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Form & function: the significance of material properties in the design of tensile fabric structures

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Abstract

Coated woven fabrics have been used in state-of-the-art structures for over 40 years yet their design is not codified and relies heavily on experience and precedent. The mechanical behaviour of fabrics is non-linear and time dependent, with assumed or highly simplified material properties commonly used for analysis. The shape of a tensile fabric canopy is fundamental to its ability to resist all applied loads in tension. Increasingly Architects are moving away from conventional fabric forms, utilising lower levels of curvature and new materials.

This paper considers the importance of material properties and structural geometry in the design and analysis of tensile fabric structures. Three typical tensile forms are examined: the conic, hyperbolic paraboloid ('hypar') and barrel vault. Whilst the barrel vault demonstrates the expected result that minimally curved structures are inefficient and highly sensitive to changes in materials properties, the hypar exhibits more complex behaviour with the structural action varying dramatically with changes in geometry, material properties and patterning (fabrication) direction. For conic structures the feasible geometries that can be attained using 'soap film' form-finding is established, which combined with checks for ponding provide a range of geometric parameters for the efficient design of conic structures.

Keywords: fabric structure, form-finding, tensile, membrane, efficient design, hypar, conic, barrel vault.

1. Introduction

1.1. Tensile membrane structures: form and function

Architectural fabrics are lightweight (0.7 to 1.3 kg/m^2), waterproof and have negligible bending and compression stiffness. By ensuring that the fabric membrane remains in tension at all times, these materials can act as both structure and cladding to efficiently cover large areas, most notably for sports stadia and airports. The shape of the fabric canopy is fundamental to its ability to resist all applied loads in tension. To resist both uplift and down-forces the surface of the canopy must be double-curved (anticlastic curvature) and prestressed [1, 2]. Boundary conditions determine the fabric shape and stress distribution; ideally a uniform prestress is applied to the fabric. To achieve a uniform prestress the fabric must take the form of a minimal surface [3]; early work on tensile structures used soap films to determine this form in a process known as form-finding [4]. The minimal surface joins the boundary points with the smallest possible membrane area and has uniform in-plane tensile stresses throughout.

The traditional aesthetic of fabric membranes comprised highly curved surfaces reminiscent of vernacular tents. These dramatically curved structures have enjoyed continued popularity from their inception in 1955 by Frei Otto with a small bandstand for the Federal Garden Exhibition in Kassel, Germany [5] to the present day, and have resulted in outstanding works of Architecture for sports stadia, airports and retail (Figure 1). Recently there has been a significant move towards flatter forms [6] driven by changing aesthetic criteria, not least the desire to differentiate new structures from the existing body of work.

The Millennium Dome in London (now The O2 Arena) is a prime example of this change in emphasis and aesthetic, with a synclastic surface being generated by a cable net clad in almost flat fabric panels (Figure 2). More recently, the London 2012 Olympic Stadium was designed for with a fabric roof and facade, both of which were to be largely flat on aesthetic grounds (Figure 3). It should be noted that in both instances the spanning capability of the membrane, and its ability to resist loads and provide positive drainage, were still influential in the design of the complete structural system, rather than the membrane being conceived simply as flat cladding without structural utilisation.

Figure 1. Fabric structures with high levels of anticlastic curvature: Denver International Airport, 1995, (top) inspired by the snow-capped Rocky Mountains (© Sox23 reproduced under the Wikipedia Creative Commons Share Alike 3.0 license), Ashford Designer Outlet, Kent, 2000, (bottom left) – a dramatic and extensive fabric canopy which combines mast supported high points with ridge and valley cables (© Ben Bridgens), Hampshire Cricket Club, 2001, (bottom right) – classic tensile architecture consisting of multiple conic canopies (© Buro Happold / Mandy Reynolds).

Due to the inherent efficiency of membrane structures, there is an increasing tendency to consider both fabric and single layer ETFE membranes as cost-effective substitutes for conventional roof and facade materials such as glazing or polycarbonate. In these instances completely flat membrane panels are now being proposed. The performance and structural action of the membrane material differs from that of the rigid construction that it replaces, and also differs from the behavior of curved fabric structures, hence careful specialist engineering analysis and design is required to successfully realise these elements.

Aesthetic requirements rather than a careful balance of form and function are increasingly driving membrane forms, and the use of anticlastic-synclastic combinations suggest a move away from pure tensile architecture. An example currently under construction is the Nuvola project in Rome, which

includes perforated silicone coated glass-fibre fabric as the cladding, with substantial secondary steelwork being required to force the minimal surface into a closer approximation to the desired geometry.

Figure 2. Millenium Dome / The O2 Arena, London, 1999, comprises virtually flat fabric panels supported by a suspended cable net (© Buro Happold/Adam Wilson)

Figure 3. London 2012 Olympic Stadium: tensile fabric stadium roof with low levels of curvature, top (© London 2012) and proposed facade wrap composed of twisted, minimally curved, fabric panels , bottom (© London 2012 / Populous).

It has been noted that completely flat panels can actually behave better than those with only slight curvature. Flat or negligibly curved panels - such as those over most of the O2 Arena roof - do not de-stress under load, and hence exhibit fewer fatigue issues than would otherwise be expected. However, the

large movements required to resist the applied loads need careful consideration, particularly in instances where fluctuating wind load or dynamic effects may be experienced.

1.2. Coated woven fabric material behaviour

Architectural fabrics consist of woven yarns to provide strength, with an impermeable coating to provide waterproofing and stabilise the weave (Figure 4). The most common material combinations are PVC coated polyester yarns and PTFE coated glass-fibre yarns, with silicone coated glass-fibre fabric becoming increasingly popular and PTFE coated ePTFE (*Tenara*) providing very high light transmission. Despite the use of architectural fabrics in state-of-the-art structures (Figures 1 & 2) for over 40 years, broad assumptions are made in both material testing and analysis. A combination of non-linear stress-strain response of the component materials (yarn and coating) with the interaction of orthogonal yarns, results in complex (non-linear, hysteretic, anisotropic) material behaviour [7]. Elastic moduli, Poisson's ratios and shear stiffness are independent and are not constrained by conventional limits and relationships for isotropic materials. Full quantification of the response of coated woven fabrics to in-plane loading (biaxial and shear) is time consuming and costly, and arguably has not yet been achieved. It is common practice to use assumed stiffness values for a given fabric material [2], but the actual stiffness may differ by a factor of between two and five from these assumed values [8].

Figure 4. Architectural fabric cross-sections showing highly crimped, woven yarn bundles encased in PTFE or PVC coating.

Even when detailed biaxial stiffness testing is carried out (typically for large or complex projects), techniques to utilize this data in structural analysis are in their infancy. The intuitive method used by engineers who are accustomed to linear-elastic isotropic materials, is to measure the gradient from the stress-strain curves at each stress ratio, but this does not provide any information about Poisson's ratio.

As virtually all membrane analysis software utilises single values for the elastic constants, the multiple stiffness values that are calculated must be averaged, or engineering judgement used to choose the most appropriate value.

The only standardised method for interpreting biaxial test data is described in the Commentary to the Membranes Structures Association of Japan (MSAJ) Testing Method for Elastic Constants of Membrane Materials [9], and has been adopted in the American Society of Civil Engineers standard for Tensile Membrane Structures [10]. This method finds the best fit elastic constants to the complete data set, but depending on exactly how it is applied it can yield a wide range of values:

“Various applicable methods should be examined and the most satisfactory combination of elastic constants must be determined” (p.20) [9]

At best this method provides a linear (planar) approximation to the non-linear stress-stress-strain data [11]. The values of elastic constants provided by this ‘best fit’ method typically includes values of Poisson’s ratio greater than one, which reflects the very significant interaction of woven warp and fill yarns [8]. Use of a more advanced multi-linear approximation to the test data demonstrates significant variability in fabric stiffness values at different stress ratios and magnitudes, and still ignores time and load history dependent behaviour.

1.3. Design criteria for fabric structures

Large displacements and a reliance on geometric stiffness has led to the adoption of a stress factor approach as the basis of a permissible stress design methodology for tensile fabric structures. However, even within Europe, the magnitudes, natures, and combinations of stress factors are not harmonised; when including ASCE, IASS, and Japanese standards, the range of stress factors increases further [10, 12]. Most recently, CEN TC250 Working Group 5 has been established to write a standard for Membrane Structures for inclusion in Eurocode 10. Against this backdrop of uncertainty and lack of design guidance, the design of membrane structures is informed by experience, engineering judgement and pragmatism.

The two structural main performance criteria for fabric structures are stress and deflection. Due to the low shear stiffness of woven fabrics compared to their tensile stiffness, maximum stresses will usually occur in the weave directions (warp and fill). Note that ‘stress’ in structural fabrics is defined as force per unit width, as fabrics do not have a consistent thickness. Stresses in warp and fill directions for each load case are compared to the fabric strip ultimate tensile strength [13] divided by an appropriate stress factor, typically between 4 and 8 to account for the severe reduction in fabric strength in the presence of a small tear [14].

Fabric structures do not have strict deflection limits such as those imposed on conventional building structures, but limits are defined by the need to avoid ‘ponding’, and in some cases to avoid clashes between the deflected fabric form and the supporting structure or other objects. Ponding is the build-up of snow, ice or water on the fabric canopy, in particular of melt water in hollows formed by snow loading. It is

therefore vital to ensure that fabric structures maintain positive drainage under all load conditions. This check is carried out following analysis with unfactored loads (due to geometric non-linearity), but this means that the subsequent design may only just avoid ponding with a factor of safety close to one, with no consideration of uncertainty and variability in the material properties, prestress levels, construction tolerances and the analysis itself. In addition, it is best practice to avoid inversion of the fabric curvature, as this may result in flapping and creasing of the material with subsequent damage and reduction in strength.

2. Methodology

2.1. Form

The significance of material properties and geometry on structural performance (stress levels and displacements) has been assessed using parametric studies of ‘typical’ fabric forms. Three fundamental fabric structure forms can be developed by manipulating the boundary conditions of an initially flat geometry within a square plan (Figure 5).

Figure 5: Manipulation of boundary conditions within a square plan and subsequent form finding enables three fundamental tensile forms to be developed.

Any generalisation of fabric forms is difficult – the appeal of fabric architecture is the ease with which unique, complex forms can be achieved. Fabric structures are also utilised across a wide range of scales, from a few metres (e.g. canopies over footpaths and bicycle racks) to hundreds of metres (sports stadia).

Fortunately the structural action of tensile structures depends on curvature rather than span, hence their efficiency for large-span structures. By consideration of ratios of geometric parameters and curvature (rather than span) the results of the analyses can be considered to be independent of scale. To reduce the number of parameters all structures are square on plan; Table 1 provides a summary of the geometric parameters used for each structural form. A constant value of prestress of 3 kN/m in warp and fill directions has been used throughout for all structures. In all cases an increase in prestress will reduce deflections and increase stresses, with the converse being true if prestress is reduced. In addition to typical membrane structures, the efficient design of tensioned edge cables has also been considered.

Table 1. Model parameters

Structure type	Parameters (variable)	Parameters (constant)	
Edge cable	Dip to span ratio (all feasible values from zero to 0.5)	Span (10m), membrane tension (1 kN/m)	
Conic	Ring height (zero to maximum feasible - §3.3), ring diameter (zero to 14m)	Base: fixed edges, 14m x 14m Ring: fixed (not suspended)	Base: size, aspect ratio (square) Fabric prestress (3 kN/m in warp and fill directions)
Hypar	Corner height (h in Figure 5, zero to 8m) Patterning direction (diagonal or orthogonal)	Cable supported edges (constant cable prestress) Symmetrical structures, two diagonally opposite high points Base: 7.07m x 7.07m	
Barrel vault	Fabric radius of curvature varied from zero (flat panel) to 6.25m, controlled by varying the arch radius of curvature.	Base: fixed edges, 10m x 10m	

2.1. Material properties

A wide range of elastic constant values has been used to investigate the effect of large variations in material behaviour which have been observed at different stress ratios and magnitudes [8], and at different shear angles [15]. Due to software limitations, the fabric properties are described by linear elastic constants (Young's moduli, Poisson's ratios and shear modulus), which is consistent with current industry best practice [12]. Whilst not providing an accurate representation of the fabric behaviour, this approach is sufficient to assess the sensitivity of different structural forms to wide variations in material behaviour.

2.2. Loading

Critical loadcases for fabric structures are usually wind and snow loading. Even if wind-structure interaction and dynamic effects are ignored, accurate determination of wind loading for fabric structure forms is difficult. Despite significant recent research in this area [16-19], there is currently very limited design guidance with wind tunnel testing used routinely for major projects [12, 20, 21].

For this work a simplified approach has been adopted: all structures have been analysed for uniform wind uplift (1.0 kN/m^2) and uniform snow load (0.6 kN/m^2). These values are broadly representative of typical values in the UK. Wind load is a suction force that acts perpendicular to the fabric surface, and has been applied using deformed, local coordinates, i.e. the wind load direction will be updated during the analysis as the structure deforms. This is consistent with the geometrically non-linear analysis (§2.3). Snow load is applied as uniform downward force acting on the projected plan area of the canopy.

2.3. Modelling of tensile fabric structures

The modelling and analysis of membrane structures is a two-stage process – form-finding followed by load analysis - requiring specialist analysis software. For the first stage, boundary conditions (support geometry, fixed or cable edges) and form-finding properties (fabric and edge cable prestress forces) are defined. Form finding is independent of the fabric material properties. A soap-film form-finding analysis [22] provides the membrane geometry and prestress loads. A new model is created with this updated ('form-found') geometry that is used for the analysis stage. Subsequently the fabric material properties are defined, loads are applied (wind, snow, prestress) and a geometrically non-linear (large displacement) analysis is carried out using membrane elements with zero bending and compression stiffness. Due to geometric non-linearity, results from different loadcases cannot be combined or factored; each combination case (e.g. prestress + wind uplift) is analysed separately and a permissible stress approach is used to assess the required membrane strength [12].

The analysis for this work was carried out using Oasys Software's 'GSA (General Structural Analysis) Fabric' software¹. GSA is a finite element package tailored to the requirements of civil and structural engineering design, and is available with the 'Fabric' add-on that provides membrane elements and form-finding for the design of tensile fabric structures. The orthotropic plane-stress linear elastic model uses geometrically non-linear eight-noded quadrilateral membrane elements.

3. Results & discussion

3.1. Efficient edge cable design

Prestressed cables within a welded fabric pocket are an elegant and popular edge detail for fabric structures. Edge cable curvature and cable tension are related by a simplified form of the Young-Laplace equation [23] for a structure spanning in one direction, which defines a linear variation of tension with curvature: *tension = uniform applied load × radius of curvature*. However, for architectural design it is the 'dip' (Figure 6) that is significant, as this determines the level of coverage and aesthetics of the canopy, and is easier to visualise than the radius of curvature. A dip to span ratio of 1:6 is often quoted as a rule of thumb for efficient and aesthetically pleasing design.

Figure 6: Edge cable curvature

Using the simplifying assumption that the weight of the edge cable is negligible compared to the applied load and hence the cable forms a circular arc, the cable tension, T , can be written in terms of the end reactions (H & V):

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¹www.oasys-software.com/products/structural/gsa/gsa_fabric.shtml

$$T = \sqrt{H^2 + V^2}, \quad H = \frac{ws^2}{8d}, \quad V = \frac{ws}{2} \quad (1)$$

Refer to Figure 6 for nomenclature. Substituting for the reactions (H & V) gives an expression for cable tension (T) in terms of dip (d), span (s) and applied load (w), and the resulting relationship between cable force and dip is shown in Figure 7.

$$T = \sqrt{\left(\frac{ws^2}{8d}\right)^2 + \left(\frac{ws}{2}\right)^2} \quad (2)$$

Figure 7: Variation of cable force with cable curvature

A dip to span ratio greater than 0.1 ('C', Figure 7) ensures low cable force and hence an efficient transfer of fabric stress back to the supporting structure. This will result in smaller diameter cables, smaller end fittings, and consequently smaller, more elegant connection details and supporting steelwork. This is consistent with the 'rule of thumb' of a dip to span ratio of 1:6 (= 0.17) that would give a very efficient design. A dip to span ratio of 0.05 to 0.1 ('B') may be desirable for architectural or functional reasons, for example to provide increased coverage, but it should be noted that over this range the cable force doubles. A dip to span ratio less than 0.05 ('A') should clearly be avoided as the cable force increases dramatically.

3.2. Hypar

Hyperbolic paraboloids or 'hypars' are characterised by alternating high and low points, usually with cable-supported edges. The simplest hypar consists of two high and two low points, but more complex forms can be achieved with multiple support points (Figure 8). A hypar can be designed with two different fabric orientations or patterning directions (Figure 9). A square hypar structure acts principally in tension between diagonally opposite corners, for the orthogonally patterned hypar this means that the fabric is acting in shear. As the shear stiffness of architectural fabrics is typically low (tensile elastic modulus divided by 20 is commonly used as a rule of thumb) an orthogonally patterned hypar will exhibit high deflections (Figure 9). However, this form of construction is popular as it allows greater shear deformation during installation to enable highly curved forms to be achieved without wrinkling. This introduces a common design conflict in membrane structures where optimum properties for installation are different to those for long term performance of the structure.

For a simple square hypar, similar to the example modelled here, it is straightforward to use diagonal patterning to ensure that the material is loaded along the warp and fill weave directions to avoid high shear stresses and strains. With diagonal patterning the behaviour is not sensitive to variations in shear stiffness, hence a single diagonal patterning case is shown in Figure 9. However, for more complex multi-point hypars (Figure 8, middle & bottom) it can be difficult to avoid mobilising the shear resistance of the material, and the exact orientation of the cutting patterns can severely influence the load paths within the membrane and hence the support reactions.

Figure 8. Hypar structures are characterised by alternating high and low points with cable supported edges. Hemmingsway Hotel, Kenya, 2002 (top), a classic 4 point hypar; Cayman International School, Cayman Islands, 2006 (middle), a 6 point hypar in which the choice of patterning direction will severely influence the support reactions; Chesterford Research Park, Cambridge, 2005 (bottom), multiple hypars with orthogonal patterning. All images © Architekten-Landrell Associates.

Figure 9: Square hypar, variation of maximum membrane deflection due to wind uplift with form (height / side length) and shear modulus (G) for orthogonal patterning, and a single value of G for diagonal patterning. Model parameters: elastic modulus $E_{\text{warp}} = E_{\text{fill}} = 1000 \text{ kN/m}$, Poisson's ratio = 0.8.

The complex structure and composite nature of architectural fabrics means that the elastic moduli, Poisson's ratios and shear stiffness are independent.

As the shear modulus increases towards the value for an isotropic material, equation (3), the shear stiffness tends towards the elastic stiffness in warp and fill directions, and hence the displacements tend towards the values for the diagonally patterned structure (Figure 9).

$$G = \frac{E}{2(1 + \nu)} \quad (3)$$

Where G = shear modulus, E = elastic modulus, ν = Poisson's ratio.

This effect is less significant for flatter structures (i.e. as height / side lengths tends to zero) when the fabric panel will be acting primarily as a two way spanning flat panel (left hand side of Figure 9). As the corner height and fabric curvature increases (right hand side of Figure 9), the structure must span between diagonally opposite corners and the effect of fabric orientation and shear stiffness on deflections becomes pronounced. The same effect is seen with the variation of fabric stress level with varying shear stiffness. For highly curved, orthogonally patterned hypars a modest change in shear stiffness from 50 kN/m to 25 kN/m gives a 25% increase in tensile stress (Figure 10). This is concerning as fabric shear

behaviour is rarely tested and poorly understood, assumed values are almost always used in design, and it would generally be assumed that highly curved structures would be less sensitive to changes in material properties.

Figure 10: Square hypar, orthogonal patterning, variation of fabric stress (fill direction) due to uniform wind uplift (1 kN/m^2) with varying form (height / side length) and shear modulus (G)

A wide range of fabric tensile stiffnesses have been modelled, from 100 kN/m through more typical values of 400 kN/m to 2000 kN/m, to a maximum of 5000 kN/m. For hypars with a high level of fabric curvature the sensitivity of the maximum fabric deflection to changes in elastic modulus are very low (right hand side of Figure 11). For flat panels and hypars with minimal curvature the structure must deform to enable the applied loading to be resisted as tension in the plane of the fabric, so clearly the deflection will increase with decreasing tensile stiffness (left hand side of Figure 11). The striking result is that the greatest sensitivity to material stiffness is shown by structures with a height to side length ratio of 0.2 to 0.4. Taking a ratio of 0.4, and considering a realistic range of material stiffnesses from 250 kN/m to 1000 kN/m (commonly exhibited by a single material under differing load conditions), the maximum fabric deflection varies by a factor of three. The consequence is that deflection checks may underestimate the tendency of the structure to pond, potentially resulting in failure.

Figure 11: Square hypar, diagonal patterning, variation of deflection due to wind uplift with varying form (height / side length) and elastic modulus (E_w = warp modulus, E_f = fill modulus)

The maximum fabric stresses in hypars with low levels of curvature are highly sensitive to changes in tensile stiffness, with stiffness having limited influence at height to side length ratios over 0.4 (Figure 12). In the fill direction, which is unloaded under wind uplift, a dramatic change in stress levels can be seen when the structure changes from acting as a two-way spanning flat panel to when it starts to act as a curved hypar and the fill stress becomes low under wind uplift.

Poisson's ratio was varied (from 0.1 to 0.9) but was found to not have a significant effect on stress or deflection levels in the structures. Poisson's ratio values as high as 2 or 3 are commonly required to represent the behaviour of architectural fabrics, but use of values greater than 0.9 is not possible in the analysis software used.

Figure 12: Square hypar with diagonal patterning, variation of fabric stress due to wind uplift with varying form (height / side length) and tensile stiffness (E_w = warp modulus, E_f = fill modulus)

3.3. Conic

A true minimal surface cannot be formed between all boundary conditions. For a conic, as the distance between the base and top ring increases the minimal surface will 'neck': a point is reached where a minimal surface cannot be formed between the rings (Figure 13). A "pseudo-minimal" surface can be developed for a fabric membrane by accepting increased stresses in the region where the soap film would have failed, reducing the limitations on the forms that can be created. However, as the desired shape moves away from the minimal surface the stress variations increase and the structure becomes less

efficient. The feasible bounds of conic geometry have been investigated for a prestress ratio of 1:1, i.e. with the intention of determining the limitations of true minimal surface forms (Figure 14). The feasible geometries have been determined by trial and error by carrying out soap-film form-finding using *Oasys GSA Fabric* for numerous boundary conditions until the limiting combinations of 'ring height / side length' and 'ring diameter / side length' were established. The results are applicable to any scale of structure; hence the geometric properties are described as ratios of ring diameter and ring height to base edge length. The result when the ring diameter is equal to the base side length of 0.49 (right hand side of Figure 14) can be compared with the theoretically derived value of ring height / ring diameter for a catenoid with identical rings of 0.66 [24]. The square based conic collapses at a lower ring height due to the difficulty in achieving a minimal surface which transitions from a square base to a circular ring.

Figure 13: Limitations of soap-film surfaces are particularly significant for conic structures

Figure 14: Limiting values of conic geometry for form-finding (1:1 prestress) and ponding under uniform snow load. Warp direction is radial, fill direction is circumferential. The forms on the right hand side are at the limit of feasibility, and may not be desirable for aesthetic and functional reasons.

Conic structures with a low ring height are prone to ponding, i.e. formation of a hollow near the corners under snow load that leads to collection of melt-water and subsequent failure. Structures have been checked for ponding by identifying areas that do not have positive drainage to the edge of the structure under uniform snow load. The ponding check is dependent on the material properties and level of prestress: two ponding boundaries are shown in Figure 14 based on typical PVC coated polyester and PTFE coated glass-fibre materials (with elastic moduli of 600 kN/m and 1200 kN/m respectively), with prestress of 3 kN/m in warp and fill directions. The envelope of feasible structures can (and often is) extended by increasing the radial to circumferential prestress ratio, but this leads to higher fabric stresses, and the prestress levels will no longer be uniform throughout the structure. It is sometimes necessary to include radial belts or cables to provide sufficient stiffness in the radial direction to prevent ‘necking’ of the conic and/or ponding at the corners. This enables much more architectural freedom, but at the expense of the simplicity and efficiency of a true minimal surface.

Within the ‘feasible zone’ (Figure 14) the variation of fabric stress with conic geometry is quite small given the wide range of ring diameters and moderate variation in tensile stiffness (Figure 15). The suggestion

from this small sample of structures is that if the designer accepts the constraints of true minimal surface form-finding, then the structural form that is developed will be reasonably efficient. Further studies would be required to thoroughly validate this assertion. For one conic form the stress levels were checked over a wide range of stiffness values (Figure 16), and this does show that underestimating the fabric stiffness could result in under-specification of the required fabric strength. This is significant as most fabrics exhibit extremely high tensile stiffness at particular stress ratios.

Figure 15: Variation of maximum warp (radial) and fill (circumferential) stress due to wind uplift with ring diameter & elastic modulus; 3.5m ring height, 14m side length. Datapoints correspond to points on line A-B in Figure 14.

Figure 16. Variation of maximum stress with elastic modulus for wind uplift; conic with 3.5m ring height, 5.0m ring diameter. Warp direction is radial and fill direction is circumferential.

3.4. Barrel vault

A rigid frame consisting of two straight sides and two curved sides (Figure 5) provides the boundary conditions for a 'barrel vault' with a similar double curved 'saddle' shape to a hypar, but the fundamental difference is that the membrane will tend to span between the edges rather than between the corners. The barrel vault form has been analysed with a combination of elastic moduli values and fabric curvatures. A small fabric radius (i.e. highly curved) provides an efficient structure with low values of stress and deflection (Figure 17). As the barrel vault flattens and tends towards a flat panel, the fabric stresses and deflections increase by a factor of between 2 and 3. At the same time, the sensitivity to variation in fabric stiffness become much more significant as the curvature is reduced.

Figure 17: Barrel vault, variation of maximum fabric deflection (top) and fabric stress (bottom) with variation in fabric curvature & elastic modulus; applied loading is uniform wind uplift, warp direction spans between straight edges.

4. Conclusions

Lightweight tensile structures are used worldwide for high profile, large-scale structures, in particular sports stadia and airports, as well as for myriad smaller applications to efficiently provide shelter from rain, sun and wind. Lack of design codes or guidance, coupled with complex material and structural behaviour, can limit the use of fabric structures or prevent their full utilisation. Introduction of new materials is hindered by the reliance of designers on past experience and intuition. There is also concern that as Architects move away from 'conventional' highly curved forms and use flat or minimally curved fabric panels, past experience may become less relevant and, for example, structures may become more sensitive to inherent variability in material properties. A profusion of graphical design tools for tensile structures which are aimed at Architects enable membrane forms to be explored and generated with ease; it is important to temper this creative ease and freedom with a clear understanding of the limits to efficient, functional forms. Efficiency in lightweight structures does not just mean being able to specify a lighter grade of fabric, hence giving a small reduction in the overall project cost. Much more important is to minimise the reaction forces, to enable elegant connection details to be designed which are in keeping with the lightweight nature of the structure.

Three typical fabric forms have been analysed with varying geometric parameters, to try to establish rules of thumb for the safe, efficient design of tensile fabric structures. The results for the barrel vault clearly show that highly curved structures work efficiently and robustly, with low stresses and deflections, which show minimal variation for a wide range of material stiffness values. Similarly for edge cables the greater the level of curvature the lower the cable force. For cables the relationship is highly non-linear, and for dip to span ratios less than 0.1 the cable force increases dramatically. This leads to larger cable sizes, and more importantly larger connection details that are at odds with the lightweight, minimalist aesthetic.

Interpretation of the results for hypars are not straightforward as the structural action varies from a two-way spanning flat panel to one-way spanning between opposite corners, depending on a combination of geometry, tensile stiffness and shear stiffness. The results show that patterning direction is critical for the performance of hypars. For four-point structures patterning should be diagonal (between high points), and for multi-point hypars it must be understood that the patterning direction is fundamental to the behaviour of the membrane and hence to the support reactions. Extremely high curvature is required to avoid hypar displacements being very sensitive to changes in tensile stiffness; a typical hypar should be analysed with a wide range of possible stiffness values to ensure that ponding (build up of rain water due to lack of positive drainage under snow load) does not occur. The hypar stress values follow a similar pattern to the barrel vault results – low curvature means sensitivity to changes in material properties and high stress levels. For a highly curved hypar with orthogonal (side to side) patterning, the stress is very sensitive to variation in shear stiffness.

The conic results are dominated by the fact that true minimal surface form-finding, combined with avoidance of ponding at corners, gives a limited range of forms that can be achieved. Within this range the structures work efficiently and only exhibit a modest dependence on material properties, increased stiffness resulting in increased stress levels. The bounds of feasible conic forms can be extended with asymmetric prestress ratios, and even inclusion of radial belts and cables, such that virtually any form can be achieved, but with decreasing efficiency as the form moves away from the ideal 'soap film' minimal surface originally described by Frei Otto [4].

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