Bit-Interleaved Turbo-Coded Modulation using Shaping Coding

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***Abstract*— We present a new and simple method for combining constellation shaping and bit-interleaved turbo-coded modulation (BITCM). By considering the example of a 3-bit/dim 16-PAM BITCM, it is shown that this technique can provide shaping gains of 0.64 dB, and error performance within 1.51 dB of the continuous-input channel capacity limit is achieved.**

***Index Terms*— Bit-interleaved coded modulation, shaping code, turbo code.**

I. INTRODUCTION

For bandwidth-limited applications over Gaussian channels, it has recently been proposed to combine constellation shaping and turbo-coded modulation by employing either a multi-level coding (MLC) approach [1], [2] or a bit-interleaved coded modulation (BICM) approach [3]. The combination of shaping and channel coding is a more challenging issue in BICM than it is in MLC because the separability of shaping and coding algorithms, which is an inherent property of MLC, does not apply to BICM schemes [2]. Due to the importance of the BICM approach for many applications, it is necessary to address the issue of combining shaping and BICM in an efficient way. In the bit-interleaved turbo-coded modulation (BITCM) scheme described in [3], the shaping method relies on a clever mapping technique that converts equiprobable binary words generated by the turbo-encoder into non-equiprobable constellation signal points.

In this Letter, we present another method for combining shaping and BITCM. Our approach is based on a shaping technique in which the basic constellation is partitioned into several equal-sized sub-constellations of increasing average energy [4]. A shaping code is then used to specify the sequence of sub-constellations so that low-energy signals are transmitted more frequently than high-energy signals. The partitioning method preserves the Gray mapping provided that the basic constellation is only divided into two sub-constellations. This compatibility between shaping and Gray mapping constitutes a crucial point since it is well known that BITCM schemes perform optimally when Gray mapping is used to label constellation signal points [5].

Manuscript received July 7, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. Chi-chao Chao.

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Digital Object Identifier 10.1109/LCOMM.2005.03003.

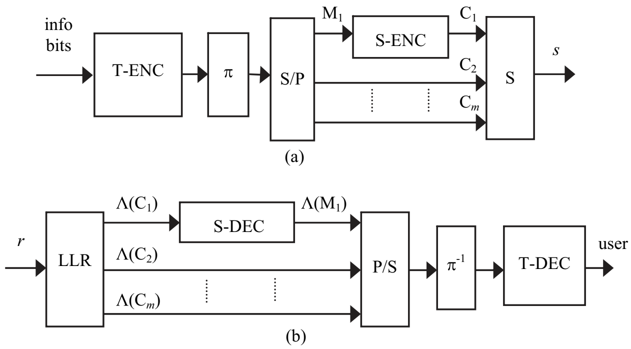

Fig. 1. Structure of the proposed BITCM scheme: (a) transmitter, (b) receiver.

Throughout this work, we assume a Gaussian channel, and only consider the case of 2^m-ary one-dimensional (1-D) constellations, hereafter referred to as 2^m-PAM constellations.

II. STRUCTURE OF THE PROPOSED BITCM SYSTEM

A. BITCM Transmitter Structure

The proposed BITCM transmitter structure is shown in Fig. 1.a. The sequence of information bits is encoded by a rate-R_c binary turbo encoder (T-ENC). The corresponding sequence of coded bits is, after interleaving (π), broken into blocks of N bits that are further divided into m binary vectors by a serial-to-parallel (S/P) converter. The first vector M_1, composed of k bits, is fed into a shaping encoder (S-ENC) which generates a corresponding codeword C_1 of n bits, with $n > k$. The rate of this encoder is thus $R_s = k/n$. The other (m-1) vectors C_j, $j \in \{2, ..., m\}$, present at the S/P converter output, are composed of n bits each. Finally, a vector $(c_{i,1}, c_{i,2,...,}c_{i,m})$, where $c_{i,j}$ denotes the ith bit of vector C_j, is mapped onto a signal point of a 2^m-PAM constellation, called S, according to Gray labeling. Therefore, the transmission of N channel-encoded bits is performed by emitting n successive 1-D signal points s_i, $i \in \{1, ..., n\}$, i.e. an n-D signal point denoted s.

The data rate R obtained with such system is given by $R = R_c(R_s + m - 1)$ bits/dim, which is less than the rate $R' = m.R_c$ bits/dim obtained with an equivalent BITCM scheme without shaping code. This loss in data rate can be compensated for by increasing the turbo code rate R_c.

The constellation S is partitioned into two sub-constellations S_0 and S_1 so that S_0 contains the 2^{m-1} signal points of lowest energies, whereas S_1 is composed of the 2^{m-1} signal points of highest energies [4]. The Gray mapping is performed in such a way that bits $c_{i,1}$, which are the bits generated by the shaping encoder, are used to select one of these sub-constellations. Assuming that $c_{i,1} = 0$ leads to the selection of S_0, the shaping

Signal	-15	-13	-11	-9	-7	-5	-3	-1	1	3	5	7	9	11	13	15
$c_{i,1}$	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
$c_{i,2}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$c_{i,3}$	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
$c_{i,4}$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1

Fig. 2. Gray mapping for 16-PAM constellation. The bit $c_{i,1}$ is used to partition the constellation into two sub-constellations S_0 and S_1.

encoder is designed so that $\Pr\{c_{i,1} = 0\} > \Pr\{c_{i,1} = 1\}$, i.e. S_0 is emitted more frequently than S_1.

B. BITCM Receiver Structure

The block diagram of the proposed BITCM receiver is depicted in Fig. 1.b. The received n-D signal r is a vector of n channel samples r_i, $i \in \{1, ..., n\}$, expressed as $r_i = s_i + n_i$, where n_i is a Gaussian noise sample with zero mean and variance σ^2. From sample r_i, the log-likelihood ratio (LLR) $\Lambda(c_{i,j})$ associated with each bit $c_{i,j}$, $j \in \{1, ..., m\}$, is computed using well-known expressions. For more details, the reader is referred to [5]. For each received n-D signal r, the LLR computation block produces m vectors $\Lambda(C_j) = (\Lambda(c_{1,j}), \Lambda(c_{2,j}), ..., \Lambda(c_{n,j}))$, $j \in \{1, ..., m\}$. The shaping decoder (S-DEC) uses the MAP algorithm to decode vector $\Lambda(C_1)$, and generates an estimate $\Lambda(M_1) = (\Lambda(m_{1,1}), \Lambda(m_{2,1}), ..., \Lambda(m_{k,1}))$ of the vector $M_1 = (m_{1,1}, m_{2,1}, ..., m_{k,1})$ encoded at the transmitter side. We can show that the MAP decoding algorithm consists of evaluating, for $q \in \{1, ..., k\}$, the expression

$$\Lambda(m_{q,1}) = \ln\left[\frac{\sum\limits_{C_1 \in \Omega_q^1} \Pr\{C_1 \mid \Lambda(C_1)\}}{\sum\limits_{C_1 \in \Omega_q^0} \Pr\{C_1 \mid \Lambda(C_1)\}}\right], \quad (1)$$

where Ω_q^t, $t \in \{0, 1\}$, denotes the set of all codewords C_1 obtained after encoding of the vectors M_1 for which $m_{q,1} = t$. The term $\Pr\{C_1 \mid \Lambda(C_1)\}$ is given by

$$\Pr\{C_1 \mid \Lambda(C_1)\} = \prod_{i=1}^{n}\left(\frac{\exp\{t_i \Lambda(c_{i,1})\}}{1 + \exp\{\Lambda(c_{i,1})\}}\right), \quad (2)$$

where $t_i \in \{0, 1\}$ is the value of the ith bit in the codeword C_1 under consideration. By combining (1) and (2), we obtain the final expression of the LLRs $\Lambda(m_{q,1})$:

$$\Lambda(m_{q,1}) = \ln\left[\frac{\sum\limits_{C_1 \in \Omega_q^1} \exp\left\{\sum\limits_{i=1}^{n} t_i \Lambda(c_{i,1})\right\}}{\sum\limits_{C_1 \in \Omega_q^0} \exp\left\{\sum\limits_{i=1}^{n} t_i \Lambda(c_{i,1})\right\}}\right]. \quad (3)$$

This equation shows that the computational complexity of the decoding algorithm increases exponentially with k since $|\Omega_q^1| = |\Omega_q^0| = 2^{k-1}$, but remains reasonable for low values of k and n.

Finally, a parallel-to-serial (P/S) converter combines successive vectors $\Lambda(M_1)$ and $\Lambda(C_j)$, $j \in \{2, ..., m\}$, to produce a sequence of LLRs which is, after de-interleaving (π^{-1}), used by the binary turbo decoder (T-DEC).

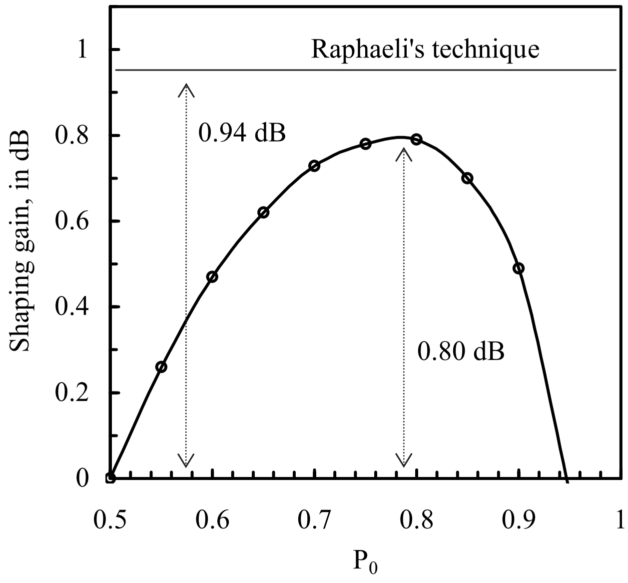

Fig. 3. Variation of the achievable shaping gain as a function of P_0 for the proposed system, when $C = 3$ bits/dim and $m = 4$ (16-PAM).

III. Example

In this Section, we consider the example of a 3-bit/dim 16-PAM BITCM scheme. The Gray mapping for 16-PAM constellation is indicated in Fig. 2.

A. Theoretical Shaping Gain

The achievable shaping gains can be determined by evaluating the gains in terms of channel capacity limit obtained with the proposed shaping algorithm. Let $s \in S$ and r denote respectively the transmitted signal and the corresponding received signal. It can be shown that the capacity C, in bits/dim, for the system described in Section II is given by

$$C = E_{s,r}\left[\log_2\left(\frac{2^{m-1}.\exp\left\{-\frac{(r-s)^2}{2\sigma^2}\right\}}{\sum\limits_{x \in S} \Pr\{x\} \exp\left\{-\frac{(r-x)^2}{2\sigma^2}\right\}}\right)\right], \quad (4)$$

where $E_{s,r}$ denotes expectation with respect to s and r. The term $\Pr\{x\}$ is equal to P_0 for signals $x \in S_0$ and equal to $(1 - P_0)$ for signals $x \in S_1$, where P_0 designates the average probability that $c_{i,1} = 0$ at the shaping encoder output. Using (4), it is possible to determine the achievable shaping gain for different values of P_0. Fig. 3 shows the variation of the shaping gain as a function of P_0, when $C = 3$ bits/dim and $m = 4$ (16-PAM). For reference purposes, the shaping gain achievable with the Raphaeli's shaping technique [3] is also indicated. It is seen that the simple partitioning method considered in this Letter can offer shaping gains larger than 0.7 dB provided that the shaping code is designed so that P_0 ranges from 0.69 to 0.85. The maximal value of the shaping gain, which is equal to 0.80 dB, is obtained when $P_0 \approx 0.78$. This is only 0.14 dB less than that obtained with Raphaeli's method which is, nevertheless, based on a partition of the 16-PAM constellation into four sub-constellations.

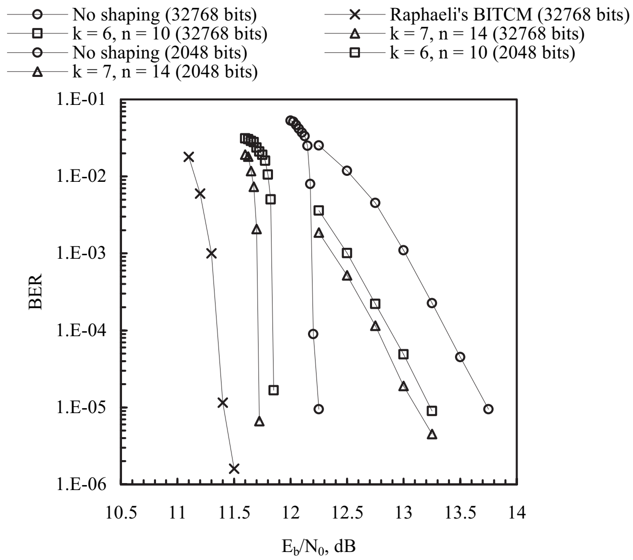

Fig. 4. Performance comparison over Gaussian channel between several 3-bit/dim 16-PAM BITCMs for 32768-bit and 2048-bit interleaving. The turbo codes based on 32768-bit and 2048-bit interleaving use 18 and 10 decoding iterations, respectively. The BITCMs with shaping code employ either the $(R_c = 5/6, k = 6, n = 10)$ configuration or the $(R_c = 6/7, k = 7, n = 14)$ configuration, while the BITCMs without shaping use a rate-3/4 turbo code.

B. Simulation Results

We simulated the error performance of several 3-bit/dim 16-PAM BITCMs based on two configurations, which are $(R_s = 3/5, R_c = 5/6)$ and $(R_s = 1/2, R_c = 6/7)$. Only low-to-moderate complexity shaping codes for which $k < 10$ were considered.

The value of P_0 for the rate-3/5 shaping codes under consideration ranges from 0.775 (for $k = 3$ and $n = 5$) to 0.818 (for $k = 9$ and $n = 15$). As for the rate-1/2 shaping codes, the value of P_0 varies from 0.750 (for $k = 1$ and $n = 2$) to 0.854 (for $k = 9$ and $n = 18$). Hence, both configurations can potentially achieve shaping gains larger than 0.7 dB in all cases. The rate-5/6 and -6/7 turbo codes are obtained by puncturing a rate-1/3 turbo code built from two parallel-concatenated 16-state recursive and systematic convolutional (RSC) codes with polynomials (23, 31) [6]. We adopted the puncturing patterns proposed in [7]. The MAP algorithm is used for the decoding of each RSC code.

Fig. 4 shows graphs of BER versus E_b/N_0 obtained with several 3-bit/dim 16-PAM BITCM schemes, when the size of the pseudo-random interleaving separating both RSC codes is equal to either 32768 bits or 2048 bits. We found that the best results are achieved when using the $(k = 6, n = 10)$ and $(k = 7, n = 14)$ shaping codes. Fig. 4 shows the BER curves corresponding to these shaping codes as well as those obtained with an equivalent 3-bit/dim 16-PAM BITCM system without shaping. From Fig. 4, it is seen that the use of a shaping code results in a significant error performance improvement for both interleaving sizes. At a BER of 10^{-5}, we obtain, with the $(k = 7, n = 14)$ code, shaping gains equal to 0.53 dB and 0.64 dB, for interleaving sizes of 32768 bits and 2048 bits, respectively. It is interesting to compare the error performance of our system to that displayed by the Raphaeli's 3-bit/dim 16-PAM BITCM scheme which is based on 32768-bit interleaving [3]. We observe that, at a BER of 10^{-5}, this BITCM scheme outperforms our system by 0.31 dB. The main reason for this performance gap is that, in our method, the basic 16-PAM constellation can only be divided into 2 sub-constellations, whereas the technique in [3] partitions it into 4 sub-constellations.

The capacity limit of the continuous-input Gaussian channel for a 3-bit/dim application is 10.21 dB. Fig. 4 indicates that, when a BER of 10^{-5} is taken as a reference, our BITCM scheme is able to perform within 1.51 dB of this capacity limit. Such error performance is approximately 0.4 dB away from that obtained with the more complex 32-D multi-level turbo-coded modulation scheme described in [2] which is, at the time of writing, the most powerful 3-bit/dim 16-PAM coding system ever designed.

IV. Conclusion

We have presented a simple technique to combine constellation shaping and BITCM. Simulation results show that a 3-bit/dim 16-PAM scheme designed using a moderate-complexity shaping code can achieve a shaping gain of 0.64 dB, and perform within 1.51 dB of the channel capacity. We believe that some performance improvement is possible by incorporating the LLR computation block and the shaping decoder inside the iterative decoding loop (as was done in [3]).

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