Elimination of Subharmonics in Direct Look-Up Table (DLT) Sine Wave Reference Generators for Low-Cost Microprocessor-Controlled Inverters

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Abstract—This paper investigates distortion of an inverter reference waveform generated using a direct look-up (DLT) algorithm. The sources of various distortion components are identified and the implications for application to variable speed drives and grid connected inverters are described. Harmonic and subharmonic distortion mechanisms are analyzed, and compared with experimental results. Analytical methods are derived to determine the occurrence of subharmonics, their number, frequencies and maximum amplitudes. A relationship is established identifying a discrete set of synthesizable frequencies which avoid sub-harmonic distortion as a function of look-up table length and a practical method for calculation of the look-up table indices, based on finite length binary representation, is presented. Real time experimental results are presented to verify the analytical derivations.

Index Terms—Distortion, harmonic, inverter reference waveform, subharmonic.

NOMENCLATURE

N	Number of table elements.
F_s	Sampling rate.
Δ	Table increment.
f_o	Synthesized frequency.
n	Sample number.
Φ	Initial phase angle.
W	Integer part of table increment.
L, M	Integers, L and M have no common factors, i.e., they are relative prime.
$|\Delta f|$	Frequency deviation.
k	Harmonic number.
A	Harmonic amplitude.
f_{base}	Base frequency.
DLT	Direct look-up table.
SNR	Signal to noise ratio.
TD	Total distortion.
THD	Total harmonic distortion.
PLL	Phase locked loop.
LSB	Least significant bit.

I. INTRODUCTION

INCREASING usage of inverters sourcing power to motors, power grids and other sensitive loads has led to concerns over the quality of power generated by them and numerous articles have been published presenting harmonic distortion analyzes of power generated by inverters. In majority of the cases, the sources of harmonic distortion are associated with inverter control, measurements and device nonidealities. However, it is clear that the spectral purity of the inverter output can only be as good as its [1]. Therefore factors affecting the quality of the inverter reference waveform are of increasing importance.

It is also noted that this problem has been adequately dealt with in precision waveform synthesis with application to communications, acoustics and speech recognition [2]–[4]. However the implications of this in the context of inverter power quality have not been investigated. This paper investigates different types of distortions resulting in the degradation of the inverter reference waveform, their implications and methods of suppression.

A. Sources of Harmonic Distortion

Among the many different methods used to synthesize reference waveforms for inverters (treated in detail in Section II), the DLT method has wide spread application. Three main factors are found to influence the harmonic content of a waveform synthesized via the DLT method.

- *Finite word length of the table values*:
  This affects the vertical quantization of waveform samples [2]. Processors with adequate word length (24 or 32 b) reduce the error to a very low level. Detailed analysis of this for shorter word lengths has been presented in [5] and is not covered further in this paper.
- *Effect of table length N:* This has an overall effect on the waveform and is treated in Section III.
- *Non-uniform sampling effect*: This is caused by table index truncation and is the most significant source of distortion. It is treated in Sections III and IV.

B. Implications for Drives and Power Systems

Two main areas where distorted waveforms from inverters cause problems are variable speed drives (VSD) and grid connected power sources. In VSDs a sinusoid with varying frequency is required as a reference to the inverter. Harmonically distorted reference waveforms (generated via DLT method) would obviously deteriorate the quality of output power feeding

the motor. As a consequence, increased heating in inverter fed machines would lower efficiency and decrease operational life. Another problem caused by harmonics is torque oscillations in machine shafts. In multi-mass drive systems with low damping, torsional resonance could lead to fatigue and shaft breakage. An example is given in [6] where an interharmonic of 350 kW was responsible for fatigue failure of a 775 MVA Kramer drive. In power systems, grid connected inverters are now widely used to supplement power requirements. These inverters work in a similar way to that used in VSDs except that they feed power to a synchronised grid. Harmonics produced by grid connected inverters excite resonance and result in overvoltages and overcurrents, accelerating thermal ageing of insulation in devices connected to the grid. A more significant problem is caused in power systems by sideband frequency components occurring below the fundamental (subharmonics). Modern line transformers (especially armophorous core type) with high levels of permeability are very sensitive to even small levels of dc [7]. Therefore dc and low frequency components injected to the grid will result in line transformer saturation.

Current standards limit dc currents injected to the grid to, 5 mA (G77 [8]) and 0.5% (IEEE [9]) of the fundamental current component. Here, it is worth noting that, as the number and power ratings of grid-connected inverters increase, the damage caused by low amplitude current subharmonics may be considerable.

II. DIGITAL REFERENCE WAVEFORM SYNTHESIS

In the widely used PWM scheme for single phase inverters, a reference sinusoid is compared with a triangular carrier wave and is used to generate the switching signals for the inverter bridge. Different techniques to generate the sine reference are proposed in the literature. Detailed analysis of these techniques and their relative merits is outside the scope of this paper but the interested reader could find them in [10], [11]. Table I of [10] identifies a trade-off between three main criteria, which are fundamental to the reference waveform synthesis problem. They are:

1) waveform quality;
2) run-time efficiency;
3) allocated memory.

Comparing these methods, the DLT method has been shown to provide a good compromise between efficiency and accuracy. Therefore this method is yet by far the most commonly used method for reference waveform generation in inverters, particularly those employing low-cost microcontrollers.

III. DIRECT LOOK-UP TABLE (DLT) METHOD

A table of N values of a complete sine wave cycle with an angle resolution $(2\pi/N)$ is stored in memory. The sinusoidal waveform is generated by accessing the table at a given rate f_s Hz. The frequency of the synthesised waveform is determined by the amount (Δ) by which the table index is incremented in each sample period. Suppose the table index is accessed in in-

crements of Δ at the rate of f_s Hz. This would produce a sine wave of fundamental frequency

$$f_o = \frac{\Delta \cdot f_s}{N} \text{ Hz}, \quad \Delta \le \frac{N}{2}. \tag{1}$$

The maximum value of this is bounded by $N/2$, which is the limit of Δ to avoid aliasing. The synthesized waveform is a set of discrete values, given by

$$X(n) = \text{Sin}\left[\Phi + \text{int}(n \cdot \Delta) \cdot \frac{2 \cdot \pi}{N}\right]. \tag{2}$$

Note that although Δ is in general a real number, the actual table index used must be an integer.

A. Non-Integer Δ Implementation

Consider a particular case of an inverter sourcing power to the grid. The synchronised inverter will be prone to grid frequency fluctuations, typically of $\pm 2\%$. The synchronization device (say a PLL) determines the grid voltage frequency and demands a sinusoidal reference to be generated via the look-up table. However, since the table access algorithm can only work with integer index values, a truncation of Δ will be done to the nearest whole number. Therefore, table values will be clocked out with a nonuniform phase increment resulting in a harmonically distorted sinusoidal waveform. Table I and Fig. 1 show a case where a look-up table is accessed in noninteger increments. The truncated index column of the table shows the effect of nonuniform sampling due to the noninteger Δ value.

B. Analysing the Non-Uniform Sampling Effect

Referring to Table I, one could observe that frequencies which result in noninteger Δ values, produce the effect of nonuniform sampling of a sinusoid.

A noninteger Δ can be decomposed to

$$\Delta = \left[W + \frac{L}{M}\right] \tag{3}$$

where W, L, M are integers and L, M are relative prime numbers (i.e., they share no common factors)
for the case in Table I: $\Delta = 1.25, W = 1, L = 1, M = 4$.

Consideration of Table I shows that the truncated index coincides with the ideal index with a period of M samples (or M/F_s s). This interesting observation explains that, as M is increased, the frequency of the output samples being equal to that of the ideal waveform becomes less, thus increasing the level of distortion of the synthesized waveform. The distortions manifest itself in the form of different frequency components. These frequency components are split in to 4 groups, which are defined in Table II, [12].

The concept of total distortion (TD) has been used to account for distortion due to different frequency components in the reference waveform. (TD should not be confused with THD where only harmonic components account for waveform distortion). TD is defined as the ratio of the sum of powers of all distortion frequency components to the power of the fundamental frequency as (4) shown at the bottom of the next page. A denotes the amplitudes of various frequency components.
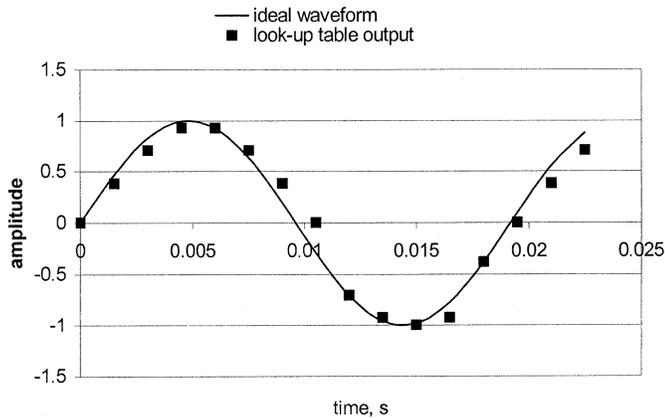
delta = 1.25, f=52.08281 Hz, Fs=666.666Hz, N=16

——— ideal waveform
■ look-up table output

Fig. 1. Comparision of synthesised waveforms for a noninteger table increment (Δ) value. Solid line shows the exact value and the symbols show the result obtained from the truncated index.

A mathematical derivation describing SNR as a function of M and N was provided by Jenq in [2].

The formula has been modified to accommodate TD instead of SNR to measure the quality of the signal

$$\text{TD} = 10 \cdot \log_{10}\left[\frac{1 - |A_o(M,N)|^2}{|A_o(M,N)|^2}\right] \qquad (5)$$

where

$$|A_o(M,N)| = \left[\frac{\sin(\pi/N)}{(\pi/N)}\right] \cdot \left[\frac{(\pi/M \cdot N)}{\sin(\pi/M \cdot N)}\right] \qquad (6)$$

is the amplitude of the fundamental component.

Substituting $M = 1$, i.e., for an integer value of Δ

$$|A(1,N)|^2 = 1. \qquad (7)$$

This verifies that distortion due to nonuniform index sampling in the synthesized waveform is not expected for an integer value of Δ.

IV. EXPERIMENTAL RESULTS

An experimental set-up based on a DLT algorithm implemented in C code and run on a TMS 320 C31 platform was used to verify the theoretical concepts in Section III. Distortion analysis was done using the real time FFT function of a HP 54602B oscilloscope. TD was experimentally determined by substituting measured distortion component amplitudes in (4). Fig. 2 shows the observed TD of the waveform as a function of M for a table of length $N = 64$. It is seen that TD of the waveform remains constant at -21 dB for values of M greater than 4. Fig. 3 shows TD of the waveform with relation to the word

TABLE I
TABLE LOOK-UP SIMULATION FOR f $= 52.082\,81$ Hz,
$f_s = 666.6$ Hz, N $= 16, \lambda = 1.25$

Time sec	Ideal index	ideal value	Truncated index	Truncated value	absolute error
0	0.00	0	0	0	0
0.0015	1.25	0.471396	1	0.382683	0.088713
0.003	2.50	0.831469	2	0.707106	0.124363
0.0045	3.75	0.995185	3	0.923879	0.071306
0.006	5.00	0.92388	5	0.92388	0
0.0075	6.25	0.634395	6	0.707108	0.072713
0.009	7.50	0.195093	7	0.382686	0.187593
0.0105	8.75	-0.290282	8	0.000003	0.290285
0.012	10.00	-0.707104	10	-0.707104	0
0.0135	11.25	-0.956939	11	-0.923878	0.033061
0.015	12.50	-0.980786	12	-1	0.019214
0.0165	13.75	-0.773013	13	-0.923881	0.150868
0.018	15.00	-0.382688	15	-0.382688	0
0.0195	0.25	0.098012	0	0	0.098012
0.021	1.50	0.555565	1	0.382683	0.172882
0.0225	2.75	0.881918	2	0.707106	0.174812

▨ - index value with zero error in output

TABLE II
DEFINITION OF THE VARIOUS HARMONIC FREQUENCY COMPONENTS

Harmonic	$f = h.f_o$	h integer > 0
DC	$f = 0$	$f = h.f_o$, h=0
Interharmonic	$f \neq h.f_o$	h integer > 0
Subharmonic	f>0 and f<f_o	

Where f_o is the fundamental frequency component

length of the table length, N for $M = 4$. (With the increase in N, the differences between subsequent table entry values decrease, reducing waveform distortion. The experimental data is in good agreement with the predicted curves of (5) and (6).

$$\text{TD} = 10 \cdot \log_{10}\left[\frac{\sum \left(A_{\text{harmonic}}^2 + A_{\text{interharmonic}}^2 + A_{\text{subharmonic}}^2\right)}{A_o^2}\right], \text{dB} \qquad (4)$$
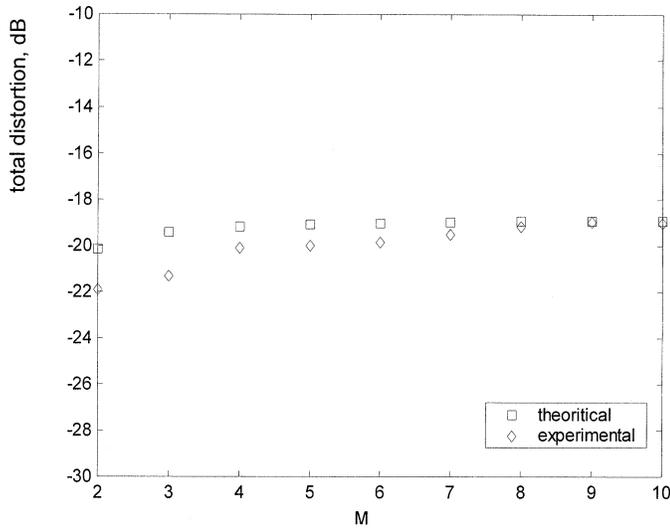
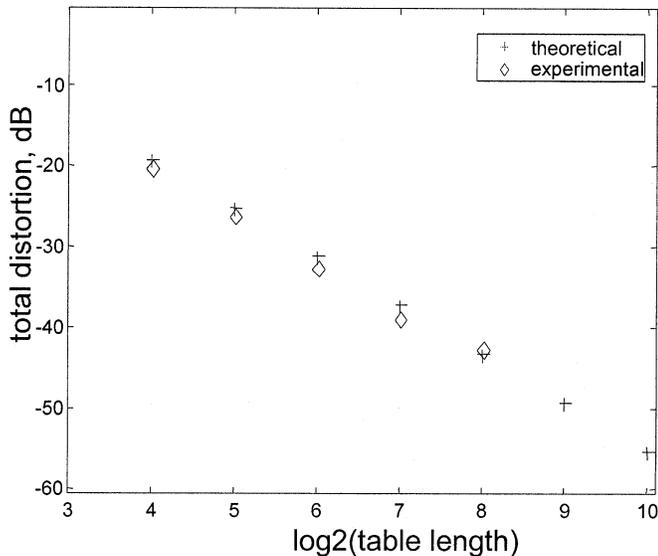Fig. 2. Comparison of theoretical and real time total distortion data (TD) as a function of M for $N = 64$.

Fig. 3. Comparison plots of theoretical and real time total distortion data (TD) as a function of table length, N, for $M = 4$.

A. Interharmonic Distortion

The generation of interharmonics can be explained as follows: Consider a waveform being generated through Table I. It is clear that only each Mth sample is error free. Therefore the resulting disparity in the rate of sampling produces a sinusoid with a dynamic ripple (or jitter effect) seen in the time domain. The repetition interval of the jitter is $M \cdot N/f_s$ s. This jitter effect results in interharmonic distortion and interharmonic components can be seen separated by f_s/M Hz in the frequency domain. A sample of the waveform stored in the oscilloscope for 2 s is shown in Fig. 4. Note that a look up table of 32 elements was used to deliberately exaggerate the jitter effect.
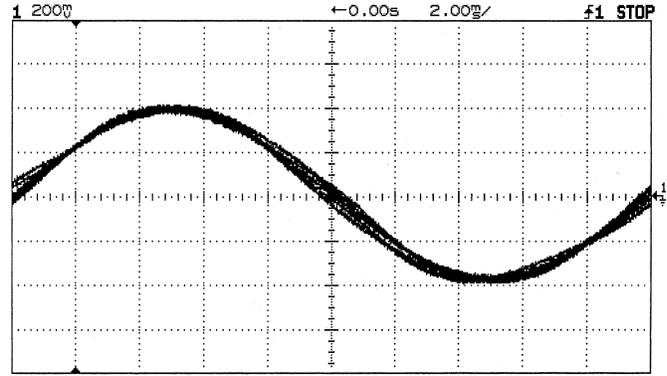

Fig. 4. Multi-cycle trace of a waveform with harmonics and sidebands. $f = 51$ Hz, $\Delta = 2.44, N = 32, M = 25$.

B. Subharmonic Distortion

Interharmonics occur above and below existing harmonics, of which a particular case would be an interharmonic below the fundamental frequency. These subharmonics are key factors in assessing low frequency distortion of inverter power sources. A subharmonic of significant amplitude below the line frequency would be highly undesirable. Higher order harmonics are less problematic since they are absorbed by the low pass filter characteristics of motors and power grids sourced by inverters and can in any case be easily removed by filtering. In this section, analytical formulas are derived and a method is proposed to determine occurrence of subharmonics, their numbers, relative positions and amplitudes.

1) Sufficient Condition for the Occurrence of Subharmonics: As shown by Jenq [2], each side-band/harmonic is separated by f_s/M Hz. Thus the frequency of the kth harmonic may be written as

$$f_k = \left(f_o + \frac{k}{M} \cdot f_s \right) \tag{8}$$

where $k \in (-\infty, +\infty)$ is an integer for subharmonic frequency components

$$-f_o < f_k < f_o. \tag{9}$$

Combining (1), (3), (8), and (9)

$$\left(\frac{-2 \cdot (M \cdot W + L)}{N} \right) < k < 0. \tag{10}$$

The condition for at least one frequency component to be below the fundamental is therefore

$$N < 2 \cdot (MW + L). \tag{11}$$

For given values of $f_o, f_s,$ and N (3) can be used to calculate values of M, W, and L. Subharmonics occur if the inequality given in (11) holds true.

2) Number of Subharmonic Components: Substituting values in (10), a range for k can be found. The finite number of integer values within this set represents all of the subharmonic components occurring for given $f_o, N,$ and f_s.

3) Relative Positions in the Frequency Spectrum: Given values for k, M, f_o, and f_s, the value of f_k can be found from (8).

4) Maximum Harmonic/Interharmonic Component Amplitudes: A full mathematical model of the harmonic structure of the synthesised waveform was derived by Jenq [2] as

$$A(k, L, M, N) = \frac{1}{M} \cdot \sum_{m=0}^{M-1} e^{-j(\frac{2 \cdot \pi}{M \cdot N})[\langle mL \rangle_M + kMN]} \quad (12)$$

where $A(k, L, M, N)$ is the amplitude of the kth harmonic and $\langle mL \rangle_M$ is the value of $m \cdot L$ modulo M.

The proof is given by Jenq [2] that, for $L \neq 1$, the subharmonic components in the frequency domain are merely interchanged. The scrambling of these harmonics/interharmonics due to the change in $L \in \{0 \ldots (M-1)\}$ will cause some frequency components to lie in the subharmonic region. Fine harmonic structure analysis in [2] shows that the maximum value of $|A(k, 1, M, N)|$ occurs when $k = 1$.

Therefore a simplified version of (12), for $L = 1$, is used as the base to calculate maximum possible amplitudes of the harmonic/interharmonic/subharmonic components

$$|A(k, 1, M, N)_{\text{sub}}| = \left| \frac{\text{Sinc}\left[\left(k + \frac{1}{N}\right) \cdot \pi\right]}{\text{Sinc}\left[\left(k + \frac{1}{N}\right) \cdot \frac{\pi}{M}\right]} \right|. \quad (13)$$

Using the observations made in [2], i.e., TD increases with M and for reasonably large N, (13) may be reduced to

$$|A(1, 1, N)|_{\text{max}} \approx \left| \left(\frac{\pi}{M \cdot N}\right) \frac{1}{\left[1 + \frac{1}{N}\right] \cdot \frac{\pi}{M}} \right| \approx \frac{1}{N}. \quad (14)$$

Equation (14) provides a useful estimate of the maximum amplitude of harmonic/interharmonic/subharmonic components as a function of table length N.

In most cases where inverters feed drives, power grids or other sensitive loads, regulations to limit subharmonic distortion are strict. Effective suppression of subharmonics can clearly be achieved by increasing table length N. However, this becomes infeasible in cost effective inverters based on micro controllers where memory requirements are restricted. A solution is therefore proposed to identify the minimum length of the look-up table suppressing subharmonic distortion.

V. ELIMINATION OF SUBHARMONIC DISTORTION

The fundamental criterion for the elimination of subharmonics from a sinusoid generated via nonuniformly sampled look-up table may be deduced from (11)

$$N > 2 \cdot (M \cdot W + L). \quad (15)$$

Although (15) provides an absolute determination of N, values of M and L are not deterministic and may vary over a virtually infinite range. It is evident from (15), that there is only a discrete set of synthesizable frequencies, free from subharmonic distortion. All other synthesized waveforms of frequencies violating (15) would contain subharmonic frequency components. It is therefore desirable to seek a method that allows determination of N as a function of required frequency (f_o), independent of L and M. This can only be achieved if values taken by L and M are restricted in accordance with (15). A convenient and a very
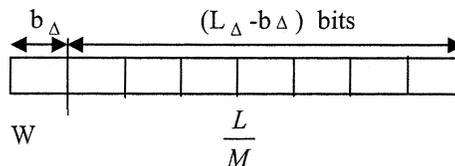

Fig. 5. Binary representation of incremental table index Δ.

practical method for restricting the values of M is to represent Δ as a binary fraction.

(15) is re-written using (1) as

$$N > 2 \cdot (M\Delta) \quad (16)$$

Taking logarithms on both sides, an inequality relating word lengths of N, M, and Δ is obtained as

$$\log_2(N) > 1 + \log_2(M \cdot \Delta) \quad (17)$$

$$L_N > (1 + L_\Delta) \quad (18)$$

where, word length $(N) = L_N = \log_2(N)$, Word length $(M \cdot \Delta) = L_\Delta = \log_2(M \cdot \Delta)$. L_Δ contains the number of bits required to represent the fractional binary value of Δ. An example of binary representation of Δ (3), in a 8 b micro-controller is shown in Fig. 5. Allocation of bits for the integer part (b_Δ) and fractional part ($L_\Delta - b_\Delta$) is arbitrary.

Given the above representation it is clear that, occurrence of sub-harmonics can be avoided in a sinusoid, generated via a look-up table accessed in nonuniform increments, if:

The minimum word length describing the table length N, is 1 bit more than the word length of the incremental table index Δ.

VI. EFFECT ON SYNTHESIZABLE FREQUENCY RESOLUTION, $|\Delta f|_{\text{min}}$

It is important to have adequate frequency resolution to produce a reference waveform with an accurate frequency relative to the demanded inverter frequency. Mismatch in frequencies would obstruct the smooth flow of power between the inverter and the grid. The results of the previous section demonstrate that effective suppression of subharmonics can be achieved by restricting the synthesisable frequencies to a discrete set. Using the binary representation described above the number of discrete frequencies is limited to half the number of table elements. Although a larger number of frequencies can always be achieved by increasing table length, this is not always a cost-effective solution. Therefore, a method is proposed to determine the maximum frequency resolution achievable with a given table length whilst retaining full suppression of subharmonic components.

Consider a typical application of a sinusoid generated via a look-up table as a reference signal for a grid connected inverter. f_s and N are chosen so that the table is accessed in single steps ($\Delta = 1$), generating a base frequency of say 50 or 60 Hz. In a real grid the frequency rarely stays constant and may typically be an arbitrary value within ± 1 Hz (2% deviation). A synchronising device, say a PLL, will extract information of the varying frequency of the grid and generate a sinusoidal reference signal to the inverter. For this case

$$\text{if the base frequency is } f_{\text{base}} = \frac{f_s}{N} \quad (19)$$

where

$$\Delta_{\text{base}} = 1 \quad \text{for a uniformly sampled look-up table}$$

and

$$f_o = \frac{\Delta_0 \cdot f_s}{N}. \tag{20}$$

Using (19) and (20)

$$|\Delta f| = |f_{\text{base}} - f_0| = \left[\frac{(1 - \Delta_0) \cdot f_s}{N} \right]. \tag{21}$$

The minimum frequency deviation relative to the base frequency is obtained as

$$|\Delta f|_{\min} = \left[\frac{(1 - \Delta_0)_{\min} \cdot f_s}{N} \right]. \tag{22}$$

Referring to Fig. 5, it is seen that the LSB of the binary word determines the resolution of change in Δ

$$(1 - \Delta_0)_{\min} = \frac{1}{2^{(L_\Delta - b_\Delta)}}. \tag{23}$$

Substituting (23) in (22), an expression for frequency resolution is found as

$$|\Delta f|_{\min} = \left[\frac{f_s}{2^{(L_\Delta - b_\Delta)} \cdot N} \right]. \tag{24}$$

Equation (24) relates the frequency resolution $|\Delta f|_{\min}$, N and the word length of Δ of a DLT generated reference sinusoid free from subharmonic distortion.

VII. EXAMPLE

Consider the PLL of a grid connected inverter, locked in to a grid frequency of 50.5 Hz. Assume the sinusoidal reference waveform is generated using a look-up table of 32 elements accessed at 1.6 kHz. The length of the look-up table is purposely chosen low to highlight subharmonic components.

- Using (1), $\Delta = 1.01 = 1 + (1/100)$, i.e., $W = 1, L = 1, M = 100$.
- Using (11), $2 \cdot (M \cdot W + L) = 202 > N(=32)$
  Therefore subharmonics would occur.
- Using (10), $-6.3125 < k < 0$, i.e., $k = -1, -2 \ldots -6$
  Number of subharmonics = 6.
- Using (8), to calculate subharmonic frequencies

$$f(-1) = \left| 50.5 + \frac{(-1)}{100} \cdot 1600 \right| = 34.5 \text{ Hz},$$

$$f(-2) = \left| 50.5 + \frac{(-2)}{100} \cdot 1600 \right| = 18.5 \text{ Hz},$$

$$f(-3) = \left| 50.5 + \frac{(-3)}{100} \cdot 1600 \right| = 2.5 \text{ Hz},$$

$$f(-4) = \left| 50.5 + \frac{(-4)}{100} \cdot 1600 \right| = 13.5 \text{ Hz},$$

$$f(-5) = \left| 50.5 + \frac{(-5)}{100} \cdot 1600 \right| = 29.5 \text{ Hz},$$

$$f(-6) = \left| 50.5 + \frac{(-6)}{100} \cdot 1600 \right| = 45.5 \text{ Hz}$$

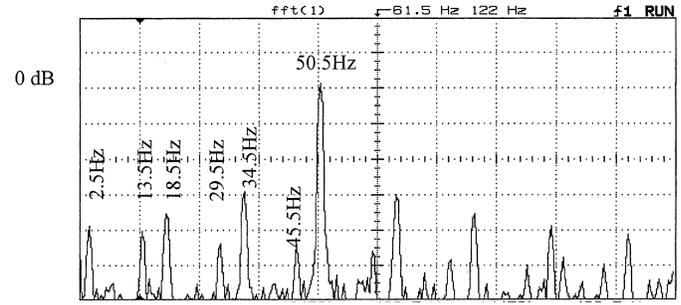maximum subharmonic amplitude $A(-1) = -30$ dB.

Fig. 6. Frequency spectrum of a synthesized waveform violating condition (11) resulting in subharmonic distortion. $f_0 = 50.5$ Hz, $N = 32$, $f_s = 1.6$ kHz, observation period $T_{\text{obs}} = 2$ s.
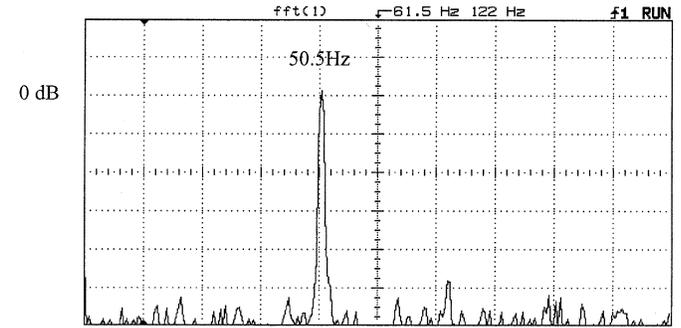

Fig. 7. Frequency spectrum of a synthesized waveform satisfying condition (11), hence no subhanrmonic distortion. $f_0 = 50.5$ Hz, $N = 256$, $f_s = 1.6$ kHz, observation period $T_{\text{obs}} = 2$ s.

The Frequency spectrum of the waveform generated in real-time is shown in Fig. 6. Harmonic amplitudes are normalized to the fundamental (0 dB) and the vertical scale is 10 dB/div. All of the subharmonic frequencies shown are in agreement with the calculated values.

- *Mitigation of Subharmonic Components*
  Using $N = 256$, condition (11) is satisfied and therefore the frequency spectrum of the sythesized waveform, shown in Fig. 7, is free from subharmonic distortion.
- *Subharmonic Amplitudes*
  Tests were performed for four different table lengths and the experimental values of maximum subharmonic amplitudes determined. Fig. 8 shows the maximum amplitudes plotted as a function of table length along with the theoretical data obtained from (14). Close agreement between the experimental and theoretical data verifies the utility of (14) as a design tool.
- *Frequency Resolution*
  If now, the user needs to achieve a frequency resolution of say $\Delta f_{\min} = 0.02$ Hz without the generation of subharmonics, then, by re-arranging (24), the minimum length of the look-up table $N_{\min}$ may be determined

$$N_{\min} = 2^{\frac{\log_2 \left[\frac{f_s}{\Delta f_{\min}} \right] + b_\Delta}{2}}$$

$$N_{\min} = 512. \tag{25}$$

where W, the integer part of Δ is either 0 or 1 and therefore in the worst case $b_\Delta = 1$.
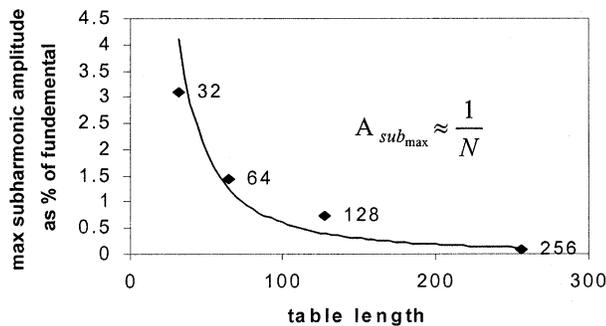
Fig. 8. Experimental variation of maximum subharmonic amplitude with table length N. Solid line shows theoretical variation given by (14).

VIII. CONCLUSION

This paper has investigated harmonic, interharmonic and subharmonic distortion in inverter sinewave reference waveforms due to nonuniform sampling of a direct look up table. The effect of table length on the maximum amplitude of the distortion component has been established and a mathematical model describing the occurrence of subharmonics, their numbers and frequencies has been derived. Although, in general, the occurrence of subharmonic components is not deterministic, a constrained set of sythesizable frequencies may be found that is free of subharmonic distortion. The conditions for complete subharmonic suppression have been determined in terms of the binary word lengths of look-up table parameters and applied to determine the minimum look-up table length that would avoid subharmonic distortion whilst achieving a given frequency resolution. A detailed example with experimental results has been used to verify the theoretical analysis. The results of this paper will be of use in specifying look-up tables for reference waveform generation in inverters, particularly those based on low cost micro-controllers.

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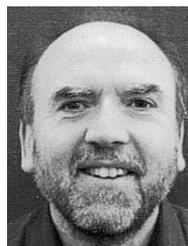
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