Extrapolation Technique for Improving the Effective Resolution of Position Encoders in Permanent-Magnet Motor Drives

Zhaodong Feng and Paul P. Acarnley

Abstract—This paper describes an extrapolation scheme for high-resolution rotor-position prediction using data from a low-resolution encoder. The predictor approximates the previous rotor movement profile by a polynomial equation, which is then used to predict the subsequent rotor motion. The low- and transient-speed characteristics are examined, and it is shown experimentally that the technique can produce effective resolution improvements by a factor of 9. When the rotor velocity reverses, large errors can arise, but it is demonstrated that this problem can be overcome by including motor current data in the position predictor.

Index Terms—Encoder, motor drive, resolution, signal processing.

I. INTRODUCTION

SUCCESSFUL operation of permanent-magnet machines requires the current flow in the machine windings to be synchronized to the instantaneous position of the rotor. For permanent-magnet machines in which the phase flux linkage versus rotor position characteristic is sinusoidal, the phase currents should be modulated sinusoidally with respect to the rotor position to ensure that the drive produces a high output torque with low ripple [1]. Therefore, the current controller must receive high-resolution information about the instantaneous position of the rotor. Machines with trapezoidal flux linkage waveforms (also referred to as brushless dc machines) have a less onerous position information requirement when operating below the base speed, because dc current is applied in “blocks” of duration 120 electrical degrees, so the three-phase machine requires position data with resolution of 60 electrical degrees. However, if the trapezoidal machine runs at speed above the base speed, then field weakening is needed and the phase of the current must be advanced in relation to the rotor position [2]. In these circumstances, higher resolution position information is needed.

The measurement of rotor position requires either an auxiliary device attached to the motor shaft (e.g., an optical encoder or resolver) or a device integrated into the motor (e.g., a Hall-effect sensor or a capacitive transducer). The high-resolution position measuring device always needed for sinusoidal permanent-magnet machines can add substantially to the overall drive package cost, so many researchers have attempted to eliminate the position sensors using estimation techniques [3]. However, a number of authors have suggested that encoder cost can be reduced by using a low-cost, low-resolution encoder, together with a position extrapolation algorithm, implemented in the drive control processor [4]–[7]. Hardware approaches involving a phase-locked loop [8]–[10] are feasible for a drive running at near-constant velocity, but may be unable to deal with transient velocity operation. A computationally intensive Kalman filter [11] is successful in dealing with velocity transients, but is susceptible to mismatch of parameters between the filter’s model and the machine/load. This paper describes the principles of a position extrapolation scheme using a polynomial approximation to the position/time relationship, and using experimental methods, illustrates its performance during steady-state and transient-drive operating conditions.

II. EXTRAPOLATION TECHNIQUE

A. Characteristics of Rotor Motion

The extrapolation scheme receives low-resolution position information from a set of three sensors, which define absolute rotor position with a resolution of 60 electrical degrees (Fig. 1). This sensor arrangement is typical of that used in a three-phase trapezoidal machine. From the sensor data, an appropriate polynomial approximation to the position/time relationship can be deduced, and this approximation is then used to generate a continuous estimate of rotor position.

The mechanical equation of motion for a motor drive can be written as

\[
j \frac{d^2 \theta}{dt^2} = T - T_L \tag{1}\]

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where $J$ is the system inertia, $θ$ is the rotor position, $T$ is the motor torque, and $T_L$ is the load torque. When the motor and load torques are equal, the rotor acceleration is zero and the drive runs at constant speed. The solution to (1) can then be written in the form

$$θ = a_0 + a_1 t$$ \hfill (2)

where $a_0$ and $a_1$ are constants, with $a_1$ being the rotor velocity. However, if the motor and load torques are unequal, the drive accelerates or decelerates. If the motor and load torques are constant, the position/time relationship is

$$θ = a_0 + a_1 t + a_2 t^2$$ \hfill (3)

where $a_0$, $a_1$, and $a_2$ are constants, with $2a_2$ being the rotor acceleration. Therefore, while the drive is operating with motor and load torques that are substantially constant between low-resolution position sensor inputs, the position/time relationship can be approximated by a quadratic function.

### B. Quadratic Fit of Position/Time Data

A high-resolution signal is generated by approximating the input position/time data using a quadratic equation of the form shown in (3).

The coefficients of the quadratic equation are derived from the low-resolution signals. If $θ_k$, $θ_{k-1}$, and $θ_{k-2}$ are the data from sensors at times $t_k$, $t_{k-1}$, and $t_{k-2}$, respectively, as shown in Fig. 2, then substituting into (3) yields the three equations

$$\begin{align*}
θ_k &= a_0 + a_1 t_k + a_2 t_k^2 \\
θ_{k-1} &= a_0 + a_1 t_{k-1} + a_2 t_{k-1}^2 \\
θ_{k-2} &= a_0 + a_1 t_{k-2} + a_2 t_{k-2}^2
\end{align*}$$ \hfill (4)

which can be solved to give the quadratic coefficients

$$\begin{align*}
a_0 &= \frac{D_1}{D} \\
a_1 &= \frac{D_2}{D} \\
a_2 &= \frac{D_3}{D}
\end{align*}$$ \hfill (5)

where

$$D = \begin{bmatrix} 1 & t_k & t_k^2 \\ 1 & t_{k-1} & t_{k-1}^2 \\ 1 & t_{k-2} & t_{k-2}^2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} θ_k & t_k & t_k^2 \\ θ_{k-1} & t_{k-1} & t_{k-1}^2 \\ θ_{k-2} & t_{k-2} & t_{k-2}^2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 1 & θ_k & t_k^2 \\ 1 & θ_{k-1} & t_{k-1}^2 \\ 1 & θ_{k-2} & t_{k-2}^2 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 1 & t_k & θ_k \\ 1 & t_{k-1} & θ_{k-1} \\ 1 & t_{k-2} & θ_{k-2} \end{bmatrix}.$$

### III. EXPERIMENTAL INVESTIGATION OF POSITION EXTRAPOLATION

#### A. Experimental System

Experimental results were obtained using a novel system, as shown schematically in Fig. 3. A high-resolution incremental encoder generating 1024 pulses per revolution quadrature signals provides the reference rotor position. The encoder signals are passed to a 10-bit up/down counter, which generates a 10-bit digital word indicating the rotor position to a resolution of 0.3°. The direction of the up/down count is defined by a forward/reverse direction signal derived from the quadrature encoder signals. An analog position reference signal is produced by feeding the 10-bit digital word into a digital-to-analog (D/A) converter.

The extrapolation technique is tested first by deriving some low-resolution encoder signals by decoding the 10-bit position reference using a logic circuit. Thus, the “synthesized” low-resolution signals are synchronized to the position reference.
These low-resolution signals are used to test the position extrapolation routine implemented in the digital signal processor (DSP). The output of the DSP is the estimated rotor position, with 10-bit resolution, which is passed via the external data bus and a latch to another D/A converter. An analog estimated position signal is therefore available for comparison with the reference.

For the experimental system, the DSP used was an Analog Systems DSP 21061 super harvard architecture single-chip computer (SHARC) floating-point processor with a 40-MHz clock. This DSP was chosen for convenience of programming and flexibility: a commercial implementation of the extrapolation techniques described here could use a lower cost processor. The extrapolation algorithm, based on (3) and (5), was implemented with a program cycle time of 75 µs. The DSP has an interval timer based on a 32-bit counter, which is incremented during each program cycle and reset by the discrete position signals generated every 60° rotation. The interval timer must not be allowed to overflow, and therefore, a limit is placed on the lowest continuous operating speed: the rotor must move 60° in a minimum time of $2^{32} \times 75 \times 10^{-6}$ s, equivalent to $3 \times 10^{-5}$ r/min.

B. Experimental Results

The extrapolation scheme has been tested for a variety of steady-state and transient operating conditions with the results shown in Fig. 4.

1) Transient Velocity: Comparisons of the actual and estimated rotor positions with the motor undergoing transitions between high and low speeds are shown in Fig. 4(a) and (b). In both cases, the motor is running in reverse direction with the position continuously reducing. However, the vertical position scale in these graphs has been reset after every revolution of the rotor, so the position data are constrained within the range $0$–$360°$. Note in particular the shape of the curve around $t = 0.6$ s in Fig. 4(a), where the quadratic form of the position/time relationship is apparent. On the position and time scales used in the graphs, there is a little discernable difference between the reference and the estimated data: information about the integral of absolute error is tabulated later.

2) Sinusoidal Variation of Velocity: A sinusoidal time variation of the rotor position and velocity is used in the standard two-axis “circle test” [12] for machine tools, because it requires the drives to operate over their full range, including speed reversals. Two tests of this type have been performed using the position extrapolator.

Fig. 4(c) shows the results obtained for a large-amplitude sinusoidal variation in rotor velocity. The velocity amplitude (3000 r/min) is sufficient to cause movement of about 20 revolutions in each direction between reversals in direction, which occur at $t = 0.3$ s and $t = 1.1$ s.

Significant position estimation errors are apparent at these reversal points. The reason for these errors lies in the assumption, made in Section II, that the motor torque is constant from the
beginning of position/time data acquisition at \( t = t_{k-2} \) and the instant of position estimation. At the reversal, the rotor velocity is zero, but the acceleration (and therefore, the motor torque) has increased to its peak value. Furthermore, the low rotor velocity causes a large time separation of the low-resolution encoder signals, so the motor torque changes substantially over the relevant time interval. The effect of the changing motor torque can only be taken into account when more low-resolution signals appear after the direction reversal.

The effect is even more noticeable in Fig. 4(d), which illustrates operation with a much lower maximum velocity, causing reversal of the direction at every revolution. To aid clarity in this graph, the rotor position has not been constrained to the usual range of \( 0^\circ \)–360°. Major estimation errors occur at almost every reversal, though the size of these errors is limited by comparing the estimated position values to the low-resolution encoder signals. If a position estimate lies outside the 60° range defined by the encoder signals, then the estimate is limited to the next expected encoder-defined value.

3) Quantifying Estimation Error: The quality of the position estimation can be assessed by evaluating the integral absolute error (IAE)

\[
\text{IAE} = \int |\theta_{\text{act}} - \theta_{\text{est}}| \, dt. \tag{6}
\]

Values of IAE from six experimental tests are shown in Table I. Because the tests are of variable duration, the IAE values have been normalized to a 1-s-duration test. Included in Table I are the results for operation at constant speeds of 390 and 4200 rpm (not shown in Fig. 4).

The results in Table I indicate that the extrapolation scheme has an average error of 3.5° when the drive is operating at constant velocity or changing velocity, but without reversing direction. Larger errors occur when there is a speed reversal, such as that occurring with small-amplitude sinusoidal velocity variations. For comparison, the low-resolution signals produced by a three-sensor system would have instantaneous position error ranging between 0° and 60° with a typical IAE of 30°. Therefore, the extrapolation scheme reduces IAE reductions of at least 2.4 and as much as 9. Nevertheless, the large instantaneous errors of up to 60° observed in Fig. 4(d) at changes of direction are unacceptable, particularly in a vector-controlled sinusoidal drive, where there would be a direct impact on instantaneous torque production, as discussed in further detail in Section IV-B.

### IV. Improving Performance During Speed Reversals

#### A. Motor Movement Model

In this section, the problem of excessive errors during speed reversal is addressed by using a drive control signal as an input to the position estimator. The basis of this development can be understood by considering the relationships between position, velocity, acceleration, and motor torque. At times when the rotor is changing direction, its instantaneous velocity is zero, but acceleration is at a maximum (positive or negative) value. If the motor load is purely inertial, the motor torque is at its maximum value. Therefore, estimation errors can be reduced by including information about the changing torque command in the position estimator at times when the rotor is moving slowly.

In situations where the load torque is proportional to velocity, (1) can be written as

\[
J \frac{d^2 \theta}{dt^2} = T - B \frac{d\theta}{dt} \tag{7}
\]

where \( B \) is a coefficient of friction. If the motor torque is constant, (7) can be integrated to give

\[
J \frac{d\theta}{dt} = Tt - B\theta + c_1 \tag{8}
\]

and this differential equation can be solved

\[
AB\theta = ATt + Ac_1 - T + c_2 e^{-At} \tag{9}
\]

where \( A = B/J \) and \( c_1 \) and \( c_2 \) are arbitrary constants, which can be evaluated by substituting data from known position/time data. If at time \( t = 0 \), the position of the rotor is 0, then substituting into (9), we get

\[
0 = Ac_1 - T + c_2
\]

so

\[
c_1 = \frac{T - c_2}{A}. \tag{10}
\]

Also, the velocity of the rotor at time \( t = 0 \) can be evaluated by differentiating (9)

\[
AB \frac{d\theta}{dt} \bigg|_{t=0} = AT - Ac_2
\]

and substituting from (10)

\[
B \frac{d\theta}{dt} \bigg|_{t=0} = Ac_1. \tag{11}
\]

Equations (9)–(11) are used to investigate the relationship between a motor torque change and the subsequent rotor position. Fig. 5 illustrates a rotor movement in which the motor torque is initially \( T \), causing the rotor to travel distance \( \theta_1 \) in time \( t_1 \). However, if the motor torque reduces to \( T_1 \), at time \( t = 0 \), the rotor takes a longer time \( t_2 \) to travel distance \( \theta_1 \). The equation of motion of (7) is modified after time \( t = 0 \) to

\[
J \frac{d^2 \theta}{dt^2} = T_1 - B \frac{d\theta}{dt}. \tag{12}
\]
This new rotor motion can be described using equations similar to (9)–(11)

\[ AB\theta = AT_1 t + Ac_1 + T_s + c_2 e^{-At} \]  
\[ c_1 = \frac{T_s - c_2}{A} \]  
\[ B \frac{d\theta}{dt} \bigg|_{t=0} = Ac_1. \]  

With torque \( T \), the rotor takes time \( t_1 \) to travel distance \( \theta_1 \) but with torque \( T_s \), the rotor travels the same distance in time \( t_2 \). Substituting into (9) and (13) and equating the left-hand side (LHS) of the two equations, we obtain the following:

\[ AT_1 + Ac_1 - T + c_2 e^{-At_1} = AT_2 t_2 + Ac_1 + T_s + c_2 e^{-At_2}. \]  

This expression can be simplified by noting that at time \( t = 0 \), the instant when the torque changes, the rotor velocity in (11) and (15) has the same value, so \( Ac_1 = Ac_1 \) and (16) becomes

\[ AT_1 - T + c_2 e^{-At_1} = AT_2 t_2 - T_s + c_2 e^{-At_2}. \]  

In (17), the quantities on the LHS, \( t_1 \) and \( \theta_1 \), would be calculated by the polynomial estimator using low-resolution signals prior to time \( t = 0 \). If the information about the motor torque change from \( T \) to \( T_s \) can be obtained from the drive current (torque) command, then (17) can be used to calculate the time \( t_2 \) at which the rotor is expected to arrive at the next low-resolution sensor position.

For example, consider a motor with \( J = 0.1 \text{ kg} \cdot \text{m}^2 \), \( B = 0.01 \text{ N-m-s} \), and \( T = 5 \text{ N-m} \), operating at an instantaneous velocity that would cause the rotor to move 60° in 0.1 s if the torque remained constant. If the torque changes to \( T_s \) at the moment \( t = 0 \), the relationship between \( T_s \) and \( t_2 \) is shown in Fig. 6.

If the torque \( T_s > T \), the time for the same movement is reduced, as shown in Fig. 6(a). On the contrary, in Fig. 6(b), when torque \( T_s > T \), the elapsed time is increased. If the torque is reduced to a negative value (braking torque), the time taken to travel 60° has two values, indicating that the rotor will move past 60° reverse direction and return to the 60° position at a later time, e.g., with \( T_s = -2 \text{ N-m} \), the rotor passes the 60° position after 0.16 s and returns to it after 0.72 s. For large values of braking torque \( (T_s < -3.2 \text{ N-m}) \), the rotor fails to reach the 60° position.

It is not possible to extract an explicit analytic expression for \( t_2 \) from (17), but for a motor drive with defined values of inertia \( J \) and viscous friction constant \( B \), it is possible to use (17) to produce a lookup table relating \( t_2 \) to \( t_1 \), \( T_s \), and \( T \). In comparison to the extrapolation technique described in Section III, this modification requires the knowledge of system mechanical parameters, notably the viscous friction constant \( (B) \), which will inevitably have an uncertain value. However, the effect of this uncertainty is minimal, because the modification is applied only at velocity reversals, where the magnitude of the velocity, and therefore, the torque due to viscous friction \( (B \frac{d\theta}{dt}) \), is small.

B. Experimental Results

As an illustration of the technique for incorporating the motor torque command into the position estimation procedure, the experimental system described in Section III was used with the motor executing a high-frequency, low-amplitude sinusoidal position variation. The initial results [Fig. 7(a)] used the second-order polynomial estimator without the torque command input. As expected, there are large errors (upto 60°) at positions where the velocity changes sign. However, when the torque command is included and the lookup table, derived from (17), is used to counteract estimation errors produced by large torque changes, the accuracy of position estimation is much improved, as shown in Fig. 7(b), where the maximum instantaneous position error is 20°. Such a position error would impact the instantaneous torque production in a vector-controlled sinusoidal drive, by introducing a transient change in the torque angle from 90°.

Fig. 5. Rotor movement change resulting from modified motor torque.

Fig. 6. Relationship between modified motor torque \( T_s \) and time to travel 60°, \( t_2 \). (a) \( T_s > T \). (b) \( T_s < T \).
The quadratic extrapolation technique described in this paper is able to take standard 60° resolution Hall-effect signals and produce a position estimate with a typical accuracy of 3.5°. Provided the drive’s rotation is unidirectional, Errors arise during speed reversals, because the interpolator does not receive any information about changing acceleration from the low-resolution input signals, but these errors can be reduced by including torque command information in the estimation procedure.

V. CONCLUSION

The quadratic extrapolation technique described in this paper is able to take standard 60° resolution Hall-effect signals and produce a position estimate with a typical accuracy of 3.5°, provided the drive’s rotation is unidirectional. Errors arise during speed reversals, because the interpolator does not receive any information about changing acceleration from the low-resolution input signals, but these errors can be reduced by including torque command information in the estimation procedure.

REFERENCES


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