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Adoption of a reliability approach for membrane structure analysis

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Abstract

Membrane structures, including tensioned fabric and pressurized foil, feature in high profile and large span projects throughout the world for well-appreciated architectural and functional reasons. From an analysis and design perspective these types of structure exhibit high levels of material and geometric non-linearity. Current design practice is use of a permissible stress approach using stress reduction factors with values typically between 4 and 8. Design guidance is limited and the magnitude and nature of these stress factors varies widely.

Recently CEN TC250 Working Group 5 has been established to write a standard for Membrane Structures for inclusion in Eurocode 10. In this paper we apply the principles of BS EN 1990:2002 “Eurocode - Basis for Structural Design” to membrane structures and explain the implications for analysis and design. The key feature of this approach is the mathematical and numerical rigor and consistency in predicting the safety of membrane structures as an explicit function of inherent uncertainty in applied loading, structural form and material properties. This new approach to membrane structure analysis is demonstrated through two examples, which show the application of reliability analysis in achieving the objectives of “Eurocode - Basis for Structural Design” and highlight the further work that is required before this approach can be fully utilized by industry.

Keywords: Membrane structure; tensile fabric; stress factor; reliability analysis; safety index; Eurocode

1 MEMBRANE STRUCTURE DESIGN – CURRENT PRACTICE

1.1 FUNDAMENTALS OF FABRIC STRUCTURE DESIGN

Fabric canopies are one of the earliest forms of roofing, and have been used for traditional forms of construction for thousands of years. However, modern fabric structures using synthetic materials have only been in use for about 50 years. A fabric membrane acts as both structure and cladding, thereby reducing the weight, cost and environmental impact of the construction. These structures combine striking architectural forms with high levels of structural stability and durability, with expected lifespans in excess of thirty years depending on the type of material used (Figure 1). Key to achieving these high levels of performance is accurate modeling of the form and behaviour of the structure, including incorporation of the inherent uncertainties in all aspects of the design process.

Architectural fabrics have negligible bending and compression stiffnesses, and therefore fabric structures must be designed with sufficient curvature to enable wind and snow loads to be resisted as tensile forces in the plane of the fabric [1, 2]. The low tensile and shear stiffnesses of the membrane material result in large strains and significant changes to the structural form under load. The modified membrane curvature results in a change in the geometric stiffness of the structure, and hence an iterative large-displacement geometrically non-linear analysis is required to determine the equilibrium configuration of the structure for each load case.

To resist both uplift and down-forces (typically due to wind and snow respectively) the surface of the canopy must be anticlastically double-curved, i.e. saddle shaped rather than dome shaped. Fabric structures are pre-tensioned (usually referred to as ‘prestressed’) to ensure that the fabric remains in tension under all load conditions and to reduce deflections. The low weight of the fabric means that gravity or ‘self weight’ loading is often negligible. Consequently, tensile fabric is frequently more structurally efficient and cost-effective for large span roofs than conventional construction methods.

Figure 1. Tensile fabric has been used for diverse, architecturally striking structures for over 50 years, examples include Schlumberger Research Centre, Cambridge, UK, 1983 (top), a complex cable supported fabric form [© Andrew Dunn (www.andrewdunnphoto.com)]; Hampshire Cricket Club, UK, 2001, (centre) – classic tensile architecture consisting of multiple conic canopies [© Buro Happold / Mandy Reynolds]; and Dalton Park, UK, 2002 (bottom) – multiple conic canopies provide a light but sheltered environment for shoppers [© Ben Bridgens].

Unlike conventional structures, the shape of a tensile structure cannot be prescribed, but must take a ‘form-found’ shape determined by equilibrium and geometric constraints. Prior to the development of

numerical methods, experimental techniques that exploited the properties of surface tension of a soap film were used to visualize and determine feasible membrane structure forms (Figure 2) [4-6]. The 'form found' shape ideally takes the form of a minimal surface which joins the boundary points with the smallest possible membrane area and has uniform in-plane tensile stresses throughout [7, 8]. True minimal surfaces cannot be formed between all boundary conditions, but a pseudo-minimal surface can be developed for a fabric membrane by accepting increased stresses in the region where the soap bubble would have failed. This reduces the limitations on the forms that can be created but results in reduced structural efficiency.

The design of membrane structures is not codified in Europe, and only recently has limited design guidance been compiled [9]. International standards do exist [10] but provide broad design principles rather than a detailed analysis and design methodology.

Figure 2. Soap-film model of a uniformly stressed membrane structure with cable boundaries (image reproduced with permission of Institut für Leichtbau, University of Stuttgart).

Architectural fabrics consist of woven yarns to provide strength, with an impermeable coating to provide waterproofing and stabilise the weave. Warp yarns run along the length of the roll and are typically fairly straight, with highly crimped weft (or fill) yarns woven across the roll (Figure 3). The most common material combinations are PVC coated polyester yarns and PTFE coated glass-fibre yarns, with silicone coated glass-fibre fabric becoming increasingly popular and PTFE coated ePTFE (*Tenara*) providing very high light transmission. A combination of non-linear stress-strain response of the component materials (yarn and coating) with the interaction of orthogonal yarns, results in complex (non-linear, hysteretic, anisotropic) material behaviour [11]. Full quantification of the response of coated woven fabrics to in-plane loading (biaxial and shear) is time consuming and costly, and arguably has not yet been achieved. It is common practice to use assumed stiffness values for a given fabric material [2], but the actual stiffness may differ by a factor of between two and five from these

assumed values [12]. Elastic moduli, Poisson's ratios and shear stiffness are independent and are not constrained by conventional limits and relationships for anisotropic materials.

Figure 3. Architectural fabric cross-sections showing highly crimped, woven yarn bundles encased in PTFE or PVC coating (left); interaction of warp and fill yarns in woven fabric results in highly non-linear stress-strain behaviour dependent on the ratio and magnitude of the applied in-plane biaxial stress.

1.2 DESIGN REQUIREMENTS

There are four fundamental design requirements for membrane structures:

- stress limits based on material strength and stress factors,
- deflection,
- avoidance of ponding,
- avoidance of slackness and/or wrinkling.

Fabric strength is routinely determined by manufacturers from uniaxial tensile tests [13], however it is well known that the presence of a flaw such as a tear or a severe crease can dramatically reduce the in-service strength of the fabric. A 40mm tear in a 400mm wide uniaxially loaded panel may reduce the strength of the fabric in a direction normal to the tear by 75% [14]. However, the specific reduction in strength is dependent on a number of factors including coating & yarn types, weave, temperature, moisture content, and loading rate. The significance of these factors has not yet been established. Transportation and deployment on site can also have significant impacts on the strength of the canopy, particularly for fabrics with brittle glass-fibre yarns.

Large displacements (e.g. +/- 500mm) are acceptable in membrane structures and it is difficult to define meaningful displacement limits. Typical design requirements are that the membrane does not clash with other parts of the structure and that the curvature of the membrane does not invert. The analysis results may show acceptable (positive) tensile stresses, but if the curvature has inverted the membrane will have passed through a state of slackness en route to the equilibrium position. This may result in flapping and crease-fold damage to the fabric, and even structural instability.

Ponding is the accumulation of rain, snow or melt-water in hollows in the membrane surface, which can lead to fabric failure. It is vital to ensure that fabric structures maintain positive drainage under all load conditions. This check is carried out following analysis with unfactored loads (due to geometric non-linearity), but this means that the subsequent design may only just avoid ponding with a factor of safety close to one, with no consideration of uncertainty and variability in the material properties, prestress levels, construction tolerances and the analysis itself. Whilst deflection limits are typically considered to be a serviceability condition, clashes with structural elements, ponding and slackness could all result in failure of the membrane structure, hence deflection should be considered to be an ultimate limit state requirement for this type of structure.

Due to the negligible compressive and out-of-plane bending stiffness of architectural fabrics, when the tensile stress reduces to zero in any direction the membrane surface will start to buckle and wrinkling will subsequently appear with further compressive strain. Wrinkling is not typically considered in analysis, but is a common problem during installation when the mobilization of shear deformation is not sufficient to achieve the required level of curvature. Wrinkling typically results in alternating regions of high and low stress, with potentially much higher stresses than predicted by the membrane analysis. Wrinkling is therefore unacceptable for both aesthetic and functional reasons, and can be considered to be both a serviceability and ultimate limit state. Wrinkling criteria have been proposed in terms of either principal stresses or principal strains, but it has been found that for geometrically non-linear analysis with anisotropic materials a combined stress and strain criteria is more appropriate [15] (Table 1).

Stress state	Wrinkling	Membrane state
Minimum principal stress > 0	No	Taut
Maximum principal strain ≥ 0 and minimum principal stress ≤ 0	Uniaxial	Wrinkled
Maximum principal strain ≤ 0	Biaxial	Slack

Table 1. Wrinkling criteria based on principal strain and stresses [15]

1.3 CURRENT DESIGN PRACTICE: STRESS FACTOR APPROACH

The geometrically non-linear behaviour of membrane structures renders the limit state method employed in the design of conventional structures (e.g. steel and concrete) to be inappropriate. A 'stress factor' or 'permissible stress' approach is employed in which the fabric strength is reduced by a factor (frequently incorrectly described as a 'factor of safety') to give a permissible stress value. The maximum fabric stress values from an analysis with un-factored (characteristic) loading are compared to this permissible stress value. The stress factor must account for all uncertainties in the structure – including variability of material properties, loading, long term material damage and degradation and construction tolerances. Any lack of confidence in the results of the analysis must also be reflected in this single factor.

“With development in science and technology, the element of ignorance can be largely eliminated, while the uncertainties, being changed in form and magnitude, can never be removed” Freudenthal, The Safety of Structures [16].

Current practice assumes that these uncertainties are accounted for via a suitable combination of individual stress factors to give a single ‘composite factor’. Engineering groups across a range of countries have adopted alternative stress factors that have been derived using a number of different approaches (Table 2). Whilst it is not within the scope of this paper to discuss the merits of each approach, it is clear that the range of stress factors is large and reflects a lack of consensus at both European and international levels. Recent work on development of a European standard for Membrane structures for inclusion in Eurocode 10 via CEN250 Working Group 5 highlights the need for harmonization of these disparate design practices.

Whether or not it is explicitly stated, the magnitude of all of these stress factors is driven by the knowledge that fabric strength is severely reduced by the presence of a tear [14]. The large magnitude of the factors, combined with the common misnomer of referring to them as ‘safety factors’, gives false comfort to the Design Engineer that a large margin of safety has been incorporated in the design, and that this will accommodate any other uncertainties that may not have been explicitly considered. For a given design case the behaviour and performance of a membrane structure may be affected by a series of uncertainty sources, each with different individual and combined influences, making the effective selection of a single value (or narrow range) very difficult. If the stress factor does not account for all of these uncertainties, the ‘safety factor’ may be in fact be one or less.

Catastrophic failures of a small number of tensile fabric structures have shown that in some cases this approach is not reliable. In the 4th February 1999 issue of New Civil Engineer a special feature appeared under the title *“Creative tension – wiring into tented structures”*. The main article opened with the following - *“Are giant tents domed? Tensile fabric structures are among the most spectacular of the 20th century – and the most controversial”*. This statement was made with reference to the collapse of the new fabric roof over Montreal’s Olympic Stadium in early 1999 (New Civil Engineer, 28th January 1999). The actual cause of the failure was identified as design based and not arising from accidental damage.

Source	Stress factors	Stated uncertainty incorporated within the stress factor
International Association of Shell and Spatial Structures Working Group 7 recommendation for pneumatic structures [9]	4.2 – 6.0 (warp) 5.0 -7.0 (fill)	Fabric and testing consistency; Calculation accuracy; Loading, fabrication and installation uncertainty; Environmental degradation; Unforeseen factors
American Society of Civil Engineers 55-10 [10]	4.0 – 7.8	Life cycle factor; Strength reduction factor; Load combination factor
French Design Guide [9, 17]	4.5 – 7.0	Fabric & seam quality; Structure scale (probability of defects); Pollution & environmental degradation (including quality of finite element analysis)
German standard (DIN 4134) [18] with reduction factors according to Minte [9, Section 6.2.3, 19]	Fabric 4.9 – 6.4 Permanent load 2.9 – 3.2 Wind load 4.4 – 5.1 Snow load Seams 6.7 – 9.5 Permanent load 3.5 Wind load 4.9 Snow load	Uniaxial strength, modified depending on whether structure is loaded biaxially or uniaxially; Load factor; Material factor, accounting for seams and connections; Load duration; Pollution and degradation; Temperature
Draft Italian Code [9] [20]	4.5 wind 3.75 snow	No details provided
Chinese Technical Standard [21]	5	Factor of 2.5 for simultaneous wind and snow loading
Membrane Structures Association of Japan guide for 'Specific Membrane Structures' [9]	8 for sustained loads 4 for temporary loads	No details provided

Table 2. Stress reduction factors for tensile fabric structure design according to different organisations and countries.

2 APPLICATION OF RELIABILITY ANALYSIS TO MEMBRANE STRUCTURES

BS EN 1990:2002 “Eurocode - Basis for Structural Design” [22] provides the underlying principles for the Structural Eurocodes and the assumptions on which they are based. The theoretical base for the Eurocodes is reliability theory, with reliability defined as the ‘*ability of a structure or a structural member to fulfil the specified requirements, including the design working life, for which it has been designed. Reliability is usually expressed in probabilistic terms*’ [22]. The development of a standard for membrane structure design within CEN250 Working Group 5 falls within the context of Eurocode “Basis for Structural Design”. Use of reliability analysis for membrane structures would bring their design in line with ‘conventional’ building materials, would fit within the widely accepted Eurocode framework, and would allow closer integration of the design of fabric structures and the supporting structure. Reliability requirements are defined according to the importance and design life of the structure. Depending on the consequence of structural failure and design life, membrane structures can be categorized according to Table 3 and hence the required safety index, β , can be determined. For each reliability class (RC) there is a specified annual probability of failure, which results in differing safety indexes for 1 year and 50 year reference periods. These are typical timescales used for temporary and permanent structures throughout the Eurocodes.

Consequences Classes or Reliability Classes	Annual failure probability [23]	Reference period	Recommended minimum values for reliability index β (ultimate limit states)
RC3: High consequence for loss of human life, <i>or</i> economic, social or environmental consequences very great .	10^{-7}	1 year 50 years	5.2 4.3
RC2: Medium consequence for loss of human life, economic, social or environmental consequences considerable .	10^{-6}	1 year 50 years	4.7 3.8
RC1: Low consequence for loss of human life, economic, social or environmental consequences small or negligible .	10^{-5}	1 year 50 years	4.2 3.3

Table 3. Target reliabilities for different building categories, from “Eurocode - Basis for Structural Design” combining Table B1 *Definition of consequences classes* and Table B2 *Recommended minimum values for reliability index β (ultimate limit states)* [22].

If used as a basis for structural design, the principle of structural reliability must be followed through the entire fabric structure design process. The safety or reliability requirements of a structure are related to different types of limit state – e.g. ultimate, fatigue, and serviceability (Table 4) – as used in other structure types.

Limit State	Target reliability index, β , for reference period of
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	1 year	50 years
Ultimate	4.7	3.8
Fatigue	-	1.5 – 3.8*
Serviceability	2.9	1.5
* Depends on degree of inspectability, reparability and damage tolerance.		

Table 4. Target reliabilities of RC2 building type by limit states, reproduced from “Eurocode - Basis for Structural Design” Table C2 Target reliability index β for Class RC2 structural members [22].

Calculation of membrane structure reliability will require a substantial amount of material uncertainty information, and a robust and efficient reliability estimation approach which can reasonably combine the material and geometric information with the characteristics of the membrane structure performance. In addition to high quality material information, of equal importance is the essential requirement for a high fidelity analysis capability. A comparative ‘round robin’ analysis has recently been carried out by the *Tensinet Analysis & Materials Working Group* (www.tensinet.com; tensinet.amwg@ncl.ac.uk), in which twenty organisations independently analysed four simple, precisely defined tensile fabric structures to identify epistemic uncertainty associated with the analysis [24]. The paper concludes that: “*The results show very high levels of variability in terms of stresses, displacements, reactions and material design strengths, and highlight the need for future work to harmonise analysis methods and provide validation and benchmarking for membrane analysis software*” [24].

The reliability of a structure is based on performance requirements or limit state functions. Within the context of membrane structures, these concepts are not new with “limit state functions” such as maximum permissible stress or maximum displacements routinely used in design. A typical design question currently used may be of the type: “*Has the stress at any point within the structure as predicted by a numerical model exceeded the (factored) permissible stress?*”. The equivalent reliability-analysis-based question would be of the type: “*What is the probability that the stress at any point within the structure as predicted by a numerical model has exceeded the fabric strength?*”. The data arising from fabric strength tests may be in the form of a stated minimum value or a statistical representation comprising a mean, standard deviation, and higher statistical moments or coefficients depending on the best-fit distribution. The principal difference between the stress associated with the stress factor approach ($\hat{f}_{FOS}$) and the reliability-based approach ($\hat{f}_{Pf}$) is not the means by which the values are determined from the material tests, but in the representation and application of the data. For example, in the case of the stress factor approach, $\hat{f}_{FOS}$ would be either the minimum stated value or the mean value perhaps reduced by a pre-selected number of standard deviations. This value is then further reduced by a stress factor as described above (Table 2). In contrast, when undertaking a reliability analysis, the full statistical information which describes the performance of the fabric can be used. A substantial programme of testing will be required to provide statistically meaningful distributions of results, and in addition existing test data (for example manufacturers’ databases of test results) will be able to be utilized more fully.

Similar questions to those regarding the fabric strength may also be posed relating to deflections, ponding and wrinkling. Formulation of these limit state functions are not as straightforward as strength criteria. Deflection of the fabric due to wind loading may be considered to be a serviceability limit state, with consequences such as noise or occupant distress rather than structural failure. However, if deflections are limited to avoid clashes with structural elements, this would become an ultimate limit state with varying limits at different points on the structure. Similarly, deflections due to snow loading represent an ultimate limit state when ponding may occur which would result in structural failure. Ponding checks require an assessment of slope rather than absolute deflection, which presents a further complexity for the analysis. Clearly in addition to development of reliability analysis tools, substantial understanding and experience is required to formulate the correct limit state functions. Straightforward examples of strength and deflection criteria are given in Table 5.

Design criteria	Design Philosophy	
	Stress factor	Reliability
Strength (warp & fill)	$\hat{\sigma}^{MAX} \delta \hat{\sigma}_{FOS}$	$\hat{\sigma}_{pf} - \hat{\sigma}^{MAX} = 0$
Deflection (e.g. 160mm at a given location to prevent clash with steelwork)	$\delta^{MAX} \delta 160\text{mm}$	$160\text{mm} - \delta^{MAX} = 0$
$\hat{\sigma}^{MAX}$ = maximum stress; δ^{MAX} = maximum deflection; $\hat{\sigma}_{FOS}$ = permissible stress; $\hat{\sigma}_{pf}$ = limiting stress		

Table 5. Example comparability between stress factor & reliability principles.

In current fabric structure design practice stress factors are assumed to account for a range of uncertainties. Unlike the use of partial factors in the design of steel and concrete structures, the loads remain un-factored and uncertainties arising from variability of the loading are included in the “permissible” stress value. The complete probabilistic content of the analysis is replaced by a factored deterministic analysis. Structural performance is assessed after the analysis has been completed, with performance criteria assessed by the Design Engineer. Assessment of the contribution made by each of the input parameters (e.g. strength, biaxial and shear stiffness etc.) to the performance of the structure is only possible with a time consuming parametric study involving multiple analyses. The performance of the structure is considered on a point-by-point basis – for example the maximum stress at any location is checked against the permissible stress. A structure with large areas of high stress would be treated in the same manner as a structure with only a localized area of high stress.

In contrast to the stress factor approach, a reliability-based analysis requires the statement of performance criteria before the analysis is undertaken. A reliability-based analysis makes use of statistical information to quantify the uncertainty content of each parameter. This statistical information is obtained directly from test data. As more tests are carried out on a particular material, an increasingly accurate statistical description of the material will be developed. New uncertainties may be introduced as information becomes available to refine the quality of the structural simulation, for example by inclusion of time-dependent behavior including creep and relaxation. The statistical information will typically be in the form of mean and standard deviation values. The coefficient of variation is defined as the ratio of the standard deviation to the mean, giving a non-dimensionalised

transferable measure of uncertainty. In a reliability-based analysis, the uncertainty measures are combined as part of the analysis methodology to assess the most onerous design condition for the structure. The combination of the uncertainty measures is a function of the uncertainty measures themselves and the structure, and will change as the uncertainty and structural parameters (e.g. geometry, loading, etc.) change.

The output from the reliability analysis is a probability of failure, p_f , or safety index, β , and a *design point* [25]. The probability of failure may be compared with the acceptance criteria specified within “Eurocode - Basis for Structural Design” (Table 3 & Table 4). The *design point* is the unique set of values of the uncertain variables that combine to make the most onerous condition for the structure when assessed against the stated engineering performance criteria (or limit state function), for example strength, deformation, etc. The physical location of this point on the structure is also identifiable from the evaluation of the limit state function.

The probability of failure may also be read as a safety index, β , which is the number of standard deviations that the *design point* is located away from the mean design. For example, in “Eurocode - Basis for Structural Design”, the minimum value of β for an ultimate limit state is 3.8 (Table 4). In other words, following the reliability analysis if the *design point* were more than 3.8 standard deviations away from the mean then the structure would not meet the requirement of the code. Placing this within a general engineering context, the same principles are used in manufacturing where the concept of the *six-sigma* approach is applied, in which a product must perform within ± 3 standard deviations of the mean to be acceptable.

For complex structures, explicit mathematical solutions of the probability of failure are not normally available. Approximate reliability methods are required to make the problem tractable [25]. These include first and second-order reliability methods (FORM and SORM [26], Figure 4), and other approximation methods, such as response surface approaches [27]. Coupled with the Hasofer-Lind transformation [28], FORM has emerged as one of the most effective reliability analysis methods in estimating the probability of failure.

Figure 4. Schematic sketch of FORM and SORM approximation in the standard normal space, reproduced from Valdebenito et al [25].

In a FORM analysis, the values of the limit state functions may be obtained in the usual way through a deterministic structural analysis, with derivatives of $G(X_s)$ calculated either analytically or numerically, effectively forming a sensitivity analysis of the structural performance with respect to uncertainty. Therefore, as with a Monte Carlo simulation, the FORM reliability analysis makes use of an existing deterministic analysis tool, but with the benefit of drastically improved efficiency. In essence, the FORM reliability analysis may be viewed as a computational tool that “wraps around” a standard analysis – there is no requirement to rewrite existing deterministic codes to be able to undertake a FORM analysis.

The limit state function is derived from the state equations, typically in terms of displacements or stresses. The stresses are functions of strains which are non-linear functions of the displacements and, for the cases presented in this paper, material constants which are independent of the deformation state. The limit state function, therefore, will typically be non-linear in terms of displacements. Derivatives of the limit state function are obtained with these non-linearities explicitly included as variables and are obtained using the chain rule, for example. Within the FORM algorithm the limit state function and associated derivatives are evaluated at equilibrium of the structure. The non-linearities are, therefore, captured explicitly at this state and contribute to the FORM calculations.

Apart from an immediate application to FORM, the sensitivity analysis is also important in enabling the effects of the random variables on the membrane structure response to be elucidated. For variables that have significant effects on structural failure, any epistemic uncertainty may be reduced by collecting additional information through testing, thereby increasing the proportion of aleatoric (intrinsic) uncertainty and producing a more accurate statistical description. High sensitivity values may also be used to direct design changes, influence material selection, or to inform manufacturing tolerances and specifications.

Variables having little influence on structural reliability may be considered to be deterministic to save computational effort, without compromising the accuracy of the reliability estimate. The influence of this strategy on the safety index β at the MPP can be readily assessed with a single subsequent analysis. Sensitivities may be calculated either analytically if possible (subject to availability of access to the analysis coding), semi-analytically if explicit analytical sensitivities cannot be derived mathematically, or using approximate numerical methods such as finite differences.

Combining sensitivity analyses and FORM to estimate the probability of failure of a system with an implicit limit state function uses computed information about the value and gradient of the limit state function at the *design point* at convergence of the analysis, and an iterative optimisation scheme to determine the safety index β [29]. The advantage of this approach is that solving the limit state equation $G(X_s) = 0$ is avoided, so that the reliability analysis can be undertaken for a structure with a complicated or implicit limit state function, given that the partial derivatives of $G(X_s)$ with respect to the uncertain variables are available.

3 MEMBRANE STRUCTURE RELIABILITY ANALYSIS: EXAMPLES & DISCUSSION

In this section two examples of the application of reliability analysis to membrane structures are presented. These analyses have been carried out using Fortran code developed for this work, based on the principles described above. The first example is a hyperbolic paraboloid, commonly referred to as a 'hypar', the simplest form of membrane structure and commonly used in practice (Figure 5). The hypar has been used as a straightforward but realistic structure that can be used to efficiently explore the effect of differing levels of variability in the analysis parameters. The second example is a case study based on an existing fabric structure, with the probabilistic analysis compared to the deterministic analysis results which were used for the actual design. The purpose of this case study is to examine the safety factors currently applied to fabric structure design consistent with the reliability requirement of the Eurocode 0. It also demonstrates how to apply the reliability analysis tool to the realistic design of a fabric structure.

3.1 HYPERBOLIC PARABOLOID ('HYPAR')

Figure 5. Hypar structures are characterised by alternating high and low points with cable supported edges. Hemmingsway Hotel, Kenya, 2002 (top), a classic 4 point hypar; Cayman International School, Cayman Islands, 2006 (bottom), an elegant 6 point hypar. All images © Architen-Landrell Associates.

In this simple but realistic example the probability of failure of a pre-tensioned hypar constructed from lightweight PVC coated polyester fabric is considered. A combination of deterministic (single valued) parameters (geometry, cable properties and prestress levels, Figure 6) and statistically defined parameters (loading and fabric properties, Table 6) has been adopted. The choice of which values to describe statistically can be decided for each structure and will depend on available statistical information.

Figure 6. Hypar example

X_j	Distribution	Mean value	Standard deviation (COV = 0.1)
E_f (kN/m)	Normal	1000	100
E_w (kN/m)	Normal	1000	100
G_{wf} (kN/m)	Normal	30	3.0
$v_{wf} = v_{fw}$	Normal	0.1	0.01
F (kN/m ²)	Normal	1.0	0.1
F			
σ_{per} (kN/m)	Normal	10	1.0

Table 6: Distributions and parameters of the random variables for the hypar example

An assumed material strength of 10 kN/m has been used, equivalent to a lightweight PVC coated polyester fabric with an ultimate tensile strength of 50 kN/m with a typical stress reduction factor of 5 applied. In this example the hypar has been subjected to uniform wind uplift only, hence for the chosen patterning direction the fill stress will always be greater than the warp stress. It follows that the limit states for this structure can be defined as:

Limiting warp stress	$G_1(X_s) =$	not required for this example
Limiting fill stress	$G_2(X_s) =$	$\int_{per} - \int_{max}^f \quad \int_{per} = \int_{ult} / 5 = 10 \text{ kN/m}$
Wrinkling criteria	$G_3(X_s) =$	$\int_{min}^p - \int_{per}^p \quad \int_{per}^p = 0$
Deflection criteria	$G_4(X_s) =$	$D_{all} - D_{max} \quad D_{all} = 35 \text{ mm}$

The allowable displacement (D_{all}) is an arbitrary choice that would usually be defined by specific issues (clashing with steelwork or ponding). In this example the value of 35mm is based on the span of the structure (4m) divided by 120.

For this example all random variables are assumed to be normally distributed with the standard deviation of each variable initially set to be 10% of the mean value (i.e coefficient of variation = $COV = 0.1$). This gives safety indices of 2.30 for fabric failure (first column of Table 7), 1.98 for wrinkling (Table 8) and 3.09 for displacement (Table 9). Safety index requirements from the Eurocodes (Table 3 & Table 4) are typically 3.8 for ultimate limit states and 1.5 for serviceability. The safety index of 2.3 for fabric failure clearly does not meet this requirement. Wrinkling is acceptable if it is considered to be a serviceability limit state, and the safety index for displacement is acceptable for either limit state.

A conventional, deterministic analysis of the same structure (i.e. $COV=0$ for all variables) gives a maximum fabric stress (fill direction) of 7.6 kN/m and a maximum displacement of 11.5 mm.

Comparing the maximum stress to the material strength with a stress reduction factor of 5 gives a factor of safety of 1.32. The displacement is well within the limit of 35mm. It is significant that using a deterministic analysis the structure would have been deemed to be acceptable, whereas with the introduction of variability ($COV=0.1$) in the analysis parameters the structure does not meet the fundamental requirement of the Eurocodes.

X_j	Safety indices					
	COV = 0.1	COV = 0.15	COV = 0.20	COV = 0.25	COV = 0.30	COV = 0.40
E_w	2.301	2.286	2.251	2.231	2.192	2.089
E_f	2.301	2.271	2.235	2.219	2.146	2.055
G_{wf}	2.301	2.301	2.301	2.300	2.300	2.299
v_{wf}	2.301	2.301	2.301	2.301	2.301	2.301
F	2.301	2.149	1.984	1.814	1.777	1.403
σ_{per}	2.301	1.596	1.213	0.977	0.817	0.616

Table 7: Safety indices for the fabric failure limit state with changing parameter variability

X_j	Safety indices					
	COV = 0.1	COV = 0.15	COV = 0.20	COV = 0.25	COV = 0.30	COV = 0.40
E_w	1.976	1.904	1.831	1.662	1.542	1.316
E_f	1.976	1.762	1.563	1.401	1.262	1.041
G_{wf}	1.976	1.976	1.975	1.975	1.973	1.973
v_{wf}	1.976	1.976	1.976	1.976	1.976	1.976
F	1.976	1.479	1.182	0.971	0.828	0.633

Table 8: Safety indices for the wrinkling failure limit state with changing parameter variability

X_j	Safety indices					
	COV = 0.1	COV = 0.15	COV = 0.20	COV = 0.25	COV = 0.30	COV = 0.40
E_w	3.086	3.067	3.061	3.054	3.045	2.996
E_f	3.086	2.486	2.183	1.725	1.515	1.167
G_{wf}	3.086	3.086	3.084	3.082	3.076	3.071
v_{wf}	3.086	3.084	3.083	3.081	3.079	3.069
F	3.086	2.382	1.956	1.681	1.426	1.057

Table 9: Safety indices for the deflection limit state with changing parameter variability

Not all of the random variables will have the same level of effect on the reliability for a specific limit state. In certain cases, only a subset of the random variables (e.g. loading and material strength) play significant roles in the reliability analysis, and the reliability is not sensitive to the other variables even though they may have high levels of uncertainty. To investigate the sensitivity of the probability of failure to the variability of each analysis parameter, the coefficient of variability (COV) of each random variable has been increased in turn whilst leaving the COV of all variables at the base-line value of 0.1. The resulting safety indices are given in Table 7, Table 8 and Table 9 for the three limit states.

As might be expected, the safety index for the failure limit state is more sensitive to the variability of permissible stress and applied load rather than the fabric elastic properties. For the wrinkling and serviceability failure criterion the permissible stress is not considered. For both of these limit states the dominant variables are the applied load and the fill stiffness, with variability of other parameters having minimal effect on the performance of the structure. Further analysis of a wide range of 'typical' fabric forms will enable key parameters to be identified for reliability analysis of different structural types, enabling the costly and time consuming process of determining statistical information for analysis parameters to be focused on the most significant variables.

3.2 IMPLEMENTATION IN REAL STRUCTURES: DONCASTER EDUCATION CITY CRECHE CANOPY

The Doncaster Education City Creche Canopy is a double-conic canopy designed by Arup and fabricated by Base Structures (Figure 7) in Doncaster, UK. The structure is a modest size twin conic with an area of 425m², but has a complex form with high levels of curvature that required radial cables to achieve the required form without excessive membrane stress or deflection that could result in ponding. The one millimetre thick PVC coated polyester fabric is supported by two steel head rings suspended from inclined masts, with radial booms supporting articulated corner clamp plates and tensioned boundary cables to the perimeter. The membrane prestress is 1.5kN/m in warp and fill directions.

The original design was carried out using *Oasys GSA (General Structural Analysis) Fabric* (www.oasys.com). This industry standard software enables simulation of the initial membrane form-finding followed by a deterministic geometrically non-linear analysis. In line with current industry practice stress reduction factors of between 5 and 10 were used to calculate the required fabric strength based on the stress results from the un-factored load combination cases.

Figure 7. Doncaster Education City Creche Canopy – general arrangement drawings (top) [courtesy of Arup] and images of the completed structure (bottom) [courtesy of Base Structures].

Figure 8. Finite element simulation using 166 linear strain triangle elements, with example displacement results.

The reliability based analysis of the membrane (Figure 8) requires definition of limit state functions and statistical descriptions of all parameters. Four limit state functions can be defined for the canopy:

Limiting warp stress	$G_1(X_s) =$	$\int_{ult} - \sigma_{max}^w$
Limiting fill stress	$G_2(X_s) =$	$\int_{ult} - \sigma_{max}^f$
Wrinkling criteria	$G_3(X_s) =$	$\sigma_{min}^p - \sigma_{per}^p \quad \sigma_{per}^p = 0$
Deflection criteria	$G_4(X_s) =$	$D_{all} - D_{max} \quad D_{all} = -160 \text{ mm}$

$G_1(X_s)$ and $G_2(X_s)$ require that the maximal warp and fill stresses (σ_{max}^w and σ_{max}^f) at any location in the membrane are less than the measured, statistically defined, strength of the fabric, $\int_{ult}$. Limit state function $G_3(X_s)$ describes a basic wrinkling criterion in which the minimum principal stresses, σ_{min}^p , at all locations are positive definite as given by σ_{per}^p . A simple deflection criteria is represented by $G_4(X_s)$ in which an allowable displacement, D_{all} is compared with the (local) maximum D_{max} . These four limit states are sufficient for this example; a more extensive and detailed set of criteria could be developed if required to consider local issues such as ponding.

Carrying out sufficient testing to determine the mean and standard deviation values for all material properties would be uneconomic for any given project. Some manufacturers carry out extensive

testing in-house for quality control and development purposes, but this data is commercially sensitive and is not published. A move towards reliability based analysis will generate demand for manufacturers to provide more detailed material properties on their data sheets, including mean and standard deviation information. This move would be motivated by the competitive advantage in showing that one manufacturer's product was more consistent than another. Currently there is little benefit in demonstrating consistency of manufacture, but once this information is included in the analysis then specifications will be able to demand tight manufacturing tolerances and consistent material performance.

For this work it was not practicable to undertake a statistically significant set of tests for all relevant parameters. To demonstrate the reliability analysis methodology, mean values based on limited test data have been used, with standard deviation values based on the test values and an assumed normal distribution (Table 10). This approximate method for interpreting small numbers of data values may be very useful as a pragmatic approach for filling in gaps in the available statistical information about a material.

Variable	Statistical distribution	Mean value	Sample standard deviation	Coefficient of variability	Values used in original design
Ultimate tensile strength, f_{ULT} (undamaged material) (kN/m)	Weibull	76.9	2.7	0.035	Warp = 84, Fill = 80 ^a
Torn fabric strength (kN/m)	Assumed normal	21.7	1.8	0.083	$f_{ULT} / 4$ ^b
Biaxial warp stiffness, E_w (kN/m)	Assumed normal	850	246	0.29	600 ^c
Biaxial fill stiffness, E_f (kN/m)	Assumed normal	924	109	0.12	600 ^c
In-plane shear stiffness (kN/m)	Assumed normal	15.0	5.0	0.33	30 ^c
Poisson’s ratio (warp-fill)	Assumed normal	0.45	0.06	0.13	0.8 ^c
Poisson’s ratio (fill-warp)	Assumed normal	0.53	0.21	0.40	0.8 ^c
Prestress	1.5 kN/m				
Applied snow load (kN/m ²)	-0.55 kN/m ²				
Applied wind uplift (kN/m ²)	0.6 kN/m ²				
Structural steelwork tolerances; fabric patterning & fabrication tolerances	Not considered				
Notes					
^a Tensile strength values provided by the manufacturer,					
^b Generally accepted 75% strength reduction with a small tear,					
^c Typical elastic constants used for PVC coated polyester.					

Table 10. Statistical description of membrane analysis parameters for Doncaster Creche Canopy analysis

The strength of the fabric with a small tear has been used [14], based on six wide panel tear tests. This is a conservative approach that is consistent with the current use of stress factors – it is assumed that the fabric is torn at any given location. Further development of the reliability analysis presented here would enable the spatial distribution and probability of a tear occurring to be included in the analysis: what is the likelihood that there will be pre-existing damage in the area of maximum stress? This will enable benefits from specifying a high level of inspection and maintenance to be incorporated in the design with a reduced likelihood of damage at a given location at a given time. Conversely, for inaccessible structures the implications of limited inspection and maintenance can be understood.

Biaxial stiffness values for architectural fabrics are typically closely repeatable between tests, but the values vary significantly dependent on the applied stress ratio, magnitude and history [11, 30]. Current best practice is to consider all stress ratios together to give a single set of elastic constants which approximate the non-linear fabric behavior [31] and are compatible with standard analysis codes. By considering best-fit elastic constants to different stress ratio ‘zones’ the variability of the stiffness parameters has been explored (Table 11) and incorporated in the reliability analysis. This method could be further generalized by considering elastic constants derived from small ‘patches’ based on a surface fit to the test data.

Biaxial stress ratios used to calculate elastic constants	Warp modulus (kN/m)	Fill modulus (kN/m)	Poisson's ratio (warp-fill)	Poisson's ratio (fill-warp)	Quality of fit of elastic constants to test data (R^2)
All stress ratios	841	931	0.38	0.65	0.973
0:1, 1:3, 1:2	370	895	0.23	1.16	0.996
3:4, 1:1, 4:3	986	1030	0.32	0.37	0.995
2:1, 3:1, 1:0	1132	967	0.51	0.17	0.997
1:3, 1:2, 3:4	693	957	0.32	0.68	0.996
4:3, 2:1, 3:1	1051	950	0.45	0.31	0.996
Mean	845	955	0.37	0.56	
Sample standard deviation	280	44	0.10	0.36	
<i>Note: best-fit elastic constants determined using differential strain minimization [31] for ‘increasing-load’ data only, with residual strain during test removed [32] and with the reciprocal relationship not applied (i.e. 4 independent elastic constants).</i>					

Table 11. Best-fit elastic constants for varying load ratios

For this example the geometry of the supporting structure is deemed to be deterministic, i.e. the boundaries conditions are taken to be certain. In addition the level of prestress and the initial shape of the canopy (determined by the accuracy of the cutting patterns and fabrication) are considered as deterministic single values. However, if sufficient information is available each of these parameters could be described statistically. The advantages of including construction and fabrication tolerances in the analysis are two-fold. The impact of these uncertainties on the behaviour of the structure is incorporated in a single analysis, ensuring that these are considered in design without the need for multiple analyses and parametric studies to determine the worst case. The benefit of reducing tolerances and improving fabrication quality is reflected in the analysis results, providing an incentive to improve quality, resulting in lower variability, more certainty in the design and hence a more efficient, cost-effective design. The importance of quality control, management and supervision for structural reliability are discussed in detail by Carpenter [33] and the Standing Committee on Structural Safety [34].

Ongoing work at Newcastle University on shear testing of architectural fabrics has shown that shear stress-strain behavior is non-linear, hysteretic, and severely dependent on previous shear deformation [35]. Repeated shear cycles result in degradation of the shear stiffness. This is reflected in the large standard deviation shown in Table 10, but further work is required to fully elucidate the complex shear response of architectural fabrics, and then to provide a statistical description.

For design of structures under the Eurocode framework, characteristic snow and wind loads with a 50 year mean return period are calculated [36, 37] and these values are then multiplied by partial factors [22]. Due to large displacements and consequent geometric non-linearity, it is standard practice to apply un-factored characteristic loads in the analysis of membrane structures. It is important to consider the purpose of each partial factor when deciding whether it should be applied to the membrane structure loading or analysis results. In “Eurocode - Basis for Structural Design” there is a distinction between the following partial factors for loads (referred to as ‘actions’):

- γ_f *Partial factor for actions, which takes account of the possibility of unfavourable deviations of the action values from the representative values*
- γ_{Sd} *Partial factor associated with the uncertainty of the action and/or action effect model*
- γ_F *Partial factor for actions, also accounting for model uncertainties and dimensional variations ($\gamma_F = \gamma_{Sd} \times \gamma_f$).*

“Eurocode - Basis for Structural Design” distinguishes between actions (the applied load) and action effects (the resultant force, moment etc. within the structure). Brief guidance on the application of partial factors in the case of geometric non-linearity is provided for the combined factor γ_F : (p.42-43) [22]:

“(4) For non-linear analysis (i.e. when the relationship between actions and their effects is not linear), the following simplified rules may be considered in the case of a single predominant action:

a) When the action effect increases more than the action, the partial factor γ_F should be applied to the representative value of the action.

b) When the action effect increases less than the action, the partial factor γ_F should be applied to the action effect of the representative value of the action.

NOTE Except for rope, cable and membrane structures, most structures or structural elements are in category (a)."

Case (a) refers to P- Δ effects where structural deformation has a destabilizing effect. In this case the partial factor should be applied to the load, to give the worst case P- Δ effect. For membrane structures structural deformation typically increases the stiffness of the structure and makes it work more efficiently. Taking the simplest example of a cable with a uniform linear load forming a semi-circular arc between two supports, the tension is equal to the load multiplied by the radius of curvature [38]. If the cable is extensible and the load remains constant, the radius of curvature will decrease as the cable extends and hence the cable tension will reduce, as described in case (b). In this case the conservative approach is to apply the partial factor to the action effect, and this represents current practice where loading uncertainty is usually incorporated in the stress factor (Table 2).

The safety indices provided by the reliability analysis for the fabric failure limit state (1.3 and 1.0 for warp and fill directions) are significantly lower than the typical Eurocode requirement of 3.8. A deterministic analysis would have shown that the design was (just) acceptable with a minimum factor of safety of 1.02, but with the inclusion of the high levels of variability used for this example the design does not meet Eurocode requirements. In addition to using the best estimate of parameter variability (Table 10), the Doncaster Creche Canopy has also been analysed with uniform, lower levels of variability applied to all parameters (right hand side of Table 12). With a coefficient of variability of 10% the minimum safety index increases to 4.0, which is acceptable under the Eurocode framework. Further decreases in variability to 5% and 1% give corresponding increases in safety index.

Material direction	Deterministic analysis using stress factor approach					Safety index from reliability analysis, using torn material strength			
	Maximum stress (kN/m)	Ultimate tensile strength - provided by manufacturer (kN/m)	Stress reduction factor	Allowable stress (kN/m)	Factor of safety	Best estimate of variability (Table 10)	COV = 0.1	COV = 0.05	COV = 0.01
Warp	16.4	84	5	16.8	1.02	1.3	5.2	11.2	44.3
Fill	10.7	80	5	16.0	1.50	1.0	4.0	8.6	33.8

Table 12. Comparison of deterministic stress-factor approach and reliability based analysis for fabric failure limit states. Doncaster Creche Canopy with prestress plus wind uplift applied.

There are two reasons why the safety indices using ‘best estimate’ variability are well below the Eurocode requirements – high levels of variability in the fabric stiffness values and the use of the torn fabric strength. Stiffness values taken from different zones of the non-linear response surfaces result in highly variable elastic moduli and Poisson’s ratios (COV up to 40%). This high level of variability is appropriate if elastic constants are used to approximate the material behavior. However, this level of variability has been shown to be unacceptable under the Eurocode framework as the resulting safety indices are too low. The alternative is to use a more complex, non-linear material model (for example using look-up tables of strain-stress data, or using neural network response surfaces). This would lead to a step change in the variability measure as it would only have to account for the variability between tests, not between different load conditions within each test.

The use of the torn fabric strength as the permissible stress in the reliability analysis is overly pessimistic and arguably does not allow the true safety index to be determined. This approach assumes that there is a 100% likelihood of a tear being present at any given location on the structure. Combined with other uncertainties this results in an unacceptably low safety index. The true benefit of the reliability analysis can only be realized when we have an understanding of the probability of a tear being present at any given location. A meaningful assessment of this probability should be based on surveys of existing structures, and should include consideration of structure and material type, maintenance, age and fabric handling during installation. This approach will enable benefits from specifying a high level of inspection and maintenance to be incorporated in the design with a reduced likelihood of damage at a given location at a given time. Conversely, for inaccessible structures the implications of limited inspection and maintenance can be understood.

4 CONCLUSIONS

A key parameter for the design of fabric structures is the ‘stress reduction factor’ which reflects the severe reduction in fabric strength in the presence of a tear, combined with any other uncertainties such as degradation and material variability (Table 2). A wide range of values are used by different countries and organizations, making consistent design and checking problematic. A major benefit of reliability based analysis of fabric structures is that it removes the need for this single, all-encompassing stress factor. The value and analytical elegance of a reliability analysis means that uncertainties in all analysis parameters can readily be included in a mathematically consistent manner as the test or simulation data becomes available. The individual contribution of uncertainty in each analysis parameter to the probability of failure of the structure can be assessed, and probabilities of failure can be compared against widely accepted values given in “Eurocode – Basis for Structural Design” to assess the adequacy of the design (Figure 9).

Figure 9. Reliability analysis principles and practice, after Melchers [39]

Within the context of recently begun work of CEN250-WG5 in contributing to Eurocode 10, the preceding examples demonstrate the application of reliability concepts to the analysis of membrane structures and the feasibility of adopting these principles. A clear step that must be taken is to provide guidance for situations where reliability analysis capabilities are not available. This may be achieved, in principle, through the normal route of code calibration (Figure 10). The most significant differences in applying this process to membrane structure design is in replacing partial factors with stress factors and in the myriad of realised forms that membrane structures can take. In adopting the philosophy of “Eurocode - Basis for Structural Design”, it is anticipated that as a minimum it may be necessary to classify structures by geometrical form (e.g. conic, hyper, barrel vault [3]) and support system (e.g. masts, cables, arch) to achieve consistency in the definition of appropriate stress factors. This approach may prove sufficient for certain classes of membrane structures, whereas for more unusual or hybrid forms it is expected that a full reliability analysis will be required and that the analysis will be subject to a safety index constraint.

Figure 10. Process for calibration of code safety-checking format, reproduced from Melchers [39]

For consistent application of Eurocode principles to fabric structures there must be consensus on whether each limit state is classed as ultimate or serviceability, with very different safety index requirements for the two limit states (Table 4). Deflection criteria are typically considered to be a serviceability limit state, but for membrane structures excessive deflections can result in failure. In addition a rigorous ponding criteria must be established, with checks for ‘hollows’ in the membrane surface required rather than overall deflection measures, and wrinkling checks must be incorporated in the analysis.

For the Doncaster Creche Canopy analysis presented above the safety indices fall well below the acceptable Eurocode values for two reasons – high levels of variability in the fabric stiffness values, and the use of the torn fabric strength. This clearly points to the developments that are required before tensile fabric structures can be designed under a reliability framework. Improvements in fabric material models, assessment of tear damage to existing structures, and increased material testing are required to provide consistent and meaningful statistical descriptions of the key analysis parameters. These requirements should drive a range of improvements across the industry: improved analysis capabilities, increased consistency of design and manufacture, routine material testing, and better specified installation and maintenance.

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