The Signaling Game Model under Asymmetric Fairness-Concern Information

Yanhong Qin[ab*],Guangxing Wei[c], JingXin Dong[d]
[a] School of Management and Economics , University of Electronic Science and Technology of China, Sichuan, 611731, China
[b] School of Management, Chongqing University of Technology, Chongqing, 400054, China
Email: qinyanhong24@163.com
[c] School of Management, Chongqing Jiaotong University, Chongqing, 400074, China
Email: wgx777@126.com
[d]Business School, Newcastle University, Newcastle upon Tyne, UK, NE1 4SE,
Email: jingxin.dong@ncl.ac.uk

Abstract：Under the wholesale price contract, we analyze the influence of the retailer's fairness-concern information on the wholesale price, order quantity, the profit of each party and the supply chain in SI (Symmetry Information) condition and AI (Asymmetry Information) condition respectively. Then, we compute the value of retailer's fairness-concern information to supplier, and we prove that the profit of all members and supply chain is decreasing with retailer's fairness concern and the profit in SI condition is always higher than that in the AI condition. Then, we set the signaling game model to reveal the transmission mechanism of retailer's fairness-concern information, and we analyze the potential separating equilibrium and pooling equilibrium existing in signaling model under asymmetric fairness-concern information. We prove that only when the signal transmission cost is different between retailers with different fairness-concern degree, the signaling model can effectively reveal the role and type of retailers. Finally, we provide some suggestions improve fairness-concern information transmission and optimize supply chain operation by discussing the condition of each separating equilibrium results.
Keywords：Fairness concern; Asymmetry information; Wholesale price contract; Signaling game model

Introduction

Fairness concern has become an important factor in the research of supply chain contract, which can provide a solid foundation for the supply chain optimization and management. More and more scholars have introduced the fairness-concern into the research field of supply chain contract, and analyzed the influence of fairness preference on the value of contract parameters, coordination and operation efficiency of supply chain (Cui, 2007；Loch and Wu, 2008；Katok and Pavlov, 2013； Katok, 2014; et al.). Notably, wholesale price contracts, especially linear wholesale price contracts, are simple to use in practice as it will incur no extra execution cost and require no sharing of sale information between suppliers and retailers (Lariviere and Porteus，2001). Under a wholesale price contract, the retailer determines the retail price and order quantity based on the wholesale price predetermined by the supplier, and disposes any unsold inventory products bearing the full market risk himself (Cachon and Lariviere，2001; Cachon, 2005). This might be the reason why the above studies have all focused on the role of wholesale price in supply chain coordination.

There are a lot of researches on the wholesale price contract, which can be divided into two stages:

① the wholesale price contract under symmetry information of fairness preference, such as Cui et al.(2007), Caliskan-Demirag et al.(2010), Ozgun (2010), Du et al. (2014a，2014b), Hoe et al. (2014), Zhang and Ma (2016), and so on；② the wholesale price contract under asymmetry information of fairness preference, such as Kalkanci et al.(2011), Katok et al.(2014), Walter & Arnald (2015), Choi and Messinger (2016), and so on. Cui et al. (2007) proved that simple constant wholesale price above wholesaler's marginal cost can lead to maximisation of channel profit and utility in a conventional dyadic channel with fairness concern. Caliskan-Demirag et al. (2010) extend the research of Cui et al. (2007) by considering a few typical nonlinear demand functions and obtain similar results. Ozgun (2010) further extend the the research of Cui et al. (2007) by considering exponential demand function, and prove that only when the retailer care about the fairness or both the supplier and retailer cared about the fairness, the wholesale price could achieve the supply chain coordination, but when only the supplier care about the fairness, the wholesale price can not do that, so the Ozgun (2010) further verify the conclusion in Cui et al. (2007) and Caliskan-Demirag et al. (2010) that the fairness preference can change the coordination virtue of wholesale price contract. Bi et al. (2013) set the mathematical model and adopt the numerical simulation to analyse the compact of retailer's fairness concern on all decisions in supply chain and coordination under wholesale price contract. Katok and Pavlov (2013) compare performance of three types of contract: wholesale price, buyback and revenue sharing; and suggest that although the performance of buyback contract and revenue sharing contract is better than wholesale price contract, the performance gap is insignificant. Du et al. (2014a, 2014b) proved that fairness preference can significantly change the equilibrium outcome of supplier-retailer game, and the wholesale price contract can achieve competitive supply chain coordination under certain conditions. Wu and Niederhoff (2014) check the traditional research on the supply chain under fairness concern in more demand distribution functions and prove that the retailer's fairness concern can improve supply chain efficiency only in high uncertain demand and retailer's profit distribution ratio exceeding a certain threshold. Qin and Wei (2015) classify the game into four stages according to the retailer's fairness types and symmetry or asymmetry information, and provid the comparative analysis of the decision variables and the profit of retailer equity preference behavior impact on supply chain members of the decision in the symmetry and asymmetry information. Zhang and Ma (2016) show that the fairness-concern behavior of retailers has a significant impact on the wholesale price, retail price and all the marketing effort level through the mathematical model and numerical analysis, and fairness-concern behavior of retailer can increase their bargaining power in the supply chain.

② The wholesale price contract under asymmetry information of fairness preference, such as Kalkanci et al.(2011), Katok et al.(2014), Walter & Arnald (2015), Choi and Messinger (2016), and so on. Kalkanci et al. (2011) add the fairness preference into the wholesale price contract to study the contract design under the asymmetric demand information, and they explained the phenomenon in the real market why the most supply chain contracts are simple linear structure but not the complex nonlinear structure. Katok (2014) study the influence of retailer's fairness concern on both decision making and supply chain coordination efficiency through mathematical model and behavioral experiment. Walter & Arnald (2015) prove that the decision maker may actively display his own real fairness-concern information so as to lessen the limited knowledge of the fairness preference, but he will not compromise the profit distribution in the supply chain, which can decrease the efficiency of supply chain coordination and may cause some wrong conclusions. Choi and Messinger (2016) apply the experiment method to verified the conclusion in Walter & Arnald (2015) and analyze the compact of fairness concern on the each partner's decision in the competitive supply chain and the whole supply chain performance. Cao and Hou (2016) establish a principal-agent model and study the retailer's fairness concern on the supply chain decision under asymmetric information about the fairness concern. Qing and Li (2016) show that the fairness-concern behavior of retailer has a significant impact on the wholesale price, retail price and all the marketing effort level through the mathematical model and numerical analysis, and fairness-concern behavior of retailer can increase their bargaining power in the supply chain. Qin et al. (2016) analyze the influence of fairness concern on the supply chain performance by assuming the cost information is private information.

It can be seen that there are more and more researches referring the effect of fairness concern on the wholesale price contract coordination, but most of them are focusing on the " symmetry information " and only few research start to study the "asymmetric information", because the fairness concern is objective and private information is generally not symmetric, and the supply chain members only can identify the accurate information about fairness concern by some efforts. Although there are few literatures referred to the supply chain wholesale price contract under symmetric coordination, but mainly concentrated on the asymmetric information about supply chain cost information or promotional effort information, none referred to the fairness concern information. Secondly, the existing research on the asymmetric information in the supply chain under the wholesale price contract only pay attention to the impact of fairness concern on all decisions and profits, but none analyze and compute the fairness preference information value, nor establish a signaling game model to reveal the fairness preference information of supply chain members. As well known to us, the authenticity problem of fairness concern information is a more basic and important problem in the contract design of supply chain, and only when the fairness information is authentic, the conclusions and suggestions obtained in the supply chain contract design by considering fairness concern will be right and meaningful for real operation management.

The remainder of the article is structured as follows. Section 2 will set the simple two level supply chain made up of one neutral supplier and one fairness concern retailer under wholesale price contract to analyze the retailer's fairness concern on the optimal wholesale price, the retailer's optimal order policy and profits of retailer, supplier and the supply chain. In section 3, we will compare the profit and decision variables in SI (Symmetry Information) condition and AI (Asymmetry Information) condition so as to compute the information value of retailer's fairness concern to the supplier and thus the supply chain. Section 4 will set the signaling game model to reveal the transmission mechanism of retailers' fairness-concern information, and we analyze the potential separating equilibrium and pooling equilibrium existing in signaling model under asymmetric fairness-concern information. We prove that only when the signal transmission cost is different between retailers with different fairness preferences, the signaling model can effectively reveal the role and type of retailers. Further, we provide some suggestions in term of fairness preference information transmission and optimization of supply chain by discussing the condition of each separating equilibrium results. Section 5 will conclude our research and point out the future directions.

2 The Model

2.1 **Notation description**

As the traditional researches, the supply chain includes a supplier and a retailer, the supplier produces only one kind of product and sell the product by retailer. Market demand X is random variable, and probability density function and distribution function is $f(x)$, $F(x)$ ($x \geq 0$). $F(0)=0$, $\overline{F}(x)=1-F(x)$. $F(x)$ is differentiable, strictly increasing and increasing generalized failure rate (IFGR). c is the unit production cost of supplier, and w is the wholesale price of supplier providing to retailer. Before sale season, the retailer decides to order the q from the supplier, and expected sale quantity is $S(q)$ ($S(q)=q-\int_0^q F(x)dx$). If the ordered product can't meet the market demand, the retailer only loss of sale profits, and thus we can assume the shortage cost is zero for both supplier and retailer.

In the end of sale season, if there are some surplus inventories, the retailer will deal with excess inventory and bear the market risk by himself completely. It is assumed that the unit residual value of surplus products is v ($c>v$). Otherwise, the supplier has a profit motive to repurchase the remaining inventory at the retailer. p is the sale price and is determined by the external market under perfectly competitive market. π denote the profit and u denote the utility.

The subscript "r, s, sc" represents retailer, supplier, and supply chain respectively, such as π_s is the supplier's profit. Superscript "SI, AI" denote symmetric information and asymmetric information, " FC" denote fairness concern and "*" represents the optimal value. For example, u_r^{SI-FC*} means optimal utility of the retailer when he pays attention to fairness and this information is asymmetric.

In the case of centralized decision making, supply chain as a whole to determine sales volume, and the total profit function of the supply chain is $\pi_{sc}(q)=pS(q)-cq+v[q-S(q)]$. It easy to compute $\dfrac{d\pi_{sc}(q)}{dq}=(p-v)(1-F(q))-c+v$, let $\dfrac{d\pi_{sc}(q)}{dq}=0$, and we can get the unique and optimal order quantity as following equation (1):

$$q_{sc}^{*}=F^{-1}\left(\frac{p-c}{p-v}\right) \tag{1}$$

2.2 Wholesale price contract

Due to the supplier acting in a dominant position in the game, we do not consider the fairness-concern behavior of supplier. The retailer is a follower in the game and thus in a weak position of profit allocation in supply chain, and therefore the retailer more care about the profit allocation and more care about fairness, so here we consider the retailer make order decision to maximize the total utility u_r including his own profit and equity (Cui et al. 2007).

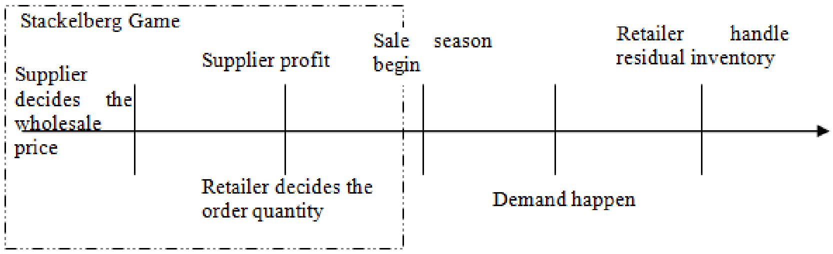

Fig.1 The time series of the game under the wholesale price contract

The actions of supplier and retailer under the wholesale price contract are as follows:

① The supplier determines the optimal wholesale price w according to the principle of maximizing profit;

② At the time of the sale season, the retailer determines the order $p^*(w)$ from the supplier according to the market demand forecast, and the supplier can know the response function $p^*(w)$ so as to decides the optimal wholesale price w^*. Then, the retailer pays the price to the supplier and supplier can obtain the revenue immediately.

③ When demand occurs in the sale season, the retailer sells the product according to the actual market and obtains the market sale income. The action process can be shown in fig.1.

From fig.1, the game process between the supplier and the retailer is a typical two-stage Stackelberg game, which can be denoted as $<\{S,R\},(w,q),(\pi_s,u_r),H>$. S denotes the supplier, R denotes the retailer, and both them are game players. w is the supplier's strategy selection and q is the retailer's strategy selection. The supplier takes the profit π_s maximization as the goal, while the retailer takes the total utility u_r including profit and fair utility as the decision goal. We can solve the perfect Nash equilibrium of sub game by backward induction $(w^*, p^*(w^*))$.

Under the wholesale price contract, given the wholesale price w provided by supplier, the profit function of retailer and supplier is corresponding as:

$$\pi_r(w,q) = pS(q) - wq + v[q - S(q)] \quad (2)$$

$$\pi_s(w,q) = (w-c)q \quad (3)$$

The retailer determines the optimal order quantity from the expected profit maximization based on the wholesale price w. For $\frac{d^2\pi_r(w,q)}{dq^2} = -(p-v)f(q) < 0$, the retailer has the unique optimal response function subject to $\frac{d\pi_r(w,q)}{dq} = (p-v)[1-F(q)] - w + v = 0$, so it can be denoted as

$$q_r^*(w) = F^{-1}\left(\frac{p-w}{p-v}\right) \quad (4)$$

For the supplier has the first decision advantage in the supply chain and thus can obtain more profit than retailer, many literatures have proved that although retailer is more close to the market but he is in the disadvantaged condition in the profit distribution, so the retailer often would pay attention to compare his own profit and supplier's profit, which is called as fairness preference. The utility function containing the retailers' profit and fair disutility can be denoted as following:

$$u_r(w,q) = \pi_r(w,q) - \lambda[\pi_s(w,q) - \pi_r(w,q)] \quad (5)$$

In above formula, λ is the retailer's fairness concern and $\lambda \geq 0$. $\lambda = 0$ indicates that retailer would not care about fairness, and only consider his own profit maximization, and the utility function (5) will be reduced to the simple profit function $u_r(w,q)=\pi_r(w,q)$. $\lambda>0$ indicates that retailer will pay attention to fairness, and the total utility maximization is including profit and fair disutility in the decision making. The bigger λ means the retailer care more about the profit allocation, and the unit profit difference between supplier and retailer fair can bring greater negative equality effect.

Take equations (2) and (3) into (5), we can get the utility function as

$$u_r(w,q)=(1+\lambda)[pS(q)-wq+v(q-S(q))]-\lambda(w-c)q$$

Proposition 1

(1) The retailer of fairness concern has unique optimal order $q_r^{FC*}(w)$ subjected to

$$q_r^{FC*}(w)=F^{-1}\left(\frac{p-w+\lambda(p-2w+c)}{(1+\lambda)(p-v)}\right)$$

(2) $q_r^{FC*}(w)$ is a decreasing function and thus the profit function of retailer, supplier and supply chain is decreasing in λ, and further deviate from the optimal order quantity of the supply chain.

Proof. For $\frac{d^2u_r(w,q)}{dq^2}=-(1+\lambda)(p-v)f(q)<0$, the retailer has the unique optimal order quantity $q_r^{FC*}(w)$ subjected to $\frac{du_r(w,q)}{dq}=0$.

$\frac{\partial q_r^{FC*}(w)}{\partial\lambda}=-\frac{\partial^2u_r/\partial q\partial\lambda}{\partial^2u_r/\partial q^2}\Big|_{q=q_r^{FC*}(w)}$, $\frac{\partial^2u_r(w,q)}{\partial q^2}<0$, and for $\frac{\partial^2u_r(w,q)}{\partial q\partial\lambda}=(p-2w+c)-(p-v)F(q)$ from $\frac{du_r(w,q)}{dq}=(1+\lambda)[(p-v)(1-F(q))-w+c]-\lambda(w-c)$, we can take $q_r^{FC*}(w)=F^{-1}\left(\frac{p-w+\lambda(p-2w+c)}{(1+\lambda)(p-v)}\right)$ into $\frac{\partial q_r^{FC*}(w)}{\partial\lambda}=-\frac{\partial^2u_r/\partial q\partial\lambda}{\partial^2u_r/\partial q^2}\Big|_{q=q_r^{FC*}(w)}$ and get $\frac{\partial^2u_r(w,q)}{\partial q\partial\lambda}\Big|_{q=q_r^{FC*}(w)}=\frac{c-w}{1+\lambda}<0$, and thus $\frac{\partial q_r^{FC*}(w)}{\partial\lambda}=-\frac{\partial^2u_r/\partial q\partial\lambda}{\partial^2u_r/\partial q^2}\Big|_{q=q_r^{FC*}(w)}<0$, which means $q_r^{FC*}(w)$ is a decreasing function in λ.

Similarly, we can get $\frac{\partial\pi_s}{\partial\lambda}=\frac{\partial\pi_s}{\partial q}\cdot\frac{\partial q}{\partial\lambda}\Big|_{q=q_r^{FC*}(w)}<0$, $\frac{\partial\pi_r}{\partial\lambda}=\frac{\partial\pi_r}{\partial q}\cdot\frac{\partial q}{\partial\lambda}\Big|_{q=q_r^{FC*}(w)}<0$.

Finally, for the supply chain profit is strictly concave function and is maximized in $q_{sc}^*=F^{-1}\left(\frac{p-c}{p-v}\right)$, so when $q_r>q_{sc}^*$, supply chain profit is decreasing in order quantity. By comparing $q_r^{FC*}(w)$ and q_{sc}^*, $q_r^{FC*}(w)<q_{sc}^*$ indicates $F(q_r^{FC*}(w))-F(q_{sc}^*)=\frac{-\lambda(w-c)}{(1+\lambda)(p-v)}<0$, and thus we can get $\frac{\partial\pi_{sc}}{\partial q}\Big|_{q=q_r^{FC*}(w)}>0$, leading to $\frac{\partial\pi_{sc}}{\partial\lambda}=\frac{\partial\pi_{sc}}{\partial q}\cdot\frac{\partial q}{\partial\lambda}\Big|_{q=q_r^{FC*}(w)}<0$.

Similarly, we can prove $q_r^{FC*}(w)<q_r^*(w)<q_{sc}^*$, which denotes $q_r^{FC*}(w)$ will further deviate from the optimal order quantity of the supply chain.

After the supplier knows the fairness concern of retailer, he will decide the wholesale price according to $q_r^{FC*}(w)=F^{-1}\left(\frac{p-w+\lambda(p-2w+c)}{(1+\lambda)(p-v)}\right)$ as following:

$$w^{SI-FC}=\frac{(1+\lambda)p+\lambda c-(1+\lambda)(p-v)F(q)}{1+2\lambda} \tag{6}$$

Take equation into supplier's profit function (3), we can denote the supplier's profit by q as

$$\pi_s^{SI-FC}(q)=(w-c)q=[\frac{(1+\lambda)p+\lambda c-(1+\lambda)(p-v)F(q)}{1+2\lambda}-c]q$$

$$\frac{d\pi_s^{SI-FC}(q)}{dq}=\frac{(1+\lambda)[p-c-(p-v)(F(q)+f(q)q)]}{1+2\lambda}$$

$$\frac{d^2\pi_s^{SI-FC}(q)}{dq^2}=\frac{-(1+\lambda)[(p-v)(2f(q)+f'(q)q)]}{1+2\lambda}<0$$

Although the supplier can't determine the order quantity directly, he can change the wholesale price to affect the retailer's optimal order quantity. For the supplier, his preference for the optimal order quantity is $q_s^{SI-FC*}=\arg\max(\pi_s^{SI-FC}(q))$, which should subject to equation (7).

$$\frac{p-c}{p-v}=F(q_s^{SI-FC*})+f(q_s^{SI-FC*})q_s^{SI-FC*} \tag{7}$$

The retailer chooses the wholesale price under the "*IS-FC*" as following:

$$w^{SI-FC}(q_s^{SI-FC*})=\frac{(1+\lambda)p+\lambda c-(1+\lambda)(p-v)F(q_s^{SI-FC*})}{1+2\lambda} \tag{8}$$

Proposition 2 Under the condition of fairness concern and symmetry information, the wholesale price of supplier selection decreases with fairness preference, but is always higher than its own cost.

Proof. $\frac{dw^{SI-FC}}{d\lambda}=\frac{-(p-c)-(p-v)F(q_s^{IS-FC*})}{(1+2\lambda)^2}<0$, $w^{SI-FC}(q_s^{SI-FC*})-c=\frac{(1+\lambda)[p-c-(p-v)F(q_s^{SI-FC*})]}{1+2\lambda}>0$.

3. The information value under asymmetric fairness concern

Because the supplier has the leading advantage in the supply chain, and the supplier has the right to decide the wholesale price under the wholesale price contract. When the retailer knows that the fairness preference information about itself, but the supplier does not consider the fairness-concern information of retailer and thus supplier makes decision under the assumption that the retailer is neutral and does not pay attention to their own profit gap with supplier.

Since this part is based on the retailer's fairness concern but the supplier considers him to be neutral, so the fairness-concern information is asymmetric, denoted as" $AI-FC$ ".

According to the optimal response function $q_r^*(w)=F^{-1}\left(\frac{p-w}{p-v}\right)$, we can get the one-to-one correspondence between the optimal order quantity of the retailer and the wholesale price of the supplier, i.e. $w=p-(p-v)F(q)$, which can be taken into equation (3), and the supplier's profit can be denoted by q as following:

$$\pi_s^{AI-FC}(q)=(w-c)q=[p-(p-v)F(q)-c]q \tag{9}$$

In equation (9)

$$\frac{d\pi_s^{AI-FC}(q)}{dq}=p-(p-v)F(q)-c-(p-v)F'(q)q$$

$$\frac{d^2\pi_s^{AI-FC}(q)}{dq^2}=-(p-v)[2f(q)+f'(q)]<0$$

For the supplier, the optimal order quantity for him is $q_s^{AI-FC*}=\arg\max(\pi_s(q))$ and $\frac{d\pi_s(q)}{dq}=0$, which is equal to

$$\frac{p-c}{p-v}=F(q_s^{AI-FC*})+f(q_s^{AI-FC*})q_s^{AI-FC*} \tag{10}$$

Based on the response function, the supplier would choose the wholesale price as following

$$w^*(q_s^{AI-FC*}) = p-(p-v)F(q_s^{AI-FC*}) \tag{11}$$

In fact, according to the Stackelberg game, retailer has the following two types of decisions according to his own type: When the retailer cares about fairness, he will consider the response function made up of his own material gain and fair negative utility to decide the optimal order quantity $q_r^{FC*}(w) = F^{-1}\left(\frac{p-w+\lambda(p-2w+c)}{(1+\lambda)(p-v)}\right)$, but not based on $q_r^*(w) = F^{-1}\left(\frac{p-w}{p-v}\right)$ to make decision.

Proposition 3 Under the asymmetric information of the retailer's fairness concern, all the indicators such as order quantity, each party's profit and the total profit of the supply chain decreased with the retailer's fairness preference except the wholesale price.

Proof. For $w^*(q_s^{AI-FC*}) = p-(p-v)F(q_s^{AI-FC*})$, $\frac{p-c}{p-v} = F(q_s^{AI-FC*}) + f(q_s^{AI-FC*})q_s^{AI-FC*}$, and $F(x)+f(x)x$ is strictly increasing, so $w^*(q_s^{AI-FC*})$ is not relative to fair preference degree and is unique and thus equal to $q_r^{FC*}(w) = F^{-1}\left(\frac{p-w+\lambda(p-2w+c)}{(1+\lambda)(p-v)}\right)$. According to proposition 1, $q_r^{FC*}(w)$ decreases with fair preference.

Here, we can apply numerical analysis and assume that $p=20$, $c=5$, $v=3$, and the market demand function is uniform distribution $X \in U[20,80]$, i.e. the probability density function of demand function $f(x) = \begin{cases} \frac{1}{60} & 20 \le x \le 80 \\ 0 & otherwise \end{cases}$, and the corresponding distribution function is $F(x) = \begin{cases} \frac{x-20}{60} & 20 \le x \le 80 \\ 1 & x > 80 \end{cases}$. The numerical analysis on the optimal order quantity, the profits of supplier, retailer and supply chain is in table1 including the two conditions where retailer's fairness-concern information is symmetric and asymmetric. It is easy to compute the optimal order quantity $q^o = 70$ and the optimal profit of supply chain is $\pi_{sc}^o = 1050$.

From table1, we can get some trends for each party as following：

① For supplier. The supplier’s profit in the SI case is better than the AI case, and the wholesale price in SI case is lower than that in AI case. The wholesale price decreases with retailer’s fairness concern both in SI and AS cases. When the retailer's fair preference information is asymmetric, wholesale price is always constant, because the supplier considers that the retailer is neutral and thus when he decides the wholesale price $w = p-(p-v)F(q)$, he would not consider fairness. Besides, Fig. 2 indicates the information value of the retailer's fairness concern for the supplier, namely IV (Information Value), which is the supplier's profit difference after an accurate understanding of the retailer's fair preference information.

② For retailer, both order quantity and the profit in SI case is higher than that in AI case, and the order quantity decreases with fairness concern both in SI and AI case.

③ For the whole supply chain, the supply chain profit is higher in IS case than that in AI case, and the supply chain profit is also decreases with fairness concern both in SI and AI case.

Thus, all the propositions 1, 2 and 3 are confirmed.

Additionally, from table 1, we can find that the decreasing extent of order quantity, supplier’s profit, retailer’s profit and supply chain profit in AI case are all greater than that in SI case. Therefore, it can improve the profit of supply chain members and supply chain by revealing the fairness-concern information.

Table1 The retailer's fairness concern in SI and AI case

λ	Wholesale price		Order quantity		Retailer's profit		Supplier's profit		Profit of supply chain	
	SI	AI	SI	AI	SI	AI	SI	AI	SI	AI
0	15.50	15.5	35.00	35.00	123.75	123.75	367.50	367.50	491.25	491.25
0.2	14.00	15.5	33.33	29.17	173.33	118.65	300.00	306.25	473.33	424.90
0.4	13.17	15.5	32.14	25.00	197.53	108.75	262.50	262.50	460.03	371.25
0.6	12.64	15.5	31.25	21.88	211.13	97.91	238.64	229.69	449.77	327.60
0.8	12.27	15.5	30.56	19.44	219.50	87.45	222.12	204.17	441.62	291.62
1	12.00	15.5	30.00	17.50	225.00	77.81	210.00	183.75	435.00	261.56
1.2	11.79	15.5	29.55	15.91	228.78	69.08	200.74	167.05	429.51	236.13
1.4	11.63	15.5	29.17	14.58	231.47	61.22	193.42	153.13	424.90	214.35
1.8	11.39	15.5	28.57	12.50	234.94	47.81	182.61	131.25	417.55	179.06
2	11.30	15.5	28.33	11.67	236.08	42.08	178.50	122.50	414.58	164.58
2.5	11.13	15.5	27.86	10.00	237.97	30.00	170.63	105.00	408.60	135.00
3	11.00	15.5	27.50	8.75	239.06	20.39	165.00	91.88	404.06	112.27

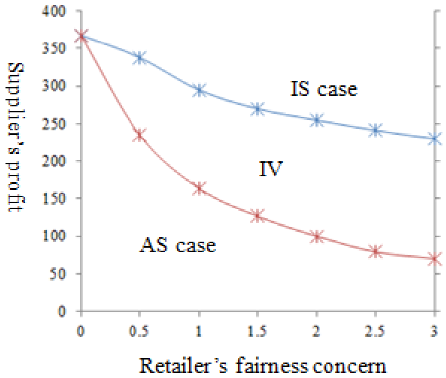

Fig. 2 The information value of fairness concern

From table 1 and Fig. 2, we can get the following conclusions.

Conclusion1. All the profit of each party and supply chain in SI case is higher than that in AI case, and thus it is necessary to reveal the information about the retailer's fairness concern so as to improve the profit of all members and supply chain.

Conclusion2. Whether the fairness information is symmetric or asymmetric, all the profit of each party and supply chain decreases with fairness preference, and besides, the decreasing speed in AI case is more faster and thus the information value increment increases with fairness concern.

4. **Signaling game model**

4.1 Model description

Because the retailer's fair preference behavior can affect the supplier's profit, as the IV shown in Fig. 2, so it is necessary set the signal transmission game model to reveal the fairness-concern information. We assume there are two types of retailer θ, i.e. H type denoting the retailer with high degree of fairness concern λ_H, and L, λ_L has the opposite meaning. If the supplier can't distinguish the each retailer's fairness concern information, then he will provide the average wholesale price for both retailers in the game equilibrium. Thus, H type retailer with intense fairness concern will actively send some signals about his own fairness preference to the supplier to different himself from L type retailer with lower fairness degree so as to obtain cheaper wholesale price which is more matched with his location, contribution and revenue in the supply chain. For the equity reference standard of retailer is the direct supplier's profit, the marginal profit is the advance of the total profit and the operation scale of each enterprise is different, so it is difficult to analyze the compact of fairness concern on the game process between supplier and retailer by comparing the total profit between supplier and retailer. Compared with total profit, it is easy to compute and compare the marginal profit. For example, the marginal product profit often be calculated to determine whether the cooperation in the process of supplier-retailer game, and enterprise's profit and revenue can be always calculated based on the marginal product profit and product sale prospect. The main focus of this paper is to study the signal transmission of retailer's fairness preference, and thus we simplify the retailer's utility function as made up of two parts: the negative utility caused by the difference of marginal product profit $(w-c)-(p-w)$ and the marginal product profit $p-w$, so the marginal utility of retailer is $\Delta u_r=(1+\lambda)(p-w)-\lambda(w-c)$.

At the same time, because the retailer fairness-concern information will affect the supplier's benefit, as the information value of fairness information illustrated in Fig.2. Here, we will set the signaling game model to reveal the retailer's fairness information, and the fig.3 describes the signaling game model, where supplier can allow the retailer to choose the wholesale price so as to reveal retailer's fairness type.

The fairness preference as the retailer's subjective and psychological preference, which is a kind of private information, and the probability distribution of retailer's type is $P\{\theta=H\}=\alpha$ and $P\{\theta=L\}=1-\alpha$, which is the common knowledge between supplier and retailer.

The timing of the game between supplier and retailer is as following:

(1) "Natural" choose retailer type according to the probability distribution of retailer's type, i.e. $P\{\theta=H\}=\alpha$ and $P\{\theta=L\}=1-\alpha$, and we assume the fairness-concern degree of H type retailer is λ_H , and the fairness-concern degree of L type retailer is λ_L , $\lambda_H>\lambda_L$.

(2) After the retailer knows his own fairness preference type θ , he will send the desired wholesale price information to the supplier, i.e. w_H or w_L , where w_H denotes the higher wholesale price and w_L denotes the lower wholesale price, $w_H>w_L$. If the supplier does not consider the retailer's fairness concern, the supplier will offer the higher wholesale price to the retailer from the second part, and thus the supplier can get more profits by his first mover advantage in the game, i.e. $p-w_H<w_H-c$, which means when the wholesale price is higher, the retailer's marginal profit will be lower than the supplier. Besides, we assume $p-w_L>w_L-c$, otherwise the signal is of no meaning for the supplier. Based on $p-w_L>w_L-c$, signal transmission enables retailer to gain more profit by obtaining lower wholesale price, and change the distribution of marginal profit between supplier and retailer to improve their profit distribution in the supply chain. In addition, the retailer sending the lower signal w_L can advertise his contribution and status in the supply chain, such as the cooperation and later division between Gome and GREE, which prove his strong fairness preference once again. We assume the signaling cost is F_L for the H type retailer, who is easy to display his own fairness concern, and F_H for the L type retailer, which means the retailer with weak fairness preference will pay more cost to transfer the same strong signal of fairness preference, i.e. $F_H>F_L$. Because the retailer of strong fairness concern has a more influence on the supplier, so it can be assumed that when the supplier cooperate with the retailer of strong fairness concern, the marginal profit is $\Delta\pi_s=k_H(w-c)$, where k_H denote the influence of retailer with strong fairness concern on supplier's profit. Similarly, when the supplier cooperate with the retailer of weak fairness preference, the marginal profit is $\Delta\pi_s=k_L(w-c)$, $k_H>k_L$. This is why the supplier must recognize the retailer's fairness concern

information, for its strong fairness preference of large retailer has more impact on the supplier's profit, and from Figure 2, the retailer with strong fairness concern will bring the greater value information for the supplier, so it is necessary to identify the fairness preference of retailer.

(3) After the supplier observe the wholesale price signal, the supplier will judge the type of retailer fairness type, and "Y" denote the acceptance and "N" denote the refusal in the extended representation of dynamic game in fig.3. Whether or not to accept, the two sides will get some pay in the end of the game, denoted as $(\Delta u_{ri}, \Delta\pi_{si})$, where the subscript i denote the i th result in the game tree, $i = 1,2,3,4,5,6,7,8$. Based on the above assumptions and the game process, we can get the dynamic game tree of signal transmission in fig.3.

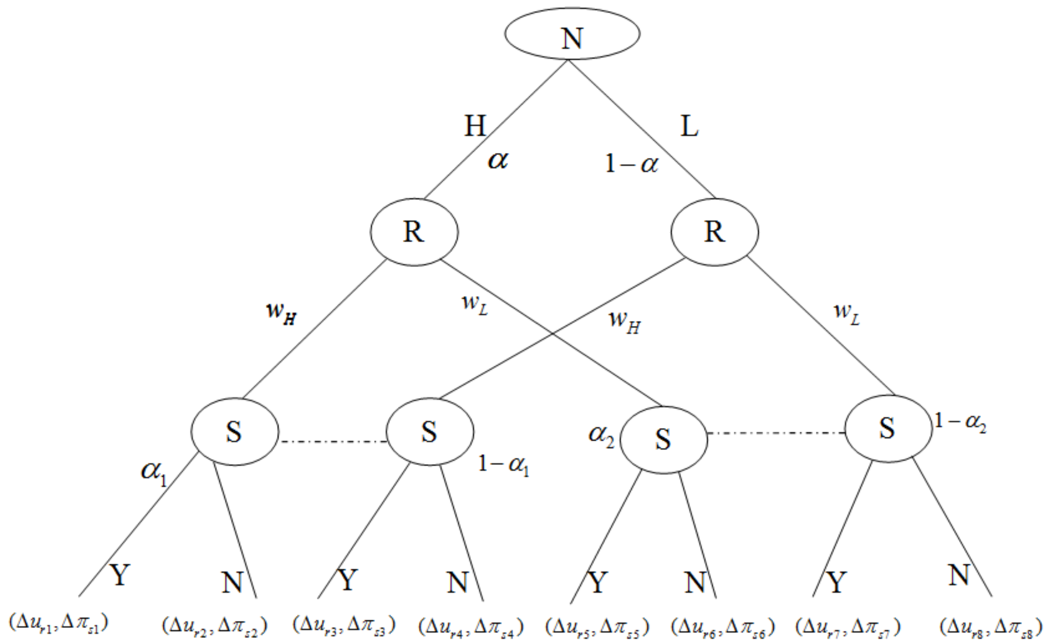

Fig.3. The extended representation of fairness-concern signaling

In fig.3, N denote the "Nature", the supplier-retailer game process can be transformed from incomplete information game to complete but imperfect information game by introducing the "N" to choose the retailer type. "R" is the retailer, and "S" is the supplier.

The marginal utility of the retailer and marginal profit of supplier in each final point of game tree in Figure 3can be calculated in table2. In fig.3, if supplier refuses the retailer and thus the two parts do not cooperate, i.e. $i = 2,4,6,8$, the retained payment of the H type retailer is $\Delta\pi_r$ and the the retained pay of the L type retailer is $\Delta\pi_r'$. Because H type retailer tend to think that his contribution to the supply chain is greater, and even if he can't cooperate with the supplier, there are still a lot of opportunities for cooperation, and thus his retained payment is higher, i.e. $\Delta\pi_r > \Delta\pi_r'$. The retained payment of supplier is $\Delta\pi_s'$. Even if the retailer does not cooperate with the retailer after the signal is transmitted, the profit of the retailer with strong fairness concern is higher than that of the weak fairness concern, i.e. $\Delta\pi_r - F_L > \Delta\pi_r' - F_H$.

Table 2 The bilateral payment in the signaling game

	Retailer	Supplier
$i = 1$	$\Delta u_{r1} = (1+\lambda_H)(p - w_H) - \lambda_H(w_H - c)$	$\Delta\pi_{s1} = k_H(w_H - c)$
$i = 2$	$\Delta u_{r2} = \Delta\pi_r$	$\Delta\pi_{s2} = \Delta\pi_s'$

$i=3$	$\Delta u_{r3}=(1+\lambda_L)(p-w_H)-\lambda_L(w_H-c)$	$\Delta\pi_{s3}=k_L(w_H-c)$
$i=4$	$\Delta u_{r4}=\Delta\pi_r\,'$	$\Delta\pi_{s2}=\Delta\pi_s\,'$
$i=5$	$\Delta u_{r5}=(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)-F_L$	$\Delta\pi_{s5}=k_H(w_L-c)$
$i=6$	$\Delta u_{r6}=\Delta\pi_r-F_L$	$\Delta\pi_{s6}=\Delta\pi_s\,'$
$i=7$	$\Delta u_{r7}=(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)-F_H$	$\Delta\pi_{s7}=k_L(w_L-c)$
$i=8$	$\Delta u_{r8}=\Delta\pi_r\,'-F_H$	$\Delta\pi_{s8}=\Delta\pi_s\,'$

In the signaling model of fig.3, the two decision nodes of the dotted line belong to the same set of information, which means that the supplier can only observe the retailer's hope for the wholesale price, without knowing the retailer's fairness preference type, i.e. although supplier see the signal w_H , he does not know whether the H type retailer sending or the L type retailer sending. So only after the supplier observe the wholesale price requirement, he can infer the detail type of retailer according to the original information of retailer, and then make decision maximizing his profit.

4.2 Model analysis

For the fig.3 and the game description, the signal transmission game model of fairness preference is an imperfect information and dynamic game, and therefore we can solve the perfect Bias equilibrium strategy in fig.3, the steps are as follows: 1) seek the perfect Bias equilibrium dependent on the belief of supplier "S"; 2) seek the perfect Bias equilibrium dependent on the belief of retailer "R"; 3) seek the perfect Bias equilibrium of both sides. The computation result is shown in table 3 and table 4, and the detail computation process can be seen in appendix.

Table 3 The perfect Bias pooling equilibrium strategy (PE) of both sides

Equilibrium	Retailer's strategy	Supplier's strategy	Belief
$PE1$	$s_1(\theta)=(w_H,w_H)$	$s_2(w)=(Y,Y)$	$\alpha=\alpha_1$， $1\ge\alpha_2>\frac{\Delta\pi_s\,'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$
$PE2$	$s_1(\theta)=(w_L,w_L)$	$s_2(w)=(Y,Y)$	$1\ge\alpha_1>\frac{\Delta\pi_s\,'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$， $\alpha=\alpha_2$
$PE3$	$s_1(\theta)=(w_H,w_H)$	$s_2(w)=(N,N)$	$0\le\alpha_1\le\frac{\Delta\pi_s\,'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$， $\alpha=\alpha_2$
$PE4$	$s_1(\theta)=(w_H,w_H)$	$s_2(w)=(N,Y)$	$\alpha=\alpha_1$， $1\ge\alpha_2>\frac{\Delta\pi_s\,'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$
$PE5$	$s_1(\theta)=(w_L,w_L)$	$s_2(w)=(N,Y)$	$0\le\alpha_1\le\frac{\Delta\pi_s\,'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$， $\alpha=\alpha_2$
$PE6$	$s_1(\theta)=(w_H,w_H)$	$s_2(w)=(Y,N)$	$\alpha_1=\alpha$， $0\le\alpha_2\le\frac{\Delta\pi_s\,'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$
$PE7$	$s_1(\theta)=(w_L,w_L)$	$s_2(w)=(Y,N)$	$1\ge\alpha_1>\frac{\Delta\pi_s\,'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$， $\alpha=\alpha_2$

Table 4 The perfect Bias separating equilibrium strategy (SE) of both sides

Equilibrium	Retailer's strategy	Supplier's strategy	Belief	Constraints
$SE1$	$s_1(\theta)=(w_L,w_H)$	$s_2(w)=(Y,Y)$	$\alpha_1=1$ $\alpha_2=0$	$\frac{F_L}{1+2\lambda_H}\le w_H-w_L<\frac{F_H}{1+2\lambda_L}$

$SE2$	$s_1(\theta)=(w_L,w_H)$	$s_2(w)=(N,Y)$	$\alpha_1=1$ $\alpha_2=0$	$\Delta\pi_r'+F_H>(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)$ $\Delta\pi_r+F_L\le(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)$
$SE3$	$s_1(\theta)=(w_L,w_H)$	$s_2(w)=(Y,N)$	$\alpha_1=1$ $\alpha_2=0$	$\Delta\pi_s'<k_L(w_H-c)$ $\Delta\pi_s'\ge k_H(w_L-c)$

From table 3, table 4 and appendix, there are 7 kinds of pooling equilibrium (PE) in the dynamic game of imperfect information between the supplier and the retailer under the fairness-concern information. i.e. $PE1$(as ① in the appendix), $PE2$ (②), $PE3$ (④), $PE4$ (⑤), $PE5$（⑥）, $PE6$ (⑧), $PE7$ (⑨) and 3 kinds of separating equilibrium (SE), i.e. $SE1$ (③)、$SE2$ (⑦)、$SE3$ (⑩). In the pooling equilibriums, the wholesale price signal sent by the retailer can't transmit more information about the retailer type to the supplier, i.e. the signal does not update the information on the probability of probability distribution of natural retailer's fairness-concern type, so the supplier still can't distinguish between retailers of different fairness concern. However, under the 3 separation equilibriums, the supplier can distinguish the retailers with different fairness concern through the wholesale price signal transmitted by the retailer. By comparison of three equilibriums, we can find:

For $SE1$: $s_1(\theta)=(w_L,w_H)$, $\alpha_1=1$, $\alpha_2=0$, $s_2(w)=(Y,Y)$, the retailer with strong fairness concern require low wholesale prices and the retailer with weak fairness concern accept high wholesale price. Once the supplier sees w_L , he will think that the retailer is a strong fairness-concern type and accepts the retailer's low price requirement, and when he sees w_H , he will think that the retailer is a weak fairness-concern type and accepts the retailer's high price requirement.

For the prerequisite of $SE1: \frac{F_L}{1+2\lambda_H}\le w_H-w_L<\frac{F_H}{1+2\lambda_L}$, in the actual operation of the supply chain management, the signaling cost of strong fairness preference F_L is lower, the greater the degree λ_H of fairness concern (corresponding to larger k_H), or signaling cost of weak fairness preference F_H is high, the fairness-concern degree λ_L is smaller (the corresponding smaller k_L) appear, the possibility of $SE1$ appearing is bigger , and at the same time, the possibility of $SE2$ and $SE3$ is smaller.

For $SE2$: $s_1(\theta)=(w_L,w_H)$, $\alpha_1=1$, $\alpha_2=0$, $s_2(w)=(N,Y)$, the retailer with strong fairness concern require low wholesale price and the retailer with weak fairness concern accept high wholesale price. Once the supplier sees w_L , he will think that the retailer is a strong fairness-concern type but reject the retailer's low price requirement, and when he sees w_H , he will think that the retailer is a weak fairness-concern type and accepts the retailer's high price requirement. In this equilibrium, the supplier does not want to cooperate with the retailer of strong fairness concern, and does not want to make profit sharing with retailer. For retailer, the higher wholesale prices will lead to greater negative fairness utility, and thus the retailer of strong fairness concern will not accept a higher wholesale price, so there will be no cooperation between supplier and retailer.

For $SE3$: $s_1(\theta)=(w_L,w_H)$, $\alpha_1=1$, $\alpha_2=0$, $s_2(w)=(Y,N)$, the retailer with strong fairness concern require low wholesale price and the retailer with weak fairness concern accept high wholesale price. Once the supplier sees w_L , he will think that the retailer is a strong fairness-concern type and accepts the retailer's low price requirement, and when he sees w_H , he will think that the retailer is a weak fairness-concern type and rejects the retailer's high price requirement. Contrary to $SE2$, the supplier is not only concerned with more profit brought by higher wholesale prices but he is more concerned with cooperation of powerful retailer, who can bring long-term profit by the strong market influence. So the supplier can tolerate retailer's lower wholesale price requirement, reduce his marginal product profit, increase the retailer's marginal product profit and utility, bring more efforts to encourage retailer selling product, so as to increase the total profit. The supplier will not cooperate with the retailer of weak fairness concern who has little influence on the supply chain, even the retailer can accept higher wholesale price. So the $SE3$ can explain the real business, where supplier would prefer to cooperate with the large retailer, endure the retailer's lower price and the various fees but do not choose to cooperate with the small retailers.

Among $SE1$, $SE2$ and $SE3$, the separating equilibrium $SE1$ is the equilibrium state that supply chain wish, which means the retailer with strong fairness concern often has more strong competitiveness and more bargaining ability in the supply chain, and thus the supplier does not want to be revenged by retailer caused by unfair revenue distribution, leading to worsening the profit of both sides, so the supplier is willing to offer lower wholesale price contract for the retailer with strong fairness concern. At the same time, the retailer with strong fairness concern feels more equitable distribution to make more hard effort to sell product due to retailer's strong contribution, status and large market influence in the supply chain, and finally to achieve the Pareto improvement of supply chain revenue. On the contrary, the retailer with weak fairness concern has no sufficient ability to influence the profit of supplier due to the weak negotiation ability, status and small market influence in the supply chain, and thus the supplier can provide a higher wholesale price for the retailer with weak fairness concern so as to earn more profits by his first mover advantage. And then, if the separating equilibrium $SE1$ appear, retailer with strong fairness concern will establish a cooperation relationship with supplier, and it is easy to find that the profit of supply chain is reduced by F_L compared with the condition of symmetric information, and F_L is just the loss of supply chain efficiency caused by asymmetric information of retailer's fairness concern, which is just the influence of incomplete information on the efficiency of supplier-retailer game.

So we can get the conclusion 3 as follows:

Conclusion3: When one or both of the following conditions are met, the supplier can distinguish the fairness type of retailers and provide the appropriate wholesale price for the retailers with different fairness concern, and promote the stable and harmonious development of the supply chain channel.

i) When the signal transmission cost difference between retailers with different fairness concern is obvious, i.e. the retailer with weak fairness concern has so high signal transmission cost to just obtain higher wholesale price and the retailer with strong fairness concern has so low signal transmission cost to get the lower wholesale price.

ii) When the market influence of the retailer with strong fairness concern is far greater than that of retailer with weak fairness concern, the retailer with strong fairness concern can get the lower wholesale price.

5. Conclusion

We analyze the influence of the retailer's fairness concern on the wholesale price, order quantity, the profit of each party and the supply chain in SI condition and AI condition respectively under the wholesale price contract. Then, we compute the value of retailer's fairness-concern information to supplier, and we prove that the profit of all members and supply chain is decreasing with retailer's fairness concern and the profit in SI condition is always higher than that in the AI condition. Then, we set the signaling game model to reveal the transmission mechanism of retailers' fairness preference information, and we analyze the potential separating equilibrium and pooling equilibrium existing in signaling model under asymmetric fairness-concern information. We prove that only when the signal transmission cost is different between retailers with different fairness preferences, the signaling model can effectively reveal the role and type of retailers. Finally, we provide some suggestions in term of fairness-concern information transmission and optimization of supply chain by discussing the condition of each separating equilibrium results so as to provide some scientific reference value for the actual operation in the supply chain. For example, we proved that when the signal transmission cost difference between retailers with different fairness preference is obvious, i.e. the retailer with weak fairness concern has so high signal transmission cost to just obtain higher wholesale price and the retailer with strong fairness concern has so low signal transmission cost to get the lower wholesale price. By the signaling model, the supplier can distinguish the fairness type of retailers and provide the appropriate wholesale price for the retailers with different fairness preference, and promote the stable and harmonious development of the supply chain channel. Besides, we explain the real business, where supplier would prefer to cooperate with the large retailer, endure the retailer's lower price and the various fees but do not choose to cooperate with the small retailers.

The authenticity problem of fairness preference information is a more basic problem compared with the coordination problem of supply chain, and only when the fairness preference information is true, the conclusions obtained in supply chain coordination under fairness preference is meaningful. Although this paper computed the information value of fairness concern by numerical analysis and we set the signaling model to solve the adverse selection problem of asymmetric information under retailer's fairness concern, which can enrich and expand the research of supply chain contract. But this paper uses so many parameters and symbols in order to get revealing mechanism of fairness preference types, and thus it is difficult to apply numerical analysis about the signaling model to get the intuitive management implications, so future research can simplify the model parameters so as to enhance the maneuverability of the model.

Acknowledgements

The research is supported by Chinese Social Science Fundation "Contract optimization and coordination of supply chain based on social preference and its cognitive dynamic evolution" under Grant Number 16CGL017.

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Appendix:

1) Seek the perfect Bias equilibrium dependent on the belief of supplier "S".

First, we will provide the initial judgment of the two information nodes when the supplier is just in the decision node "S" which includes two corresponding information nodes, as shown in fig.3.

This paper first gives the initial judgment of the two information section for supplier "S" who contains two information codes. α_1 and $1-\alpha_1$ is the probability referred by supplier about the retailer with strong fairness concern and weak fairness concern accordingly when the supplier sees the high wholesale price requirement w_H. Similarly, α_2 and $1-\alpha_2$ is the probability referred by supplier about the retailer with strong fairness concern and weak fairness concern accordingly when the supplier sees the low wholesale price requirement w_L. Then, when the supplier gets the signal w_H, i.e. $w = w_H$, the maximizing decision problem of supplier is as follows:

$$\begin{aligned} &\max\left\{\alpha_1\Delta\pi_{s1}+(1-\alpha_1)\Delta\pi_{s3},\alpha_1\Delta\pi_{s2}+(1-\alpha_1)\Delta\pi_{s4}\right\} \\ &=\max\left\{\alpha_1 k_H(w_H-c)+(1-\alpha_1)k_L(w_H-c),\Delta\pi_s'\right\} \end{aligned}$$

We can get the belief dependent strategy of supplier is

$$\alpha_1(w_H)=\begin{cases} Y & if\,\alpha_1 > \dfrac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)} \\ N & if\,\alpha_1 \le \dfrac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)} \end{cases}$$

Similarly, when the supplier gets the signal w_L, i.e. $w = w_L$, the maximizing decision problem of supplier is

$$\begin{aligned} &\max\left\{\alpha_2\Delta\pi_{s5}+(1-\alpha_2)\Delta\pi_{s7},\alpha_2\Delta\pi_{s6}+(1-\alpha_2)\Delta\pi_{s8}\right\} \\ &=\max\left\{\alpha_2 k_H(w_L-c)+(1-\alpha_2)k_L(w_L-c),\Delta\pi_s'\right\} \end{aligned}$$

Which is equal to the equation: $\alpha_2(w_L)=\begin{cases} Y & if\,\alpha_2 > \dfrac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)} \\ N & if\,\alpha_2 \le \dfrac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)} \end{cases}$

Then, We will get the perfect Bias Nash equilibrium strategy $\alpha = (\alpha_1(w_H),\alpha_2(w_L))$ dependent on the belief of supplier by discussing the combination of $\alpha_1(w_H)$ and $\alpha_2(w_L)$, as follows:

$$\alpha=(\alpha_1,\alpha_2)\in D_1=\left\{1\ge\alpha_1>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)},1\ge\alpha_2>\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}\right\},s_2(w)=(Y,Y) \quad (1)$$

$$\alpha=(\alpha_1,\alpha_2)\in D_2=\left\{0\le\alpha_1\le\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)},0\le\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}\right\},s_2(w)=(N,N) \quad (2)$$

$$\alpha=(\alpha_1,\alpha_2)\in D_3=\left\{0\le\alpha_1\le\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)},1\ge\alpha_2>\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}\right\},s_2(w)=(N,Y) \quad (3)$$

$$\alpha=(\alpha_1,\alpha_2)\in D_4=\left\{1\ge\alpha_1>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)},0\le\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}\right\},s_2(w)=(Y,N) \quad (4)$$

2) Seek the perfect Bias equilibrium dependent on the belief of retailer "R".

We will get the perfect Bias Nash equilibrium strategy dependent on the belief of retailer "R" in each regions D_j ($j = 1,2,3,4$) respectively. Specifically, in the first phase of the game, the retailer's equilibrium strategy $s_1(\theta)$ based on his own fairness type corresponding to each supplier's strategy $s_2(w)$ in each $\alpha = (\alpha_1, \alpha_2) \in D_j$ ($j = 1,2,3,4$).

2.1) $\alpha = (\alpha_1, \alpha_2) \in D_1$

When $\alpha = (\alpha_1, \alpha_2) \in D_1$, $s_2(w) = (Y, Y)$ by formula (1), i.e. whether the retailer sends out the signal $w = w_H$ or $w = w_L$, the supplier always accept the wholesale price requirement and will deal with both types of retailers. Under $s_2(w) = (Y, Y)$, if $\theta = H$ denoting the retailer is of strong fairness concern, then his decision problem is as follows:

$$\max\{\Delta u_{r1}, \Delta u_{r5}\} = \max\{(1+\lambda_H)(p - w_H) - \lambda_H(w_H - c), (1+\lambda_H)(p - w_L) - \lambda_H(w_L - c) - F_L\}$$

Based on the assumption that the supplier always chooses "Y", the retailer will compare the utility of sending signals $w = w_H$ and $w = w_L$, i.e.

$$\alpha_1(H) = \begin{cases} w_H & if w_H - w_L < \dfrac{F_L}{1+2\lambda_H} \\ w_L & if w_H - w_L \ge \dfrac{F_L}{1+2\lambda_H} \end{cases} \tag{5}$$

Similarly, we can compute the decision problem when the retailer is of weak fairness concern $\theta = L$ under $s_2(w) = (Y, Y)$, i.e.

$$\max\{\Delta u_{r3}, \Delta u_{r7}\} = \max\{(1+\lambda_L)(p - w_H) - \lambda_L(w_H - c), (1+\lambda_L)(p - w_L) - \lambda_L(w_L - c) - F_H\}$$

Similar to the equation (5), the strategy dependent on the belief of retailer with weak fairness concern is as follows:

$$\alpha_1(L) = \begin{cases} w_H & if w_H - w_L < \dfrac{F_H}{1+2\lambda_L} \\ w_L & if w_H - w_L \ge \dfrac{F_H}{1+2\lambda_L} \end{cases} \tag{6}$$

According to equation (5), (6) and the hypothetical condition $F_H > F_L$ and $w_H > w_L$, we can get the separating equilibrium (shorted as SE) and pooling equilibrium (PE) of supplier-retailer game by discussion in each situations.

① when $w_H - w_L < \dfrac{F_L}{1+2\lambda_H}$, $w_H - w_L < \dfrac{F_H}{1+2\lambda_L}$ is always correct, and therefore the retailer will have PE strategy denoted as $s_1(\theta) = (w_H, w_H)$, which means both the retailers will choose to send the signal w_H whether the type of retailer is strong fairness concern or weak fairness concern.

② when $w_H - w_L \ge \dfrac{F_H}{1+2\lambda_L}$, $w_H - w_L \ge \dfrac{F_L}{1+2\lambda_H}$ is always correct, and therefore the retailer will have PE strategy $s_1(\theta) = (w_L, w_L)$ which means both the retailers will choose to send the signal w_L whether the type of retailer is strong fairness concern or weak fairness concern.

③ when $\dfrac{F_L}{1+2\lambda_H} \le w_H - w_L < \dfrac{F_H}{1+2\lambda_L}$, the retailer will have SE strategy $s_1(\theta) = (w_L, w_H)$, which means the retailer with strong fairness concern will send the signal w_L and the retailer with weak fairness concern will choose to send the signal w_H .

2.2) $\alpha=(\alpha_1,\alpha_2)\in D_2$

When $\alpha=(\alpha_1,\alpha_2)\in D_2$, $s_2(w)=(N,N)$ by formula (2), i.e. whether the retailer sends out the signal $w=w_H$ or $w=w_L$, the supplier always rejects the wholesale price requirement and will refuse to deal with both types of retailers. Under $s_2(w)=(N,N)$, if $\theta=H$ denoting the retailer is of strong fairness concern, then his decision problem is as follows:

$$\max\{\Delta u_{r2},\Delta u_{r6}\}=\max\{\Delta\pi_r,\Delta\pi_r-F_L\}=\Delta\pi_r$$

It is easy to get $\alpha_1(H)=w_H$, and then the retailer with strong fairness concern will send the signal w_H under $s_2(w)=(N,N)$.

if $\theta=L$, then the decision problem of retailer is:

$$\max\{\Delta u_{r4},\Delta u_{r8}\}=\max\{\Delta\pi_r',\Delta\pi_r'-F_L\}=\Delta\pi_r'$$

It is easy to get $\alpha_1(L)=w_H$, and then the retailer with weak fairness concern will send the signal w_H under $s_2(w)=(N,N)$.

④ when $\alpha=(\alpha_1,\alpha_2)\in D_2$, we can get the PE strategy, i.e. $s_1(\theta)=(w_H,w_H)$.

2.2) $\alpha=(\alpha_1,\alpha_2)\in D_3$

When $\alpha=(\alpha_1,\alpha_2)\in D_3$, $s_2(w)=(N,Y)$ by formula (3), i.e. if the supplier sees the signal w_H, he will reject, and if supplier sees the signal w_L , he will accept. If $\theta=H$, the retailer with strong fairness concern will decide the following decision problem:

$$\max\{\Delta u_{r2},\Delta u_{r5}\}=\max\{\Delta\pi_r,(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)-F_L\}$$

And then

$$\alpha_1(H)=\begin{cases}w_H & if\,\Delta\pi_r+F_L>(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)\\ w_L & if\,\Delta\pi_r+F_L\le(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)\end{cases}\tag{7}$$

Similarly, when $\theta=L$, the retailer with weak fairness concern will decide the following decision problem:

$$\max\{\Delta u_{r4},\Delta u_{r7}\}=\max\{\Delta\pi_r',(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)-F_H\}$$

Similar to equation (7), we can get

$$\alpha_1(L)=\begin{cases}w_H & if\,\Delta\pi_r'+F_H>(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)\\ w_L & if\,\Delta\pi_r'+F_H\le(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)\end{cases}\tag{8}$$

According to equation (7), (8) and the hypothetical condition $F_H>F_L$, $w_H>w_L$, $p-w_L>w_L-c$ and $\Delta\pi_r>\Delta\pi_r'$, we can get the retailer's strategy as follows:

⑤ when $\Delta\pi_r+F_L>(1+\lambda_H)(p-w_L)-\lambda_H(w_L-c)$, the retailer will have PE strategy: $s_1(\theta)=(w_H,w_H)$.

⑥ when $\Delta\pi_r'+F_H\le(1+\lambda_L)(p-w_L)-\lambda_L(w_L-c)$, the retailer will have SE strategy $s_1(\theta)=(w_L,w_L)$.

⑦ when $\Delta\pi_r' + F_H > (1+\lambda_L)(p-w_L) - \lambda_L(w_L - c)$ and $\Delta\pi_r + F_L \le (1+\lambda_H)(p-w_L) - \lambda_H(w_L - c)$, the retailer will have PE strategy: $s_1(\theta) = (w_L, w_H)$.

2.2) $\alpha = (\alpha_1, \alpha_2) \in D_4$

When $\alpha = (\alpha_1, \alpha_2) \in D_4$, $s_2(w) = (Y, N)$ by formula (4), i.e. if the supplier sees the signal w_H, he will accept, and if supplier sees the signal w_L, he will reject. If $\theta = H$, the retailer with strong fairness concern will decide the following decision problem:

$$\max\{\Delta u_{r1}, \Delta u_{r6}\} = \max\{(1+\lambda_H)(p-w_H) - \lambda_H(w_H - c), \Delta\pi_r - F_L\}$$

Then

$$\alpha_1(H) = \begin{cases} w_H & if\,\Delta\pi_r - F_L < (1+\lambda_H)(p-w_H) - \lambda_H(w_H - c) \\ w_L & if\,\Delta\pi_r - F_L \ge (1+\lambda_H)(p-w_H) - \lambda_H(w_H - c) \end{cases} \tag{9}$$

If $\theta = L$, the retailer with weak fairness concern will decide the following decision problem:

$$\max\{\Delta u_{r3}, \Delta u_{r8}\} = \max\{(1+\lambda_L)(p-w_H) - \lambda_L(w_H - c), \Delta\pi_r' - F_H\}$$

$$\alpha_1(L) = \begin{cases} w_H & if\,\Delta\pi_r' - F_H < (1+\lambda_L)(p-w_H) - \lambda_L(w_H - c) \\ w_L & if\,\Delta\pi_r' - F_H \ge (1+\lambda_L)(p-w_H) - \lambda_L(w_H - c) \end{cases} \tag{10}$$

According to equation (9), (10) and the hypothetical condition $F_H > F_L$, $w_H > w_L$ and $\Delta\pi_r > \Delta\pi_r'$, we can get the retailer's strategy as follows:

⑧ when $\Delta\pi_r - F_L < (1+\lambda_H)(p-w_H) - \lambda_H(w_H - c)$, the retailer will have PE strategy: $s_1(\theta) = (w_H, w_H)$.

⑨ when $\Delta\pi_r' - F_H \ge (1+\lambda_L)(p-w_H) - \lambda_L(w_H - c)$, the retailer will have PE strategy: $s_1(\theta) = (w_L, w_L)$.

⑩when $\Delta\pi_r - F_L \ge (1+\lambda_H)(p-w_H) - \lambda_H(w_H - c)$ and $\Delta\pi_r' - F_H < (1+\lambda_L)(p-w_H) - \lambda_L(w_H - c)$, the retailer will have SE strategy: $s_1(\theta) = (w_L, w_H)$.

3) Seek the perfect Bias equilibrium of both sides.

According to the strategies $s_2(w)$ and $s_1(\theta)$ dependent of supplier's and retailer's belief, we can further solve the perfect Bayesian equilibrium strategies in each region D_j ($j = 1,2,3,4$) for dynamic game under the imperfect fairness information.

①when $\alpha = (\alpha_1, \alpha_2) \in D_1$, $s_2(w) = (Y, Y)$ and $w_H - w_L < \dfrac{F_L}{1+2\lambda_H}$, $s_1(\theta) = (w_H, w_H)$, the deduction or belief about the type of retailer's fairness concern should meet: $1 \ge \alpha_1 = \alpha > \dfrac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $1 \ge \alpha_2 > \dfrac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, and then if $\alpha > \dfrac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $1 \ge \alpha_2 > \dfrac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, we can get the $PE1$ under the imperfect fairness information.

$PE1$: $s_1(\theta) = (w_H, w_H)$, $\alpha = \alpha_1$, $1 \ge \alpha_2 > \dfrac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, $s_2(w) = (Y, Y)$

② when $\alpha = (\alpha_1, \alpha_2) \in D_1$, $s_2(w) = (Y, Y)$ and $w_H - w_L \ge \dfrac{F_H}{1+2\lambda_L}$, $s_1(\theta) = (w_L, w_L)$. The belief about the type of retailer's fairness concern should meet: $1 \ge \alpha_1 > \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $1 \ge \alpha_2 = \alpha > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$. Then, if $1 \ge \alpha_1 > \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $\alpha > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, then we can get the $PE2$ as follows:

$PE2$： $s_1(\theta) = (w_L, w_L)$, $1 \ge \alpha_1 > \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $\alpha = \alpha_2$, $s_2(w) = (Y, Y)$

③ When $\alpha = (\alpha_1, \alpha_2) \in D_1$, $s_2(w) = (Y, Y)$, $\frac{F_L}{1 + 2\lambda_H} \le w_H - w_L < \frac{F_H}{1 + 2\lambda_L}$, $s_1(\theta) = (w_L, w_H)$, The belief about the type of retailer's fairness concern should meet: $\alpha_1 = 1 > \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $\alpha_2 = 0 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$. if $\frac{F_L}{1 + 2\lambda_H} \le w_H - w_L < \frac{F_H}{1 + 2\lambda_L}$, there $SE1$（Separating Equilibrium，SE）:

$SE1: s_1(\theta) = (w_L, w_H)$, $\alpha_1 = 1$, $\alpha_2 = 0$, $s_2(w) = (Y, Y)$

④ When $\alpha = (\alpha_1, \alpha_2) \in D_2$, $s_2(w) = (N, N)$， $s_1(\theta) = (w_H, w_H)$. The belief about the type of retailer's fairness concern should meet: $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $0 \le \alpha = \alpha_2 \le \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$. Then, if $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$ and $\alpha \le \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, we can get $PE3$:

$PE3$: $s_1(\theta) = (w_H, w_H)$, $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $\alpha = \alpha_2$, $s_2(w) = (N, N)$

⑤ When $\alpha = (\alpha_1, \alpha_2) \in D_3$, $s_2(w) = (N, Y)$, and if $\Delta\pi_r + F_L > (1 + \lambda_H)(p - w_L) - \lambda_H(w_L - c)$, $s_1(\theta) = (w_H, w_H)$, then the belief is $0 \le \alpha = \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $1 \ge \alpha_2 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$ and $\Delta\pi_r + F_L > (1 + \lambda_H)(p - w_L) - \lambda_H(w_L - c)$, and the $PE4$:

$PE4$: $s_1(\theta) = (w_H, w_H)$, $\alpha = \alpha_1$, $1 \ge \alpha_2 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$, $s_2(w) = (N, Y)$

⑥ When $\alpha = (\alpha_1, \alpha_2) \in D_3$, $s_2(w) = (N, Y)$, and if $\Delta\pi_r' + F_H \le (1 + \lambda_L)(p - w_L) - \lambda_L(w_L - c)$ $s_1(\theta) = (w_L, w_L)$, then the belief is $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $1 \ge \alpha = \alpha_2 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$. Then under the constraints: $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $1 \ge \alpha_2 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$ and $\Delta\pi_r' + F_H \le (1 + \lambda_L)(p - w_L) - \lambda_L(w_L - c)$, we can get $PE5$:

$PE5: s_1(\theta) = (w_L, w_L)$, $0 \le \alpha_1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, $\alpha = \alpha_2$, $s_2(w) = (N, Y)$

⑦ when $\alpha = (\alpha_1, \alpha_2) \in D_3$, $s_2(w) = (N, Y)$, and if $\Delta\pi_r' + F_H > (1 + \lambda_L)(p - w_L) - \lambda_L(w_L - c)$, $\Delta\pi_r + F_L \le (1 + \lambda_H)(p - w_L) - \lambda_H(w_L - c)$, $s_1(\theta) = (w_L, w_H)$. Then the belief is $\alpha_1 = 1 \le \frac{\Delta\pi_s' - k_L(w_H - c)}{(k_H - k_L)(w_H - c)}$, and $\alpha_2 = 0 > \frac{\Delta\pi_s' - k_L(w_L - c)}{(k_H - k_L)(w_L - c)}$. So when $\Delta\pi_r' + F_H > (1 + \lambda_L)(p - w_L) - \lambda_L(w_L - c)$ and $\Delta\pi_r + F_L \le (1 + \lambda_H)(p - w_L) - \lambda_H(w_L - c)$, we can get $SE2$:

$SE2: s_1(\theta)=(w_L,w_H)$, $\alpha_1=1$, $\alpha_2=0, s_2(w)=(N,Y)$

⑧ when $\alpha=(\alpha_1,\alpha_2)\in D_4$, $s_2(w)=(Y,N)$, and if $\Delta\pi_r-F_L<(1+\lambda_H)(p-w_H)-\lambda_H(w_H-c)$, $s_1(\theta)=(w_H,w_H)$, Then the belief is $1\ge\alpha_1=\alpha>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$ and $0\le\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$. So when $\alpha>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$, $0\le\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$ and $\Delta\pi_r-F_L<(1+\lambda_H)(p-w_H)-\lambda_H(w_H-c)$, we can get $PE6$:

$PE6$: $s_1(\theta)=(w_H,w_H)$, $\alpha_1=\alpha$, $0\le\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$, $s_2(w)=(Y,N)$

⑨ when $\alpha=(\alpha_1,\alpha_2)\in D_4$, $s_2(w)=(Y,N)$, and if $\Delta\pi_r'-F_H\ge(1+\lambda_L)(p-w_H)-\lambda_L(w_H-c)$, $s_1(\theta)=(w_L,w_L)$, Then the belief is $1\ge\alpha_1>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$ and $0\le\alpha=\alpha_2\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$. So when $\Delta\pi_r'-F_H\ge(1+\lambda_L)(p-w_H)-\lambda_L(w_H-c)$, $1\ge\alpha_1>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$ and $\alpha\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$, we can get $PE7$:

$PE7$: $s_1(\theta)=(w_L,w_L)$, $1\ge\alpha_1>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$, $\alpha=\alpha_2$, $s_2(w)=(Y,N)$

⑩ when $\alpha=(\alpha_1,\alpha_2)\in D_4$, $s_2(w)=(Y,N)$, and if $\Delta\pi_r-F_L\ge(1+\lambda_H)(p-w_H)-\lambda_H(w_H-c)$, $\Delta\pi_r'-F_H<(1+\lambda_L)(p-w_H)-\lambda_L(w_H-c)$ and $s_1(\theta)=(w_L,w_H)$, Then the belief is $\alpha_1=0>\frac{\Delta\pi_s'-k_L(w_H-c)}{(k_H-k_L)(w_H-c)}$ and $\alpha_2=1\le\frac{\Delta\pi_s'-k_L(w_L-c)}{(k_H-k_L)(w_L-c)}$, we can get $SE3$:

$SE3: s_1(\theta)=(w_L,w_H)$, $\alpha_1=1$, $\alpha_2=0$, $s_2(w)=(Y,N)$.