Flood Inundation Modeling with an Adaptive Quadtree Grid Shallow Water Equation Solver

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Abstract: Flood risk studies require hydraulic modeling in order to estimate flow depths and other hydraulic variables in the floodplain for a wide range of input conditions. Currently there is a need to improve the computational efficiency of fully two-dimensional numerical models for large-scale flood simulation. This paper describes an adaptive quadtree grid based shallow water equation solver and demonstrates its capability for flood inundation modeling. Due to the grid dynamically adapting to dominant flow features such as steep water surface gradients and wet-dry fronts, the approach is both efficient and accurate. The quadtree model is applied to a realistic scenario of flood inundation over an urban area of 36 km², resulting from the flood defenses breaching at Thamesmead on the River Thames, UK. The results of the simulation are in close agreement with alternative predictions obtained using the commercially available software TUFLOW.

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**Introduction**

Flood risk analysis typically involves numerical modeling to estimate flood depths (or other hydraulic variables associated with the harmful impacts of flooding) for a wide range of input conditions (Dawson et al. 2005; Dawson and Hall 2006). Sampling the hydraulic boundary conditions, failure states of the flood defense system and other uncertainties sufficiently densely to ensure a converged risk estimate typically involves large numbers (hundreds) of runs of a hydraulic model. It is therefore essential to develop accurate and robust numerical models for accurate prediction of flood flows. In coastal and lowland fluvial inundation, the flow routing is generally two-dimensional and may even be three-dimensional when strong shear layers occur due to interactions between flows in the main channel and the floodplain (Yu and Lane 2006a). Two-dimensional depth-averaged models based on the shallow water equations have been widely reported for simulating local or large-scale flood inundation (e.g. Galland et al. 1991; Bates et al. 1995). However, in the past, the application of two-dimensional inundation models for risk analysis was restricted by the computational power required to undertake large numbers of model runs, and by the lack of topographic information for model parameterization and field data for validation. In recent years, the data constraint has generally been resolved due to the availability of rich sources of high-resolution data provided by remote sensors and airborne scanning laser altimetry (LiDAR) (Mason et al. 2003). Also thanks to improving computer speed, a number of fully two-dimensional numerical models,
such as TELEMAC-2D and TUFLOW, have been used to simulate large-scale flood events (see e.g. Horritt and Bates 2002; Mignot et al. 2007; Néelz and Pender 2007).

Nowadays, flood risk analysis is being applied on an increasingly strategic scale. For example, van Mierlo et al. (2007) interpret water level interactions in the Meuse delta with a view to understanding the effectiveness, in terms of risk reduction, of potential sites for flood detention basins. This type of broad scale analysis requires the ability to resolve accurately the flow at critical points in the system (for example to calculate inflow into the detention basin) while simultaneously having the capacity to predict water level variations on a scale of 10s of km. Furthermore, these analyses need to be repeated for large numbers of boundary conditions and potentially also for large numbers of design options. Therefore, there is a need to improve the computational efficiency of hydraulic models for flood risk analysis. With this in mind, simplified two-dimensional diffusion-wave models have been developed (Bates and De Roo 2000; Bradbrook et al. 2004; Yu and Lang 2006a;b). Typically, these models simulate the flow over the floodplain using a two-dimensional diffusion-wave scheme, whilst the channel flow is assumed to be one-dimensional. The diffusion-wave method (Cunge et al. 1976) moves the two-dimensional flood wave according to the Manning equation and by examining the difference of water surface elevation between any two adjacent cells. The method satisfies mass conservation, but is based on a highly simplified representation of momentum conservation (Yu and Lane 2006a). It aims to predict primarily the flood inundation extent and ignores most dynamical aspects of flood waves in order to simplify the governing equations and thus to reduce the computational cost. The flood depth at an array of points in the floodplain is typically the most important hydraulic indicator variable in flood risk analysis. Convolution of
a probability distribution of flood depth with a function describing the relationship between economic damage and flood depth, forms the basis of commonly used flood risk analysis methods (USACE 1996; Hall et al. 2003). However, other variables, such as the arrival time of the wetted front and the local flow speeds, are significant for analysis of the risks to people and for planning of evacuation schemes (HR Wallingford 2003). Without accurate representation of the hydrodynamic effects, accurate prediction of these variables is beyond the scope of simplified diffusion-wave models.

In this paper, we describe the use of an adaptive quadtree grid shallow flow model for flood inundation modeling. On the quadtree grid, local refinement and dynamical adaptation are easy to achieve so that the detailed terrain of a floodplain is accurately represented and key flow features, such as vortices and bores, are captured, without resorting to a sub-grid-scale method. At the same time, less complicated flow regions are modeled on larger grid cells, providing the capacity to model water level interactions in two dimensions over broad scales. Based on a Godunov-type finite-volume scheme, the model also automatically simulates transcritical flows and predicts wetting and drying. In the following section, the shallow water equation solver is described in outline, with reference to more detailed descriptions in the literature. In Section 3 the code is illustrated by application to a real flood risk analysis problem in the estuary of the River Thames, UK, which has previously been modeled with a commercial 2D code. The merits of the proposed approach are discussed in Section 4 and the conclusions are listed in Section 5.
Adaptive Quadtree Grid Based Shallow Water Equation Solver

The two-dimensional shallow water equations can be used for simulating flood flows, as the horizontal scale of a floodplain is normally much larger than the water depth and so the vertical acceleration of the fluid particles can be neglected and the pressure taken to be hydrostatic. In matrix form, the hyperbolic conservation law formed by the two-dimensional non-linear shallow water equations may be written as

\[
\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} = \mathbf{s},
\]

(1)

where \( t \) denotes time, \( x \) and \( y \) are the Cartesian coordinates, and \( \mathbf{u}, \mathbf{f}, \mathbf{g} \) and \( \mathbf{s} \) are the vectors representing conserved variables, fluxes in \( x \) and \( y \)-direction, and source terms, respectively. The vector terms are given by

\[
\mathbf{u} = \begin{bmatrix} \eta \\ uh \\ vh \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} uh \\ u^2 h + \frac{1}{2} g (\eta^2 - 2\eta z_b) \end{bmatrix},
\]

(2)

\[
\mathbf{g} = \begin{bmatrix} vh \\ v^2 h + \frac{1}{2} g (\eta^2 - 2\eta z_b) \end{bmatrix}, \quad \text{and} \quad \mathbf{s} = \begin{bmatrix} 0 \\ -\frac{\tau_{bx}}{\rho} - g \eta \frac{\partial z_b}{\partial x} \\ -\frac{\tau_{by}}{\rho} - g \eta \frac{\partial z_b}{\partial y} \end{bmatrix},
\]

In the above, as shown in Figure 1, \( \eta \) is the surface elevation above the datum; \( z_b \) is the bed elevation above the datum; the water depth \( h = \eta - z_b \). \( u \) and \( v \) are the depth-averaged velocity components in the two Cartesian directions. \( g = 9.81 \text{ m/s}^2 \) is the gravitational acceleration. \( \tau_{bx} \) and \( \tau_{by} \) are the bed friction stresses. \( \rho = 1000 \text{ kg/m}^3 \) is the density of water. \( \frac{\partial z_b}{\partial x} \) and \( \frac{\partial z_b}{\partial y} \) are the bed slopes in the \( x \) and \( y \)-directions, respectively. The bed stress terms \( \tau_{bx} \) and \( \tau_{by} \) represent the effect of bed roughness on the flow and may be estimated using the following empirical formulae.
\[ \tau_{bx} = \rho C_f u \sqrt{u^2 + v^2}, \text{ and } \tau_{by} = \rho C_f v \sqrt{u^2 + v^2}. \]

The bed roughness coefficient \( C_f \) can be evaluated using \( C_f = g n^2 / h^{1/3} \), where \( n \) is the Manning coefficient. It should be noted that the shallow water equations (1) and (2) are derived by Liang and Borthwick (2007) using a generalized flux-source term balancing technique proposed by Rogers et al. (2001; 2003). A Godunov-type solver based on the present formulation predicts shallow flow hydrodynamics over uneven bed bathymetry without any need for specialized numerical treatment of the source terms. It should be noted that Rogers et al. (2001; 2003) represent the flow depth variables in terms of the free surface elevation above the still water level, whereas in the present formulation the water depth is determined from the free surface elevation and the bed elevation above a given datum. This subtle but important difference facilitates the application of the present model to cases involving wet-dry fronts (see Liang and Borthwick 2007).

The hyperbolic conservation law of the shallow water equations (1) and (2) is discretized on a dynamically adapted quadtree grid using a Godunov-type finite volume scheme with interface fluxes evaluated by the HLLC approximate Riemann solver. Overall second-order accuracy is achieved using the MUSCL-Hancock method. Liang et al. (2004) and Liang and Borthwick (2007) provide a detailed description of the numerical scheme.

To construct the quadtree grid, the problem area under consideration is first fitted into a square domain with the boundary geometry and those areas in need of high-resolution meshing defined by seeding points. An initially irregular quadtree grid is then generated by recursively subdividing the initial square domain into smaller
quadrant cells according to certain criteria. The criterion used is that a cell is divided into four quadrant cells when it contains two or more seeding points and its subdivision level is less than the maximum specified. The quadtree grid is then regularized to ensure that no cell has a side length more than twice that of its neighbors and hence restrict the number of hanging nodes (vertices of cells located along the cell interface of a neighboring cell). The side length \( ds \) of a cell can be calculated via

\[
ds = \frac{L}{2^{\text{lev}}},
\]

where \( L \) is the dimension of the square domain, and \( \text{lev} \) is the subdivision level of the grid cell.

The grid generation algorithm can be illustrated more clearly using the simple example shown in Figure 2. Four seeding points are located at (0.30, 0.16), (0.30, 0.16), (0.57, 0.91) and (0.91, 0.57) inside the unit square domain, as illustrated in Figure 2(a). Here the coordinates of the bottom left hand corner are set to (0, 0). The domain is firstly divided into four equal quadrant cells in Figure 2(b). Then, in Figure 2(c), the subdivision procedure begins by checking the cell containing seeding point 1 for another seeding point. In this example, seeding points 1 and 2 are coincident so that they are always in the same cell. The cell is divided repeatedly until the maximum subdivision level is reached, which is 4 in this case. Next, the checking procedure goes to seeding point 2. Even though another seeding point (seeding point 1) is found, the cell cannot be subdivided further because the maximum subdivision level has already been met. The same procedure is applied to the rest of the seeding points. Figure 2(d) depicts the irregular grid obtained after all the seeding points have been checked. Figure 2(e) shows the final mesh generated after regularization.
In practice, seeding points can be created in any region of interest to obtain a locally fine mesh. However, the seeding points used for describing boundary geometries need to be arranged in closed loops in order to identify whether or not a cell is inside the water domain. Only grid cells within the water domain are regularized, in order to minimize the total number of grid cells. All cells outside the water domain are flagged and excluded from hydrodynamic computation. Setting up the quadtree grid therefore requires user input solely to identify the flow geometry that is specified in digitized form through seeding points.

Generating a quadtree grid is cheap and automatic, and the grid information is stored in a simple hierarchical data structure that facilitates identification of neighboring cells. It is straightforward to add and remove grid cells according to user-specified criteria at intervals during the simulation, thus obtaining a locally high resolution, dynamically adaptive grid. In the present work, in order to capture accurately the complicated flow situation and floodplain topography at the same time, the quadtree grid is adapted according to the following criterion,

$$\Theta = \max \left\{ \sqrt{\left( \frac{\partial \eta}{\partial x} \right)^2 + \left( \frac{\partial \eta}{\partial y} \right)^2}, \sqrt{\left( \frac{\partial \eta}{\partial x} \right)^2 + \left( \frac{\partial \eta}{\partial y} \right)^2}, \sqrt{\left( \frac{\partial \eta}{\partial x} \right)^2 + \left( \frac{\partial \eta}{\partial y} \right)^2} \right\},$$

based on the magnitude of the free surface and bed elevation gradients. During the simulation, grid enrichment is applied up to the maximum subdivision level, when $\Theta$ is greater than a prescribed upper critical value. Grid coarsening is implemented when all four child cells of a given cell have $\Theta$ less than a prescribed lower limit and the subdivision level of these child cells are at least 1 level higher than the minimum
subdivision level of the grid. In dry areas, the value of $\Theta$ is set to zero and no grid adaptation is necessary as the water depth is zero and no flow is predicted.

The dynamical grid adaptation provides an efficient solution to the problem of local cell resolution and is ideal for large-scale flood simulations where the floodplain topography may be locally very complicated and the induced flood flow could be rapidly changing. With a quadtree grid, the geometry of a floodplain can be clearly represented with locally fine grid cells inserted in areas containing small-scale man-made structures, steep bed slopes, changes in vegetation characteristics, etc. Using the routine outlined above, the quadtree grid can also easily adapt to the moving wet-dry interface and other flow features, thus accurately predicting the flood wave front and its routing. Furthermore, compared with other unstructured grid methods (e.g. the unstructured triangular grid used in TELEMAC-2D), a quadtree grid consists of axis-oriented Cartesian cells of different sizes and the governing equations are solved directly in Cartesian coordinates. The benefits of accurate spatial derivatives and high-order cancellation of discretization errors may be exploited.

**Application to Flood Inundation in Thamesmead**

The aforementioned quadtree grid based solver has been intensively validated elsewhere (see Liang et al. 2004; Liang and Borthwick 2007). Herein the numerical solver is applied to simulate flood inundation at Thamesmead, UK, where a processed 10m bare earth Digital Terrain Model (DTM) is available for model setup and alternative results from the commercial software TUFLOW are available for comparison. TUFLOW (e.g. Syme 1991) is a finite difference, fixed grid, alternating direction implicit (ADI) scheme for solving the 2D shallow water flow equations. The
performance of TUFLOW for simulating realistic flood inundation has been evaluated elsewhere in the literature (see e.g. Wicks et al. 2004; Tarrant et al. 2005). After carrying out a comparison with other flood simulation tools, Tarrant et al. (2005) found that TUFLOW provided the modeling system of least risk and so recommended it be used for real time simulation.

Thamesmead is a low-lying area located on the south bank of the estuary of the River Thames, UK. It is densely populated and is protected from tidal flooding by a system of flood defenses constructed to a design water level corresponding approximately to a 1 in 1000-year flood. Hydraulic modeling is required in order to assess the risk to the occupants of the floodplain from breaching or overtopping of the flood defenses. The site has been subject of previous model studies by consultancy companies and researchers, whose results are available for comparison, and so is a suitable case study site.

The simulation is carried out for the 9 km × 4 km Thamesmead site shown in Figure 3, and the four-point stars indicate the gauge point where time history of water depth are recorded and presented later. Seeding points are produced to approximate the geometry of the flood defenses and a disused railway embankment that crosses the site. Figure 4 shows the initial quadtree grid, of highest and lowest subdivision levels of 10 and 6. The side lengths of the largest and smallest grid cells are about 140.6 m and 8.8 m. The level 10 finest grid cells are located in zones close to the initial domain boundary, breach for inflow, and at the railway embankment, where grid resolution needs to be high as these boundary and structural features can have a
significant affect on the interior flow dynamics. Finest grid cells are also generated at those gauge points where temporal change of water depth is recorded.

Bed elevation values from the 10 m DTM are stored in a matrix, and interpolated onto the quadtree grid during grid generation using natural neighbor interpolation. It is noted that, when a new cell is produced during grid adaptation, the bed elevation value is also required at the cell centre and it is directly interpolated from the original matrix (rather than from neighbor cells) again using natural neighbor interpolation to enable more accurate description of bed topography. The lateral boundaries of the floodplain are assumed to be transmissive. However, the boundary conditions do not affect the numerical predictions as the bed elevation at the domain boundary is much higher than the surrounding floodplain and no flow can actually get through. Manning’s coefficient is set to $0.035 \text{ s/m}^{1/3}$ for the whole computational domain throughout the simulation. In order to reproduce previous simulations by TUFLOW, a breach in the flood defense of 150 m in width is simulated, centered at $(545855, 181040)$ as indicated in Figure 3, where the starting coordinates at the left bottom origin point are $(543000, 178000)$. A time series of inflow discharge, given in Figure 5, is input at the breach. For grid adaptation, the upper and lower limits of $\Theta$ are 0.002 and 0.0008. The Courant number takes the value of 0.95 for all the tests.

Simulation is carried out for 10 hours after breaching and results are presented in Figure 6 and Figure 7. Figure 6(a) and Figure 7(a) present the inundation extent and water depth at $t = 1.67 \text{ h}$ and $t = 10 \text{ h}$, comparing with the corresponding predictions by TUFLOW shown in Figure 6(b) and Figure 7(b). Adapted quadtree grids are presented in Figure 6(c) and Figure 7(c). The results agree very well with previous
simulations using TUFLOW, obtained on a resolution equivalent to a 10m uniform
grid. Results from the quadtree model properly reproduce most of the small-scale
effects caused by the complex floodplain topography. The quadtree predictions are
slightly more diffusive at the flood fronts than those predicted by the TUFLOW
model, but this does not have much effect on the overall results in terms of inundation
extent and water depth. The adapted quadtree grids are consistent with the evolution
of the flood flow, and ensure the simulation is of high-resolution while maintaining
optimum numbers of grid cells. From Figure 8, it can be seen that the number of cells
increases rapidly from about 15,000 at 1.5 h to about 45,000 cells at $t = 5.5$ h. The rate
of additional cell generation then declines as the flow becomes less dynamic due to
the cessation of further inflow from the breach. The CPU time for running the case is
about 4 hours on a PC of Pentium IV 3.2GHz with 1GB RAM. However, to obtain
comparative resolution on a uniform 10 m grid, 360,000 cells have to be used to cover
the 36 km$^2$ floodplain, far more than required by the adaptive quadtree grid. Therefore,
running such a flow simulation on a uniform grid of similar resolution is
computationally much more demanding.

Figure 9 further compares the results from the quadtree grid model with those
produced by TUFLOW in terms of water depth history for Gauge 1 located near the
breach at (546025, 180905), as illustrated in Figure 3. As shown in Figure 9(a), the
flow depth increases rapidly to a maximum of about 2.7 m at $t = 1.75$ h, remains
nearly constant until $t = 3.5$ h, and decreases slowly from then on. Figure 9(b)
presents the absolute difference between the temporally varying water depth
calculated by the quadtree model and that predicted by TUFLOW. The absolute
difference is less than 15 cm. The RMSE of 15 cm is equal to the vertical accuracy of
the LiDAR data from which the digital elevation model (DTM) was generated (Bates 2004). Therefore the quadtree predictions again compare well with the results provided by TUFLOW.

The mass balance error should be properly addressed in a hydraulic model (de Roo and Bates 2000). In this work, an approach similar to the zero mass error method proposed by Brufau et al. (2004) is used in dealing with the wetting and drying (Liang and Borthwick 2007). During grid adaptation, the bed elevation at a newly generated cell is directly interpolated from the background DTM while the value for surface water level is interpolated from neighboring cells using natural neighbor interpolation. The surface water level is then locally modified in order to ensure that volume is conserved. This process creates local but not global disturbances to the solution, as suggested by Krámer and Józsa (2007). Figure 10 plots the relative mass balance error between the water stored in the floodplain (there is no outflow in this case) and the mass transfers from the breach due to the inflow. The time history of the relative error oscillates at the beginning of the simulation and settles down to a smooth curve after $t = 2$ hours. Apart from a short period at the beginning, the mass error is within 0.3%.

The convergence property of the adaptive quadtree based simulation is examined by testing the sensitivity of the results to the choice of adaptation criteria. Further simulations are therefore undertaken with half and double the limiting values for $\Theta$, i.e. $\Theta_{\text{min}} = 0.001$; $\Theta_{\text{max}} = 0.004$ and $\Theta_{\text{min}} = 0.004$; $\Theta_{\text{max}} = 0.016$, respectively. Figure 11(a) indicates that the temporal change in cell numbers tend to converge as the limiting values for $\Theta$ reduce. Figure 11(b) compares the time series of water depth at Gauge 2 located at (545835, 179985) as shown in Figure 3. The gauge point is at a
location near the steady wet-dry front where the predicted water depth may be sensitive to local grid resolution achieved by dynamical adaptation. However, Figure 11(b) demonstrates that the change of water depth at this location is identical for all three cases and is not sensitive to the adaptation criteria. This is to be expected since the highest resolution quadtree grid cells are initially generated near to the breach. As the front of the flood wave travels across the floodplain, the dynamically adapting grid retains high resolution at the wet-dry front as well as at rapidly varying flow features. The flow simulation therefore should converge provided sufficiently smaller limiting values are selected for $\Theta$. It should be noted that the results are unlikely to be overly dependent on the exact values chosen, provided they are within a sensible range.

**Discussion**

As indicated by Wicks et al. (2004), the drawbacks of existing two-dimensional floodplain modeling tools are long run time, computational difficulties with supercritical flow, steep water surface gradients, and wetting and drying. Obviously, two-dimensional numerical tools take much longer to run compared against one-dimensional approaches. However, the benefits of the two-dimensional modeling are the potentially more accurate prediction, more reliable flood routing, and the detailed information on flood extent and velocity distribution. An adaptive quadtree grid two-dimensional model is computationally much more efficient than its counterpart on uniform fixed grids as far fewer grid cells are required in order to obtain results of similar resolution (Liang et al. 2007).
The present quadtree model is a second order high-resolution scheme in terms of both time and space. However, in large-scale simulations of real flood inundation, the influence of the high-resolution scheme on the predictions may not be significant when compared with uncertainties in topographic data and information available for calibration of friction parameterization. If a first order scheme were to be implemented, the quadtree grid model should provide further significant gains in computational efficiency.

Horritt and Bates (2001) argue that, given the typical resolution and accuracy of topographic and calibration data, it can be hard to distinguish between the results from 2D diffusion wave inundation models and 2D finite element shallow water solvers. However, judgement as to what level of computational accuracy is justified depends upon the complexity of the flow situation in question and the variables of interest in the analysis. A requirement for accurate velocities and arrival time for the flood wave will increase the attractiveness of full 2D schemes. Moreover, recent years have seen rapid improvements in the quality of topographic data, remotely sensed observations of roughness and remotely sensed observations of flooding (Mason et al. 2003; Néelz et al. 2006), providing renewed motivation to achieve commensurate accuracy in hydrodynamic simulations. Increasing interest in the effect of levee breaches which may suddenly cause supercritical flow conditions in the floodplain provides further motivation for the use of full 2D schemes.

**Conclusions**

A validated adaptive quadtree grid based shallow water equation solver has been developed for flood inundation modeling. The model was applied to the breaching of
a flood defense at Thamesmead, U.K. Close agreement has been achieved with previously obtained results, building confidence in the model’s predictive capability.

In the quadtree model, the governing shallow water equations are solved using a Riemann based Godunov-type scheme. Complicated flow conditions such as supercritical flow, steep water surface gradients or even bore-like fronts, and wetting and drying are thereby automatically and accurately modeled. Furthermore, the numerical model is based on a new formulation of governing equations that properly balance the flux gradient and source terms. This new formulation enables the present numerical model to simulate correctly flow over complicated topographic profiles which are often encountered in practice.

Use of flood inundation for flood risk analysis involves a compromise between predictive accuracy, often over broad scales, and computational efficiency. Because of constraints applied by the need for computational efficiency, 2D shallow water equation solvers have seen rather limited practical application for probabilistic risk analysis. The use of adaptive quadtree models provides considerable computational efficiency gains whilst being able to resolve local flow processes and broad scale water level interactions at the same time. Though the use of quadtree models for very broad scale inundation studies (for example at the scale of an entire river delta) has yet to be proven, this paper has demonstrated considerable potential.
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Notation

_The following symbols are used in this paper:_

\( \tau_{bx}, \tau_{by} \) \hspace{1cm} \text{bed shear stress components in the } x \text{- and } y \text{-directions}

\( \Theta \) \hspace{1cm} \text{free surface gradient adaptivity parameter}

\( \Delta t \) \hspace{1cm} \text{time step}

\( \rho \) \hspace{1cm} \text{water density}

\( C_f \) \hspace{1cm} \text{bed roughness coefficient}

\( ds \) \hspace{1cm} \text{side length of a grid cell}

\( \mathbf{f}, \mathbf{g} \) \hspace{1cm} \text{flux vectors}

\( g \) \hspace{1cm} \text{acceleration due to gravity}

\( h \) \hspace{1cm} \text{total water depth}

\( L \) \hspace{1cm} \text{dimension of the initial square domain for grid generation}

\( \text{lev} \) \hspace{1cm} \text{subdivision level of a quadtree grid}

\( n \) \hspace{1cm} \text{Manning coefficient}

\( \mathbf{s} \) \hspace{1cm} \text{vector of source terms of the shallow water equations}

\( t \) \hspace{1cm} \text{time}

\( \mathbf{u} \) \hspace{1cm} \text{vector of conserved flow variables}

\( u, v \) \hspace{1cm} \text{depth-averaged velocity components in the } x \text{- and } y \text{-directions}

\( x, y \) \hspace{1cm} \text{horizontal and vertical Cartesian co-ordinates in physical plane}
\( z_b \)  \hspace{1cm} \text{bed elevation above the datum}

\( \eta \)  \hspace{1cm} \text{water surface elevation above the datum}
References:


Figure 1  Definition sketch of bed topography for the shallow water equations.

(a)  (b)  (c)

(d)  (e)
Figure 2  Quadtree grid generation: (a) seeding points in a square domain, indicated by ‘+’; (b) grid after first level subdivision; (c) grid after checking the first two seeding points; (d) grid before regularisation; (e) regularised grid.

Figure 3  DTM image of the Thames floodplain at Thamesmead.

Figure 4  Initial quadtree grid.

Figure 5  Inflow hydrograph through the breach.
Figure 6  Flood inundation at $t = 1.67h$: (a) Water depth predictions by the adaptive quadtree based solver; (b) Corresponding results from TUFLOW; (c) Adapted quadtree grid.

Figure 7  Flood inundation at $t = 10h$: (a) Water depth predictions by the adaptive quadtree based solver; (b) Corresponding results from TUFLOW; (c) Adapted quadtree grid.

Figure 8  Time history of the total number of cells in the adaptive quadtree grid.

Figure 9  Temporal change in water depth at the gauge point: (a) time history; (b) absolute difference between the adaptive quadtree grid and TUFLOW solutions.
Figure 10  Mass conservation error with time.

(a)     (b)

Figure 11  Sensitivity to the adaptation parameter $\Theta$: (a) temporal change in cell numbers; (b) time history of water depth at the gauge point.
Figure 1
Figure 2(a)
Figure 2(b)
Figure 2(c)
Figure 2(d)
Figure 3
Figure 5
Figure 6(a)
Figure 6(c)
Figure 7(a)
Figure 7(b)
Figure 7(c)
Figure 8
Figure 9(a)
Figure 9(b)
Figure 10
Figure 11(a)

Number of cells

\[ \Theta_{\text{min}} = 0.001; \Theta_{\text{max}} = 0.004; \]
\[ \Theta_{\text{min}} = 0.002; \Theta_{\text{max}} = 0.008; \]
\[ \Theta_{\text{min}} = 0.004; \Theta_{\text{max}} = 0.016 \]
Gauge 2

Figure 11(b)