Nonlinear sloshing in rectangular tanks under forced excitation

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1. Introduction

Recently, the exploitation of natural gas fields located in deep waters has become desirable due to improvements in liquid natural gas storage techniques. The Floating Liquefied Natural Gas (FLNG) system, which is a ship-shaped floating hull equipped with liquefaction plants and LNG storage tanks, is attractive for its high economic efficiency in allowing the exploitation of scattered and remote gas fields. The FLNG platform is subject not only to complex sea conditions, but also to the sloshing loads in LNG tanks (Zhao et al., 2011, 2013). Pressure on tanks caused by sloshing may have unexpected effects on the motion and structural safety of FLNG vessels. The variation of filling level during the exploration and offloading operation makes the prediction of sloshing loads challenging. Thus, an efficient and reliable method to predict sloshing in LNG storage tanks is essential for the design and operation of FLNG systems.

Sloshing in liquid tanks and the effects on vessels has been the subject of numerous past studies. Computational Fluid Dynamics (CFD) method based on solving Navier–Stokes equations can deal with violent sloshing with wave breaks and multiple phase flow. Kim (2001) and Lee et al. (2007) simulated the liquid sloshing in LNG tanks in the time domain by a Navier–Stokes solver. Celebi and Akyildiz (2002) used finite difference approximations to solve Navier–Stokes equations and used volume of fluid technique to determine the free surface. Ransau and Hansen (2006) carried out numerical simulations of sloshing in rectangular tanks using the commercial CFD software FLOW3D. Kim (2007) applied both finite-difference method and Smoothed-Particle-Hydrodynamics (SPH) method to the simulation of violent sloshing. Though CFD methods

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A numerical code is developed based on potential flow theory to investigate nonlinear sloshing in rectangular Liquefied Natural Gas (LNG) tanks under forced excitation. Using this code, internal free-surface elevation and sloshing loads on liquid tanks can be obtained both in time domain and frequency domain. In the mathematical model, acceleration potential is solved in the calculation of pressure on tanks and the artificial damping model is adopted to account for energy dissipation during sloshing. The Boundary Element Method (BEM) is used to solve boundary value problems of both velocity potential and acceleration potential. Numerical calculation results are compared with published results to determine the efficiency and accuracy of the numerical code. Sloshing properties in partially filled rectangular and membrane tank under translational and rotational excitations are investigated. It is found that sloshing under horizontal and rotational excitations share similar properties. The first resonant mode and excitation frequency are the dominant response frequencies. Resonant sloshing will be excited when vertical excitation lies in the instability region. For liquid tank under rotational excitation, sloshing responses including amplitude and phase are sensitive to the location of the center of rotation. Moreover, experimental tests were conducted to analyze viscous effects on sloshing and to validate the feasibility of artificial damping models. The results show that the artificial damping model with modifying wall boundary conditions has better applicability in simulating sloshing under different fill levels and excitations.

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have good applicability in solving complicated sloshing cases, the calculation tends to have low efficiency. For the advantage of high computing efficiency, potential flow theory that assuming the flow is irrotational and inviscid has been widely used in many researches. Based on assumption adopted in potential flow, many analytical solutions of sloshing problem can be obtained. Abramson (1966) presented analytical results of sloshing in different kinds of tanks. Faltin sen (1974, 1978) presented solutions of two-dimensional nonlinear sloshing in rectangular tanks in both analytical and numerical methods. A multimodal approach was adopted by Faltinsen and Timokha (2001) and Rognebakke and Faltinsen (2003) to investigate sloshing in liquid tanks and the effects on ship motions. Frandsen (2004) derived an analytical solution of sloshing in excited tanks; the stability properties of sloshing under vertical excitation were analyzed. Equations of sloshing in cylindrical tanks were derived by Takahara and Kimura (2012) based on the variation principle. Compared with analytical solution, numerical solution is more capable in dealing with complex sloshing model, especially in the considering of sloshing nonlinearity. Using the Finite Element Method (FEM), Wu et al. (1998) analyzed sloshing excited by translational excitation in three dimensional tanks, and Siram et al. (2006) investigated sloshing problems in random excitations. The BEM discretized the fluid domain only in the fluid boundaries and less mesh were produced. Chen et al. (2007) simulated sloshing behaviors in liquid tanks subject to harmonic and seismic excitation by using the BEM. Numerical simulation of sloshing based on coupled FEM-BEM was carried out by Saghi and Ketabdar (2012), Zhao et al. (2014) coupled BEM solved tank sloshing with ship motions in the time domain. Resonant sloshing in wedged tanks due to translational excitations were numerically studied by Zhang (2016) using the BEM. Chen et al. (2017) employed a new boundary integral method (BIM) to simulate sloshing problems, this method was simple and singularity integrals were avoided.

Studies mentioned above mainly focus on sloshing under translational excitations, fewer studies have been conducted on rotationally excited liquid tanks. Nakayama and Washizu (1980, 1981) applied the BEM and the FEM to calculate sloshing in containers subjected to forced rotational excitation. Ning et al. (2012) and Stephen et al. (2016) investigated two dimensional sloshing under combined excitations. La Rocca et al. (2000) derived the analytical solution of sloshing in rotating containers. An experimental investigation of sloshing loads on tanks under rotational excitation was conducted by Akylidiz and Unal (2005), Bouscasse et al. (2014a, b) conducted numerical and experimental research on the energy dissipation of sloshing in a fully angular motion system. Additional studies on the sensitivity of sloshing to rotational excitation parameters are still required.

One disadvantage of potential flow theory is no viscosity is accounted, sloshing responses tend to be over predicted and no transient process occurs. To account for energy dissipation caused by liquid viscosity, artificial damping models have been introduced in some numerical models. Faltinsen (1978) used a device suggested by Rayleigh to simulate the effects of viscous damping, a force opposing the particle velocity was assumed on the free surface. Malenica et al. (2003) considered the damping effects in sloshing motion by modifying the boundary conditions on the wet tank walls. These artificial damping models could not and did not intend to correctly model the complicated sloshing phenomena. But with the help of the artificial damping model, energy dissipation caused by viscous effects can be predicted and more reliable sloshing loads can be modelled more effectively. Nevertheless, the artificial damping value still needs to be verified through experimental researches.

To date, reliable and efficient methods to predict liquid sloshing in FLNG systems are still limited. The main motivation of this study was to develop a numerical program to efficiently capture the internal free-surface elevation and sloshing loads on rectangular LNG tanks in FLNG systems. Due to safety reasons, the motion response of both FLNG and LNG carriers are restricted. Hence, offloading of liquid natural gas can only be conducted in mild sea conditions. As a result, violent sloshing in the tanks will be avoided. From this perspective, potential flow theory combined with artificial damping model is adopted. In this research, wave elevations under different excitations are analyzed in both time domain and in frequency domain. Hydrodynamic pressures on specific positions are calculated and validated by corresponding experimental tests. Two phase flow including fluid and air flow is not considered and interaction between air and liquid flow is not included, which may have nonnegligible effects on liquid flow when violent sloshing occurs. This paper is organized as follows. The mathematical formulation is firstly presented in section 2. Then, numerical method used in solving the governing equations is described in section 3. In section 4, sloshing in rectangular and membrane tank under different excitations is simulated; viscous effects on sloshing are investigated based on numerical and experimental results. Finally, main conclusions are drawn in section 5.

2. Mathematical formulation

Numerical code developed in this paper simulates sloshing in a three dimensional rectangular liquid tank with length B, width W and water depth D. As shown in Fig. 1, space-fixed coordinate system XYZ and tank-fixed coordinate system xyz are defined. The two coordinate systems are at the same position in the initial condition, with an origin is located in the middle position of undisturbed free surface, the xy plane is in parallel with the free surface and the z-axis is positively upward. Under forced excitation, the tank has translational displacement $\mathbf{d}(t) = [d_x(t), d_y(t), d_z(t)]$ in space-fixed coordinate system XYZ and rotational motion $\mathbf{R}(t) = [\theta_x(t), \theta_y(t), \theta_z(t)]$ around the center of rotation. The translational and angular velocity of the tank can be obtained as follows:

$$\mathbf{V} = [u, v, w] = \frac{d [d_x(t), d_y(t), d_z(t)]}{dt}$$

(1)

$$\mathbf{\omega} = \left[ \omega_{xy}, \omega_{yz}, \omega_{zx} \right] = \frac{d [\theta_x(t), \theta_y(t), \theta_z(t)]}{dt}$$

(2)

The velocity potential $\phi$ is then decomposed as $\phi = \psi + xu + yv + zw$ to simplify the wall boundary condition by eliminating translational excitation on the tank. But this method...
cannot be used to cope with rotational excitation as no irrotational potential can be found to satisfy rotational condition on the walls. Boundary value problem of velocity potential in tank-fixed coordinate system can be derived as:

In the fluid domain

\[ \nabla^2 \phi = 0 \]  

On \( S_B \)

\[ \frac{\partial \phi}{\partial n} = (\omega \times \mathbf{r}) \cdot \mathbf{n} \]  

(4)

where \( \mathbf{n} \) is the outward vector and normal to the tank walls, \( \mathbf{r} \) is the distance from the center of rotation to the boundary.

On \( S_F \)

\[ \frac{\partial \phi}{\partial t} - \omega \times \mathbf{r} \cdot (\nabla \phi + \mathbf{V}) + x \frac{d u}{d t} + y \frac{d v}{d t} + z \frac{d w}{d t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + k_0 \mathbf{g} \cdot \mathbf{r} = 0 \]  

(5)

\[ \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial \phi}{\partial z} \frac{\partial^2 \phi}{\partial z^2} - \omega \times \mathbf{r} \cdot \nabla \phi = 0 \]  

(6)

Initial conditions on the free surface can be expressed as \( \phi = 0 \) and \( \zeta(x, y, 0) = 0 \), or equivalently \( \phi = -xu - yv - zw \) and \( \zeta(x, y, 0) = 0 \) in tank-fixed coordinate system.

Following the solution of velocity potential, hydrodynamic pressure in wall boundaries can be achieved according to Bernoulli equation:

\[ \frac{P}{\rho} = \frac{\partial \phi}{\partial t} - \omega \times \mathbf{r} \cdot (\nabla \phi + \mathbf{V}) + x \frac{d u}{d t} + y \frac{d v}{d t} + z \frac{d w}{d t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + k_0 \mathbf{g} \cdot \mathbf{r} \]  

(8)

To determine the hydrodynamic force, the value of \( \frac{\partial \phi}{\partial t} \) in boundary is required. In this study \( \phi_t = \frac{\partial \phi}{\partial t} \) is defined and solved as another boundary value problem. Compared with using difference scheme of \( \phi \) in time, this method could improve numerical stability and computational efficiency (Tanizawa, 1995; Bandyk and Beck, 2011). The boundary value problem of \( \phi_t = \frac{\partial \phi}{\partial t} \) can be derived as:

In the fluid domain

\[ \nabla^2 \phi_t = 0 \]  

On \( S_F \)

\[ \phi_t = -x \frac{d u}{d t} - y \frac{d v}{d t} + z \frac{d w}{d t} - \frac{1}{2} \nabla \phi \cdot \nabla \phi - k_0 \mathbf{g} \cdot \mathbf{r} + (\omega \times \mathbf{r}) \cdot (\nabla \phi + \mathbf{V}) \]  

(10)

On \( S_B \)

\[
\frac{\partial \phi_t}{\partial n} = \mathbf{n} \cdot \left[ \omega \times \mathbf{r} + 2 \omega \times \mathbf{V} + \omega \times (\omega \times \mathbf{r}) - (\nabla \phi \cdot \nabla \phi) \right]
+ \mathbf{n} \cdot (\omega \times \mathbf{r}) \cdot (\nabla \phi + \mathbf{V}) \]

(11)

where \( \mathbf{V}_r \) is the velocity of fluid observed in a tank-fixed coordinate system. The dot notation indicates derivation with respect to time.

After the solution of hydrodynamic pressure, calculations can proceed to the next time step. The time derivative of potential on the free surface is:

\[
\frac{d \phi}{d t} = \frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial x} \frac{d x}{d t} - \frac{\partial \phi}{\partial y} \frac{d y}{d t} - \frac{\partial \phi}{\partial z} \frac{d z}{d t} - \frac{1}{2} \nabla \phi \cdot \nabla \phi - k_0 \mathbf{g} \cdot \mathbf{r}
+ (\omega \times \mathbf{r}) \cdot (\nabla \phi + \mathbf{V})
\]

(12)

For the convenience of mesh updating during calculation, free surface elevation is updated vertically in the tank-fixed coordinate:

\[
\frac{d \zeta}{d t} = \frac{\partial \zeta}{\partial y} - \frac{\partial \alpha_\phi}{\partial x} \frac{\partial \phi}{\partial x} - \frac{\partial \alpha_\phi}{\partial y} \frac{\partial \phi}{\partial y} + (\omega \times \mathbf{r}) \cdot \nabla \zeta
\]

(13)

3. Numerical procedure

The governing equations of both velocity potential and acceleration potential derived in Section 2 are boundary value problems, which are solved with the boundary element method in this study. By applying Green’s second identity (Hunter and Pullan, 2001), the boundary integral equation can be derived:

\[
c(p) = \int \left( \frac{\partial \phi(p)}{\partial n} G(p, q) - \phi(q) \frac{\partial G(p, q)}{\partial n} \right) d\Gamma
\]

(14)

where the solid angle coefficient \( c(p) \) stands for the ratio of that angle occupied by the fluid to \( 4\pi \); \( p \) and \( q \) are the source and field points in the respective boundaries. \( \Gamma \) denotes the boundaries of the fluid domain. \( G(p, q) \) denotes Green’s function, and the fundamental solution for the three dimensional Laplace equation can be written as follows:

\[
G(p, q) = \frac{1}{4\pi r}
\]

(15)

where \( r = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2 + (z_p - z_q)^2} \) is the distance between point \( p \) and point \( q \).

The direct method to obtain \( c(p) \) is based on the unit normal vector on the elements around point \( p \) (Teng et al., 2006). In this study, the solid angle is solved using an indirect method. A non-zero constant potential is assumed over the boundaries \( \Gamma \) in (14), and no flux is produced over the boundaries. Then, the integral to solve \( c(p) \) can be obtained by writing (14) as follows (Ning et al., 2010):
The boundaries are then discretized into continuous and non-overlapping small elements. Triangular and rectangular elements are used in the discretization of free surfaces and wall boundaries, respectively, as shown in Fig. 2. This type of discretization is convenient for conducting mesh regeneration in the time-domain calculation and capturing a curved free-surface profile. During the regeneration of mesh in each step, nodes in free surface are updated vertically as described in (13). In this study, double and triple nodes are used in the boundaries and corners of the tank model, respectively (Grilli and Svendsen, 1990). Based on discretized elements (14) can be rewritten as follows:

$$c(p) = - \int_{\Gamma} \frac{\partial G(p, q)}{\partial n} d\Gamma$$

(16)

where $N_e$ is the total number of elements on the wetted boundary.

Each element in (17) is mapped onto a two-dimensional plane $(\xi, \eta)$, and the physical parameters in the element can be interpolated using a shape function as $\psi(\xi, \eta) = \sum_{k=1}^{K} N_k(\xi, \eta) \psi_k$, where $\psi$ represents the physical parameters such as coordinates $x, y, z$ and the velocity potential $\phi$. The shape function $N_k(\xi, \eta)$ can be expressed in triangular and rectangular elements as follows:

for triangular element:

$$\begin{align*}
N_1(\xi, \eta) &= 1 - \xi - \eta \\
N_2(\xi, \eta) &= \xi \\
N_3(\xi, \eta) &= \eta
\end{align*}$$

for rectangular element:

$$\begin{align*}
N_1(\xi, \eta) &= (1 - \xi)(1 - \eta)/4 \\
N_2(\xi, \eta) &= (1 + \xi)(1 - \eta)/4 \\
N_3(\xi, \eta) &= (1 + \xi)(1 + \eta)/4 \\
N_4(\xi, \eta) &= (1 - \xi)(1 + \eta)/4
\end{align*}$$

Then (17) can be written in the following form:

$$c(p_i) \psi(p_i) + \sum_{j=1}^{N_e} \int_{\Gamma_j} \psi(p) \frac{\partial G(p, q)}{\partial n} d\Gamma = \sum_{j=1}^{N_e} \int_{\Gamma_j} \frac{\partial \psi(q)}{\partial n} G(p, q) d\Gamma$$

(17)

where $N_e$ is the total number of elements on the wetted boundary. For nodes on the free surface, the fluid velocities on the boundaries can be obtained by computing the spatial derivatives of $\phi$ as:

$$\begin{bmatrix}
\frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial y} \\
\frac{\partial \phi}{\partial \eta}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial \phi}{\partial \xi} & \frac{\partial \phi}{\partial \eta} & \frac{\partial \phi}{\partial \eta}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{\partial \psi_k}{\partial \xi} \\
\frac{\partial \psi_k}{\partial \eta} \\
\frac{\partial \psi_k}{\partial \eta}
\end{bmatrix}$$

(24)

Similarly, spatial derivatives of wave elevation in (13) can be computed as:

$$\begin{bmatrix}
\frac{\partial \zeta}{\partial x} \\
\frac{\partial \zeta}{\partial y} \\
\frac{\partial \zeta}{\partial \eta}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial \zeta}{\partial \xi} & \frac{\partial \zeta}{\partial \eta} & \frac{\partial \zeta}{\partial \eta}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{\partial \zeta_k}{\partial \xi} \\
\frac{\partial \zeta_k}{\partial \eta} \\
\frac{\partial \zeta_k}{\partial \eta}
\end{bmatrix}$$

(25)

4. Results and discussion

In this section, convergence analysis and validation of the
Numerical code are carried out. Then, sloshing properties under translational and rotational excitation are investigated in both the time and frequency domain. Finally, the viscous effects on sloshing are studied with the help of an artificial damping model and experimental results.

Natural sloshing frequencies of the liquid in a three-dimensional rectangular tank can be expressed analytically as:

$$\omega_{mn} = \sqrt{\frac{m^2}{B^2} + \frac{n^2}{W^2}} \tanh\left(\sqrt{\frac{m^2}{B^2} + \frac{n^2}{W^2}}\right) \left(\frac{2}{C_0}\right)^{0.578}$$

where, $B$, $W$ and $D$ represent the length, width and water depth of the tank; $m$ and $n$ are positive integers. In the following analysis, except in the cases involving experimental results, $D$ is set as $1m$ and other parameters are nondimensionalized based on water depth $D$, water density $\rho$ and gravity $g$, where: $(B, W, A_x, A_y, \eta, a, b) \rightarrow (B, W, A_x, A_y, \eta, a, b)/D, \tau \rightarrow \tau / \sqrt{g/D}, P \rightarrow P/\rho gD$; Power Spectrum Density (PSD) of wave elevation in the frequency domain analysis is nondimensionalized as $PSD \rightarrow PSD_{010}/D^2$. Excitations on the tank are exerted in sinusoidal form as $d(t) = A_0 \sin(\omega t)$ for translational excitation, or $\theta(t) = A_0 \sin(\omega t)$ for rotational excitation. Where $A_0$ and $\omega_0$ are the amplitude of translational and rotational excitation, $\omega_0$ stands for excitation frequency. $\Delta T$ stands for the time step in the time domain calculation.

4.1. Convergence studies and validation

Convergence studies were conducted to determine the mesh number and time step used in the numerical simulation. In different mesh models, the coordinates of the eight corners and the middle position of the free surface are fixed and do not change with mesh number. For triangular meshes in the free surface, distances among nodes and angles of each mesh are optimized to avoid a low quality mesh and to increase the accuracy of subsequent calculations. In the convergence studies, sinusoidal excitation in x direction is exerted on the tank. Parameters are chosen as $B = D = 2$, $A_0 = 0.002$ and $\omega_0 = \omega_{10}$ (5.316 rad/s). Firstly, the time step is fixed to $\Delta T = 0.0443$, two mesh models are selected with mesh numbers along the tank dimensions, where the length, width and water depth are $14 \times 14 \times 8$ and $20 \times 20 \times 12$, respectively. Wave elevation histories at the free surface corner $(B/2, -D/2)$ and wave profiles along $y = -W/2$ at $\tau = 39.46$ are plotted in Fig. 3(a). Good agreement between results of these two mesh models shows that mesh number $14 \times 14 \times 8$ is sufficient for mesh convergence. In the following cases, the mesh number of $14 \times 14 \times 8$ is adopted for tank with $B = W = 2$, and particular mesh numbers are indicated for tanks with different sizes. Then, time step convergence is conducted. Two time steps with $\Delta T = 0.0313$ and $\Delta T = 0.0188$ are chosen. Fig. 3(b) shows good agreement in both wave elevation histories at corner $(B/2, -D/2)$ and wave profile along $y = -W/2$. In the following calculations, except extreme cases with very large sloshing amplitude, time step $\Delta T = 0.0313$ is adopted for better computational efficiency.

Validations of the present numerical program are conducted by comparing numerical simulation results with previous studies. Firstly, results of sloshing under translational excitation are validated. A two dimensional experimental sloshing case conducted by Liu and Lin (2008) is selected. The two dimensional tank has the size $B = 0.57m, D = 0.15m$. The horizontal displacement of the tank is in a sinusoidal form written as $d(t) = A_0 \sin(\omega_0 t)$, where $A_0 = 0.005m$ and $\omega_0 = 6.0578 rad/s$. In this case, a numerical tank with $B \times W \times D = 0.57m \times 0.2m \times 0.15m$ is built to simulate two dimensional sloshing. Mesh numbers are set as $24 \times 10 \times 8$ and a time step of $\Delta T = 0.0188$ is chosen. Linear analytical results are also obtained for comparison. Results of wave elevation at corner $(B/2, D/2)$ are presented in Fig. 4. It can be seen that the numerical results agree with the experimental data. Resonant phenomena can be observed when the excitation frequency is equal to the first natural frequency of the liquid tank. Compared with linear analytical results, obvious nonlinear properties can be observed in the experimental and numerical results. Three dimensional sloshing results are compared with calculation results of Wu et al. (1998) using the FEM method. Parameters of tank are $B = W = 4$, sinusoidal displacement of tank in x and y directions are in the form of $d_x(t) = A_0 \sin(\omega_0 t)$ and $d_y(t) = A_0 \sin(\omega_0 t)$ with $A_0 = 0.0372$, $\omega_0 = \omega_{10}$, $A_0 = 0.0182$, $\omega_y = \omega_{01}$. Wave elevation histories at the two corners are presented in Fig. 5 and good agreement is obtained. Compared with mesh number adopted by Wu et al. (1998) $(40 \times 40 \times 12)$, a smaller mesh number $(14 \times 14 \times 5)$ is needed in the current
Additionally, sloshing results under rotational excitations are validated. A two-dimensional tank subjected to rotational excitation solved by Nakayama and Washizu (1980, 1981) is chosen. The tank has parameters of $B = 0.9\,m$ and $D = 0.6\,m$. Rotational excitation on the tank is in written in the cosine form as $A_{q_x}(t) = \theta_x \cos(\omega_{q_x}t)$, where $\theta_x = 0.8\,\text{deg}$ and $\omega_{q_x} = 5.5\,\text{rad/s}$. In the present numerical simulation, a numerical tank with $B \times W \times D = 0.9\,m \times 0.3\,m \times 0.6\,m$ is built and the mesh number is set as $18 \times 6 \times 12$. Wave elevation histories at the corner $(B/2, -D/2)$ are presented in Fig. 6 and good agreement is obtained. Moreover, sloshing loads on the tank wall due to rotational excitation are validated. Experimental and numerical studies conducted by Chen et al. (2013) are chosen for comparison. Dimensions of the tank are $B \times W \times D = 1\,m \times 1\,m \times 0.3\,m$ and the mesh number is set as $15 \times 15 \times 5$ in the present simulation. Rotational excitation on the tank is in sinusoidal form as

![Fig. 4. 2D sloshing free surface elevation histories at the corner.](image)

![Fig. 5. 3D sloshing free surface elevation histories.](image)
\[ A_{\theta x}(t) = \theta_x \sin(\omega_{\theta x} t) \] with \( \theta_x = 5\text{deg} \). Two cases with \( \omega_{\theta x} = 0.95\text{rad/s} \) and \( \omega_{\theta x} = 3.09\text{rad/s} \) are selected. Pressure histories on the left wall, 0.1m from the initial free surface are captured. Fig. 7 shows that present numerical results in \( \omega_{\theta x} = 0.95\text{rad/s} \) match well with experimental and numerical results obtained by Chen et al. (2013). When excitation increases to \( \omega_{\theta x} = 3.09\text{rad/s} \), larger sloshing is induced and pressure vibration appears. The general tendency of amplitude of sloshing pressure are obtained, which validates the present numerical program.

It should be noted that the applicability of the present numerical program is limited to sloshing with single value free surface profile. Violent sloshing that includes wave breaking and overturning is not considered in the present study.

### 4.2. Sloshing under translational excitation

In this subsection, translational excitation corresponding to vessel surge, sway and heave motions are exerted on the tank, and sloshing response properties are studied.
4.2.1. Sloshing under surge and sway excitation

First, surge excitation on the tank in x direction is considered. To determine excitation frequency influences on sloshing, simulations with, \( \omega_x = 0.8\omega_{10}, \omega_x = \omega_{10}, \omega_x = 1.3\omega_{10}, \omega_x = \omega_{20}, \omega_x = \omega_{30} \) are conducted. Parameters are set as \( B=W=2, A_x = 0.002. \) Wave elevation histories at the free surface corner \( (B/2, -D/2) \) and corresponding spectrum are plotted in Fig. 8. It can be seen that resonant sloshing is induced in cases where \( \omega_x = \omega_{10} \) and \( \omega_x = \omega_{30}. \)

As the corner \( (B/2, -D/2) \) is the standing point of the second order mode profile, a small response is induced in the case where \( \omega_x = \omega_{20}, \) in which the first and third natural frequencies are the dominant response frequencies. In cases where \( \omega_x = 0.5\omega_{10}, \omega_x = 0.8\omega_{10} \) and \( \omega_x = 1.3\omega_{10}, \) amplitude modulated waves are induced. These envelope curves are formed by \( \omega_x \) and \( \omega_{10}, \) which are the dominant response frequencies as shown in Fig. 8(b). Small responses in \( \omega_{30} \) are also induced in these cases.

![Fig. 8. Wave elevation histories (a) and corresponding power spectra density (b) of sloshing due to surge excitation.](image)

![Fig. 9. Time history (a) and spectrum (b) of tank's displacement in white noise excitation.](image)
To have a better understanding of frequency properties of sloshing under surge excitation, a white noise excitation is exerted on the tank in the x direction. The displacement of the tank under such excitation can be expressed as a linear superposition of a series of sinusoidal excitations:

$$dx(t) = \sum_{i=1}^{N} A_x \sin(\omega_x t + \sigma_x),$$

where $A_x$ is the excitation amplitude and has a constant value in the white noise excitation, $N$ is the total number of sinusoidal excitations, $\omega_x$ are excitation frequencies that distribute evenly within the frequency range, $\sigma_x$ are phases of each sinusoidal excitation and are randomly distributed over the range $[-\pi, \pi]$. In this study, $A_x = 0.0001$ is selected and excitation frequency ranges from 2 rad/s to 18 rad/s with a frequency step of 0.1 rad/s. This frequency range is sufficient to capture the main frequencies of the tank. Time history and spectra of the tank displacement are presented in Fig. 9.

It can be seen that power density is distributed uniformly in the range of 2 rad/s to 18 rad/s. Sloshing responses including wave elevation histories and power spectrum are plotted in Fig. 10. Fig. 10(a) shows that wave elevation at the free surface corner $(B/2, -D/2)$ increases gradually and reaches a steady state after $\tau = 60$. Nevertheless, the wave elevation at $(0, 0)$ is small. The wave elevation spectrum at $(B/2, -D/2)$ shows that peak values appear in the first three odd natural modes $\omega_{10}$, $\omega_{30}$, and $\omega_{50}$. As point $(0, 0)$ is the standing point of these three odd natural modes, a much smaller wave elevation is induced. Fig. 11 shows the wave elevation contour along the boundary $y = -W/2$ in the time domain in cases where $\omega_x = \omega_{10}$ and $\omega_x = \omega_{30}$. A shallow water case is also included with parameters $B = D = 10$, $A_x = 0.01$, and $\omega_x = \omega_{10}$. As predicted, the wave elevation around the standing points is not precisely zero and small amplitude travelling waves can be found. In the case of shallow water, the nonlinearity is clearer and significant travelling waves are induced. In addition, it is known that second order resonance can occur when the sum or difference of excitation frequencies is equal to the natural modes (Wu, 2007).

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Fig. 12. Wave elevation at tank corners due to surge and sway excitations. $\tau = 3.132 \quad \tau = 9.396 \quad \tau = 11.902$

Fig. 13. Wave profiles at different time steps.

Fig. 14. Wave elevation at tank corners due to different surge and sway excitations.
Fig. 15. Stability map for vertically excited sloshing.

Fig. 16. Wave elevation histories due to stable and unstable vertical excitations, $A_{z} \omega_{z}^{2}/g = 0.6$

Fig. 17. Three tank models with different initial disturbances.
But such responses cannot be found from the spectrum in Fig. 10(b), which proves that second order resonance is less obvious in comparison to the first order behavior.

Then, sloshing under coupled surge and sway excitation are studied. For rectangular tank, sloshing excited by surge and sway motion share similar properties. In the particular case where $B = D = 2$, $A_x = A_y = 0.01$, $\omega_x = 0.8\omega_{10}$, $\omega_y = 0.8\omega_{01}$, as shown in Fig. 12, sloshing induced by surge and sway excitation will have the same phase at corner $(B/2, -D/2)$, $(-B/2, -D/2)$ where wave elevation will be doubled and have opposite phase at corner $(B/2, D/2)$, $(-B/2, -D/2)$ where wave elevation is cancelled out. Representative wave profiles are presented in Fig. 13. But the wave elevation $(B/2, D/2)$, $(-B/2, -D/2)$ and $(0, 0)$ are not precisely zero due to the sloshing nonlinearity. When excitation parameters differ in surge and sway direction, wave elevations will have two excitation frequency contents. Fig. 14 shows wave elevation histories in the case where $B = D = 2$, $A_x = 0.002$, $A_y = 0.01$, $\omega_x = 0.9\omega_{10}$, $\omega_y = 0.7\omega_{01}$. It can be seen that two periodic modulated waves are excited at corners $(B/2, D/2)$ and $(-B/2, -D/2)$, which are formed by the two excitation frequencies and the first natural frequency. However, at corners $(B/2, -D/2)$ and $(-B/2, D/2)$, modulated waves between the two excitation frequencies are induced. Considering that the first natural mode and excitation frequencies are the dominant response frequencies, corners on the diagonal will have similar wave elevation amplitudes and opposite phases.

**4.2.2. Sloshing under heave excitation**

Vertical excitation on an LNG tank corresponds to the heave motion of the vessel. According to the linear theory, pure vertical excitation on a tank cannot induce sloshing. But when nonlinearity of sloshing is considered, resonant sloshing can occur under certain excitations (Frandsen, 2004; Zhang, 2016). Faraday (1831) discovered this resonant sloshing through experimentation. Benjamin and Ursell (1954) mathematically investigated this resonant phenomenon and found that the solution of vertically excited sloshing can be described by Mathieu’s equation as $\frac{d}{dt} (\delta + 2\varepsilon \cos(2t)) \dot{\xi} = 0$, where $\delta$ and $\varepsilon$ are terms related to excitation frequency and amplitude and can be expressed as $\delta = 2A_\varepsilon^2 \omega_n^2 \omega_0^2$. The stability properties of Mathieu’s equation have been investigated extensively (Abramowitz and Stegun, 1966). For the convenience of analysis of the influence of excitation parameters, parameters $(\delta, \varepsilon)$ are transformed as $(A_0\omega_0^2 / g, \omega_0 / \omega_n)$. Natural frequencies $\omega_n$ for two dimensional rectangular tanks correspond to $\omega_{0\alpha}$ or $\omega_{0\beta}$ in three dimensional tanks. A stability map is shown in Fig. 15, and the first three instability regions for each $\omega_n$ are selected in this study. Test cases considered in the following analysis are marked on the map.

To check sloshing responses in the stable and unstable regions, two cases that are located on the two sides of the boundary line in the stability map are selected (shown as $\bigtriangleup$ in Fig. 15). Excitation parameters are selected as $\omega_{0\alpha} = \omega_{10}$, $A_0\omega_0^2 / g = 0.4$, $\omega_0 / \omega_n$ for the two cases are set as 0.44 and 0.46, respectively. A sinusoidal disturbance has parameters of $A_\varepsilon = 0.001$, $\omega_\varepsilon = 0.8\omega_{10}$ is applied in...
the initial 1s of simulation, which has no effects on vertically excited sloshing stability. Wave elevation histories at corner \(B/W, -W/2\) are plotted in Fig. 16. The two cases have similar wave elevation at the initial stage, in which the initial disturbance is the
Fig. 22. Wave elevation histories (a) and corresponding power spectra density (b) of sloshing due to rotational excitation.

Fig. 23. Sensitivity of rotationally induced sloshing to rotation center for $A_q x = 1\text{deg}$, $u_q x = 1\times 10^2$.

Fig. 24. Wave elevation histories at corner $(B/2, -W/2)$ for $A_q x = 5\text{deg}$, $u_q x = 0.2\times 10^2$.
main contribution to sloshing. Later, resonant sloshing can be observed from the case where \( \omega_n/\omega_z = 0.46 \). As no energy dissipation occurs in potential flow, the free surface elevation in the case where \( \omega_n/\omega_z = 0.44 \) shows little change as the simulation proceeds. Even though these two cases are similar in their excitation parameters, significant differences in sloshing responses are induced for the different stability properties. The existence of the initial disturbance will not affect the stability properties of sloshing. However, for a 3D tank, the disturbance direction will affect the resonant sloshing modes excited.

In the discussion of initial disturbance effects on resonant sloshing modes, three cases with different initial disturbances are adopted. As shown in Fig. 17, the first two cases have parameters where \( B=W=2, \omega_n = \omega_{10}, A_2\omega_z^2/g = 0.6, \) and \( \omega_n/\omega_z = 0.5 \). Sinusoidal disturbance mentioned above is used for the surge direction for case 1 and for the surge and sway directions for case 2. For case 3, tank width \( W = 1 \), and the same disturbance is applied in the sway direction. Under such conditions, sloshing in the disturbance direction of case 1 and 2 are within the instability region. No disturbance is applied in surge direction of case 3, which is also located in the instability region. Wave elevation histories and typical wave profiles are presented in Fig. 18 and Fig. 19. For case 1, first order resonant sloshing is induced in the surge direction. For case 2, first order resonant sloshing is induced in both the surge and sway directions. The same excitation parameters in these two directions cause cancellation at corner (B/2, -W/2) and double frequency at (B/2, W/2). This is similar to sloshing under coupled surge and sway excitation. For case 3, disturbance is applied in the direction that sloshing lies within the instability region and no disturbance is applied in the direction that sloshing lies within the instability region. As a result, no obvious sloshing is induced in this case.

Sloshing properties in instability regions are studied based on excitation parameters. Sinusoidal disturbance in the surge direction with \( A_2 = 0.001, \omega_n = 0.8\omega_{10} \) are used. Firstly, three cases that are located in different instability regions are considered. Parameters are selected as \( \omega_n = \omega_{10}, A_2\omega_z^2/g = 0.6, \omega_n/\omega_z = 0.5 \) (shown as ○ in Fig. 15). Wave elevation histories at the corner (B/2, -D/2) and corresponding spectra are plotted in Fig. 20. For these three cases, all resonant responses can be observed as excitation parameters are located in the instability regions. Resonant sloshing in instability region 1 is much rapidly induced compared with that in instability region 2, and resonant sloshing in instability region 3 is induced slower. Spectrum results show that dominant frequencies of all these three resonant cases are \( \omega_{10} \). For different instability regions in Fig. 15, under the same \( \omega_n \), resonant sloshing in the upper instability regions (corresponding to a low vertical excitation frequency) can only be excited by large excitation amplitudes. Resonant sloshing in the lower instability regions (corresponding to a high vertical excitation frequency) can be excited by excitation amplitudes in a wide range. The effects of \( \omega_n \) on resonant sloshing are analyzed. Two cases with \( \omega_n = \omega_{10} \) and \( \omega_n = \omega_{30} \) are included. Other parameters are chosen to satisfy \( A_2\omega_z^2/g = 0.6 \) and \( \omega_n/\omega_z = 0.5 \) as in test case 1. Histories and corresponding spectrum of wave elevation at corner (B/2, -D/2) are presented in Fig. 21. Fig. 21(a) reveals, as in test case 1, rapid resonant sloshing is induced in the case where \( \omega_n = \omega_{30} \). The spectrum in Fig. 21(b) shows that resonant sloshing frequencies in these two cases all equal the corresponding natural frequencies. For the case where \( \omega_n = \omega_{20} \), no obvious resonant sloshing can be observed. \( \omega_{10} \) and \( \omega_{30} \) are the dominant response frequencies. It can be seen that sloshing spectra in Fig. 21 are similar with these under excitations in the surge direction that shown in Fig. 8(b), except no significant response in \( \omega_{30} \) can be found in case \( \omega_n = \omega_{20} \).
4.3. Sloshing under rotational excitation

In this subsection, sloshing properties under rotational excitation are investigated. Sloshing sensitivity to excitation frequency, amplitude and rotation center location are studied. $d_{\text{axis}}$ is used to denote the distance from rotational axis to the tank bottom.

First, sloshing due to different rotational frequencies is simulated. The tank is excited in a rotational motion where $\theta(t) = A_{\theta_0} \sin(\omega_{\theta_0} t)$, $A_{\theta_0} = 1$ deg. The rotational axis is parallel with the x axis and through the center of liquid where $d_{\text{axis}} = 0.5D$. Fig. 22 shows the wave elevation histories at $(B/2, -W/2)$ under different excitation frequencies as well as corresponding spectra. It can be seen that excitation frequency and first natural mode are the dominant response frequencies, which are similar to those of horizontally excited sloshing. It can be deduced that sloshing responses under coupled sway and rotational excitations are significantly affected by the phase shift between the two excitation modes. For the case where $\omega_{\theta_0} = 0.5\omega_{10}$, no amplitude modulated wave merges as shown in Fig. 22(a); and the spectrum in Fig. 22(b) shows that the sloshing response in the first natural mode is much smaller than that in excitation frequency, which is different from that of sloshing under the surge excitation as shown in Fig. 8(b). This difference can also be found when comparing the spectrum results in the case where $\omega_{\theta_0} = 0.8\omega_{10}$. This is because for sloshing due to rotational excitation, both gravity effects that are related to the inclination of the tank and the dynamic excitation on the tank can induce sloshing. Under low excitation frequency, the tank inclination rather than dynamic excitation on the tank is the main contribution to wave elevation.

Then, the sensitivity of sloshing to rotation axis location is studied. Four cases where $d_{\text{axis}}$ equals 0, 0.5D, D and 1.5D are selected. Wave elevation histories at $(B/2, -W/2)$ are plotted in Fig. 23. It can be seen that both amplitude and phase of the wave elevation are affected by rotational axis location. Considering the boundary condition described in (4), it is known that larger excitation is exerted in the normal direction of tank when rotational axis is away from the tank wall. As a result, sloshing tends to be more violent in cases where $d_{\text{axis}} = 0$ and $d_{\text{axis}} = 1.5D$. Moreover, the excitation on the tank wall will have opposite direction for wall boundaries located above and below the rotational axis. For cases where $d_{\text{axis}} = 0.5D$ and, the rotational axis through the liquid, the excitation on the liquid above and below the rotational axis will have cancellation effects, which will weaken the sloshing response. For cases where $d_{\text{axis}} = 0$ and $d_{\text{axis}} = 1.5D$, dynamic excitation on the wall boundaries are opposite in direction and a 180° phase shift appears between wave elevation histories of the two cases. When the excitation frequency is relatively low, dynamic excitation on the tank is less significant than the gravity effect that is caused by inclination of the tank. Phase shift for the change of rotation axis location will not occur. As shown in Fig. 24, in cases where $\omega_{\theta_0} = 0.2\omega_{10}$, the position of rotation axis has a slight effect on the sloshing phase amplitude. Wave elevation amplitudes in these cases are approximately equal to $\sin \theta_{\text{W}}$. These sloshing properties indicate that the tanks located away from center of the vessels roll motion tend to experience more violent sloshing. When the roll motion frequency is much smaller than the natural sloshing frequency, sloshing is no longer sensitive to the tank location and sloshing can be evaluated using a quasi-static method.

4.4. Sloshing in membrane tank

Sloshing in membrane LNG tank with prismatic shape is discussed in this section. The wedged bottom is formed by cutting off two isosceles right triangles from the low corners of rectangular tank. Mesh scheme of tank with low slopes are shown in Fig. 25. $W_s$ is the width of the bottom and is also nondimensionalized based on...
water depth $D$. In the following discussions, tank has size $B=W=2$, $W_s=1.4$ and $W_s=1$ are selected to check the effects of slope size on sloshing.

Wave elevation histories in tanks under different excitation modes are plotted in Fig. 26, wave elevations in rectangular under the same excitations are presented for comparison. Firstly, sinusoidal excitation is exerted on tank in surge and sway direction, respectively. The excitation amplitude equals 0.005, excitation frequency is set as 0.6$\omega_{01}$, 0.8$\omega_{01}$, $\omega_{01}$ and 1.3$\omega_{01}$. Wave elevation results in the free surface corner $(B/2,-W/2)$ are shown in Fig. 26 (a), (b). It can be found surge and sway motion excited sloshing responses are similar with these in rectangular tank and rarely affected by the slope size in the bottom. Fig. 26 (c) shows the resonant sloshing under instable vertical excitation. The comparison of wave elevation histories shows the existence of slope in the bottom does not change the sloshing stability and the sloshing amplitude has slight decrease in the case with larger slope size. Fig. 26 (d) shows that the sloshing amplitude under rotational excitations is also affected by the slope size. In the cases $\omega_{0k}=\omega_{01}$ and $\omega_{0k}=1.3\omega_{01}$, smaller sloshing amplitude is excited when $W_s=1$. Based on analysis above, it can be known sloshing in membrane tank has similar properties with these in rectangular tank. Cases with free surface located in the slopes are not considered for the limitation of free surface update scheme adopted in this study, related studies can refer to research conducted by Zhang (2016).

4.5. Viscous effects on sloshing

Energy dissipation due to liquid viscosity is not included in the above discussions for the flow is assumed to be irrotational and inviscid. Compared with real conditions, sloshing responses tend to be over predicted and steady state is not achievable. In this subsection, viscous effects on sloshing are discussed using artificial damping models. Two artificial damping models are included in this study, which are respectively related to the flow conditions in the free surface and wall boundaries. Rayleigh proposed an artificial damping model, in which a force that is proportional to the velocity of liquid particles and in opposite direction is assumed in the free surface (Faltinsen, 1978). The Euler equation for a water particle on the free surface can be written as:

$$\frac{D\phi}{Dt} = -\frac{1}{\rho} \nabla p - g \nabla z - \mu \nabla \phi \tag{27}$$

where $\mu$ is the damping value and a damping force is exerted on the free surface when $\mu$ is positive. Based on the assumption that energy dissipation mainly occurs on the boundary layer near a tank wall, Malenica et al. (2003) proposed a method to handle the energy dissipation by modifying the boundary condition on wet walls as:

$$\frac{\partial \phi}{\partial t} = (\mathbf{V} + \omega \times \mathbf{r}) \cdot \mathbf{n} - \frac{\mu}{g} \frac{\partial \phi}{\partial t} \tag{28}$$

An experiment on horizontally excited sloshing is conducted to verify the applicability of artificial damping models, which also provides a perceptual understanding of the sloshing mechanism. A rectangular liquid tank of size $0.65m \times 0.06m \times 0.8m$ is used in the experiment. The test platform is shown in Fig. 27. The liquid tank is made of plexi-glass and deformation of the tank due to water pressure is minimized by reinforcement. A micro servo motor that can output sinusoidal motions according to input parameters is used to produce horizontal excitation on the tank. The motor motion will not be affected by sloshing loads on the tank and no coupling between the tank motion and sloshing loads occurs. Fig. 28 shows the equipment set up. Three pressure sensors are installed in the specific position on the left side of the tank; three wave probes are used to capture free surface elevation on the left, middle and right side of the tank. Pressure and wave elevation data are recorded at a sampling frequency of 25 Hz. As violent sloshing that may induce impulsive loads is not considered in this study, this sampling frequency is sufficient to capture accurate wave elevation and pressure histories. Table 1 lists the four representative cases adopted in this study. Based on these cases, the sensitivity of the artificial damping value to excitation amplitudes, frequencies and fill levels are studied.

Artificial damping models described in (27) and (28) are used for

![Fig. 27. Test platform of sloshing experiments.](image-url)
comparison with experimental results. Mesh number distributed along the tank length and width are 24 × 4, mesh number along the water depth is adjusted according to the fill level. $\Delta \tau = 0.0313$ is selected except in case 4, in which a smaller time step $\Delta \tau = 0.0188$ is selected for large wave elevation in resonant response. Fig. 30 shows the comparison of wave elevation and pressure histories between numerical and experimental results with artificial damping applied on the free surface as in (27). It can be seen that good agreement is obtained when the value of $\mu$ is set at 0.18, 0.12

<table>
<thead>
<tr>
<th>Case</th>
<th>$B$ (m)</th>
<th>$W$ (m)</th>
<th>$D$ (m)</th>
<th>$A_x$ (m)</th>
<th>$w_{10}$ (rad/s)</th>
<th>$w_{6}$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.65</td>
<td>0.06</td>
<td>0.2</td>
<td>0.002</td>
<td>5.952</td>
<td>1.1</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.65</td>
<td>0.06</td>
<td>0.4</td>
<td>0.0035</td>
<td>6.743</td>
<td>0.9</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.65</td>
<td>0.06</td>
<td>0.4</td>
<td>0.0016</td>
<td>6.743</td>
<td>$w_{10}$</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.65</td>
<td>0.06</td>
<td>0.5</td>
<td>0.0025</td>
<td>6.831</td>
<td>$w_{6}$</td>
</tr>
</tbody>
</table>

Fig. 29. Wave elevation and pressure histories with artificial damping applied to the free surface.
and 0.11 with a fill level of 0.2\,m, 0.4\,m, 0.5\,m, respectively. This shows that the artificial damping value should be adjusted when dealing with different filling levels. Fig. 29(b), (c) show that \( \mu = 0.12 \) can predict sloshing when \( D = 0.4\,m \) in spite of variance in excitation amplitude and frequency, which indicates that the value of \( \mu \) is less affected by these excitation parameters. The damping force exerted on the free surface is proportional to the velocity of the water particle at the free surface and the influence of excitation parameters can be explained. Fig. 30 shows the results when artificial damping model is used to modify wall conditions. This model gives good prediction of the four sloshing cases with \( \mu = 0.05 \), though the agreements are not as accurate as those in Fig. 30. This kind of artificial damping model can be used to predict sloshing with different excitation parameters and fill levels. This is because the model produces energy dissipation on all of the wet surfaces on walls and fill level effects can be included. In addition, \( \Delta v/vt \) can reflect the sloshing intensity that is related to the excitation parameters. Both the two artificial damping models benefit from experimental results in determination of the artificial damping value.

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Sloshing properties are significantly affected when considering the liquid viscosity. Firstly, the existence of damping model can lead to accurate prediction of sloshing frequency properties. Wave elevation and pressure histories in Figs. 29 and 30 show that sloshing due to harmonic excitations reach steady state gradually due to damping effects. The damping will dissipate sloshing in the first mode frequency and sloshing gradually reaches a time-periodic state in excitation frequency. In case 3, resonant sloshing is induced at the initial stage, but the maximum value of the free surface elevation is limited due to the existence of damping. It also can be found that the damping force is rather small compared with the gravity force. In the case where $D = 0.2m$, steady periodic states are obtained after $\tau = 100$. In other three cases, modulated waves are still obvious after $\tau = 120$. For the same reason, viscous effects on sloshing are not significant at the initial stage of sloshing, which explains the good agreement between experimental and numerical results without introducing an artificial damping model (Fig. 4).

It should be noted that artificial damping model could not capture the viscous mechanism in real sloshing. But it’s proved to be feasible in applying potential flow theory to predicting real sloshing cases.

5. Conclusions

In the present study, a numerical code based on potential flow theory is developed to predict three dimensional sloshing responses in rectangular LNG tanks under forced excitation. Governing equations are solved numerically by using the boundary element method. Based on the numerical code developed, research on sloshing excited by different excitation models is conducted. Viscous effects on sloshing are investigated by introducing an artificial damping model and comparing them with experimental results. The following conclusions can be drawn.

1) For liquid tanks under surge or sway excitation, the first three odd natural modes tend to excite the most obvious sloshing and second order sloshing is less important. With the increase of
nonlinearity of the free surface, travelling waves at the free surface can be observed.

2) The stability properties of sloshing under vertical excitation are closely related to the excitation parameters. Slight sloshing can be induced when the excitation is located in the stability region, and resonant sloshing will occur rapidly when the excitation is located in the instability region and where initial disturbance exists.

3) Sloshing due to rotational excitation shares similar frequency properties to sloshing under horizontal excitation. The location of the rotation center of the tank is critical to the sloshing amplitude and phase.

4) Without consideration of two phase flow, sloshing responses in membrane tank with slopes in the bottom share similar properties with these in rectangular tanks; vertically and rotationally excited sloshing has decreased amplitude when the slope size increases.

5) Good prediction of sloshing in liquid tank can be obtained by applying an artificial damping model to account for energy dissipation. An artificial damping model that allows modification of the wall boundary conditions is more applicable in simulating sloshing under different fill levels and excitations.

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