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DOI link to article:

https://doi.org/10.1061/(ASCE)WW.1943-5460.0000457

Date deposited:

04/02/2018
Oblique wave effects on the hydrodynamic responses of side-by-side moored FLNG and LNGC

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Abstract: The motion responses prediction of side-by-side moored FLNG (Floating Liquefied Natural Gas) and LNGC (Liquefied Natural Gas Carrier) under oblique waves is critically important to validate operational security. This article studies the hydrodynamic interactions of side-by-side moored FLNG+LNGC under oblique waves by both numerical simulation and model testing. Artificial damping method, calibrated through gap wave elevations measured in model tests, is adopted to simulate the viscous effect in the gap region using the state-of-the-art software Hydrostar (Bureau Veritas, 2007). The hydrodynamic performances of the side-by-side system under oblique waves are investigated. Relative motions under different wave directions are also investigated and the resonant phenomena are analyzed through phase shift. The investigations indicate that motion responses of FLNG are less affected by wave directions, while the motions of LNGC at the lee side are suppressed due to the shielding effect of FLNG. Relative motions between FLNG and LNGC tend to be amplified with the out-of-phase mode when two vessels oscillate in the opposed directions, induced by gap water resonances at high frequencies, while the mode of relative motions induced by roll resonance depends on wave directions and resonance frequencies.

Key points: FLNG; hydrodynamic interactions; oblique wave; shielding effect; gap water resonance

Introduction

With the development of natural gas exploitation from offshore stranded gas reserves, FLNG has attracted considerable attention as an effective platform for exploitation, processing, and storage of the natural gas in remote offshore areas. Instead of a conventional method of gas transportation through long pipelines, the cryogenic nature of liquefied natural gas (LNG) determines a small distance between FLNG and LNGC with a side-by-side configuration in offloading operation. Nevertheless, the side-by-side offloading under unexpected weather conditions, in particular occasional oblique waves, might induce high risks of collision and large...
motions of vessels. Thus, it is necessary to implement an accurate prediction of side-by-side vessel motions to ensure safe operability. The objective of the present research is to experimentally and numerically analyze the hydrodynamic performances of side-by-side FLNG and LNGC under various oblique wave sea states.

Two problems are faced when predicting the motion of parallel arranged vessels in close proximity—hydrodynamic interactions between floating bodies and viscous effect of gap water in between. Both problems have been studied by many researchers.

For frequency-domain analysis of multi-body hydrodynamic interactions, Ohkusu (1969) firstly applied the 2-D strip theory to the calculation of hydrodynamic parameters of two circular cylinders and Kodan (1984) subsequently measured the motions of parallel barge and ship in waves and compared the 2-D results with test results. As the improvement of computational ability, 3-D method based on wave Green’s function was introduced for multi-body calculation. Van Oortmerssen (1979) carried out numerical studies for floating vertical cylinders and barges in heading wave condition and compared the numerical results with the experimental results. Fang & Chen (2001) investigated the relative motions and wave elevations between two bodies and obtained satisfactory agreement between numerical and experiment results. Lewandowski (2008) compared the 3-D boundary element method with 2-D method in the calculation of hydrodynamic parameters of twin barges in close proximity and confirmed the profound influence on the hydrodynamic forces and responses of the bodies near the critical frequencies due to resonant behavior of the water in the gap. The higher-order boundary element method (HOBEM) has been used by Choi and Hong (2002) to investigate the hydrodynamic interactions of floating multi-body system, including motion responses and wave drift forces, for the sake of computational efficiency and convergence. Sun et al. (2015) used first and second order diffraction analyses to study hydrodynamic interactions, and captured the intense fluid motions within the gap between the two vessels which are all fixed or free-floating. Xu et al. (2016) investigate the hydrodynamic interactions between three side-by-side barges with both low-order and high-order boundary element methods. Mean drift force of each barge in a head sea was evaluated with near-field and middle-field method to draw a comparison with the experimental results.

For time-domain analysis, extensive study has been done based on the impulse response
function following the original formulation of Cummins (1962). Buchner et al. (2001) developed
the numerical time domain simulation model to predict the motion response of FPSO LNG with
an alongside moored LNG carrier, which was validated by basin model tests. Kim et al. (2008)
adopted the 3D Rankine panel method to study the motion responses of multiple adjacent floating
bodies in time domain. Configuration of two adjacent Series 60 hulls and a ship-barge model in
oblique waves were investigated and numerical simulation was consolidated with experimental
results. Zhao et al. (2014) investigated the hydrodynamic characteristics of the side-by-side
moored FLNG and LNGC connected by hawsers and fenders using a time-domain simulation
code SIMO and validated the ship motions and loads on hawsers and fenders with test results.
Watai et al. (2015) studied the seakeeping problem of two ships in side-by-side configuration and
improved the convergence of the time domain Rankine panel method by incorporating the
artificial damping method.

As studied by previous research, classical potential theory always overestimates the fluid
motion at certain spaced frequencies corresponding to the wave resonance in the gap. Molin (2001,
2012) did the pioneering research on the wave propagation in a channel as the simplified model of
the gap wave resonance problem. The resonance frequencies and free-surface modal shapes
between twin floaters were further studied by Yeung et al. (2007). Recent research (Pessoa et
al., 2015; Shivaji et al., 2016) indicated that the gap can be treated as a longitudinally unbounded
moonpool sharing similar resonant modes, though not identical, with the canonical ones for
moonpool including the pumping mode and sloshing modes (Molin, 2001). Viscous effect of wave
resonance in the gap shouldn’t be ignored due to the viscous dissipative effect that actually takes
place near the bilge keels via flow separation and on the vessel walls due to friction. Many
scholars have adapted the inviscid potential theory by introducing certain damping mechanisms to
control the unrealistic wave elevations and fluid motions. A ‘rigid lid method’ was first proposed
by Huijsmans et al. (2001) to fully stifle the wave elevations in the gap and applied by Buchner et
al. (2001) in his study of side-by-side offloading system to press down drastic wave resonance. In
order to allow a wavy motion of the lid, Newman (2004) rendered the ‘flexible lid’ whose
deformation equals the free surface elevation and is described by a set of Chebychev polynomials
as the basis functions. A damping coefficient is then introduced to reduce lid deformation. Both
methods bring about disturbance of the real flow field around hulls and only serve as basic approximation of viscous effect.

Unlike the methods above, Chen (2005) applied directly a linear damping force into the original equation to emulate the nature of energy dissipation in the flow field. A modified free surface condition is derived based on the linear damping coefficient epsilon (\( \varepsilon \)) and plays an important role in predicting the behavior of water in the confined zone. The damping coefficient could be selected through matching the computed results with testing results in terms of wave elevations or second order quantities. Extensive study has been done to investigate the effectiveness of this artificial damping method. Fournier et al. (2006) validated the function of the artificial damping that a single linear damping coefficient tuned by experiment shows good correspondence in terms of wave elevations, ship motion RAOs (Response Amplitude Operators) and mean drift force transfer functions in head seas. However, Pauw et al. (2007) complemented that, for narrow gap (less than one tenth of vessel breath in the literature), no single value of artificial damping could fully cover all the first order quantities and the value should be tuned according to second order test results like wave drift forces. Bunnik et al. (2009) suggested the artificial damping on the interior body surface to eliminate irregular frequencies instead of the conventional rigid lid method, which shows a strong mesh dependency.

Model tests on the hydrodynamics of the side-by-side configuration have been applied for over one decade. To validate the time domain calculation of multi-body hydrodynamic interactions, Buchner et al. (2001) carried out model tests for coupled and uncoupled side-by-side vessels in close proximity. Hong et al. (2002) conducted model tests for side-by-side tankers to investigate the hydrodynamic interactions between two vessels with respect to their motion responses and drift forces in head sea and beam sea conditions. To provide insight into the relative motions and the forces in the mooring lines between two hulls, Valk & Watson (2005) implemented a comprehensive set of basin model tests under multi-directional wave climates and recommended that a full dynamic positioning might be preferred over the physical mooring arrangement in severe environment. Pauw et al. (2007) performed the model test with the LNGC along the basin wall under both free and fixed mode in order to obtain the value of damping parameter. Model tests of side-by-side moored FLNG and LNGC were also performed by Zhao et al. (2014) in Deep
water Offshore Basin at Shanghai Jiao Tong University for a better understanding of the hydrodynamic characteristics. The numerical method based on the time-domain code SIMO was also validated.

In this study, hydrodynamic analysis of side-by-side offloading system was carried out under different oblique waves, using the state-of-the-art software Hydrostar (Bureau Veritas, 2007) for frequency domain calculation and an in-house code to implement the time domain simulation of this multi-body system. Artificial damping method (Chen, 2005) was used to simulate the viscous effect between closely arranged vessels and modify the motion responses in the vicinity of gap resonance frequencies. Model tests were carried out to tune the damping coefficient $\epsilon$ based on wave elevations within the gap. Based on the finely tuned $\epsilon$, the hydrodynamic performances of side-by-side system under oblique wave conditions were studied by comparing numerical results and test results. Subsequently, the motion responses under different wave directions were compared and analyzed to reveal the influence of wave directions. Finally, relative motions were calculated with the developed in-house code in time domain and compared with experimental results.

### Mathematical formulation

This article aims at investigating the hydrodynamic couplings between side-by-side FLNG and LNGC system under oblique waves. Two free floating vessels are set parallel with a gap width of 6 meters. The mathematical formulation describing this problem will be deducted in this section, followed by its solving method in both frequency and time domain.

### Coordinate system

To describe the hydrodynamic problem, three coordinate systems are established (Fig. 1), including two reference coordinate systems with subscripts of A or B to represent FLNG and LNGC respectively, as well as the global coordinate system.

The origin of each reference system is fixed at the center point of water plane. The global system origin is fixed at the center point of the gap surface. The x-axis directs positively towards ship bow, while the z-axis directs vertically upwards. Regarding this double-body problem, 12
degrees of freedom termed as generalized modes need to be considered to describe the motion of two vessels in space.

**Governing equations and boundary conditions**

Based on the incompressible and inviscid potential flow, the velocity potential satisfies the Laplace equation in fluid domain.

\[ \nabla^2 \Phi = 0 \quad (1) \]

Based on the small amplitude wave assumption and perturbation procedure, the linearized boundary value problem can be derived and solved. The fluid motion is assumed to be harmonic in time with the circular frequency \( \omega \). The total periodic velocity potential in form of sinusoidal oscillation can be expressed as

\[ \Phi(x, y, z, t) = \text{Re} \left\{ \eta_w \phi_i + \phi_d + \sum_{j=1}^{12} \zeta_j \phi_j e^{i \omega t} \right\}, \quad (2) \]

where \( \eta_w \) represents the wave amplitude, \( \zeta_j \) is the complex amplitude of \( j \)-th mode motion. \( \phi_j \) is the incident potential, \( \phi_d \) is the diffraction potential induced by the presence of bodies in waves. \( \phi_j (j = 1, 2, \ldots, 12) \) represents the radiation potentials induced by forced motions in still water. The linear boundary conditions satisfied by velocity potentials \( \phi_m (m = D, o_r, j) \) in the frequency domain are as following.

1) Linear free surface condition:

\[ -\omega^2 \phi_m + g \frac{\partial}{\partial z} \phi_m = 0, \quad z = 0 \quad (3) \]

2) Sea bed condition:

\[ \frac{\partial}{\partial n} \phi_m = 0, \quad z = -h \quad (4) \]

3) Body surface condition for diffraction potential:

\[ \frac{\partial \phi_d}{\partial n} = -\frac{\partial \phi_i}{\partial n}, \text{ on } H_1 \cup H_2 \quad (5) \]

4) Body surface condition for radiation potential:
\[
\frac{\partial \phi_j}{\partial n} = n_j (j = 1, 2, \ldots, 6) \text{ on } H_1 \quad (a)
\]

\[
\frac{\partial \phi_j}{\partial n} = 0 (j = 1, 2, \ldots, 6) \text{ on } H_2 \quad (b)
\]

\[
\frac{\partial \phi_j}{\partial n} = n_j (j = 7, 8, \ldots, 12) \text{ on } H_2 \quad (c)
\]

\[
\frac{\partial \phi_j}{\partial n} = 0 (j = 7, 8, \ldots, 12) \text{ on } H_1 \quad (d)
\]

5) Radiation condition in the far field:

\[
\lim_{R \to \infty} \sqrt{R} \left( \frac{\partial \phi_m}{\partial R} - ik\phi_m \right) = 0, \quad R = \sqrt{x^2 + y^2}
\]

where \( h \) is the water depth, \( k \) is the wave number which satisfies the dispersion relation \( \omega^2 = g \tanh(kh) \). \( H_1 \) and \( H_2 \) represent the body boundaries of FLNG and LNGC respectively.

\( (n_1, n_2, n_3) = n, (n_4, n_5, n_6) = (x_1, y_1, z_1) \times n, \quad n \) is the unit vector normal to the body boundary.

**Artificial damping method**

The artificial damping method (Chen, 2005) derives from the modification of inviscid potential flow by introducing a fictitious force dependent on the fluid velocity in the momentum equation

\[
f_{\text{dmp}} = -\mu V,
\]

where \( V \) is the velocity of water particles and \( \mu \) is defined as a damping factor. Hence, the modified momentum equation is

\[
V \nabla V + \frac{\partial V}{\partial t} = -\mu V + f + \frac{1}{\rho} \nabla P,
\]

where \( f \) is the inertia force (gravity here), while \( P \) represents the fluid pressure. Due to the fact that this fictitious force not introducing any vorticity, though incorporating the viscous effect of fluid motion, the existence of velocity potential is safeguarded. Bernoulli equation derived from (9) can be expressed as
\[
\frac{P}{\rho} + gz + \frac{\partial \Phi}{\partial t} + \frac{\nabla \Phi \cdot \nabla \Phi}{2} + \mu \Phi = 0, \quad (10)
\]

where \( \rho \) is the water density, \( g \) is the acceleration of gravity. Ignoring the second order item, the wave elevation can be expressed as

\[
\eta = \frac{1}{g} \left( \frac{\partial \Phi}{\partial t} + \mu \Phi \right). \quad (11)
\]

For the linear boundary value problem, the free surface condition can be expressed as

\[
g \frac{\partial \Phi}{\partial z} + \frac{\partial^2 \Phi}{\partial t^2} + \mu \Phi = 0. \quad (12)
\]

Hence, the conventional free surface condition in equation(3) can be modified as

\[
-(1 + i\varepsilon)\omega^2 \phi_m + g \frac{\partial}{\partial z} \phi_m = 0, \quad \frac{\mu}{\omega} = \varepsilon, \quad (13)
\]

where \( \varepsilon \) represents the artificial damping coefficient ranging from 0 and 1. When \( \varepsilon = 0 \), this boundary condition degrades into the conventional one as equation(3). When \( \varepsilon = 1 \), the equivalent effect of rigid lid is achieved to suppress any free surface motion.

**Frequency-domain solution**

The abovementioned first order boundary value problem can be solved using the source distribution method. The radiation and diffraction potential can be expressed by an integral of source distribution on the boundaries consisting of all body surfaces \( H \) and free surface \( F \) as following:

\[
\phi(P) = \int_S dS \sigma(Q) G(P, Q), S = H \cup F, \quad (14)
\]

where \( \sigma(Q) \) represents the source density of source point Q on the boundaries, \( G(P, Q) \) is the Green’s function standing for the potential at field point P induced by source of unit density at point Q. \( H \) is the assemble of all body surfaces \( (H_1 \cup H_2) \) and \( F \) represents the whole free surface.

Considering the rectified free surface condition in equation(13), the source distribution \( \sigma(P) \) is determined by satisfying not only the boundary conditions on the hull \( H \), but also on the free surface \( F \), as the two following equations:
\[ 2\pi \sigma(P) + \iint_S dS\sigma(Q) \frac{\partial}{\partial n} G(P, Q) = \nu_a, p \subset H, \quad (15) \]

\[ 4\pi \sigma(P) + i\varepsilon k \iint_S dS\sigma(Q)G(P, Q) = 0, p \subset F. \quad (16) \]

Boundary conditions vary for different problems. For the diffraction potential calculation, 
\[ \nu_a = -\partial \phi / \partial n, \] for the radiation potential calculation, \[ \nu_a = n_j. \] For the purpose of suppressing unrealistic wave elevations only between two bodies, it is appropriate to apply a non-zeros \( \varepsilon \) onto the confined damping zone and leave \( \varepsilon = 0 \) on the outer free surface. What is still uncertain is the size of the slender damping zone and the spatial distribution of \( \varepsilon \) value.

Once the integral equations solved, wave forces applied on vessels can be achieved through the integral of hydrodynamic pressure over the body surfaces. The linear and harmonic motions of floating bodies are evaluated by solving a coupled motion equation at the wave frequency \( \omega \):

\[ [-\omega^2(M + a) - i\omega(b + b') + K] \xi = F, \quad (17) \]

where \( M \) is the generalized mass matrix; \( a \) and \( b \) are the added mass and potential damping coefficient matrix; \( b' \) is the linear viscous damping matrix, which is derived from decay tests (Zhao et al., 2013); \( K \) is the hydrostatic restoring force matrix; \( \xi \) is the motion response vector of two vessels; \( F \) is the wave exciting force vector consists of the F-K force and diffraction force.

Note that all the formulas presented here correspond to the theories employed in the software Hydrostar (Bureau Veritas, 2007) for frequency-domain calculation.

**Time-domain solution**

A time-domain simulation is adopted to further simulate the hydrodynamic couplings between side-by-side vessels. Based on the impulse response theory by Cummins (1962), the coupled equation in time domain can be written as

\[ [M + a(\infty)] \dot{\xi}(t) + \int_0^{\infty} [h(t - \tau)] \ddot{\xi}(t) d\tau + C_{lin} \dot{\xi}(t) + K \xi(t) = F(t), \quad (18) \]

where \( M \) and \( K \) have been defined previously in equation(17). \( a(\infty) \) is the added mass matrix at infinite frequency. \( C_{lin} \) is the linear damping matrix. \( \xi(t) \) indicates the displacement vector of
the vessels. $F(t)$ denotes the vector of wave force time traces converted from frequency domain
diffraction results through $\text{FFT}$. $h(t)$ signifies the retardation function matrix, representing the
memory effect of the free surface on the subsequent ship motions. It can be achieved from the
reverse Fourier transformation of frequency-domain coefficients as the following equation:

$$h(t) = \frac{2}{\pi} \int_0^{\infty} b(\omega) \cos \omega \tau d\omega = -\frac{2}{\pi} \int_0^{\infty} \omega[a(\omega) - a(\infty)] \sin \omega \tau d\omega,$$

(19)

where $a$ and $b$ are the added mass matrix and radiation damping matrix obtained from frequency
domain calculation.

In this study, frequency-domain damping coefficients modified by the artificial damping
method are used to calculate the retardation function. Due to the asymptotic behavior that
damping coefficients approach zero at infinite frequencies, high frequency truncation would yield
adequately accurate retardation function.

A fully coupled model is used in this calculation considering hydrodynamic interactions, in
that case the inertia term $[M + a(\infty)]$ can be expanded as

$$\begin{bmatrix}
(M + a(\omega))_{ij} & a(\omega)_{ij} \\
(a(\omega))_{ji} & (M + a(\omega))_{ji}
\end{bmatrix}\begin{bmatrix}
\xi_i \\
\xi_j
\end{bmatrix},

(20)

where the indices $i$ and $j$ refer to body $i$ and body $j$. The convolution term can be expanded as

$$\int_0^t \begin{bmatrix}
h(t-\tau)_{ij} & h(t-\tau)_{ij} \\
h(t-\tau)_{ji} & h(t-\tau)_{ji}
\end{bmatrix}\begin{bmatrix}
\xi_i \\
\xi_j
\end{bmatrix} d\tau.$$

(21)

The coupled time domain analysis was carried out with an in-house code using $\text{FORTRAN}$ to
predict the motions of side-by-side vessels under different wave directions. Fourth-order
Runge-Kutta method is used to solve the partial differential equation. Being different from single
body calculation, multi-body time domain simulation faces a great challenge of convergence
aroused by gap water resonance. The accurate calculation of retardation function is vital to the
convergence of time domain solution and will be extensively explained in the following section.
Improvement of retardation function

In the time domain analysis, retardation function gives the influence of precedent fluid domain on the present ship motions. Normally, the Fourier cosine transformation, as the first expression in equation (19), is used to calculate the retardation function based on the frequency-domain potential damping for faster convergence. Hence, the quality of potential damping decides the outcome of retardation function. The potential damping of FLNG for both multi-body ($\varepsilon=0/0.05$) and single body cases are illustrated in the Fig. 2 (a). It is observed that the curves demonstrate the shape of delta function in the multi-body case, with huge spikes at resonant frequencies induced by sloshing waves in the gap zone. Nevertheless, the introduction of non-zero $\varepsilon$ reduces the peak value of spikes.

Following the ideal fluid assumption of Lewandowski (2008), the multi-body potential damping can be represented in form of the summation of single body damping and delta function modification as

$$b(\omega) = b_1(\omega) + \sum_n C_n \omega_n \pi \delta(\omega - \omega_n), \quad (22)$$

where $\omega_n$ indicate the critical frequencies, $n=0,1,2,\ldots$, and $b_1$ is the basic damping coefficient without resonance effect, which is qualitatively similar to the single body potential damping. $C_n$ are constants associated with the peak amplitudes. The Fourier cosine transformation of (22) can be written as follows:

$$h(t) = h_1(t) + 2\pi \sum_n C_n \omega_n \cos(\omega_n t). \quad (23)$$

This means that the retardation function (ideal fluid) can be treated as the sum of sinusoid component at each critical frequency superimposed on a normal memory function $h_1$ (single body case), as shown in Fig. 2 (b) when $\varepsilon=0$.

It is observed from Fig. 2 (b) that the retardation function in the single body case fully decay within 20s, while it takes much longer for multi-body case (over 150s). This is due to the wave reflection within the gap region without viscosity. The long-lasting oscillatory sinusoid component of retardation function in multi-body cases (blue line in Fig. 2) is undesired in time-domain.
Simulation for it causes the accumulation of motion energy and divergence of results. Improvement should be made in frequency-domain results to simulate the dissipation of energy. The higher $\varepsilon$ introduced in frequency domain to suppress the resonance peak (Fig. 2 (a)), the faster the retardation function (red line in Fig. 2 (b)) will decay, making it easier for time-domain simulation to converge in avoidance of unrealistic energy accumulation. In this study, a fine frequency interval of $0.005\text{rad/s}$ and high frequency truncation up to $3\text{rad/s}$ are used in the integral of retardation function. The retardation function with $\varepsilon=0.05$ gets sufficiently damped and thus can be truncated at $300\text{s}$.

To help understand the calculation process, the flow chart of multi-body frequency and time domain simulation is made to clarify each step, as shown in Fig. 3.

**Numerical and experimental models**

**Numerical model**

The frequency-domain model, incorporating the artificial damping method, of this side-by-side configuration was established to calculate the gap wave elevation RAOS, hydrodynamic coefficients and motion responses of two vessels by using Hydrostar (Bureau Veritas, 2007). As we mainly focused on the hydrodynamic interactions under various wave headings, both vessels are floating freely in close proximity without mechanical coupling. The gap between two bodies is set as $6\text{ meters}$ constantly. The principal scantlings of the FLNG and LNGC are listed in Table 1. The wet-surface panel models of the side-by-side vessels are shown in Fig. 4; the number of elements for FLNG and LNGC are $2808$ and $1916$ after a check of the grid independence to ensure the convergence of solution.

Before the implementation of artificial damping method, the right artificial damping coefficient $\varepsilon$ and the size of free-surface damping zone needed to be determined. Nevertheless, a uniquely precise definition of the free-surface in the gap zone is often not possible particularly when the two geometries are dissimilar in the present study. The selection of $\varepsilon$ is also somewhat empirical and mainly determined through tuning with tests results. The effectiveness of this method can be justified only if the tuned $\varepsilon$, after implementation of artificial damping method,
can suppress the drastic free-surface motions near resonant frequencies.

In this study, a fixed value of $\varepsilon$ on the whole gap area following the method of Watai et al. (2015) was decided by tuning the wave elevations from numerical simulation with experimental data. The area of free-surface grid in this study covers the gap region along parallel midsection of LNGC where 144 panels are generated, as seen in Fig. 4.

In order to take into account the viscous drag of the roll motion, the linear viscous damping coefficient of roll motion was added into the motion equation in both frequency and time domain respectively. The frequency-domain roll damping coefficient can be derived from the time series of decay tests in still water according to Zhao et al. (2013). While in the time domain, the appropriate roll damping added into the time domain motion equation can be achieved by tuning the calculated decay curves with the test results according to Xu et al. (2015).

**Experimental set-ups**

To provide experimental validations and determine the important viscous damping terms that cannot be calculated, model tests of side-by-side FLNG and LNGC were carried out in Deepwater Offshore Basin at Shanghai Jiao Tong University. The model were made at a scale of 1:60 according to Froude scaling principle.

Fig. 5 illustrates the layout of two parallel vessels at a distance of 6m in full scale. The midship sections of two vessels are both at the zero point on the x-axis of the global coordinate system (see Fig. 4) so that the multi-body system is longitudinally symmetric. Each floating body was horizontally moored with four soft springs, two at bow with 45° while two at stern with 45°. Soft mooring system is used to prevent the second order drift motions and fix their headings. The stiffness of each spring is small enough to keep the natural periods of vessels’ horizontal motions far longer than wave periods. As a result, no interference between wave frequency motions and low frequency motions would be caused by mooring lines. To prevent collisions and protect the models in tests, two identical fenders (black strips in Fig. 5), without any hawser, were placed symmetrically about the midship on the interior water plane, while in numerical simulation no mechanical coupling is taken into account. The longitudinal distance between the fender and midship is 46 meters and the linear stiffness is 885.6 KN/m. The fenders were attached to FLNG
with no pretension and had little influence on the couplings between vessels because collisions, in fact, rarely occurred in the mild wave conditions.

For the environmental condition, only waves were included without wind or current. The tests were carried out in the water depth of 5m, corresponding to the water depth of 300m in reality. The white noise waves, in full scale, had significant wave height of 3 meters to meet the linear assumption and wave frequencies ranging from 0.25 rad/s to 1.25 rad/s to cover the main response frequencies of floaters. Each test was run for duration of 3 hours. The same white noise wave spectrum was adopted in model tests incoming from three different angles 135, 180 and 225°, as illustrated in Fig. 5. Calibration of white noise wave spectrum is shown in Fig. 6.

The tests included decay tests and white noise wave tests. In white noise wave tests, resistance-type wave probes were used to measure wave elevations in the gap. They were set longitudinally at the beginning, end and middle of the parallel midsection of LNGC along the gap central line, as illustrated with the tiny circles in Fig. 5. Motions of each vessel were measured by the non-contacting laser pointer finder. The response amplitude operators (RAOs) of wave elevations and motions of six degrees of freedom can be obtained through spectra analysis of measured time series. The wave elevation RAOs can be used to tune the artificial damping $\varepsilon$ to suppress unrealistic wave elevations in simulation, while the motion RAOs of both vessels can be used to validate the numerical model.

Results and discussions

Calibration of numerical model

For better accuracies, the numerical model was calibrated by test results to determine the artificial damping $\varepsilon$ and viscous damping in certain modes. Firstly, the artificial damping $\varepsilon$ should be tuned through gap wave elevations and the frequency-domain viscous damping can be obtained from decay analysis (see Table 2). Based on the frequency-domain hydrodynamic coefficients under proper $\varepsilon$, the time-domain viscous damping could then be selected with the measured decay curves.
Artificial damping method application

The selection of damping coefficient $\varepsilon$ in equation (13) is vital to suppress the resonant amplitude of gap waves to the realistic value. Lots of works have been done to study the determination of $\varepsilon$. Watai et al. (2015) found that the $\varepsilon$ fitting the test results best tends to be larger if a narrower gap width existed. Lu et al. (2010) observed that the artificial damping method generates results in reasonably good correlation with the test results for a fairly wide range of damping coefficient values, suggesting that reasonable numerical results could be obtained even if the damping coefficient was not precisely tuned.

In most studies for a given gap width, a constant artificial damping $\varepsilon$ can be used to match the test results in heading waves. Nevertheless, it is unknown that whether the value of $\varepsilon$ is susceptible to the change of wave directions, which remains to be validated in the present study.

Following the $\varepsilon$-selecting scheme presented in Chen (2011), wave elevations measured at the center of the gap can be used to tune the numerical model. Wave elevations calculated based on four values of parameter $\varepsilon(0,0.02,0.04,0.05)$ for each wave heading have been compared with test results at the gap center in present study. It is observed that (shown as Fig. 7), though under different wave directions $(135,180,225^\circ)$, $\varepsilon$ between 0.04 and 0.05 can give the numerical result which fits the experimental data best. For the convenience of further study, the $\varepsilon = 0.05$ is selected to implement calculation despite various wave headings.

From Fig. 7, one observes that the increase of $\varepsilon$ indeed suppresses the amplitude of wave elevations in certain frequencies at a constant interval, while causes less effect over the rest frequencies. Extensive works on the resonant phenomenon of gap water have been carried out by Lewandowski (2008) and Sun et al. (2015). These critical frequencies, following the definition by Lewandowski (2008), are corresponding to the natural modes of free-surface elevation in the gap zone. The amplification and cancellation are attributed to the superposition of incoming waves, diffraction waves, radiation waves of ship motions, and most importantly, resonant waves between gap exits dividing the open water and gap region.

In addition to a brief analysis of gap water resonance, some interesting phenomena are also observed in Fig. 7. Firstly, wave elevations under oblique seas (Fig. 7 (a) (c)) show small
fluctuation at frequencies of 0.48 rad/s and 0.56 rad/s, which correspond to the roll natural frequencies of FLNG and LNGC respectively. This reveals a weak coupling effect, which is even negligible in engineering, between wave elevations and roll motions of two vessels under oblique waves. Secondly, the amplitude of wave resonance under waves of 225 deg is much larger than that under waves of 135 deg. This possibly arises from the shielding effect of FLNG on the weather side under 135 deg waves, which reflects most incoming waves to keep the gap region less disturbed. Thirdly, the frequency shift between the numerical and test results at resonant frequencies is observable but only with small discrepancy (about 0.02 rad/s), which also justifies the accuracy of grid density according to Bunnik et al. (2009).

**Decay tests**

In decay tests of roll and pitch mode, moments of inertia of models have been validated through measurement of natural frequencies. Among all modes, the roll viscous damping is the most important for its value is comparable with the radiation damping and must be considered in numerical simulation. Results of roll decay analysis are shown in Table 2, where the damping coefficient refers to the linearized damping ratio with respect to the critical damping.

Based on the in-house time domain code, an appropriate roll viscous damping \( C_{\text{in}} \) in equation (18) could be determined by simulating the decay of either vessel while the other one is fixed. The accuracy of the code is also validated through the comparison between numerical and test decay results.

The decay results of model tests and numerical simulation are compared for both FLNG and LNGC in roll and pitch modes, as shown in Fig. 8. It is observed that though sharing the same periods, the calculated roll decay curves (Fig. 8 (a), (b)) without viscous damping decay slower than the test results. The application of appropriate roll viscous damping makes two vessels decay faster to match the test results precisely. The roll viscous damping coefficients are tuned by matching the calculated decay curves with the test results, which are 2.9E+8 Ns/m for FLNG and 1.2E+8 Ns/m for LNGC in time domain simulation. For the pitch decay curves, it is observed that a fairly good agreement can be directly achieved between numerical and test results in Fig. 8 (c), (d). Hence, no extra viscous damping is needed in the pitch mode in that the radiation damping
generated by pitch motion is far more important than the viscous damping, as opposed to the roll mode where the viscous damping is equally important. The natural pitch period is 10.5s for FLNG and 8.8s for LNGC.

Hydrodynamic performances under oblique waves

Frequency domain analysis is used to investigate the side-by-side system’s motion responses of all degree of freedoms. To develop a qualitative understanding of hydrodynamic interactions under oblique waves, motion responses under oblique waves are studied firstly through the comparison between numerical and test results.

A typical oblique wave direction of 135deg is selected to reveal the features of motion under oblique waves and the results are illustrated in the Fig. 9 and Fig. 10. Fig. 9 shows the surge, heave and pitch results of FLNG (on the right side) and LNGC (on the left side), respectively. While Fig. 10 shows the roll, pitch and yaw results. All motion responses are normalized by incidental wave amplitude and plotted against frequency rad/s. Numerical results of different values of $\epsilon$ are presented for the comparison with experimental results (black dot).

In Fig. 9, it is observed that motion responses show similar trend for both ships, with lower amplitude for FLNG because of its large inertia. Gap water resonance imposes little influence on the motion responses of the three modes at frequencies from 0.9 to 1.1 rad/s, with only small variations for LNGC in heave and pitch modes around 1 rad/s. For the heave motion of FLNG, it is noticeable that a slump of the curve occurs at the roll natural frequency of LNGC and vice versa. This indicates weak hydrodynamic couplings exist between the heave motion and other floater’s roll motion, which is a main difference from the motion responses in head seas. Nevertheless, an overall agreement of numerical and experimental results can be achieved despite the change of $\epsilon$.

In Fig. 10, it is indicated that the effect of gap water resonance on motion responses of sway and yaw is quite obvious at high frequencies. The application of $\epsilon$ equal to 0.05 successfully suppresses the impulsive motion responses at gap resonance frequencies, as shown in the subplots of sway and yaw motion, which justifies the effectiveness of the artificial damping method in the prediction of motion responses. Numerical model tends to slightly overestimate the peak value of roll resonance despite the linearized viscous damping has been added. This is induced by the
nonlinearity of roll viscous damping at the natural frequency (Jung, Chang, & Jo, 2006). Couplings between roll resonance and sway motion reoccur for both vessels so that a fluctuation of the sway RAO is observable at two roll natural frequencies.

Therefore, it is concluded that two features are obvious for multi-body motion responses under oblique waves, i.e. 1) the coupling effects between roll motion and other modes, 2) the strong resonant motions at high frequencies induced by gap wave sloshing. Note that weak as the coupling effects from roll resonance might be, they would induce unfavorable resonances of relative sway and heave motions between two hulls, as shown in discussions of relative motions.

Due to the different displacements of two vessels, hydrodynamic performances under symmetrical oblique waves are not identical. To illustrate the influence of wave directions, motion responses under symmetric wave directions (135deg/225deg) are further calculated based on the constant value $\varepsilon=0.05$. Comparisons of motion responses under different wave directions are plotted in Fig. 11 and Fig. 12, where single body case refers to the condition of only FLNG or LNGC in quartering seas without multi-body interaction.

It is firstly observed that the amplitudes of the transversal motions of sway, roll and yaw (see Fig. 12) under head seas are much smaller for both vessels. Thus, the head sea is the most ideal wave condition for side-by-side configuration.

For the oblique wave conditions (135/225deg), it can be seen from Fig. 11 and Fig. 12 that motion responses of FLNG are insusceptible to the change of wave directions due to its large inertia. The low frequency responses ($\omega<0.45 rad/s$) of LNGC are similar with single body under either 135deg or 225deg waves, showing no hydrodynamic disturbance. This is because low frequency waves have strong transmission effect and cause weak hydrodynamic interactions between hulls. With the increase of incoming wave frequency, LNGC bears smaller motions at the lee side (135deg), especially for heave, pitch and yaw motion. This can be explained by the shielding effect of FLNG because once FLNG at the weather side (135deg), reflection effect dominates at high wave frequencies so that less wave energy absorbed by the lee-side LNGC. The same phenomenon is also observed by Kim et al. (2008).

A special phenomenon associated with sway and yaw motions (see Fig. 12) can be found that regardless of wave directions, either LNGC at lee or weather side generate smaller motion
amplitude than that in single-body case. The huge FLNG can act as a shielding obstacle at the weather side or a quay at the lee side, both of which serves to stifle the motion responses of LNGC beside. This is a favorable behavior because LNGC in the side-by-side configuration has smaller transversal motion responses.

At resonance frequencies featured by sloshing waves in the gap, greater motion responses of FLNG and LNGC can be found under 225deg wave direction in sway and yaw modes (see Fig. 12). This behavior concurs with the trend of gap wave elevations under various wave directions. Being exposed to the 225deg waves, the motion of weather-side LNGC is significantly amplified by the drastic resonant waves at frequencies between 0.9rad/s and 1.2rad/s.

Relative motions under different wave directions

Relative motion between two hulls is an important issue in side-by-side configuration because offloading arms are sensitive to the relative motions between two vessels. Time domain calculation is implemented to study the relative motions between two vessels under oblique waves, defined in equation (24) as the motion of body $i$ with respect to body $j$ in six degrees of freedom.

$$\xi_{ij}(t) = \left[ \xi_i(t) - \xi_j(t) \right], \quad \xi(t) = \begin{bmatrix} \xi_i(t) \\ \xi_j(t) \end{bmatrix}$$

(24)

The response amplitude operators of relative motions in all modes are calculated at each frequency with the in-house code, and compared with experimental results under two typical wave directions, as shown in Fig. 13. The test results of 135deg oblique wave and head wave are presented for validation. Large discrepancies between the 180deg experimental and numerical results of sway and yaw motion are observed at low frequencies. This could be explained by the non-exact head waves in model tests, as opposed to the numerical simulation. Although held at the correct heading, two vessels are inevitably rotated by the small yaw motions in model tests to increase the transversal wave loads on hulls, as well as the consequent sway and yaw motions.

Nevertheless, Fig. 13 demonstrates an overall agreement between numerical and experimental results in terms of the trend and magnitude for most modes, except for relative surge motion where large discrepancy also occurs at low frequencies. The reason for this discrepancy is unclear. Due to the overall concurrence of absolute surge amplitude (see Fig. 9), it is conjectured...
that the phase shift associated with surge damping might be responsible. Because in model tests, the rotation of floaters owing to yaw motions would change their original surge damping, causing variation of the motion phase angle subsequently. Hence, the measured relative surge motion obviously differ from the one calculated based on linear assumption that neglect the rotation of bodies. Also this explanation can be supported by the phenomenon that with the decrease of yaw motions at high frequencies (>0.8rad/s), the calculated relative surge motion match well with the test results.

Relative motions under head sea conditions are much smaller compared with those under oblique waves (see Fig. 13), justifying the favorable wave direction among all. For example, the relative roll resonance reaches its maximum of 6.74deg under the 135deg oblique wave while only 1.75deg under the head sea. For the oblique waves, the relative motion responses of surge, heave and pitch motion demonstrate a highly damped mode. Under the wave direction of 135deg, relative pitch is smaller than that under 225deg wave due to the shielding effect of FLNG on LNGC. Spikes in relative heave and sway motion are aroused from strong couplings with roll resonance. Peaks of the relative roll motion are associated with roll resonance of both ships and the amplitudes are bigger when LNGC is on the weather-side (225deg wave).

At high frequencies, the influence of gap water resonance on relative motion is obvious for sway and yaw motions, especially when LNGC at the weather side (225deg wave). Fig. 14 shows the time series of relative sway and yaw motion for the regular wave parameter $\omega=1.05$rad/s (resonance frequency) and $\xi_w=1m$. It brings greater risks of collision when two vessels oscillate in opposed directions, which results in more intensive relative motions.

To further analyze the resonant peaks of relative motion, a phase shift analysis based on numerical results is implemented hereafter. The absolute phase shift of relative motions, together with relative motion RAOs, are presented in Fig. 15 and Fig. 16. The out-of-phase and in-phase relative motions, following the definition by Voogt & Brugts (2010), account for the natural modes of relative motions. The in-phase mode means both bodies moving together when phase shift approaching zeros, while the out-of-phase mode means they moving in opposed directions when phase shift approaching 180deg.

In Fig. 15, relative heave motions of both oblique waves show no sharp resonant peak,
though coupling effect of roll resonance is noticeable. For the relative roll motion, it is observed
that in-phase and out-of-phase resonant mode can respectively explain the peaks at frequencies
0.48rad/s and 0.56rad/s for two different oblique waves. Therefore, the roll resonance of FLNG,
whose natural roll frequency is 0.48rad/s, is quite dangerous that may give rise to a lateral
collision, especially under 225deg wave direction.

For the relative sway and yaw motions in Fig. 16, it is observed that the resonant peaks at
high frequencies induced by gap water sloshing demonstrate a dangerous out-of-phase behavior.
This could be explained that the amplified free surface elevation between two hulls tends to dispel
and draw back the two vessels in opposed directions periodically. As a result, it is safer to subject
the FLNG against the incoming oblique wave at the weather side when the resonant free surface
elevations are effectively suppressed, as well as the transversal relative motions of sway and yaw.

For the sharp peak of relative sway motion aroused from couplings with FLNG roll
resonance, the phase shifts dependent on different wave directions decide the relative motion
predominantly. Note that out-of-phase mode happens under 225deg wave direction and in-phase
mode under 135deg wave. This indicates that drastic relative sway motion will be caused when
oblique waves coming from the side of LNGC (225deg) with considerable energy around FLNG’s
roll resonance frequency.

The abovementioned resonance frequencies of relative motions are categorized and listed in
Table 3. The numbers in bold represent the roll resonance of FLNG (0.48rad/s) or LNGC
(0.56rad/s), while the rest are associated with gap water resonance. It is found that out-of-mode
relative motions are always triggered by gap water resonance under any wave direction, while the
mode of relative motion induced by roll resonance varies with different wave directions and
resonance frequencies.

**Conclusions**

A numerical model of side-by-side FLNG and LNGC is established and calibrated with
model tests. Artificial damping method is used to suppress drastic gap wave resonance and modify
the hydrodynamic coefficients. Hydrodynamic performances under various wave directions are
predicted and analyzed. Based on what, the following conclusions can be obtained.
For the artificial damping method used in this investigation, a constant value of $\varepsilon$ is applicable for different oblique wave directions from 135deg to 225deg. The amplitude of gap waves are found smaller under the 135deg wave, owning to the shielding effect of FLNG.

With the finely tuned $\varepsilon$, the numerical model is able to predict the motion responses of two vessels under oblique waves, showing good agreement with model tests. Two special phenomena present the multi-body hydrodynamics under oblique waves: 1) the couplings between roll motion and other modes, 2) strong resonant motions at high frequencies induced by gap wave sloshing.

Comparisons of motion responses under different wave directions show that motion responses of FLNG are insusceptible to the change of wave directions because of its large inertia, while motion responses of LNGC at the lee side (135deg) is stifled due to the shielding effect of FLNG. Interestingly, either LNGC at lee or weather side has smaller amplitude of sway or yaw motion than that in the single-body case, owning to the suppression effect of the FLNG aside.

Relative motions of all modes under different wave directions have been calculated and analyzed, showing good agreement with test results. Side-by-side vessels experience larger relative motions under oblique waves, particularly when LNGC is at the weather side. For the relative motion resonance, gap water resonance always enhances the relative motions with the out-of-phase mode, while the mode induced by roll resonance depends on wave directions and resonance frequencies.

To reveal the oblique wave effect on FLNG system in side-by-side offloading operation, this study focuses on the hydrodynamics of the FLNG system due to variation of wave directions. Further research will incorporate the inner-tank sloshing effect and the mechanical couplings to study the integrated hydrodynamic responses of the system under oblique waves.

Acknowledgements

This work was financially supported by the China National Scientific and Technology Major Project (2016ZX05028-002-004). This source of support is gratefully acknowledged by authors.

References


### Table 1 Principal particulars of the FLNG and LNGC carrier

<table>
<thead>
<tr>
<th>Designation</th>
<th>Unit</th>
<th>FLNG</th>
<th>LNGC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length over all, Loa</td>
<td>m</td>
<td>213.94</td>
<td>171.152</td>
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<tr>
<td>Breath, B</td>
<td>m</td>
<td>44.8</td>
<td>35.84</td>
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<td>Depth, D</td>
<td>m</td>
<td>25.5</td>
<td>20.4</td>
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<tr>
<td>Draft, T</td>
<td>m</td>
<td>10.8</td>
<td>9</td>
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<tr>
<td>Displacement weight, Δ</td>
<td>t</td>
<td>98923.1</td>
<td>52821.3</td>
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<tr>
<td>COG from keel, VCG</td>
<td>m</td>
<td>13.8</td>
<td>12</td>
</tr>
<tr>
<td>Roll radius of gyration, Rxx</td>
<td>m</td>
<td>16</td>
<td>10.2</td>
</tr>
<tr>
<td>Pitch radius of gyration, Ryy</td>
<td>m</td>
<td>60</td>
<td>50</td>
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<tr>
<td>Heave natural frequency</td>
<td>rad/s</td>
<td>0.97</td>
<td>1.04</td>
</tr>
<tr>
<td>Roll natural frequency</td>
<td>rad/s</td>
<td>0.48</td>
<td>0.56</td>
</tr>
<tr>
<td>Pitch natural frequency</td>
<td>rad/s</td>
<td>0.60</td>
<td>0.66</td>
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Table 2 Results of roll decay tests

<table>
<thead>
<tr>
<th>Designation</th>
<th>Natural period (s)</th>
<th>Non-dimensional Damping coef.</th>
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<tbody>
<tr>
<td>FLNG</td>
<td>13.08</td>
<td>0.0114</td>
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<tr>
<td>LNGC</td>
<td>11.22</td>
<td>0.0122</td>
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Table 3 Relative motion resonance frequencies

<table>
<thead>
<tr>
<th>Relative motion resonance frequency (rad/s)</th>
<th>135deg</th>
<th>225deg</th>
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<tbody>
<tr>
<td>In-phase</td>
<td>Out-of-phase</td>
<td>In-phase</td>
</tr>
<tr>
<td>Relative roll</td>
<td>0.56</td>
<td>0.48</td>
</tr>
<tr>
<td>Relative sway</td>
<td>0.48</td>
<td>0.93, 1.02</td>
</tr>
<tr>
<td>Relative yaw</td>
<td>—</td>
<td>0.97, 1.05</td>
</tr>
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