Kinematic and static GPS techniques for estimating tidal displacements with application to Antarctica

Short title: GPS Estimates of OTL Displacements

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Abstract

For several decades relative gravimetric measurements have allowed the precise observation of harmonic signals related to Earth body and ocean tides. More recently, GPS data have been shown to be precise enough to allow the determination of antenna displacements at tidal frequencies in three dimensions. In this paper I focus on a comparison between ‘kinematic’ and ‘static’ tidal displacement estimation techniques using GPS data between ~1998.5 and 2003.5 from South Pole (AMUN). The GPS estimates are compared with modelled values using the TPXO.6 and FES99 numerical tide models which themselves are found to be in agreement at the ~1/100 mm level except for $O_1$ and $N_2$. The kinematic estimates are of lower accuracy to the static estimates and the height time series is dominated by non-tidal errors. The best resolved frequencies in the kinematic analysis are solar-related constituents, suggesting the presence of GPS systematic biases. The static analysis agrees with the model estimates, generally at the sub-mm level, with larger errors evident at $S_2$, $K_1$ and $K_2$ frequencies. A time-variable behaviour of $K_2$ is demonstrated. After
combination of all daily data, high correlations (0.7-0.9) are evident between north and east components of each constituent, whilst the remainder of the correlations are less than 0.06. These correlations alter with site latitude and point to the source of the correlations being related to the non-integer ambiguities in the daily GPS estimates which are known to introduce correlations between horizontal and vertical site coordinate components and also change with site latitude. Fixing carrier phase ambiguities to integers may therefore increase the precision of harmonic parameter estimates using GPS.

Keywords: ocean tide loading displacements, static and kinematic GPS, South Pole
1. Introduction

Relative gravimetry currently represents the most accurate measurement technique for
the estimation of loading variations due to tidal mass redistribution, with ongoing
improvements through the use of new technologies [Hinderer and Crossley, 2004].

Alternatively, ocean tide loading displacements may be determined using Very Long
Baseline Interferometry (VLBI) observations with sub-mm accuracies in three
dimensions [Petrov and Ma, 2003; Schuh and Moehlmann, 1989; Sovers, 1994].

However, due to logistical and cost restrictions there are only ~50-100 sites where
either VLBI or long-term gravimetric measurements are available. Consequently there
are many locations where tidal measurements are not being made at present to, for
example, validate ocean tide models.

Recently, it has been demonstrated that GPS data may be used to estimate semi-
diurnal and diurnal tidal loading displacements with accuracies in each of three
dimensions of ~0.5-5.0 mm [Allinson et al., 2004; Khan and Scherneck, 2003; Khan
and Tscherning, 2001]. The global GPS network, including the network of the
International GPS Service, now consists of several thousand sites and offers the
possibility of much greater spatial density of tidal loading displacement measurements
made on a continuous basis (unlike VLBI). Tidal displacements at these sites are also
of interest since any mismodelled tidal signal will propagate into longer period signals
[Lambert et al., 1998; Penna and Stewart, 2003] and bias estimates of tropospheric
water vapour [Vey et al., 2002]. Importantly, GPS, like VLBI, does not require
regular calibration and hence long time series of data may be collected with little
manual intervention. These space geodetic techniques are also not sensitive to local
mass variations and hence may be located at any distance from the coast. The main
The disadvantage of GPS measurements of this type is the satellite orbital period and constellation repeat period is at \( K_2 \) and \( K_1 \) periods, respectively, meaning that systematic errors are prone to map into tidal displacement estimates at these frequencies and their higher order harmonics.

To date, GPS-based studies have concentrated on the determination of diurnal and semi-diurnal parameters, typically the 4 major diurnal constituents (\( K_1, O_1, P_1, Q_1 \)) and the 4 major semi-diurnal constituents (\( M_2, S_2, N_2, K_2 \)). Two main techniques have been applied: i) generation of sub-daily GPS coordinate time series after which a conventional tidal analysis is performed [Vey et al., 2002] or ii) explicit parameterisation at the required frequencies at the static GPS data processing stage on a daily basis followed by combination [Allinson et al., 2004; Schenewerk et al., 2001]. Generally the first technique uses 1-4 h batch coordinate solutions, and if horizontal displacements are to be estimated ambiguity parameters must be fixed to integer values to avoid aliasing of vertical signals into the horizontal estimates [King et al., 2003]. However, this requirement to fix ambiguities does not apply when coordinates are derived every measurement epoch (i.e., in a kinematic solution).

However, the accuracy of kinematic GPS solutions using long time series of onshore data are yet to be demonstrated, although ocean tide measurements have been made using a kinematic Precise Point Positioning (PPP) approach [King and Aoki, 2003] suggesting that this approach may be feasible.

In this paper I compare tidal loading displacement estimates using both the kinematic and static technique using GPS data collected between \( \sim 1998.5 \) and \( 2003.5 \) at the South Pole (-89.9978°S, 139.1822°E, \( \sim 250 \) m from the Pole), hereafter referred to by
its GPS site name, AMUN. The site moves in a northerly direction at the velocity of the ice it is attached to, which is approximately 9.98 m yr\(^{-1}\). This site is of specific interest since diurnal and semi-diurnal Earth body tides are theoretically zero at the Poles [Agnew, 1995; Knopoff et al., 1989], and are \(<0.1\) mm at AMUN. Ocean tide loading displacements do not exceed \(~2\) mm at this site, allowing a test on the accuracy of tidal displacement measurements using GPS at different tidal frequencies.

A near-perfectly symmetric GPS satellite constellation exists at AMUN, meaning assessment of the measurement techniques may be made while several potential sources of error are eliminated. However, observations are limited to satellites at low elevation angles due to the satellite orbital inclination, reducing the geometric strength of the solution and tropospheric conditions are not be typical (on a global scale) due to the extremely dry atmosphere in Antarctica.

The major semi-diurnal, diurnal and fortnightly constituents were each estimated and the results compared with modelled ocean tide loading displacement measurements from several recent ocean tide models using the SPOTL software [Agnew, 1997]. I also show the effect of time-variable \(K_2\) errors in the static approach and discuss methods for mitigating their effect on the other constituents and the impact of ambiguity resolution on the static solutions.

2. Diurnal and semi-diurnal signals

2.0 Model estimates

Ocean tide loading displacements were computed using the SPOTL software. Until recent years the Antarctic coastline has been poorly determined [e.g., Fricker et al., 2001], and consequently the care taken by D. Agnew to include an up-to-date
Antarctic coastline in SPOTL is important. Estimates from two recent models were computed, namely TPXO.6 [Egbert et al., 1994; Egbert and Erofeeva, 2002] and FES99 [Lefevre et al., 2002], and these are shown in Tables 1-3. P₁ is not included in FES99 and hence it is not shown. Agnew [1995] showed that a predecessor of FES99, FES95.2, was at the time the most accurate model for AMUN when compared against the relative gravity record there. Comparison of these more recent models with the same gravity analysis results [Knopoff et al., 1989] show these models to be in closer agreement with the gravity than FES95.2 [King et al., in press]. The largest model differences are evident for N₂ and O₁ with vector differences close to 0.2 mm. Apart from these, the agreements are at the level of a few hundredths of 1 mm.

2.1 Kinematic GPS technique

King and Aoki [2003] showed that it is possible to measure ocean tide signals using only a single GPS receiver with the Precise Point Positioning GPS data processing approach [Zumberge et al., 1997]. This technique allows the determination of site coordinates by fixing satellite orbit and clock and earth orientation parameters at values from an earlier global analysis. Site coordinates may be generated up to once per measurement epoch in a Kalman Filter (or similar), with the site motion constrained through the choice of filter process noise. Subsequent experience [e.g., Bindschadler et al., 2003] has shown that epoch-to-epoch repeatabilities are ~0.02 m and 0.05 m in the horizontal and vertical components respectively at high latitudes over snow/ice surfaces.

The same procedure was largely followed for the estimation of tidal displacement signals at AMUN using the GIPSY/OASIS software [Webb and Zumberge, 1995].
Data between 1998.5 and 2003.5 was processed in 30 h batches and to reduce edge effects only the central 24 h retained. Fiducial satellite orbit and clock products from the Jet Propulsion Laboratory (JPL) were used. An elevation cut-off angle of 7° was used and Earth body tides modelled. Site motion was loosely constrained using a random walk standard deviation [Lichten, 1990] of 4.2 mm/√h, with tropospheric zenith delay (TZD) constrained at the level of 4.8 mm/√h. Horizontal tropospheric gradients were not estimated. Site coordinates were generated every 5 min following backward smoothing, although data was decimated to hourly intervals prior to tidal analysis [Pawlowicz et al., 2002].

All constituents that could be separated by at least one complete period from their neighbouring constituents over the length of data record (i.e., a Rayleigh criterion of 1 [Godin, 1972]) were solved in this analysis (68 in total), plus a linear drift to allow for vertical motion of at the AMUN site. A different analysis solving for a reduced set of constituents gave only negligibly different results. Non-tidal (systematic or random) noise dominates the time series since tidal signals explained less than 0.1% of the time series variance. Constituent noise estimates were estimated using a coloured noise approach [Pawlowicz et al., 2002] and the vertical estimates are shown in Table 1. The level of agreement between the model and kinematic estimates is not uniform across constituents. For example, the M$_2$ estimate is almost an order of magnitude too small, even considering its uncertainty. The other constituents are in generally better agreement, although they often still differ given their uncertainties.

Solar-related constituents have the greatest signal-to-noise ratios (SNR) (defined as the square of the ratio of estimated amplitude to formal amplitude error from the
harmonic analysis), with $P_1$, $S_1$, $K_1$, $S_2$ and $K_2$ having SNRs greater than 2. Of the lunar constituents, $M_2$ had a SNR of 1.1, with the remainder of the constituents having SNRs below 1. The dominance of the solar constituents is suggestive of GPS systematic effects, especially with the presence of a 1.5 mm $S_1$ signal, other than $K_1$ the largest amplitude determined. Given that $S_1$ ocean and atmospheric tide loading displacements are several orders smaller than 1 mm at this site [Ray and Egbert, 2004; Ray and Ponte, 2003], either diurnal tropospheric gradients, orbit mismodelling or second order ionospheric disturbances [Kedar et al., 2003] are the most likely sources of this signal. The failure of the approach to detect the ~1.1 mm $M_2$ signal means that realistic uncertainty estimates for the constituents are at least comparable to the magnitude of the determined signals. On the other hand, the lack of wildly erroneous constituent estimates does show that a lower accuracy bound for the technique is approximately 1-2 mm.

The horizontal estimates are shown in Tables 2 and 3. The agreements between the models and the kinematic estimates are mixed, with the disagreement with $M_2$ again evident in the north component where the kinematic estimate is lower (as with the vertical estimate). Overall, the differences are generally larger than the kinematic estimate uncertainties.

2.2 Static GPS technique

Estimation of harmonic parameters in static GPS time series has been demonstrated by, for example, Allinson et al. [2004] and Schenewerk et al. [2001]. The approach used in this study was similar to that of King et al. [in press], using GIPSY/OASIS to estimate parameters at eight tidal frequencies for each of three dimensions, using JPL
non-fiducial orbits and clocks. The only difference was due to the high site velocity (~0.03 m/day) which, if uncorrected, could bias the harmonic estimates [King, 2004]. So, before analysis in GIPSY the horizontal velocity was first removed from the daily RINEX files [King et al., 2000] using a velocity determined using a conventional multi-year PPP solution. The solutions shown here for AMUN are therefore an improvement on those shown in King et al. [in press]. After this correction to the RINEX data the analysis continued in GIPSY. To overcome potential numerical instabilities due to the closeness of the tidal frequencies, constraints of 0.2 m and 0.02 m were placed on each initial parameter estimate in the vertical and horizontal components respectively. These constraints were chosen to be several times the largest possible constituent estimate, and hence do not overly constrain the estimates. An elevation cut-off angle of 7° was used and Earth body tides were modelled. Tropospheric zenith and horizontal gradient estimates were made at each measurement epoch (5 min after pseudorange smoothing and decimation), using random walk standard deviations of 10.2 mm/√h and 0.3 mm/√h, respectively. The daily harmonic parameter estimates were then combined together with their variance-covariance information in a Kalman Filter with zero process noise (effectively a weighted average). The daily unit variances were computed on the first iteration and on a subsequent iteration these were used to scale the daily variance-covariance matrices and obtain the final harmonic parameter estimates.

The parameter correlations (R) derived from the variance-covariance matrix for a single daily solution (harmonic parameters only) are shown in Figure 1a. Examining R shows values greater than 0.5 between $S_2$ and $K_2$ for east, north and vertical, respectively. Since these constituents are the closest in frequency of those estimated
(separated in period by only 0.03 h), a maximum R of 0.7 reflects that numerical
stability of the daily solutions. Following the combination of all daily solutions
between ~1998.5 and 2003.5, the values of R are as shown in Figure 1b. The majority
of the constituent estimates are now decorrelated, with R<0.06. For each constituent
however, the east cosine component is correlated with the north sine component and
the north cosine component correlated with the east sine component, resulting in the
pattern shown. These vary according to constituent species; for the diurnal
constituents they are all close to 0.7 and for the semi-diurnal constituents they are
close to 0.9. The existence of these correlations reflects the site latitude and GPS
satellite constellation geometry since repeating these experiments at other latitudes
produced a slightly different pattern to Figure 1b, with increased correlations between
the horizontal and vertical components (Figure 2). While further investigation is
required, these may be reflective of the presence of non-integer carrier phase
ambiguities in the daily solutions. It has been previously demonstrated that choosing
not to fix these parameters to integers may introduce correlations between the east and
vertical [Blewitt, 1989; King et al., 2003], although different correlations occur at
high latitudes [King, 2004].

The vertical harmonic parameter estimates from the combined solution are shown in
Table 1. The static estimates are closer to the model estimates than the kinematic
estimates, with good agreement in phase. In fact, apart from K1 and S2, the phase
obtained from the static GPS estimates are in agreement with the model estimates.
Given the uncertainties on the static estimates, neither model is preferred for N2 or O1.
The static S2, K2 and K1 estimates are in worse agreement, having amplitude
differences of >1 mm and as with the kinematic solutions their solar origin is
suggestive of GPS-related systematic errors. The differences for O₁, P₁ and Q₁ are also systematically too large, although for O₁ and Q₁ these are within the uncertainties. The horizontal components (Tables 2 and 3) further confirm the generally closer agreement with the models than found when comparing the kinematic estimates.

As the daily solutions are combined together in a Kalman Filter, parameter estimates may be examined following the addition of each additional day of data. The time-variation of the parameter estimates is shown in Figure 3, for amplitude and phase for each of the eight constituents, relative to the final estimates obtained after all data was added. The K₂ amplitude estimate does not stabilise during the period of the observations, whilst the other constituents obtain reliable estimates with ~900 daily solutions. In comparison, Q₁ has a similar predicted amplitude to K₂ and it stabilises on its final estimate very quickly. The slow convergence rate in K₂ amplitude suggests a time-variable behaviour to this constituent with presently unknown origin, although it is presumably related to the changes in the GPS satellite constellation made up of satellites with an orbital period being nominally at this frequency.

The time-variable behaviour in K₂ will impact on the other constituents through their covariances. To assess the possible effect, the daily static solutions were recombined, but with the addition of 1 mm² process noise to each of the K₂ parameters. The results are shown in Table 1. Due to the process noise, the K₂ estimate follows the measurements more closely and hence its estimate changes significantly from the zero process noise solution, with a corresponding increase in its uncertainty. Of the other constituents, S₂ is altered by the greatest amount, with a 0.6 mm increase in amplitude, showing the effect of the S₂/K₂ covariance. Further tests using a 25 mm²
process noise of $K_2$ yielded a further change in $S_2$ amplitude, although it still did not resemble the modelled estimates, possibly due to real solar-related variations being present in the GPS data.

3. Discussion and conclusions

I have shown that using either the static or kinematic estimation techniques tidal displacement parameters may be determined with an accuracy of ~0.5-1.0 mm at the South Pole, apart from $K_1$ and $K_2$. The GPS data were not of sufficient precision to separate FES99 and TPXO.6 for the two constituents where they differed most ($N_2$ and $O_1$).

The time-variable nature of the daily $K_2$ estimates was also demonstrated and the addition of process noise to this parameter improved the agreement of $S_2$ with the model results, although unmodelled solar-related effects are evident in both the static and kinematic results. Changes in processing strategies at JPL during the period of the orbit computation may have an influence on $K_2$ over time, as would satellite-specific phase centre mismodelling such as shown by Ge et al. [2005]. Tropospheric gradient parameters were not estimated in the kinematic solutions, and a spectral analysis of these parameters from the static solution suggested that this may be a source of some of the $S_1$ signal evident in the kinematic time series. The satellite products may have affected the relative accuracies of each of the respective techniques. Fiducial orbit and clock products were used in the kinematic analysis with fiducial-free products used in the static analysis, although the magnitude of this effect is not known. For the remainder, orbit mismodelling, multipath and higher-order ionospheric effects are likely sources of error, the later of which should be readily removed in the near future.
[Kedar et al., 2003]. Ambiguity fixing is a further avenue for improving the static technique since this will decorrelate the three coordinate components which are presently strongly correlated.

Only the major semi-diurnal and diurnal constituents have been discussed in detail here. There are significant difficulties in estimating longer-period tidal signals such as $M_f$ and $M_m$ due to propagation of mismodelled shorter period signals using conventional analyses [Penna and Stewart, 2003]. Both of the estimation strategies described here should allow the determination of longer period signals without the possibility of this problem. However, inspection of $M_f$ and $M_m$ estimates from the time series shows them to be accurate within 1-2 mm, although not significantly different from zero (the modelled estimates are ~1 mm). Before these may be determined much longer time series are required and the correction for other loading effects (e.g., atmospheric loading displacements) considered.

While the results shown here suggest GPS tidal displacement estimates are not yet as consistently accurate as those from VLBI, promised global navigation satellite system (GNSS) changes (the launch of the Galileo constellation and the replenishment of the GLONASS constellation) should in the future assist in the identification and removal of systematic errors at tidal frequencies (due to their orbital periods being at non-tidal frequencies) and add extra observations to further reduce random errors. The extension of the GPS time series as new data are collected will further reduce the random errors and allow more accurate separation of tidal constituents. As a result, GNSS measurements of tidal displacements will increase in both accuracy and precision in the coming decades.
Acknowledgements

This work would not have been possible without the foresight of Larry Hothem (USGS) to install a GPS receiver at AMUN during the early 1990s. We also thank JPL for providing GPS orbits and clock products, Frank Wu for providing modified GIPSY routines and Duncan Agnew for making his SPOTL software freely available. I also thank Hans-Georg Scherneck, two further anonymous reviewers and Editor Rüdiger Haas for their comments which helped to significantly improve this paper.

References


Figure Captions

Figure 1: AMUN harmonic parameter correlations (R) from one daily estimate (a) and after combination of all daily estimates (b). Parameter numbers 1-6 are the $M_2$ east, north, vertical cosine parameters, followed by the sine components. In the same way, these are followed by the parameters relating to $S_2$, $N_2$, $K_2$, $K_1$, $O_1$, $P_1$ and $Q_1$. 
Figure 2: Same as Figure 1, but for CAS1 (66.3°S, 110.5°E)
Figure 3: Harmonic parameter amplitude estimates after the addition of each daily solution, relative to the final estimate, for east (top), north (middle) and vertical (bottom).
Table 1: Estimates of AMUN vertical site displacement amplitude (top, in mm) and phase (bottom, in degrees) based on the TPXO.6 and FES99 models and GPS data.

Phase lags are negative. \( P_1 \) is not modelled directly in FES99. Also shown are the formal errors (one standard deviation) for the GPS-estimated amplitudes from both the kinematic and static analyses.
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<th>Constituent (East)</th>
<th>Constituent (North)</th>
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<td>$S_2$</td>
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<td>TPXO.6</td>
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<td>0.28</td>
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<tr>
<td></td>
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<td>47.2</td>
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<td>FES99</td>
<td>0.65</td>
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<td></td>
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<td>Static</td>
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<td>0.5±0.6</td>
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<td>Static + PN</td>
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<td>0.4±0.8</td>
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<td>Table 2: Same as Table 1, but for the East component.</td>
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<td>125.6</td>
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