Transactions in RCCS. In *Proceedings of 16th International Conference on Concurrency Theory (CONCUR'05)*, volume 3653 of *Lecture Notes in Computer Science (LNCS)*, pages 398–412. Springer, 2005.

[9] Vincent Danos, Jean Krivine, and Paweł Sobociński. General reversibility. *Electronic Notes Theoretical Computer Science*, 175(3):75–86, 2007.

[10] Leonard Eugene Dickson. Finiteness of the odd perfect and primitive abundant numbers with n distinct prime factors. *Amer. Journal Math.*, 35:413–422, 1913.

[11] Ivan Lanese, Claudio Antares Mezzina, and Jean-Bernard Stefani. Reversing higher-order Pi. In *Proceedings of 21th International Conference on Concurrency Theory (CONCUR'10)*, volume 6269 of *Lecture Notes in Computer Science (LNCS)*, pages 478–493. Springer, 2010.

[12] Tadao Murata. Petri nets: Properties, analysis and applications. *Proceedings of the IEEE*, 77:541 – 580, 1989.

[13] Iain Phillips and Irek Ulidowski. Reversing algebraic process calculi. *Journal of Logic and Algebraic Programming*, 73(1-2):70–96, 2007.

[14] Iain Phillips and Irek Ulidowski. Reversibility and asymmetric conflict in event structures. *Journal of Logic and Algebraic Methods in Programming*, 84(6):781–805, 2015.

[15] Iain Phillips, Irek Ulidowski, and Shoji Yuen. A reversible process calculus and the modelling of the ERK signalling pathway. In *Proceedings of 4th Workshop on Reversible Computation (RC'12)*, volume 7581 of *Lecture Notes in Computer Science (LNCS)*, pages 218–232. Springer, 2012.