Collaborative mechanism design for container sharing and pricing at ports

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Abstract

In this project, a collaboration mechanism in a centralized model will be designed and applied for empty container sharing between two firms. The developed model is based on two firms on two terminals. Both the firms accept the cargoes from consignee so both of them have the demands for the empty container. The two operators charge their customers separately once they accept the cargoes from consignee, which can affect the demand of empty container. Additionally, empty containers can be exchanged between two firms when the containers the operator has is insufficient whereas the other has surplus containers.

Keywords: Collaboration mechanism, Empty container, Centralized model

1. Introduction

With rapidly global trade and economic development of the world in the last few decades, port is playing more and more important and indispensable role for international trading. How to manage and re-allocate empty container takes a huge challenge to port managers. Typically, a reasonable solution is to divide whole huge port area into several terminals as individual operation entity and rent them to private port company to operate separately. Indeed, it is a right way to solve the problem, but it also take a new question to port management, i.e. how to establish a mechanism system to collaborate and coordinate each individual terminal to maximize profit for whole port. Xie et al. (2017) solved the problem of empty container inventory sharing and coordination by using centralized model. According to these researches, therefore, in this research, firstly, several elements are included in the model, which are (1) office of firm 1 and 2 are responsible for operation of product collection, dispatch, receive and transport in terminal A and B respectively, (2) a railway connect two terminals for exchanging empty container and (3) a central planner for operating two firms in the centralized model. Moreover, firm 1 and 2 own empty container at Container Yard (CY) in terminal A and B respectively (Xie et al. 2017). Secondly, in this research, we have two goals. We build a centralized model between two
terminal firms to develop an optimal delivery policy for repositioning empty container, which means there is a central planner operate the whole system. In addition, two variables of initial inventory of empty container and charging price ‘p’ to empty container are considered simultaneously when deciding optimal delivery quantities in centralized model. Therefore, this research is based on this model.

2. Literature review

In this part, we are going to review the literature in terms of empty container repositioning problem and inventory sharing game to support this research.

There are many valuable previous researches about empty container repositioning problem in empty container supply chain management (Florez, 1986; Crainic, Gendreau and Dejax, 1993; Shintani et al., 2007; Song and Zhang, 2010, Zhang, Ng and Cheng, 2014). Some of the results have an important value this research. For example, in the research of Xie., et al (2017), they claimed that linear programming is a most commonly way to formulate empty container repositioning problem and they investigated the sharing and coordination problem for empty container inventory in an intermodal transport. In this system, there are one liner firm in a seaport and one rail firm in a dry port operating the whole port simultaneously. By using integer program, Choong, Cole and Kutanoglu (2002) solved the planning horizon problem of minimizing container repositioning cost in container-on-barge intermodal transportation network. They pointed out that planning horizon need to be carefully considered for slow speed of barge transportation and they think that the transportation way of container-on-barge could be a viable alternative to rail and truck if a long planning horizons is used. (Choong, Cole and Kutanoglu, 2002). In 2014, Pérez-Rodríguez and Holguín-Veras (2009) explored a stochastic model to formulate empty container accumulation process in urban areas over a time expanded network to investigate the influence of profit margins, acquisition costs and storage costs for accumulation of containers at areas which can be used for urban planning and policy making. They final confirm and quantify that demand uncertainty is a key factor which can affect port terminal’s costs significantly. By literature review discussion and industrial practice observation, Song and Carter (2009) developed a research that they identified the critical factors which could affect empty container movement significantly. They adopted three main routes which are Trans-Pacific, Trans-Atlantic, Europe–Asia as an example to quantify the scale of empty container repositioning. They also use this example to contrast and evaluate different strategies of shipping lines and container operators which may reduce the cost of their empty container repositioning behaviour. Moreover, to minimize the expected total costs which includes inventory-holding costs, transportation costs and repositioning costs etc., Dong and Song (2009) investigated empty container repositioning problem in multi-vessel, multi-port and multi-voyage shipping systems considering joint container fleet sizing with circumstance of uncertain and imbalanced customer demand. They also developed a simulation-based evolutionary algorithm to solve the optimization problem which has been approved for its effectiveness by using a case studies. They finally conducted a numerical examples by using several repositioning policy which involves a heuristics repositioning policy (HRP), an evolutionary algorithm-based policy (EAP) and a non-repositioning policy (NRP) for comparison facilitation.

On the other hand, there is a very long history of the research of inventory sharing game. The most classical model of inventory sharing game is ‘Newsvendor model’ which is a basic model well known in operations research and applied economics, then the newsvendor model has been developed in many disciplines. The word of ‘newsboy’ is firstly used by Morse and Kimball (1951) in their research (Chen et al., 2016). Morse and
Kimball (1951) originally explained it as “Let us consider the case of a newsboy who is required to buy his papers at 2 cents and sell them at 3 cents and is not allowed to return his unsold papers. He has found by experience that he has on the average 10 customers a day, and that customers appear at random. How many papers should he buy?” Normally, there are two players including supplier and retailer in newsvendor model, but these two names may vary in different meanings in different research. In inventory sharing game, which is included in the discipline of supply chain management, for instances, Rudi, Kapur and Pyke, (2001) studied inventory transshipment problem between two firms as two players in a newsvendor model. They realized that each local decision makers usually prefer to make their own performance rather than a central planner to collaborate overall performance.

It is now necessary to discuss the knowledge gap between this research and the others to show why this paper is valuable for operational research. To the best of author’s knowledge of relative literature, in centralized model, there is no evidence clearly shown that there existed researches about empty container repositioning problem in the circumstance of uncertain demand for empty container caused by empty container leasing price. (i.e. the demand of empty container is stochastic, and demand is also affected by price charging to empty container). This is the knowledge gap between this paper and previous research. Therefore, we will add the variable of “p” with the meaning of “Basic price charged for leasing empty container” to the centralized model to show demand of empty container is dependent with "p" and it is stochastic. Furthermore, based on this assumption, we will develop the centralized model with a central planner to develop an optimal policy for empty repositioning problem.

3. Model information
In this section, we fully introduce the model we design in this research with more details. For clarity and convenience reading, we will divide this section into 2 parts which are “basic model design” and “centralized model design”.

3.1 Basic model design
The model includes two firms (firm 1 and firm 2) operate two terminals named A and B respectively in one port area. They can both transport and ship cargoes on land or between oversea ports individually and both firms own inventory of empty container in their terminals Container Yard (CY) (Xie et al. 2017). Generally, the empty container of one of the firms can be borrowed by the other firm in some circumstances (e.g. empty container is not enough to satisfy the demand) if it is necessary and we assume that there will be a railway to connect both terminals to deliver empty container. The demand functions (\(X_1\) and \(X_2\)) for terminal A and terminal B are denoted as below:

\[
X_1 = D_1(p) + \xi_1 = a_1 - b_1 p + \xi_1, b_1 > 0
\]

\[
X_2 = D_2(p) + \xi_2 = a_2 - b_2 p + \xi_2, b_2 > 0
\]

Where \(D_1(p)\) and \(D_2(p)\) are the demand function for firm 1 and 2, \(a_1, a_2, b_1\) and \(b_2\) are parameters. \(p\) is the basic payment to firm for leasing empty container. \(\xi_1\) and \(\xi_2\) are error term and they are denoted with probability density function (pdf) is \(f_i(x)\) and cumulative distribution function (cdf) is \(F_i(x), i=1,2\). Furthermore, we assume that there is no demand for the firm 1 at terminal 2 which means that there is no empty container accumulations and inventory at terminal 2 for firm 1 and vice versa.
On the one hand, from the perspective of firms, one firm can deliver their empty container to the other firm as long as they have extra empty container after its self-optimization. Now, it is clear to show that a transshipment game between two firms is what we are going to investigate in this paper. Now, we can show an example to clearly explain the whole process. For firm 1, the empty container inventory in terminal 1 should be firstly used for satisfying its own demand and they should unload all containers from overseas. The unloaded container of firm 1 should be added to terminal 1’s inventory. The unloaded containers of firm 2 should satisfy firm 1’s extra demand in terminal 1 if it is necessary, finally, if there are still some firm 2’s extra empty containers in terminal 1 after demand satisfaction, these empty containers should be delivered to terminal 2 immediately. Vice versa.

On the other hand, for consignors who want to transport their cargoes to overseas, they can choose the firm what they want to export their product to overseas. Firstly, the cargoes should be delivered by corresponding consignor to container freight station (CFS) in firm 1 or firm 2. Secondly, consignor should pay firm 1 or firm 2 a basic fee for leasing empty container and ocean freight cost. Lastly, the cargoes should be transported to the corresponding terminal from firm’s CFS. If the inventory of empty container in any of two firms cannot be satisfied with their own demand, then the basic price for empty container leasing will be changed to fulfill the demand in both terminals and rebalance the demand in whole system. However, there could still exist unsatisfied demands in one of the firms even if port office changing the basic price to affect demand. Therefore, at the same time, under this circumstance, the other firm should transport their extra empty containers to the firm which its own demands are not fully satisfied. In addition, for considering of simplify calculation, it is assumed that the basic price (p) for leasing empty container charged by firm 1 and firm 2 is same. Moreover, for the cargoes imported from overseas, we assume that any arriving container ship can be accepted in both terminals and unloaded regardless the ownership of the container belongs to which firm. Then, the unloaded container should be added to its corresponding terminal’s inventory. However, for example, if firm 1’s container arrive at terminal B and firm 2 has an extra demand for empty container, so this container can be borrowed by firm 2. Otherwise this container should be returned to terminal 1 immediately and vice versa. The whole process can be found in figure 1.

Now, it is better to show the notations for this model below. It has already denoted that $X_1$ and $X_2$ represent the random demands of empty container for firm 1 and firm 2 in the CFS, respectively. Denote $Y_1$ and $Y_2$ are the empty container’s random arrivals in the container freight station of terminal 1 and 2, respectively. We let $n_1$ and $n_2$ are inventory of empty containers at CY of terminal 1 and 2, respectively. Let $r_1$ and $r_2$ are revenue from shipping out per loaded container at terminal 1 and 2. At the same time, we denote $r_l$ is revenue from leasing per empty container for both firms. We also consider a holding
cost per empty container at terminal 1 and 2 are \( h_1 \) and \( h_2 \), respectively. Denote goodwill penalty per empty container at terminal 1 and 2 are \( g_1 \) and \( g_2 \). \( c_t \) is transportation cost per empty container between two terminals.

### 3.2 Centralized model design

The number of empty containers delivered between two terminals \((q)\) and basic price for leasing empty container \((p)\) are the decision variable. Basically, there is a central planner operating the whole port including firm 1 and firm 2 in centralized model. The central planner decides the basic price for leasing empty container and quantity of delivered empty container between two firms. For example, given the inventory of empty container in both terminals, the decision variable of \( q \) and \( p \) can be calculated by the central planner. In opposite, given the delivery quantity and basic price, the inventory in both terminals \((n_1 \text{ and } n_2)\) also can be fixed. It is necessary to define that the value is positive (+q) when empty container delivered from terminal 2 to terminal 1 and is negative (-q) for the opposite. Figure 2 show the whole process for centralized system.

**Figure 2 Centralized model description**

### 4. Centralized Model

In section 4, a centralized model will be developed which means there is a central planner operate two firms. According to this assumption, we are going to develop the optimal empty container delivery policy.

#### 4.1 Profit function

For terminal 1 and terminal 2:

Let \( Q_1(q,n_1,p) \) denote the satisfied demand for empty containers at terminal 1.

\[
Q_1(q,n_1,p) = \text{Emin}\{X_1, n_1 - q + Y_1\} \\
= \text{Emin}\{D_1(p) + \xi_1, n_1 - q + Y_1\} = \text{Emin}\{a_1 - b_1p + \xi_1, n_1 - q + Y_1\}
\]

Let \( I_1(q,n_1,p) \) denote the leftover inventory at terminal 1.

\[
I_1(q,n_1,p) = (n_1 - q + Y_1 - X_1)^+ \\
= [n_1 - q + Y_1 - D_1(p) - \xi_1]^+ = (n_1 - q + Y_1 - a_1 + b_1p - \xi_1)^+
\]

Let \( L_1(q,n_1,p) \) denote the unsatisfied demand at the terminal 1.

\[
L_1(q,n_1,p) = (X_1 + n_1 - q + Y_1)^+ \\
= [D_1(p) + \xi_1 + n_1 - q + Y_1]^+ = (a_1 - b_1p + \xi_1 - n_1 + q - Y_1)^+
\]

Let \( Q_2(q,n_2,p) \) denote the satisfied demand for empty containers at terminal 2.

\[
Q_2(q,n_2,p) = \text{Emin}\{X_2, n_2 + q + Y_2\} \\
= \text{Emin}\{D_2(p) + \xi_2, n_2 + q + Y_2\} = \text{Emin}\{a_2 - b_2p + \xi_2, n_2 + q + Y_2\}
\]

Let \( I_2(q,n_2,p) \) denote the leftover inventory at terminal 2.
\begin{align*}
I_2(q, n_2, p) &= (n_2 + q + Y_2 - X_2)^+ \\
&= [n_2 + q + Y_2 - D_2(p) - \xi_2]^+ = (n_2 + q + Y_2 - a_2 + b_2p - \xi_2)^+
\end{align*}

Let \( L_2(q, n_2, p) \) denote the unsatisfied demand at the terminal 2.

\begin{align*}
L_2(q, n_2, p) &= (X_2 + n_2 + q + Y_2)^+ \\
&= [D_2(p) + \xi_2 + n_2 + q + Y_2]^+ = (a_2 - b_2p + \xi_2 - n_2 - q - Y_2)^+
\end{align*}

Furthermore, the transportation cost for the empty containers is \( c_t |q| \) between terminal 1 and terminal 2. Then, the expected profit function \( \Pi(q, n_1, n_2, p) \) can be written as follows:

\[
\Pi(q, n_1, n_2, p) = (r_1 + r_l)EQ_1(q, n_1, p) + (r_1 + r_i + r_2)EQ_2(q, n_2, p) - h_1El_1(q, n_1, p) \\
- h_2El_2(q, n_2, p) - g_1El_1(q, n_1, p) - g_2El_2(q, n_2, p) - c_t |q| \]

Then, similar to that on page 9 in Cachon (2003), we can rewrite the profit function. Then, the final profit function can be stated below:

\[
\Pi(q, n_1, n_2, p) = K[M - n(M)] + L[N - n(N)] - h_1M - h_2N - g_1E(\xi_1 - Y_1) \\
- g_2E(\xi_2 - Y_2) + (r_1 + r_l)E(Y_1 + a_1 - b_1p) + (r_1 + r_i + r_2)E(Y_2 + a_2 \\
- b_2p) - c_t |q| 
\]

Where \( \overline{\Pi()} \) is the complementary loss function and \( K = r_1 + r_l + h_1 + g_1, L = r_1 + r_i + r_2 + h_2 + g_2, M = n_1 - q - a_1 + b_1p \) and \( N = n_2 + q - a_2 + b_2p \). Moreover, from equation above, it is clear that \( \Pi(q, n_1, n_2, p) \) is not differentiable at \( q = 0 \).

4.2 Optimal delivery policy

In this part, a basic analysis for centralized model will be provided. In order to explain clearly for the steps of calculation and its outcome, we adopt step-wise analysis. Firstly, we will give the proof for special case where \( n_1 \) and \( n_2 \) are given and \( p \) and \( q \) are given respectively. Then, a simultaneous decision-making process where, \( n_1, n_2, p \) and \( q \) are jointly affected will be stated.

We denote \( z_1() \) is the probability density function (pdf) for \( \xi_1 - Y_1 \) and \( Z_1() \) is the cumulative distribution function (cdf) for \( \xi_1 - Y_1 \). Denote \( z_2() \) is the probability density function (pdf) for \( \xi_2 - Y_2 \) and \( Z_2() \) is the cumulative distribution function (cdf) for \( \xi_2 - Y_2 \). Now, we are going to prove \( \Pi(q, n_1, n_2, p) \) is jointly concave in \( q \) and \( p \) where \( n_1 \) and \( n_2 \) are given. Then, based on this calculation of first order condition for the model, the optimal number of delivers between two ports and price charging level from two firms will also be developed further.

**Lemma 1** Given \( n_1 \) and \( n_2 \), \( \Pi(q, n_1, n_2, p) \) is jointly concave in \( p \) and \( q \), the unique optimal \( q^* \) and \( p^* \) are:

For \( q > 0 \):

\[
p^* = \frac{Z_1^{-1}(A) + Z_2^{-1}(B) + a_1 + a_2 - n_1 - n_2}{b_1 + b_2}
\]

\[
q^* = \frac{Z_2^{-1}(B) - Z_1^{-1}(A) + b_2(n_1 - a_1) - b_1(n_2 - a_2)}{b_1 + b_2}
\]
For $q < 0$:

$$\begin{align*}
p^* &= \frac{Z_1^{-1}(A') + Z_2^{-1}(B') + a_1 + a_2 - n_1 - n_2}{b_1 + b_2} \\
q^* &= \frac{Z_1^{-1}(B') - Z_2^{-1}(A') + b_2(n_1 - a_1) - b_1(n_2 - a_2)}{b_1 + b_2}
\end{align*}$$

where,

$K = r_1 + r_i + h_1 + g_1 \neq 0, L = r_1 + r_i + r_2 + h_2 + g_2 \neq 0, M = n_1 - q - a_1 + b_1p, N = n_2 + q - a_2 + b_2p, A = \frac{(b_1 + b_2)(K - h_1) - (r_1 + r_2 + r_i)b_2 + b_2c_t}{(b_1 + b_2)K}$

$$\begin{align*}
B &= \frac{(b_1 + b_2)(L - h_2) - (r_1 + r_2 + r_i)b_2 - b_1c_t}{(b_1 + b_2)L}, A' = \frac{(b_1 + b_2)(K - h_1) - (r_1 + r_2 + r_i)b_2 - b_2c_t}{(b_1 + b_2)K}$
\end{align*}$$

$$B' = \frac{(b_1 + b_2)(L - h_2) - (r_1 + r_2 + r_i)b_2 + b_4c_t}{(b_1 + b_2)L}$$

Lemma 1 describes the connection between basic price $p^*$ for empty container and number of deliveries $q^*$ when both inventory level in two terminals are given. It also reveals that there exists an unique optimal solution for the joint number of deliveries between two firms and charging price setting given both firms' inventory level. From Lemma 2 and satisfied demand function $Q_1(q, n_1, p)$ and $Q_2(q, n_2, p)$, it is very easy to obtain the optimal satisfied demands number for empty containers in whole system as follows:

$$Q^*(q, p, n_1, n_2) = n_1 + n_2 + E(Y_1 + Y_2) - n_i(n_1 - q^* - a_1 + b_1p^*) - n_i(n_2 + q^* - a_2 + b_2p^*)$$

**Lemma 2** Given $p$ and $q$, $\Pi(q, n_1, n_2, p)$ is jointly concave in $n_1$ and $n_2$, the unique optimal $n_1^*$ and $n_2^*$ are:

$$n_1^* = Z_1^{-1}(\frac{K - h_1}{K}) + q + a_1 - b_1p$$

$$n_2^* = Z_2^{-1}(\frac{L - h_2}{L}) - q + a_2 - b_2p$$

where,

$K = r_1 + r_i + h_1 + g_1 \neq 0, L = r_1 + r_i + r_2 + h_2 + g_2 \neq 0, M = n_1 - q - a_1 + b_1p, N = n_2 + q - a_2 + b_2p$

Now, we can easily recognize that Lemma 2 is a newsvendor problem solution. According to Lemma 2, it can be determined the optimal inventory level for each firm when the price charging for empty container and exchanging quantity are given. But there are still some differences between the problem that Lemma 2 solved and classical newsvendor problem solution. For instance, Bell and Zhang (2006) provided a classical newsvendor problem solution for maximizing systems profit by choosing a price when firms' inventory level are fixed. Moreover, Zhang et al., (2010) found that it is relative independent for inventory level in each market segments when prices are given. However, Lemma 2 gives a result for a new newsvendor model that inventory level can be determined for each firm to maximize system's profit when both charging price and the exchanging quantity of empty container between two firms are given simultaneously. Compared with Proposition 1 in Zhang, et al (2010), it is very clearly to notice that it is the value of $q$ in Lemma 2 which is a key factor that cause this difference. Furthermore, the inventory level in each firm are also not relatively independent because of $q$ existence (but the core reason is that the model of Zhang, et al (2010) was based on pricing
Corollary 2 Assume $Z_i(\xi_i - Y_i), i = 1, 2$ increase in $\xi_i - Y_i$, customer order quantity in terminal 1 increases in $r_1, r_i$ and $g_1$, but decreases in $h_1$ and customer order quantity in terminal 2 increases in $r_1, r_i, r_2$ and $g_2$, but decreases in $h_2$. The result is opposite when assuming $Z_i(\xi_i - Y_i), i = 1, 2$ decrease in $\xi_i - Y_i$.

In this case, it is necessary to discuss in different situation because we cannot determine the monotonicity of the function $Z_i(\xi_i - Y_i), i = 1, 2$. Firstly, We assume $Z_i(\xi_i - Y_i), i = 1, 2$ increase in $\xi_i - Y_i$, Corollary 2 reveals that inventory level increases in $r_1, r_i$ and $g_1$, but decreases in $h_1$ for firm 1. Inventory level in firm 2 increases $r_1, r_i, r_2$ and $g_2$, but decreases in $h_2$. Similarly, in the opposite situation, inventory level decreases in $r_1, r_i$ and $g_1$, but increases in $h_1$ for firm 1. Inventory level in firm 2 decreases in $r_1, r_i, r_2$ and $g_2$, but increases in $h_2$. It should be clearly stated that the inventory level in one of the firms is not independent from the price because of $q$ existence, which means the inventory level is related to the value of $q$ when price is given.

Lemma 1 and Lemma 2 provide a method to transform a four-decision variable problem into a two-dimensional research because of computational complexity (Zhang., et al, 2010). Next, we are going to show a unique solution in a general condition.

Lemma 3 $\Pi(q, n_1, n_2, p)$ is jointly concave with respect to $q, p, n_1$ and $n_2$.

Lemma 3 provides a solution to find a decision making among inventory level, delivery quantity and price charging. It shows that $\Pi(q, n_1, n_2, p)$ is concave in $q, p, n_1$ and $n_2$ and it also reveals that there exists a optimal group of $q, p, n_1$ and $n_2$ to satisfy a maximum $\Pi$. Therefore, the optimal transhipments and optimal price charge can be determined. However, due to the computational complexity, we will show the optimal value of each variable in the future research.

Conclusion

In this research, we consider a system including two firms operating two terminals in a port area simultaneously. At the same time, we build a centralized model which means there is a central planner to control the whole system. In this model, two variables considered are 1) the quantity of empty containers that need to be transferred between two terminals and 2) the price charged by each operator. In this decentralized model, the two operators will be treated as a single company, which will lead to perfect collaboration to find an optimal policy for delivery of empty container. Finally, we provide three results which are: (1). Given $n_1$ and $n_2$, $\Pi(q, n_1, n_2, p)$ is jointly concave in $p$ and $q$ and the unique optimal $q^*$ and $p^*$ are determined; (2). Given $p$ and $q$, $\Pi(q, n_1, n_2, p)$ is jointly concave in $n_1$ and $n_2$ and the unique optimal $n_1^*$ and $n_2^*$ are determined. (3). It has been proofed that $\Pi(q, n_1, n_2, p)$ is jointly concave with respect to $q, p, n_1$ and $n_2$ and there exists optimal group of $q, p, n_1$ and $n_2$ to satisfy a maximum $\Pi$. Overall, in a centralized model, we find a policy to determine the quantity of transhipment of empty container between two terminals and constrained with leasing price ‘$p$’ for empty container. However, in this paper, we just focus on solving empty container repositioning problem by using centralized model. It is necessary to keep on research to solve this problem by using decentralized, which means there is no central planner to operate two terminals but using a contract to determine the empty container transhipment.
Reference