
Effect of temporal aggregation on the estimate of annual maximum rainfall depths for the design of hydraulic infrastructure systems.  


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DOI link to article:  
https://doi.org/10.1016/j.jhydrol.2017.09.050

Date deposited:  
14/12/2017

Embargo release date:  
28 September 2018

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Abstract

For a few decades the local rainfall measurements are generally obtained by tipping bucket sensors, that allow to record each tipping time corresponding to a well-known rain depth. However, a considerable part of rainfall data to be used in the hydrological practice is available in aggregated form within constant time intervals. This can produce undesirable effects, like the underestimation of the annual maximum rainfall depth, $H_d$, associated with a given duration, $d$, that is the basic quantity in the development of rainfall depth-duration-frequency relationships. The errors in the evaluation of $H_d$ from data characterized by a coarse temporal aggregation, $t_a$, and a procedure to reduce the non-homogeneity of the $H_d$ series are here investigated. Our results show that for $t_a = 1$ min the underestimation is practically negligible, whereas for larger temporal aggregations with $d = t_a$ the error in a single $H_d$ can reach values up to 50% and in a series of $H_d$ in the average up to 17%. Relationships between the non-dimensional ratio $t_a/d$ and the average underestimation of $H_d$ derived through continuous rainfall data observed in many stations of Central Italy, are presented to overcome this issue. These equations allow to improve the $H_d$ estimates and the associated depth-duration-frequency curves at least in areas with similar climatic conditions. The effect of the correction of the $H_d$ series on the rainfall depth-duration-frequency curves is quantified. Our results indicate that the improvements obtained by the proposed procedure are of the order of 10%.

Keywords: Rainfall data; Temporal aggregation; Annual maximum rainfall depths; Depth-duration-frequency curves

1 Introduction
Rainfall data with relatively high time resolution are essential for many hydrologic studies, including the development of rainfall modeling (Corradini and Melone, 1989; Haile et al., 2011a), simulation of infiltration (Melone et al., 2008), representation of the mechanisms of runoff generation (Govindaraju et al., 1999), description of soil erosion (e.g., Angel et al., 2005) and even design of hydraulic infrastructure systems (Adamowski et al., 2010; Notaro et al., 2015). The last topic relies upon the determination of rainfall depth-duration-frequency relationships (Willems, 2000; Overeem et al., 2008) which require the knowledge of the annual maximum rainfall depths, $H_d$, accumulated over different durations, $d$ (Koutsouyiannis et al., 1998). The time resolution of rainfall data can play a significant role, particularly in the estimation of extreme rainfalls with short duration that are of primary importance in the design of widespread hydraulic and drainage infrastructure systems (Du Plessis and Burger, 2015).

Historical rainfall data may be available with different temporal aggregations (or time resolutions), $t_m$, linked to the progress of recording systems through time. Currently, through tipping bucket sensors, rainfall amounts are recorded in a data-logger for each tip time associated with a fixed rainfall depth (usually 0.1 or 0.2 mm). The rain event properties are then summarized by aggregating the number of tips over a selected $t_m$, that can vary from 1 min to much longer time intervals.

After this aggregation procedure, rainfall analyses at temporal scales smaller than the adopted $t_m$ cannot be derived, while for $d > t_m$ they can be affected by significant errors (Haile et al., 2011b).

This occurs because often in hydrological practice there is no access to basic metadata collected by hydrological agencies, particularly in the case of historical data derived from potentially inaccurate long-standing recording systems (e.g., paper rolls). The quantification of the errors in extreme rainfall amount caused by different values of $d$ for a fixed $t_m$ have been analyzed in several studies. It is well known that for $d$ comparable with $t_m$ the actual maximum accumulations may be underestimated (Hershfield, 1961; Weiss, 1964; Young and McEnroe, 2003; Yoo et al., 2015). Weiss (1961) observed that for $d = t_m$ the results obtained from an analysis based on actual maxima were closely approximated through a frequency analysis of $H_d$ with values multiplied by 1.13. Weiss (1964), on probabilistic grounds, under the assumption of a uniform rainfall throughout the duration of interest, developed a relationship between the sampling ratio, $t_m/d$, and the average ratio of the real maximum rainfall accumulation for a given $d$ to the maximum one deduced by a fixed recording interval, henceforth designated as sampling adjustment factor (SAF). Young and McEnroe (2003) used high temporal resolution data from 15 rain gauges located in the Kansas City metropolitan area to derive a single empirical relationship between SAF and sampling ratio. This relation was found to provide adjustments consistent with other empirical studies (Muller et al., 1973; Frederick et al., 1977; Huff and Angel, 1992). However, the length of the considered rainfall series (in the range 5.3–14.9 years, with average value of 9.6 years) was too limited to draw a conclusion of general validity. Yoo et al. (2015) extended the probabilistic approach presented by Weiss (1964) considering several not uniform rainfall temporal distributions that were found significantly related with the SAF. Overall, previous studies suggest that the SAF is dependent on both sampling ratio and $d$, with the latter that is involved because the shape of the rainfall temporal distribution is linked to it.

The first objective of this paper is to define, for a given duration, the length of a $H_d$ series, observed with a given aggregation time, that is required to derive an average adjustment factor to be applied to each series element to reduce the involved original errors. Considering the random nature of $H_d$ this is an important point, but sometimes it has not been considered in depth. We note, for example, that Young and McEnroe (2003) used series with fairly short length and did not examine the problem of their reliability in the determination of the adjustment factors. In this study we use, as a benchmark, rainfall data observed for many years with an aggregation time of 1 min. Furthermore, in the analysis performed for $t_m$ and $d$ of interest the series incorporate rainfall temporal distributions with a variety of shapes that included the different theoretical distributions supposed by Yoo et al. (2015). The second objective of this paper is to define a methodology to obtain homogeneous series of annual maximum rainfall depths from data derived through different temporal aggregations. This is a crucial issue because many rain gauge stations were installed in the first half of the twentieth century and their series of annual maximum rainfall depths are not homogeneous (Alexandersson, 1986; Hanssen-Bauer and Forland, 1994) as a result of many values derived from rainfall data with a coarse $t_m$ (e.g. when a recording system on rolling paper was adopted) and the remaining ones with $t_m = 1$ min. The third objective of this paper is to estimate the sensitivity of the rainfall depth-duration-frequency curves to the corrections of the $H_d$ series performed by the proposed methodology.

2 Methods

Following Burlando and Rosso (1996) and Boni et al. (2006) we provide the definition of annual maximum rainfall depth through the rainfall rate at time $t$, $x(t)$, measured at a specific location. The accumulated rainfall recorded over a time interval $d$, $x_d(t)$, is given by:

$$x_d(t) = \int_{t}^{t+d} x(t') dt'$$  \hspace{1cm} (1)

The annual maximum rainfall depth over a duration $d$, $H_d$, is therefore expressed as:

$$H_d = \max\{x_d(t) : t_0 < t_0 = d + 1 \text{ year}\}$$  \hspace{1cm} (2)

where $t_0$ is the starting time of each year.

To determine $H_d$ for a specific year, the knowledge of rainfall data characterized by any $t_m \leq d$ is necessary. When $d = t_m$, independently of the rainfall pulse shape, the $H_d$ value is sometimes correctly estimated (Fig. 1a) but can
also be underestimated (Fig. 1b,c) with errors up to 50% (Fig. 1c). The underestimation error adopted here is directly related to both the SAF introduced by Young and McEnroe (2003) and the correction factor of Yoo et al. (2015).

Despite the inability to correctly quantify the accuracy of a given $H_d$ value, a representation of the average error for a time series containing a large number of elements can be established.

It is well-know that for each duration $d$, a long $H_d$ series is affected by an average error depending on both $t_a$ and the shape of the rainfall pulses. In the case of rectangular pulses, the average underestimation is equal to 25%, because each error assumes with the same probability of occurrence a value in the range 0–50%. This is consistent with the theoretical results by Yoo et al. (2015). However, it is widely recognized that the $H_d$ values are determined by heavy rainfalls of erratic shape (Balme et al., 2006; Al-Rawas and Valeo, 2009; Coutinho et al., 2014). For example, Fig. 2 shows a few sample hyetographs associated with the annual maximum rainfall rates for $d = 60$ min that were recorded by a rain gauge station located in Central Italy. The hyetographs exhibit irregular shapes that can be roughly considered of triangular type.

![Fig. 1](image1.png)

**Fig. 1** Schematic representation of a rectangular rainfall pulse with duration, $d$, equal to the measurement aggregation time, $t_a$: (a) condition where a correct evaluation of the annual maximum rainfall rate of duration $d$, $H_d$, is possible; (b) condition for a generic underestimation of $H_d$; (c) condition for the maximum underestimation of $H_d$ (equal to 50%).

![Fig. 2](image2.png)

**Fig. 2** Sample hyetographs recorded at the Bastardo station (Umbria Region, Central Italy) involving annual maximum rainfall depths for $d = 60$ min. From top to bottom, moving windows starting on October 18, 2007 (2:17 p.m.), July 23, 2008 (4:28 a.m.) and June 26, 2009 (3:10 p.m.).
Under the assumption of a triangular rainfall pulse characterized by a duration \( d \), the total rainfall depth, \( R_{pd} \), is (Fig. 3a):

\[
R_{pd} = \frac{dh}{2}
\]

with \( h \) equal to the rainfall intensity peak.

When \( t_a = d \), also with a triangular pulse the underestimation error of a single \( H_d \) is within the range 0–50%. The error associated with the possible pulse positions (Fig. 3b) is displayed in Fig. 3c. Its average value, \( E_a \), obtained by integration through the pulse duration (see also Yoo et al., 2015) is given by:

\[
E_a = \frac{1}{12} t_a h
\]

This quantity may be expressed in terms of percentage of the rainfall pulse depth as:

\[
E_{a\%} = 100 \frac{E_a}{R_{pd}}
\]

For \( t_a = d \), \( E_{a\%} \) assumes the value 16.67% that agrees with the conclusions by Yoo et al. (2015).

However, an analysis of a considerable number of measured hyetographs performed for different rain gauge stations and \( d \) values highlights that in many cases before and after the peak the rainfall depth exhibits a steeper trend (Fig. 4). Therefore the actual value of \( E_{a\%} \) should be less than 16.67%.
We note that, in principle, underestimation errors in determining the $H_d$ values cannot be eliminated, independently of the adopted $t_p$. Moreover, the average error $E_{a%}$ decreases when the ratio $t_p/d$ decreases. For example, from Eqs. (3) and (5) it follows that:

$$E_{a%}(d = t_p) = \frac{1}{n} E_{a%}(d = t_p) \quad n = 1, 2, ...$$

which implies that for $t_p/d$ sufficiently small $E_{a%}$ becomes negligible.

On the basis of the aforementioned analysis, if $d = t_p = 1$ min for an extreme rainfall event of intensity equal to 300 mm/h the underestimation error becomes less than 1 mm. Further, considering that from a practical point of view the durations of interest for $H_d$ are always ≥5 min, rainfall data with $t_p = 1$ min may be considered with negligible error as continuous data.

### 3 Experimental system

Rainfall data used in this study were mainly recorded in the Umbria Region (8456 km²), located in Central Italy. This Region is characterized by a complex orography of mountainous type along the eastern side, where the Apennine Mountains exceed 2000 m a.s.l., and of hilly type with elevation ranging from 100 to 800 m a.s.l., in the central and western areas.

Mean annual rainfall, for 1921-2015, is about 900 mm but varies spatially from 650 mm to 1450 mm. Higher monthly rainfall values generally occur during the autumn-winter period, when floods caused by widespread rainfall are frequently observed.

A wide part of the study area is included in the Tiber River basin, which crosses the Region from North to South-West, receiving water from many tributaries mainly located on the hydrographic left side.

The study area is currently monitored through a dense rain gauge network (about 1 rain gauge every 90 km²) mostly with a continuous ($t_p = 1$ min) connection to a central unit by a radio link. Before 1992 a reduced rain gauge network (18 devices) was in operation with $t_p = 30$ min.

In this study only the rain gauge stations characterized by continuous rainfall data for at least 20 years (16 out of 93), are considered. Their geographic position, together with the main characteristics of the selected time series, are summarized in Fig. 5.
The selected rain gauge stations were divided into two groups: one with 12 stations used during a first phase to develop a methodology of data analysis and the other with 4 stations preserved for validation purposes.

Rainfall data from the Fabra Observatory of Barcelona (Spain) (Burgueño et al., 1994; Casas et al., 2004), with elevation 411 m a.s.l. are also considered for the validation stage. Due to the location on the northeast coast of the Iberian Peninsula, rainfall in Barcelona is rather limited with an average annual depth less than 640 mm distributed in few rainy days (55 per year), usually in late summer and autumn when advection of warm and humid air from the Mediterranean Sea can cause heavy rainfall events of convective type (Rodríguez-Solà et al., 2017). Rainfall data from Barcelona were recorded in the period 1951–1981 with $t_a = 1$ min.

### 4 Results

#### 4.1 Development of average error relationship

Starting from the continuous rainfall data of all selected stations, aggregated data with the following $t_a$ were obtained: 1 min, henceforth denoted as “Observed”; 10, 15, 30, 60, 180, 360, 720 and 1440 min, henceforth denoted as “Generated”. An example of this procedure is shown in Table 1 for rainfall data recorded at the Petrelle station.
Table 1 “Observed” and “Generated” rainfall data characterized by different temporal aggregations, \( t_a \), starting from January 1, 2006 at 0:00 a.m. at the Petrelle station (Umbria Region, Central Italy).

<table>
<thead>
<tr>
<th>“Observed” rainfall depth (mm)</th>
<th>“Generated” rainfall depth (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1’</td>
<td>10’</td>
</tr>
<tr>
<td>0.0</td>
<td>0.6</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0</td>
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<tr>
<td>0.1</td>
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</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>0.0</td>
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</tr>
<tr>
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<td>0.6</td>
</tr>
<tr>
<td>0.0</td>
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</tr>
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</tr>
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<td>0.0</td>
<td>1.7</td>
</tr>
<tr>
<td>0.0</td>
<td>1.4</td>
</tr>
<tr>
<td>0.0</td>
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</tr>
<tr>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>0.0</td>
<td>0.7</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7</td>
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<td>0.0</td>
<td>1.2</td>
</tr>
<tr>
<td>0.0</td>
<td>2.3</td>
</tr>
</tbody>
</table>

For each selected station and considering some typical values of \( d \) (≤1440 min), all \( H_d \) values may be easily determined by using both the “Observed” and “Generated” data. For each set of rainfall data, \( H_d \) can be deduced only for \( d \geq t_a \).

Assuming each \( H_d \) value obtained from the observed data as a benchmark, the \( H_d \) underestimation caused by the use of rainfall data with a coarse \( t_a \) (“Generated”) can be quantified. As representative cases, Tables 2 and 3 highlight the underestimation errors for the Bastia Umbria station considering temporal aggregations equal to 30 and 15 min, respectively. It can be seen that, for fixed \( t_a \) and \( d \), errors can randomly vary with years. The minimum underestimation error in Table 2 is practically negligible (0.32% in 1992) for \( t_a = d = 30 \) min, whereas the error increases to about 34% in 1997. It may be observed that more significant errors occur when \( t_a > d \), while they become less than 1% when \( t_a/d \leq 0.1 \). A comparison of Tables 2 and 3 shows that the error magnitude, particularly in terms of the average value for all years, is mainly related to the ratio \( t_a/d \). For example, values in the third column of Table 2 (where \( d = 60 \) min and \( t_a/d = 0.5 \)) are comparable with those in the second column of Table 3 (where \( d = 30 \) min and \( t_a/d = 0.5 \)), with a difference in terms of average values less than 1%. However, Table 4 shows that in some cases these differences become significant because the average underestimation errors depend also on \( d \). For equal ratios of \( t_a/d \) a smaller average error is obtained when \( d \) is longer because the probability to have a dry period is higher.
Table 2 Underestimation errors (in %) in the evaluation of the annual maximum rainfall depth considering rainfall data with time of aggregation of 30 min and different durations, \( d \), at the Bastia Umbra station (Umbria Region, Central Italy).

<table>
<thead>
<tr>
<th>Year</th>
<th>30</th>
<th>60</th>
<th>180</th>
<th>360</th>
<th>720</th>
<th>1440</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>0.32</td>
<td>0</td>
<td>0.41</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1993</td>
<td>16.22</td>
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<td>1.54</td>
<td>0.02</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>1995</td>
<td>23.72</td>
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<td>0</td>
</tr>
<tr>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>1998</td>
<td>30.32</td>
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</tr>
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</tr>
<tr>
<td>Average</td>
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<td>5.11</td>
<td>1.41</td>
<td>0.44</td>
<td>0.37</td>
<td>0.07</td>
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Table 3 Underestimation errors (in %) in the evaluation of the annual maximum rainfall depth considering rainfall data with time of aggregation of 15 min and different durations, \( d \), at the Bastia Umbra station (Umbria Region, Central Italy).
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</tr>
<tr>
<td>2002</td>
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<tr>
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<td>0</td>
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</tr>
<tr>
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<td>0.22</td>
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Table 4: Average underestimation errors (in %) in the evaluation of the annual maximum rainfall depth for the rainfall stations used during the first phase of this work. Different values of duration, d, are considered. The symbol $t_a$ denotes the aggregation time. In the last line the average values representative of each duration are shown.
Additional information on our results are given in Fig. 6, that indicates the absence of a link between the rain gauge location and the error magnitude.

Finally, the dependence of the average error on the length of the data series has been investigated. Fig. 7 shows the error variability with increasing the measurement period that precedes the last $H_d$ value. The results of this analysis, performed using series with a length of at least 20 years, are synthesized through a few representative cases referred to $t_a/d = 1$ and $d \gg t_a$, which determine extreme values of the average error. From Fig. 7a-f it can be seen that increasing the series dimension the average error trend is rather irregular, independently of the ratio $t_a/d$. This is an expected result considering that $H_d$ is a random variable. However, it is possible to deduce the data series length required to obtain a reliable estimation of the average error: In most cases it should be approximately greater than 15-20 years (Fig. 7a-e), but for $d \gg t_a$ (Fig. 7e) the average error magnitude is of minor importance even though much
shorter lengths are used. These results highlight a possible critical point in the earlier study by Young and McEnroe (2003) who examined data series with average length less than 10 years. However, a partial support to their study is given by the results we have obtained in a limited number of historical series for which a length approximately greater than 7 years (Fig. 7) seems to be appropriate for the average error estimation.

Fig. 7 Average errors in the evaluation of annual maximum rainfall depth, $H_d$, as a function of the number of years preceding the last $H_d$ value for different combinations of aggregation time, $t_a$ (in minutes), and duration, $d$ (in minutes): (a) Bastardo station, $t_a = 30, d = 30$; (b) Cerbara station, $t_a = 30, d = 30$; (c) Ponte Santa Maria station, $t_a = 60, d = 60$; (d) Ripalvella station, $t_a = 60, d = 60$; (e) Nocera Umbra station, $t_a = 30, d = 180$; (f) Nocera Umbra station, $t_a = 30, d = 30$.

An overall analysis of our results suggests that:

- the developments presented in ion 2 for the evaluation of errors on $H_d$ are substantially well-founded;
- in any case an average error becomes reliable if its estimation is carried out on the basis of at least 15-20 years of observed rainfall data;
- the largest average error occurs for $d = t_a$ and does not exceed 16.67%;
- for $d = t_a$ the average error is less than or equal to $(1/n)16.67%$;
- the average error depends on both $t_a/d$ and $d$;
- for each specific year the error is a random quantity with value in any case less than or equal to 50%;
- the average errors are independent of the considered rain gauge location.

4.2 Correction of $H_d$

The aforementioned results account for the effect of temporal aggregation on $H_d$ values, either for a specific year or for a long time series. Therefore, on this basis we can define a methodology to improve the homogeneity of $H_d$ series obtained from rainfall data with very different temporal aggregations.

Considering only rainfall data observed in the stations selected for the first phase of this work, Fig. 8 displays all average underestimation errors for different values of $t_a/d$, including those obtained from daily rainfall data. The
best interpolation function can be expressed as:

\[
\varepsilon_{\text{rel}} = 4.01 \left(\frac{t}{d}\right)^2 + 6.94 \frac{t}{d} \quad [\%]
\]

From Fig. 8 it can be deduced the uncertainty associated to the results obtained by Eq. (7). This should be useful in hydrological practice because the users could decide, particularly for \(t/d = 1\), which estimation to adopt depending on the level of risk they want to assume.

Furthermore, Fig. 8 shows that our results are very close to those obtained by Young and McEnroe (2003) even though, as above discussed, in general terms they considered too short series of \(H_d\) with lengths in the range 5.3–14.9 years while lengths larger than 15–20 years could be in principle more appropriate considering also the random nature of the investigated variable. The reliable curve of Fig. 8 obtained by Young and McEnroe (2003) could be therefore ascribed to a use, for \(t/d = 1\), of a significant number of series with lengths close to 15 years and to stations with rainfall temporal structure similar to that characterizing our representative station of Fig. 6f. In any case, the stations to implement to obtain acceptable data corrections through shorter series lengths cannot be identified a priori, therefore the good relation proposed by Young and McEnroe (2003) does not justify the adoption of series with too short duration. In addition, Fig. 8 highlights a significant difference between our representation of the average error by Eq. (7) and that proposed by Weiss (1964), who in his probabilistic approach assumed a uniform rainfall rate through the accumulation period. In the light of our analysis on the rainfall patterns observed in the study stations, this assumption does not appear fully justified even though the adjustment factor was applied as an average quantity to the series of annual maximum rainfall depths with a given duration. From our experimental data we deduced that the assessment of \(\varepsilon_{\text{rel}}\) could be further improved by splitting Eq. (7) on the basis of the duration of interest because of its link with the shape of the rainfall temporal distribution that influences the error magnitude (Yoo et al., 2015). Rectangular rainfall pulses were typically observed for \(d\) up to 30 min, triangular pulses for greater values of \(d\) up to 180 min and pulses representable by quadratic functions (Yoo et al., 2015) for larger values of \(d\). On this basis the following three relations, plotted in Fig. 9, were derived:

\[
\begin{align*}
\varepsilon_{\text{rel}} & = 6.14 \left(\frac{t}{d}\right)^2 + 5.96 \frac{t}{d} \quad [\%] \quad d \leq 30 \text{ minutes} \\
\varepsilon_{\text{rel}} & = 6.7 \left(\frac{t}{d}\right)^2 + 4.72 \frac{t}{d} \quad [\%] \quad 30 \text{ minutes} < d < 180 \text{ minutes} \\
\varepsilon_{\text{rel}} & = 5.2 \left(\frac{t}{d}\right)^2 + 5.57 \frac{t}{d} \quad [\%] \quad d \geq 180 \text{ minutes}
\end{align*}
\]
For each value of $t_a/d$ and $d$, Eq. (8) can be used to quantify the correction to be apply to the $H_d$ series obtained from data with a coarse $t_a$. Through a sensitivity analysis it was checked that the number of stations adopted was sufficiently high to assure the robustness of the proposed methodology. This methodology was validated using rainfall data from the 2nd group of stations located in the study area and from the Barcelona station (Burgueño et al., 1994). Each series of $H_d$ values obtained with coarse $t_a$ (10, 15, 30, ... minutes) was corrected by adding the quantity given by Eq. (8) and then compared with the “Observed” series ($t_a = 1$ min). Fig. 10 shows the corrected average $H_d$ values, for all the examined combinations of $t_a$ and $d$, against the benchmark values obtained from the rainfall data characterized by $t_a = 1$ min. The proposed methodology provides an accurate representation of the actual average $H_d$ values, with determination coefficients in respect to the bisecting line higher than 0.99.

The improvements obtained through the application of the developed methodology can also be deduced from Table 5, where positive and negative values of the residual average error after the correction by Eq. (8) indicate underestimation and overestimation, respectively, and from Fig. 11, referred to cases with $t_a/d = 1$. In many cases the residual average errors become of minor interest.

**Table 5** Errors (in %) associated with the determination of the average annual maximum rainfall depth from data with aggregation time of 30 min for two different durations, $d$. Rainfall data from the stations used in the validation phase. Positive and negative values represent underestimation and overestimation, respectively. “Uncorrected” and “Corrected” stand for the errors before and after the application of the proposed procedure, respectively.

<table>
<thead>
<tr>
<th>Rain gauge station</th>
<th>“Uncorrected”</th>
<th>“Corrected”</th>
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</thead>
<tbody>
<tr>
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</table>

Fig. 9 Average underestimation error of the annual maximum rainfall depth obtained by Eq. (8) as a function of the ratio between aggregation time, $t_a$, and duration, $d$.

Fig. 10 “Corrected” average annual maximum rainfall depths, $\mu(H_d')$, as a function of the corresponding values derived from rainfall data with $t_a = 1$ min, $\mu(H_d)_{t=1}$. Results from rain gauge stations selected for the validation phase.
Finally, the effect of the correction of $H_d$ on the rainfall depth-duration frequency curves was quantified. This issue is addressed below through the description of both the adopted procedure and the results obtained for the representative rain gauge station of Gubbio. For each duration, in addition to 24 values of $H_d$ appropriately observed with $t_a = 1\text{ min}$ (see also Fig. 5), 20 values obtained from data recorded earlier than 1992 with $t_a = 30\text{ min}$ were used. This $H_d$ series represents the uncorrected one, while a series including the 24 values of $H_d$ observed with $t_a = 1\text{ min}$ and the remaining 20 values modified by the proposed methodology is denoted as the corrected series. The statistical analysis of each random variable, $H_d$, was performed using the Generalized Extreme Value (Jenkinson, 1955; Coles, 2001) distribution function. Fig. 12 indicates that for durations up to 3 h the error expressed as a percentage of the annual maximum rainfall depth is slightly variable with both return period and duration and that the use of uncorrected $H_d$ series determines depth underestimations between 5% and 10%. Similar results were obtained for durations up to 24 h. Furthermore, we note that the above errors would experience an appreciable increase in the case the uncorrected series involving only data deduced through $t_a \gg 1\text{ min}$.

![Fig. 11](image1.png)

**Fig. 11** Average annual maximum rainfall depths, $\mu(H_d)$, versus the corresponding values derived from rainfall data with $t_a = 1\text{ min}$, $\mu(H_d)_{ta=1}$. “Uncorrected” and “Corrected” stand for the values obtained before and after the application of the proposed methodology, respectively. Rainfall data from the stations used in the validation phase. Only cases with duration equal to the aggregation time are shown.

![Fig. 12](image2.png)

**Fig. 12** Rainfall depth-duration curves for different return periods, $T_r$. Comparison of curves obtained by uncorrected $H_d$ series and corresponding series corrected by the proposed methodology. Sample rain gauge station of Gubbio.
5 Conclusions

The evaluation of rainfall depth-duration-frequency curves should be made by using $H_d$ values derived from continuously recorded rainfall depths but until few decades ago these were available only with coarse temporal aggregations. Therefore, a correction of the $H_d$ values deduced from data recorded with a significant temporal aggregation is required for hydrological applications.

In this paper we have first examined in depth a few critical points already remarked in previous works. Our study, developed through the use of a large number of rain gauge stations operative for many years with $t_s = 1$ min, emphasizes the following elements:

- $H_d$ values derived from rainfall data characterized by every $t_s$ involves underestimation errors, that for $t_s > 10$ min can become important;
- in the worst conditions, that occur for $d = t_s$, a single $H_d$ value can be affected by an underestimation error up to 50%, while the average underestimation error for a series of appropriate length is less than or equal to 16.7%;
- each $H_d$ series usually contains many values significantly underestimated. In our study area, equipped with 93 rain gauge stations, the percentage of $H_d$ values determined by rainfall data recorded with $t_s = 30$ min is equal to 34.7%, with a value of 100% for a few series.

On this basis we have shown that:

- to develop reliable relationships between the average underestimation error, $E_{t/s}$, and values of $t_s$ and $d$, data series with a length of at least 15-20 years have to be available;
- a relationship between $E_{t/s}$ and $t_s/d$ split up into three expressions associated with different duration ranges enables us to obtain very reliable $H_d$ series;
- the use of uncorrected $H_d$ series for the determination of rainfall depth-duration-frequency curves can lead to underestimations of the order of 10%.

Acknowledgment

The authors are thankful to the Umbria Region and its Functional Centre for providing the rainfall data and I. Bavicchi, L. Pesaresi and M. Stelluti for their technical assistance.

This research was mainly financed by the Italian Ministry of Education, University and Research (PRIN 2015). It also forms part of the INTENSE project through the European Research Council (grant ERC-2013-CoG-617329). Hayley Fowler is funded by the Wolfson Foundation and the Royal Society as a Royal Society Wolfson Research Merit Award (WM140025) holder.

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**Highlights**

- Rainfall data to be used in the hydrological practice is available in aggregated form.
- Aggregated form produce the underestimate of annual maximum rainfall depth ($H_0$).
- Errors in the $H_0$ evaluation from data with coarse time aggregations are investigated.
• Relationships to overcome the underestimate of $H_i$ are presented.

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