Transactions Letters

Bit-Interleaved Coded Modulation With Iterative Decoding Using Constellation Shaping

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***Abstract*—We investigate the association between constellation shaping and bit-interleaved coded modulation with iterative decoding (BICM-ID). To this end, we consider a technique which consists of inserting shaping block codes between mapping and channel coding functions in order to achieve constellation shaping. By assuming the example of a 2-b/s/Hz 16-quadrature amplitude modulation BICM-ID, it is demonstrated using computer simulations that this technique can improve the performance of BICM-ID schemes by a few tenths of a decibel.**

***Index Terms*—Bit-interleaved coded modulation with iterative decoding (BICM-ID), convolutional code, shaping code.**

I. Introduction

RECENTLY, some methods for combining constellation shaping and bit-interleaved turbo-coded modulation have been proposed for the design of bandwidth- and power-efficient communication systems over additive white Gaussian noise (AWGN) channels [1], [2]. In this letter, we investigate the application of constellation shaping to bit-interleaved coded modulation with iterative decoding (BICM-ID) [3], [4]. To this end, we consider a shaping technique which consists of partitioning the basic constellation into several subconstellations, so that the lower energy signals are transmitted more frequently than their higher energy counterparts [5]. In practice, such a technique can be implemented by inserting shaping block codes between mapping and channel coding functions. At the receiver side, an iterative decoding algorithm is used to exchange extrinsic information between the channel decoder, shaping decoder, and demapper blocks. Throughout this letter, without loss of generality, we focus on the design of a 2-b/s/Hz BICM-ID system employing a 16-ary quadrature amplitude modulation (QAM) constellation.

This letter is organized as follows. In Section II, the structures of our BICM-ID transmitter and receiver are presented. Computer simulation results are shown in Section III. Finally, conclusions are drawn in Section IV.

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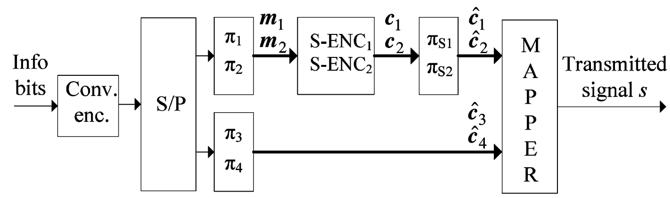

Fig. 1. Structure of the 16-QAM BICM-ID transmitter.

II. System Model

A. Transmitter Structure of BICM-ID Using Shaping Coding

Fig. 1 shows the transmitter structure of a 2-b/s/Hz 16-QAM BICM-ID scheme using shaping codes. A frame of N_b information bits is first encoded by a rate-R_c convolutional encoder. The resulting encoded sequence is divided into four parallel binary vectors by a serial-to-parallel (S/P) converter, which are then interleaved using a set of random interleavers $\pi_j,\ j \in \{1,2,3,4\}$. The outputs of the first two interleavers are broken into L successive K-bit vectors $\boldsymbol{m}_j,\ j \in \{1,2\}$. Each vector $\boldsymbol{m}_j$ is fed into a binary shaping encoder S-ENC$_j$, $j \in \{1,2\}$, which generates a corresponding N-bit codeword $\boldsymbol{c}_j$. Both shaping encoders S-ENC$_j$ are identical, and their rate R_s is given by $R_s = K/N (N > K)$. We recall that shaping encoders are designed so that the probability of a zero in codewords $\boldsymbol{c}_j$ is maximized [2], [5]. Each sequence, composed of L codewords $\boldsymbol{c}_j,\ j \in \{1,2\}$, is then randomly interleaved (π_{Sj}). For convenience, we can hereafter view the resulting sequence as a succession of L codewords $\hat{\boldsymbol{c}}_j,\ j \in \{1,2\}$.

The remaining streams available at the output of interleavers $\pi_j,\ j \in \{3,4\}$, can also be seen as composed of L successive N-bit codewords $\hat{\boldsymbol{c}}_j,\ j \in \{3,4\}$. Finally, a vector of four bits $(\hat{c}_{i,1},\ldots,\hat{c}_{i,4})$, where $\hat{c}_{i,j}$ is the ith bit in the codeword $\hat{\boldsymbol{c}}_j$, $j \in \{1,2,3,4\}$, is mapped onto a signal point of a 16-QAM constellation. The spectral efficiency η of the system is given by

$$\eta = 2R_c(R_s + 1) \text{ b/s/Hz}. \quad (1)$$

The 16-QAM constellation is divided into three subconstellations $S_x,\ x \in \{1,2,3\}$, so that S_1 contains the four signal points with lowest energies, S_2 includes the eight signal points with medium energies, and S_3 is composed of the four signal

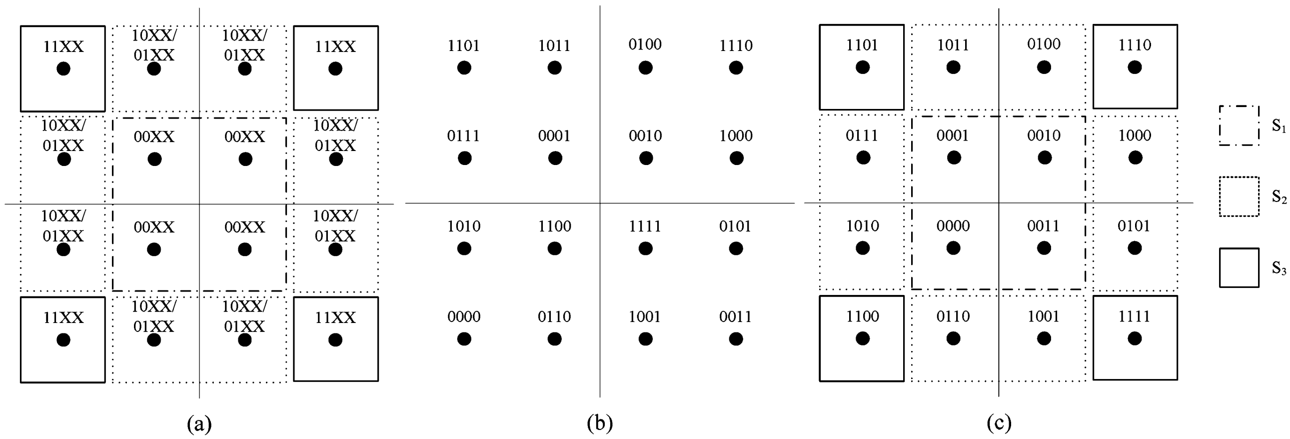

Fig. 2. Different mappings for 16-QAM. (a) 16-QAM mapping required for the BICM-ID system with constellation shaping. (b) Schreckenbach's mapping. (c) Modified Schreckenbach's mappping.

points with highest energies. Bits $\hat{c}_{i,1}$ and $\hat{c}_{i,2}$, which are originally generated by the shaping encoders, are used to select one of these subconstellations as follows. If $(\hat{c}_{i,1}, \hat{c}_{i,2}) = (00)$, then S_1 is selected, whereas S_2 is chosen whenever $(\hat{c}_{i,1}, \hat{c}_{i,2}) = (01)$ or (10). At last, the case $(\hat{c}_{i,1}, \hat{c}_{i,2}) = (11)$ leads to the selection of S_3. With such a mapping procedure, we guarantee that signals with high energies are transmitted less frequently than low-energy signals, since $\Pr\{\hat{c}_{i,j} = 0\} > \Pr\{\hat{c}_{i,j} = 1\}$, $j \in \{1, 2\}$ [2], [5]. We can easily show that when compared with 16-QAM with equiprobable signaling, the energy saving E_s, in decibels (dB), is given by

$$E_s = 10 \log_{10} \left(\frac{5}{9 - 8P_0} \right) \tag{2}$$

where P_0 is the average probability of a zero at the shaping encoder output.

Fig. 2(a) shows the 16-QAM mapping required for our system. Note that the notation "XX" in this figure denotes any pair of bits, i.e., "00," "01," "10," or "11." Therefore, there is a very large number of suitable mappings for designing our BICM-ID scheme with constellation shaping. In [6], Schreckenbach proposed a 16-QAM mapping that optimizes the error performance of BICM-ID at high signal-to-noise ratio (SNR). Such mapping is indicated in Fig. 2(b). From an error-performance viewpoint, it is obviously desirable to employ a mapping which is as close to that given in Fig. 2(b) as possible. We found that it is actually possible to obtain a labeling compatible with Fig. 2(a) by slightly modifying Schreckenbach's mapping. The resulting mapping is shown in Fig. 2(c), and is obtained by simply swapping labels "0000" and "1100," as well as labels "0011" and "1111" in Schreckenbach's mapping.

B. Receiver Structure of BICM-ID Using Shaping Coding

The receiver structure of our BICM-ID system is shown in Fig. 3. In order to optimize the error performance, the demapper, shaping decoder, and convolutional decoder exchange information in an iterative manner by employing soft-input soft-output

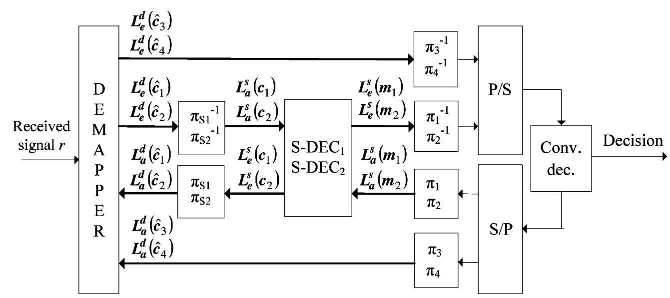

Fig. 3. Structure of the 16-QAM BICM-ID receiver.

(SISO) modules [7], [8]. For each transmitted signal s, the corresponding received signal r is expressed as $r = s + n$, where n is a Gaussian noise sample with zero mean and variance σ^2. Based on sample r and the corresponding *a priori* log-likelihood ratios (LLRs) $L_a^d(\hat{c}_{i,j})$, $j \in \{1, 2, 3, 4\}$, generated by both shaping and convolutional decoders, the demapper calculates an extrinsic LLR $L_e^d(\hat{c}_{i,j})$ associated with bit $\hat{c}_{i,j}$, $j \in \{1, 2, 3, 4\}$, by using well-known expressions [3], [4].

The LLR $L_e^d(\hat{\boldsymbol{c}}_j)$ of vector $\hat{\boldsymbol{c}}_j$, $j \in \{1, 2\}$, is deinterleaved by π_{S1}^{-1} and π_{S2}^{-1}. The resulting LLR $L_a^s(\boldsymbol{c}_j)$ is then used as *a priori* information by the shaping decoder. The purpose of the SISO shaping decoder S-DEC$_j$, $j \in \{1, 2\}$, is to generate both extrinsic LLRs $L_e^s(\boldsymbol{c}_j)$ and $L_e^s(\boldsymbol{m}_j)$ which are associated with codewords $\boldsymbol{c}_j$ and $\boldsymbol{m}_j$, respectively. By using the maximum *a posteriori* (MAP) algorithm, the extrinsic LLR $L_e^s(m_{j,k})$, $k \in \{1, \ldots, K\}$ corresponding to the kth bit in message $\boldsymbol{m}_j$ can be computed as

$$L_e^s(m_{j,k}) = \ln \left[\frac{\Pr\{m_{j,k} = 1,\ L_a^s(\boldsymbol{m}_j),\ L_a^s(\boldsymbol{c}_j)\}}{\Pr\{m_{j,k} = 0,\ L_a^s(\boldsymbol{m}_j),\ L_a^s(\boldsymbol{c}_j)\}} \right] \tag{3}$$

where $L_a^s(\boldsymbol{m}_j)$ represents the extrinsic LLR generated by the convolutional decoder at the previous iteration. We could show that (3) is actually equivalent to (4), shown at the bottom of the next page, where Ω_k^t, $t \in \{0, 1\}$, denotes the set of all messages $\boldsymbol{m}$ whose kth bit is equal to t. In this equation, $t_l \in \{0, 1\}$ is the value of the lth bit in the message $\boldsymbol{m}$ under consideration, and $t_n \in \{0, 1\}$ is the value of the nth bit in the codeword $\boldsymbol{c}$ associated with this particular message. In (4), L_n represents

TABLE I
SEVERAL POSSIBLE CONFIGURATIONS FOR THE DESIGN OF A 2-B/S/HZ 16-QAM BICM-ID SCHEME WITH SHAPING CODING. FOR EACH CONFIGURATION, THE ENERGY SAVING AND THE CORRESPONDING VALUE OF P_0 ARE INDICATED

R_c	R_s	K	N	E_s (dB)	P_0
2/3	1/2	7	14	3.63	0.854
		6	12	3.58	0.851
		5	10	3.38	0.838
3/5	2/3	6	9	2.41	0.766
		4	6	2.22	0.750
4/7	3/4	6	8	1.85	0.717
		3	4	1.55	0.688

the *a priori* knowledge, regarding the nth bit in codewords $\boldsymbol{c}_j$, available before the first decoding iteration starts. It is computed as

$$L_n = \ln\left[\frac{1-P_n}{P_n}\right] \tag{5}$$

where P_n designates the probability that the nth bit in a codeword $\boldsymbol{c}_j$ is equal to zero. By using an expression very similar to (4), it is possible to calculate LLRs $L_e^s(c_{j,n})$, $n \in \{1,\ldots,N\}$. The extrinsic LLRs $L_e^s(\boldsymbol{c}_1)$ and $L_e^s(\boldsymbol{c}_2)$ are further fed into the demapper as an *a priori* information for the next iteration, while $L_e^s(\boldsymbol{m}_1)$, $L_e^s(\boldsymbol{m}_2)$, $L_e^d(\hat{\boldsymbol{c}}_3)$ and $L_e^d(\hat{\boldsymbol{c}}_4)$ are processed by the convolutional decoder in the same iteration.

Finally, the extrinsic LLRs generated by the convolutional decoder are split into four parallel vectors and interleaved via π_j, $j \in \{1,2,3,4\}$. The first two vectors $L_a^s(\boldsymbol{m}_1)$ and $L_a^s(\boldsymbol{m}_2)$ are further fed into the shaping decoder for the next iteration, while $L_a^d(\hat{\boldsymbol{c}}_3)$ and $L_a^d(\hat{\boldsymbol{c}}_4)$, together with $L_a^d(\hat{\boldsymbol{c}}_1)$ and $L_a^d(\hat{\boldsymbol{c}}_2)$, are fed back to the demapper.

III. SIMULATION RESULTS

We now consider a 2-b/s/Hz BICM-ID system employing a 16-QAM constellation. The error performance of this system was evaluated using various low-complexity shaping codes whose characteristics are indicated in Table I [2], [5]. Since the computational complexity of the shaping decoding varies exponentially with K ($|\Omega_k^1| = |\Omega_k^0| = 2^{K-1}$ in (4)), we decided in this letter to only consider short-length shaping codes for which $K \leq 7$.

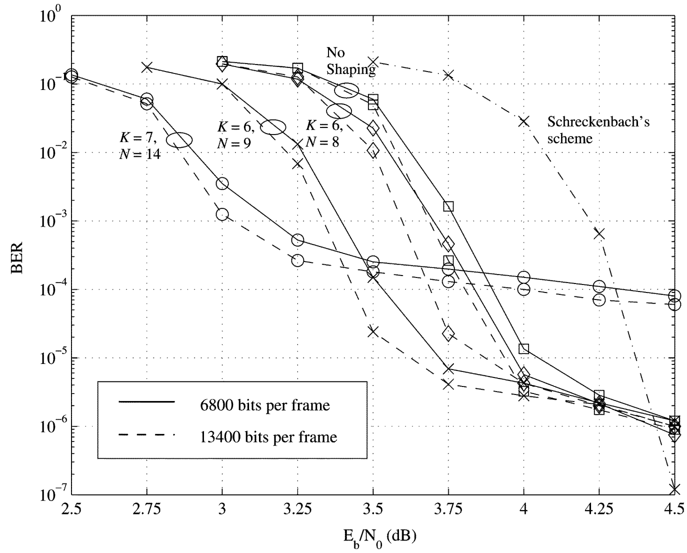

Fig. 4. Performance comparison over Gaussian channel between several 2-b/s/Hz 16-QAM BICM-ID scheme using convolutional coding with 10 iterations and frame sizes $N_b = 6800$ and 13 400 information bits per frame. Performance of equivalent BICM-ID system without shaping is also shown.

The rate-R_c channel codes are obtained by puncturing a four-state rate-1/2 recursive and systematic convolutional (RSC) code with generator polynomials (7, 5). The iterative decoding at the receiver side is performed in 10 iterations, and the Log-MAP algorithm is used for the decoding of the convolutional code.

Computer simulations were performed by selecting the shaping codes corresponding to the highest energy saving for each configuration in Table I. Fig. 4 illustrates the bit-error rate (BER) performance versus SNR E_b/N_0, where E_b is the energy transmitted per information bit and N_0 is the one-sided noise power spectral density, obtained with the 2-b/s/Hz 16-QAM BICM-ID schemes using frame sizes $N_b = 6800$ and 13 400 bits, respectively. For comparison's sake, the performance of an equivalent 2-b/s/Hz 16-QAM BICM-ID system without shaping is also plotted. It can be seen that the use of the shaping code results in a significant performance improvement for both frame sizes. For every frame size, we remark that the maximum value of the shaping gain at BER $= 10^{-3}$ is approximately equal to 0.7 dB. This substantial gain is achieved using the ($K = 7$, $N = 14$, $R_c = 2/3$) configuration. However, in this case, one can notice that there is an error-floor effect occuring at BER level below 10^{-3}. We believe that this error floor is due to the relatively poor error-correcting capabilities of a punctured rate-2/3, four-state convolutional code. Nevertheless,

$$L_e^s(m_{j,k}) = \ln\left[\frac{\sum\limits_{\boldsymbol{m}\in\Omega_k^1} \exp\left\{\sum\limits_{l=1,\, l\neq k}^{K} t_l L_a^s(m_{j,l}) + \sum\limits_{n=1}^{N} t_n\left(L_a^s(c_{j,n}) - L_n\right)\right\}}{\sum\limits_{\boldsymbol{m}\in\Omega_k^0} \exp\left\{\sum\limits_{l=1,\, l\neq k}^{K} t_l L_a^s(m_{j,l}) + \sum\limits_{n=1}^{N} t_n\left(L_a^s(c_{j,n}) - L_n\right)\right\}}\right] \tag{4}$$

such performance can still be of interest in some applications for which a BER of 10^{-3} is the target.

The error-floor level can be reduced by decreasing the channel code rate R_c, i.e., increasing the shaping code rate R_s. For instance, it is seen from Fig. 4 that the error floor only occurs below BER $= 10^{-5}$ when using the ($K = 6, N = 8$, $R_c = 4/7$) configuration. However, we notice that decreasing the channel code rate R_c results in smaller shaping gains at high BERs ($\approx 10^{-3}$). There is, therefore, a compromise between performance at high BERs and performance at medium BERs. We observe that the best tradeoff is achieved with the ($K = 6, N = 9$, $R_c = 3/5$) configuration. Using this particular configuration, we obtain shaping gains equal to 0.35 and 0.30 dB at BER $= 10^{-5}$, for frame sizes of 6800 and 13 400 bits, respectively.

It is interesting to compare our results with those achieved using Schreckenbach's mapping for a 16-QAM BICM-ID scheme without constellation shaping [6]. Note that 6800 bits per frame used in our system is equivalent to the interleaver size of 10 000 bits considered in [6]. At BER $= 10^{-3}$, our scheme using the ($K = 7, N = 14$) configuration outperforms this BICM-ID scheme by 1.1 dB, whereas at BER $= 10^{-5}$, our scheme with the ($K = 6, N = 9$) configuration is 0.75 dB better than this scheme.

IV. Conclusion

In this letter, we investigated the gain which can be obtained by applying constellation shaping to the design of BICM-ID. We showed that the error performance of a 2-b/s/Hz 16-QAM convolutional coding scheme can be improved by 0.7 dB at BER $= 10^{-3}$, and 0.35 dB at BER $= 10^{-5}$, when employing carefully selected shaping codes.

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