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A Comparison of Computational Methods to Predict the Progressive Collapse Behaviour of a Damaged Box Girder

1. Introduction

A critical strength criterion for a thin plated box girder structure, such as the mid body region of a ship, is the ability to withstand combinations of vertical and horizontal bending moments acting upon the longitudinally continuous structure. The maximum capacity of a ship's hull girder under a pure longitudinal bending moment, normally referred to as its ultimate strength, can be determined using several numerical approaches which are generally referred to as progressive collapse analysis.

The ultimate strength of an intact ship is an important measure to ensure the structure will not collapse under maximum expected load scenarios. The ultimate strength is also increasingly used as a design factor for limit state analysis and is now a requirement in some classification rules. In addition to its intact strength, a ship structure may need to be assessed in various damaged conditions. If a portion of the longitudinally effective structure is ruptured or severely damaged through collision, grounding or malicious attack, the ultimate capacity will inevitably be reduced. An assessment of residual ultimate strength in a damaged condition is thus useful for determining recoverability, seaworthiness or assessing the capabilities of a particular structural arrangement for withstanding damage.

The results obtained from physical experiments provide an invaluable resource for validating theoretical modelling approaches and demonstrating how a structure behaves under closely controlled loading conditions. However destructive testing of large scale structures, such as ships and bridges, are normally limited by size and cost constraints. These factors have placed a great emphasis on developing robust theoretical techniques to examine structural characteristics such as ultimate strength and the effects of damage. In particular, the nonlinear finite element method (FEM) has become a dominant computational approach for complex structural engineering problems.

However, the use of FEM opens many questions concerning how to best simulate specific load scenarios and also about the reliability of results from such complex analyses. A general purpose FEM package such as ABAQUS includes a range of different fundamental solution methods involving either static or dynamic equilibrium equations, and also has options to use different ways equilibrium is treated using either implicit or explicit convergence techniques. These approaches have particular advantages and disadvantages which affect their suitability for different problem types.

In addition to the complexities of the solution choice, FEM requires a rigorous definition of the material and geometric properties inherent in the structure, including an adequate representation of geometric imperfections and residual stresses. These parameters are especially critical for progressive collapse analysis, where a portion of the longitudinal structure is placed under in-plane compressive loading up to and beyond its buckling capacity. It is well known that the buckling strength of plates and stiffened panels are significantly affected by the magnitude, shape and distribution of imperfections.

The complexities of FEM in model setup together with the relatively expensive computation time means that there is a continued need for simplified analytical methodologies which are more time efficient and also provide a robust means of accounting for the nonlinearities associated with the buckling response. Therefore simplified methodologies can often be a more reliable choice for the structural engineer, depending on the type of problem and the context in which it needs to be solved.

The Smith progressive collapse method [1] is a widely known incremental approach used to predict the bending response of ship hull sections. The Smith method follows a fairly simple procedure, whereby the longitudinally effective structure is divided into elements, each element is assigned a load-end shortening curve and incremental curvature is applied at the instantaneous neutral axis. At each increment the in-plane displacement of each element is calculated and the cumulative

response over the entire section is summed to calculate the incremental change in bending moment. The Smith method has been validated and shown to provide excellent results when compared to large and small scale sections [2]. The approach can also be used to predict the damaged strength of a girder, although the limiting assumptions of the method mean that only a relatively simplistic representation of the damaged area can be modelled.

Within the context introduced above, this paper provides a comparative assessment of several FEM and simplified approaches for the analysis of box girder structures in intact and damaged scenarios. The box girders replicate actual structures tested by Gordo and Guedes Soares [3]. In this study the girders are analysed in an intact condition and with ruptured penetrations to represent damage. Firstly, intact vertical bending moment tests are carried out and compared with the original physical experiments. The theoretical analyses are then extended to deal with biaxial bending. Comparative results are presented from implicit and explicit FEM together with simplified progressive collapse analyses. The theoretical analyses correlate very closely but there are significant differences in the prediction of ultimate strength compared to the physical experiment. A hypothesis to explain the discrepancy between results is demonstrated.

The use of FEM and simplified methods for determining the residual strength of box girders which have sustained damage is then investigated. Three severe damage scenarios are simulated using explicit FEM, and the ultimate strength of the damaged girder is then calculated using several techniques. The results demonstrate the significance of the residual stresses sustained in the damage simulation.

2. Progressive Collapse Experiments

Published research concerning hull girder progressive collapse mostly falls into one or more of three broad categories: reporting of physical experiments on box girders or ship structures; derivation of

theoretical methods to estimate progressive collapse or ultimate strength; and results from theoretical modelling of sections using simplified and FEM approaches.

2.1. Physical Experiments

Experimental data pertaining to global progressive collapse of ship structures is extremely limited, primarily because of the impracticalities and expense in testing large scale girder models in primary bending. Most experimental tests use relatively small box girder models, which are more easily tested within a laboratory. For example Reckling [4] carried out seven steel box girder tests, investigating their strength under pure vertical bending moment. Experimental box girder tests have also been completed by Dowling et al. [5], Ostapenko [6], Nishihara [7], Qi et al. [8], Akhras et al. [9], Gordo and Guedes Soares [10] and Saad-Eldeen et al. [11]. There are very few physical experiments which use actual ship structures. The only laboratory test result available in open literature is that of a 1/3 scale frigate model, which was loaded up to and beyond collapse under a vertical sagging bending moment [2].

Specific cases of actual ship's failures by a progressive collapse mechanism have also proved useful. One example is the merchant vessel *Energy Concentration*, which broke its back during loading in Rotterdam harbour. The incident provides a usable case study because the circumstances of the hull collapse were unusual. The hull girder broke in still water conditions and, furthermore, the loading condition at the time of the accident was known. Rutherford and Caldwell [12] used this information to calculate the applied bending moment at the time of failure. Equivalent calculations using the progressive collapse method correlated closely to the estimated actual ultimate strength of the hull.

This paper replicates experiments on three simple multi-frame box girder structures, which were originally physically tested at the Technical University of Lisbon (IST) using a four point bending rig as pictured in Fig. 1. The girders were built simply; because it is a small scale model the stiffeners are placed on the outside of the shell to enable welding access during construction. In total three girders

were tested; all had the same longitudinal cross section (see Fig. 1) but with different transverse spacing between frames. The principal box girder dimensions are shown in Fig. 1 with spacing and thicknesses presented in Table 1.

Table 1 – Box Girder Properties

ID	Specimen Length (mm)	Frame Spacing (mm)	Plate Thickness (mm)	Stiffener Height (mm)	Stiffener Thickness (mm)
F200	1000	200	4	20	4
F300	1100	300	4	20	4
F400	1400	400	4	20	4

Each test specimen length includes an additional 100mm span at each end, which was connected to the bending rig by a heavy bulkhead. Load was applied through hydraulic jacks connected to a strong box, which in turn rests on the outer supports of the bending rig. All the supporting structure was constructed from thick high tensile steel. The test specimen was welded between the outer supports whilst the outer edges of the supports rested on the floor. The rig thus produces a four point bending load, with the central section under pure bending moment.

Fig. 1. (a) Lisbon box girder test experimental setup. (b) specimen cross section [source: [2]].

2.2. Simplified Progressive Collapse Methods

Longitudinal progressive collapse involves nonlinear buckling and collapse of compressed portions of the box girder beam. Numerical tools of the hull girder and to assess the forces at which service, damage and ultimate limits are reached. These tools are also used to assess the strength of a structure after entering service, with specific considerations of damage or age related effects. A number of simplified progressive collapse approaches have been proposed and several continue to be developed. They range in complexity and include simple closed form empirical formulae [13], interframe progressive collapse methods [1] and compartment level methods [14]. The key methodology employed in this paper is the Smith method [1], which is one of the most well established simplified approaches to progressive collapse analysis.

The Smith method has three underlying assumptions: that plane sections remain plane, buckling of panels is interframe and that the behaviour of individual elements (described by load end shortening curves) can be treated in isolation. The assumption of interframe collapse means the method can be called two dimensional because only the longitudinal structural arrangement is considered.

The Smith method has been developed into various proprietary computer codes [15–19]. These codes use the same underlying methodology but differ in their approach to derive the load-shortening curves. For example, the UK MOD code NS94 [15] uses either a special purpose FEM program, FABSTRAN, to determine the load shortening curve for the plate-stiffener combinations. Later versions include simplified bilinear curve datasets which will provide a curve for a specific stiffened panel by interpolation. Alternative methods have been developed using design formulas to determine stiffened panel behaviour [16].

This study uses the progressive collapse approach developed by Benson [19], which has several options for defining the panel load shortening curves including a semi analytical orthotropic plate method which can be used to predict buckling modes both interframe and over several frame spaces.

It was assumed throughout the present study that buckling of the IST box girders is predominantly interframe and the load shortening curves were derived appropriately using the method as detailed by Benson et al. [20].

2.3. Finite Element Progressive Collapse Methods

Nonlinear FEM is a viable option for hull girder strength assessment. However, from a design perspective, FEM requires detailed knowledge of the geometry, imperfections and residual stresses in the structure. These are not necessarily well defined, even in structures physically tested in carefully controlled laboratory environments. Furthermore, from an analysis perspective, FEM requires considerable computer time both in setting up and solving the discrete model. Elements must be sized sufficiently small to represent the local structure including stiffeners and plating adequately. The number of elements for an entire hull girder mesh can easily run to several hundred thousand, which becomes computationally expensive for nonlinear type FEM analyses.

An early application of nonlinear FEM for the analysis of hull girders is presented by Kutt et al. [21], who used the ABS nonlinear FEM program USAS to calculate the longitudinal strength of four ship hulls including a passenger ship and a tanker. Results from nonlinear FEM analyses of box girders are presented by Qi et al. [8]. Analyses of tanker structures using large scale FEM models are made by Amlashi and Moan [22], Kippenes et al. [23] and Zipfel & Lehmann [24].

Typically, nonlinear FEM analyses of hull girder progressive collapse utilise a static solver together with an equilibrium convergence iterator using either the Riks arc length method or modified Newton-Raphson method [25]. The static solver assumes that the time dependent mass and inertia effects are small and thus can be neglected [26]. This assumption implies a quasi-static structural response, which is usually valid if the loading frequency is less than a quarter of the lowest natural frequency of the structure.

The implicit static approach assumes that the box girder response can be characterised as quasi static. However, damage simulations such as are shown in this paper must be characterised as dynamic, thus including mass and damping effects in the equilibrium equation. Use of a dynamic solver has been shown to be essential for ship collision, grounding and blast loading of ship structures where the damage is sustained relatively quickly. Some research studies have also used dynamic solvers to analyse the progressive collapse of box girder structures [24,27]. This paper compares the effectiveness of using a dynamic solver for the progressive collapse analysis in addition to its application in damage simulation. All damage simulations and ultimate strength calculations using the dynamic solver have been conducted using the general approach described by AbuBakar and Dow [28].

3. Material Properties

The definition of the material properties for high tensile steel grade S690 are an important aspect of all FEM analyses presented in this paper. The material properties are identical in all analyses and are now summarised.

3.1. Stress Strain Relationship

The material characteristics, in the form of a true stress-strain curve, must be adequately represented in the finite element model to ensure an accurate treatment of plasticity. This is important for the progressive collapse analysis and also for realistically modelling rupture in the damage tests. Therefore a literature search was conducted to better define the nonlinear response for the material. The nominal yield strength of the high tensile steel S690 is 690MPa with a Young's modulus of 200GPa. No tensile test data for the steel used in the box girder experiments is available, although manufacturers quoted values are 732MPa at yield and 808MPa at 15% elongation. However, tests on specimens from 4mm plate by the same research team are reported as 680MPa at yield and 764MPa at 10.5% elongation [29]. A separate study presents a complete stress-strain

curve for S690 [30]. This curve can be described using a modified power law, which was developed to accurately represent the initial yield plateau characteristic of steel [31]. The power law uses a step function as follows:

$$\sigma_{eq} = \begin{cases} \sigma_Y & \text{if } \varepsilon_{eq} \leq \varepsilon_{plat} \\ K(\varepsilon_{eq} - \varepsilon_0)^n & \text{otherwise} \end{cases} \quad (1)$$

$$\varepsilon_0 = \varepsilon_{plat} - \left(\frac{\sigma_Y}{K}\right)^{1/n} \quad (2)$$

The constants for S690 used in this study are based on the curve by Sedlacek and Muller [30] as follows: $K=1250\text{MPa}$, $n=0.12$, $\varepsilon_{plat}=0.0124$, $\sigma_0=745\text{MPa}$ and $E=211\text{GPa}$. The stress-strain curve is shown in Fig. 2.

Fig. 2. S690 Stress-Strain Curve

3.2. Failure Model

The material failure model is used in conjunction with the material properties described previously.

The failure model permits the rupture of the box girder structure when the material exceeds the allowable or maximum strain in any direction of the shell elements during penetration of the indenter.

The material failure model is based on forming limit diagram (FLD) method which is a concept introduced by Keeler and Backofen [32] to determine the amount of deformation that a material can withstand prior to the onset of necking instability. The maximum strains that sheet material can sustain prior to the onset of necking are referred to as the forming limit strains as described in the ABAQUS documentation [26] and also in AbuBakar and Dow [28].

Considering the forming limit strains as rate independent effects in the FLD method, details of which can be found in Jie et al. [33], the following relationships are used:

$$\varepsilon_1 = \begin{cases} \frac{n}{(1 + r_\varepsilon)} & \text{if } r_\varepsilon \leq 0 \\ \frac{3r_\varepsilon^2 + (2 + r_\varepsilon)^2 n}{2(2 + r_\varepsilon)(1 + r_\varepsilon + r_\varepsilon^2)} & \text{if } r_\varepsilon > 0 \end{cases} \quad (3)$$

where $r_\varepsilon = \varepsilon_2/\varepsilon_1$ is the strain ratio. $r_\varepsilon=0$ for plane strain $r_\varepsilon=-0.5$ for simple tension and $r_\varepsilon=1$ biaxial tension, which is the basis for localized necking failure. n is the hardening constant.

For all of the simulations carried out the friction coefficient was set at 0.3 and the displacement at failure considered to be $\varepsilon_u L$. Where ε_u is ultimate strain, approximately $0.5\varepsilon_f$; where ε_f is fracture strain and L is characteristic element length. In the post necking regime the element characteristic size has a significant influence on the accuracy of the results. For shell and 2D elements, L is square root of the integration area and for 3D elements L is the cube root of the integration of volume.

Fig. 3. (a) The scaling forming limit diagram at onset necking versus element length [32]. (b) The failure strain versus element length.

The rupture of the structure depends upon FLD_0 , which is the point of minimum strain under plane strain conditions when local necking occurs. For this study a rupture strain of 0.1 was assumed, as shown in Fig. 2. The FLD_0 in relation to element length in Fig. 3a introduced by AbuBakar et al. [34] are used throughout this analysis. Fig. 3b shows the relationship of rupture strain between AbuBakar et al. [28] and Ehlers [35], where the rupture point according to the element length is alike. Ehlers also shows that the thicknesses of the plate do not give any significant effect to the rupture strain point.

4. Progressive Collapse Analysis of Intact Box Girders

To ensure a robust modelling approach, the parameters which are used to replicate various important aspects of the experimental setup were first defined using preliminary analyses where appropriate. A summary of these preliminary tests are now presented.

4.1. Base Model

A common idealisation when analysing a hull section or box girder section is to only analyse a section or slice of the complete geometry. This reduces the size and complexity of the mesh, allowing efficient use of the FEM solver. It is especially important when simulating damage

scenarios, which require very small increment sizes and may take a substantial amount of computation time to complete.

Therefore, a representation of the IST box girder test section as an isolated unit was first developed in Abaqus CAE. Bending moment is applied through rotation controls applied to one or both ends of the section. Suitable boundary conditions can be set to ensure the rotation produces a pure bending moment without introducing an eccentric longitudinal force. In this study, the section boundary conditions were set as shown in Fig. 4. One end of the section is constrained in all 6-degrees of freedom. The other end is tied to a reference point using rigid body constraints. Rotation is applied at the reference point with all other degrees of freedom unconstrained. This effectively creates the same boundary as at the fixed end but with the plane free to translate bodily. The rotation of the rigid body is then controlled in the analysis to cause progressive application of pure bending moment in the mid region, which is ensured because the end bays are shorter and thus less susceptible to interframe buckling than the central bays.

Unless stated otherwise, all the analyses presented in the following sections use the static-implicit Riks solver.

Fig. 4. Test Section Model Boundary Conditions

4.2. Mesh Size

All models were meshed using Abaqus S4R quad elements, with S3 triangular elements used where necessary to mesh round complex features such as the frames. S4R is a 4 node doubly curved finite strain element with reduced integration. The default in plane displacement hourglass control within Abaqus is employed. A mesh convergence study was undertaken to indicate the required element size to generate consistent results. Usually a more refined mesh with a greater number of elements will generate more realistic results. However, mesh size has a significant effect on computation time. As an example, box F200 was tested in vertical bending with four mesh sizes: the average element length was set at 50mm, 20mm, 10mm and 5mm. A test section model was used with average geometric imperfections.

The resulting load shortening curves are shown in Fig. 5. The computational penalty for refining the mesh is demonstrated in Table 2. A 20mm element length is shown to produce a sufficiently accurate mesh with similar bending moment results compared to the 10mm and 5mm models. The 50mm element length model is too coarse. Halving the element length from 20mm to 10mm has a minor effect on the bending moment curve whilst increasing the computation time by over 5 times. The 20mm mesh size was therefore considered sufficient. With this mesh discretisation, the stiffeners are modelled with only 2 elements over the web height. The tensile residual stress region has only a single element across its width. This is a relatively coarse discretisation and suggests that mesh size may not always need to be particularly fine to generate realistic results. This observation is particularly relevant when considering large scale analyses of full scale ship models, where the number of elements can reach the order of 10^6 .

Table 2. Comparison of mesh size and solution time (Riks solver method)

Mesh Size	Number of Elements	CPU Time* (seconds)	Time penalty (compared to 20mm mesh model)
50mm	2929	160	0.2x
20mm	10547	710	-
10mm	42017	3774	5.3x
5mm	166130	28800	40.5x

* Using a single processor on an Intel Core i7-2600@3.40GHz with 16Gb RAM

Fig. 5. Mesh Convergence Study for F200

4.3. Geometric Imperfections

Welded structures are inevitably geometrically imperfect. This is due partly to the initial out of flatness of the rolled or extruded sections, but usually the most significant contribution to out of flatness is the effect of the welding process itself. Welding causes uneven heating of the metal close to joints which typically produces a wrap up effect. Residual stresses due to welding are not modelled in this study.

The magnitude and shape of geometric imperfections are well known to have a significant effect on the strength of plates [36,37] and stiffened panels [38]. In NLFEM, initial imperfections are essential to ensure the instability of compressed portions of the structure behave realistically. A significant issue for any analysis is to ensure the idealised imperfection is sufficiently representative of the actual geometry, as the results can be markedly different depending on the imperfection chosen.

Measurement of initial imperfections of the box girders were not undertaken, although the plate was observed to have minimal fabrication induced out of flatness. The lack of specific measurement data meant that the initial imperfections had to be estimated. Statistical definitions of maximum imperfection amplitude were therefore used [39]. Imperfections are usually defined separately for the plating and stiffeners. The plate imperfection amplitude is defined as the maximum deviation from the flat plane (w_{opl}). Stiffener imperfection is defined as a combination of the maximum deviation from the flat plane of the web (w_{ow}) and the deviation of the web tip from perfectly straight (w_{os}). Furthermore, a stiffened panel has a column imperfection, which is the deviation of the entire interframe panel from the initial flat plane between the panel edges (w_{oc}).

The relationship between the imperfection amplitude at any point on the panel and the maximum imperfection values are described by Fourier series. In certain cases the Fourier decomposition of a panel imperfection shape can be derived from detailed measurements of as-built panels [40]. When such data is unavailable a two or three mode idealisation can be used. Therefore, this study uses standard Fourier series imperfection patterns using the general approach detailed in Benson [19]. Average magnitudes of imperfection are assumed following the equations proposed by Smith et al. [39]. This defines plate imperfection as a function of plate slenderness, β :

$$\beta = \frac{b}{t} \sqrt{\frac{\sigma_0}{E}} \quad (4)$$

where b is the plate width, t is the plate thickness, σ_0 is the yield stress and E the Young's modulus.

The imperfection magnitudes are calculated as follows:

$$w_{opl} = 0.1\beta^2 t \quad (5)$$

$$w_{oc} = 0.002a \quad (6)$$

$$w_{osl} = w_{ow} = 0.001a \quad (7)$$

where a is the panel length between transverse frames.

4.4. Geometric Extents and Boundary Conditions

To demonstrate the validity of the test section model for comparison with the physical experiment results, a complete representation of the 4-point bending rig was also developed. This enabled the test section to be loaded in the same manner as followed in the original experiment (Fig. 6). The model is pinned at the outermost supports and bending moment is imparted in the central section using displacement controlled constraints at the load application points.

Comparative bending moment-curvature plots in Fig. 7 show very good correlation between the two FEM model extents. Additionally, it can be seen in the example FEM mesh plots (Fig. 8) that the models buckle in a near identical pattern, with failure predominantly interframe and nucleating in the central frame space. These results confirm that the idealised boundary conditions in the test specimen model is a capable technique to model a pure bending moment in a multi stiffened girder.

Fig. 6. Complete FEM model extents

Fig. 7. IST box girder experiments comparison with FEM models: (a) F200, (b) F300 and (c) F400

Fig. 8. FEM mesh plots for F200 test: (a) specimen model, (b) complete model

4.5. Comparison with Physical Experiments

The FEM bending moment-curvature plots in Fig. 7 generally show relatively poor correlation with the physical test results. The FEM predicts lower ultimate strength by about 20% in the F200 and F300 specimens. The F400 specimen results show a much closer correlation, with the FEM predicting only a slightly decreased ultimate strength. However, the FEM results all show lower initial stiffness.

Any explanation of the discrepancy between the experiment and FEM results can only be made based on the information regarding the experiment setup contained in the original test report. A simple hypothesis for the discrepancy is proposed here, which is based on observation of the load application setup used in the physical experiments. The authors conjecture that artificial loads were created during the physical test, due to friction forces acting at the connection between the load box and the four point bending rig, which in the physical experiment uses simple half cylinder bearing surfaces at the vertical load edges. During the application of load, the top of the box girder is compressed, causing the top flange to pull in. The load edges also pull in, but the simple bearing with the strong box above may produce a frictional resistance to the pull in force. This would effectively provide a restraint to delay the buckling response of the top flange, effectively strengthening the structural model.

This could be a plausible explanation for creating the discrepancy between results, as shown in the following discussion, but as the authors did not attend the tests nor have any additional knowledge of the experiment procedure beyond that published in literature, it must be impressed that the following remains as conjecture only.

To test the hypothesis, a very basic approach is adopted for the estimation of the additional friction force magnitude and the modelling of the friction in the FEM model. Using the four point bending FEM model, an artificial friction restraint was modelled using a nonlinear spring attached to the support edges and acting horizontally. The spring stiffness was estimated by analysing the behaviour of the unrestrained four point bending models. The applied vertical force at the supports, which can be computed directly from the FEM model, is related to a potential frictional force assuming a friction coefficient of 0.3. A plot of this “virtual” friction force as a function of the horizontal displacement of the support is shown in Fig. 9 for all three test specimens. The results show that the relationship is initially linear, suggesting that the friction can be represented by a linear spring with stiffness equal to the initial gradient of the curves in Fig. 9, which are 180MN/m for F200, 160MN/m for F300 and 130MN/m for F400. The position of the spring is shown schematically in Fig. 10.

Fig. 9. Horizontal Force-displacement curves for the unrestrained four point bending model

Fig. 10. Representation of the spring boundary conditions used to model the horizontal load point friction in the IST box girder tests.

Two spring models were compared using four point bending FEM models. The first uses a simple linear spring model whilst the second uses an improved bilinear representation.

For the first friction model, the restraint is modelled with a linear spring constant applied throughout the load regime. This approach was attempted using the F200 box girder, with results shown in Fig. 11. However, the use of a linear spring was found to be highly unrealistic as the load approached the nominal ultimate strength. As the top flange of the box starts to buckle, the horizontal displacement of the support increases rapidly. With a linear spring, the restraining force becomes unrealistically high, effectively preventing the box from collapsing. The bending moment curve therefore continues upward.

Fig. 11. Representation of support friction using spring restraints for test specimen F200

A second model uses a nonlinear spring, which assumes the friction coefficient is constant up to the collapse point of the structure but then decreases back to zero in the post collapse region, as shown in Fig. 12. This assumes that, when the box begins collapsing, the frictional restraint reduces because the collapse is dynamic, thus the contact force between the support and the strong box reduces rapidly. The boundary conditions of the 4 point bending model means that the displacement at the two load application points is different, with End 1 displacing further than End 2.

Results are presented in Fig. 13 for the three box girder specimens. All show a much closer correlation between the numerical model and the experiment and similar characteristics between numerical results. The linear spring model shows an initial collapse forming at about the same bending moment as the experiment ultimate strength. However, the continued restraint at the load edges prevents the box from failing completely and the post collapse behaviour is highly unrealistic. The nonlinear spring corrects this problem by switching to negative stiffness when the box surpasses the ultimate strength point and collapses. This produces a very close correlation with the experiment data.

Fig. 12. Representation of the spring boundary conditions used to model the horizontal load point friction in the IST box girder tests.

Fig. 13. Comparative results showing the effect of spring. (a) F300 and (b) F400

4.6. Comparison of Static and Dynamic Solvers

The test report by Gordo and Guedes Soares [3] describe the dynamic nature of the box girder collapse. In particular, the F200 girder collapses quite suddenly with a quick discharge of load and large deformations in the collapsed zone. This highly dynamic failure contrasts sharply with the way the FEM analysis is solved, and therefore raises the question as to whether a dynamic solver may produce a different collapse mechanism and hence give different results. The quasi static Riks method was therefore compared to an equivalent nonlinear dynamic-explicit FEM analysis to demonstrate the suitability of the quasi-static post collapse characterisation. The small size of the IST box girders means that they are well suited to an efficient analysis without excessive computational effort.

Results for the F200 specimen are presented in Fig. 14. The static and dynamic approaches produce almost identical behaviour. The findings suggest that the quasi static Riks approach is an acceptable solver method even when handling a highly nonlinear post collapse scenario. The results are so close

as to suggest that box girder collapse is essentially quasi-static, even when the peak of the bending moment curve is sharp and the unloading portion of the curve is steep.

Fig. 14. Comparison of FEM solvers and the simplified progressive collapse method for the F200 box girder model with vertical (sag) bending moment.

4.7. Comparison with Simplified Progressive Collapse Analysis

Fig. 14 shows that the FEM analyses of the F200 IST box girder also compare closely to an equivalent interframe progressive collapse analysis using the Smith method. The progressive collapse representation is relatively simple, with hard corners extending $30t$ from the four box corners and the remaining structure represented with simple PSC elements. Because the box is very small the side plates are split into 6 elements to ensure the progression of collapse into the side shell is correctly represented. The load shortening curves used for the F200 box are as shown in Fig. 15. The bottom flange is much weaker than the side and top because it consists of only a single stiffener with relatively wide plate width, meaning that the non-dimensional plate slenderness ratio is very high ($\beta=5.7$). The side and top flanges are less slender ($\beta=2.9$ and $\beta=2.1$ respectively).

Fig. 15. Load shortening curves for the F200 box girder

An example interaction plot (Fig. 16) shows the relationship between horizontal and vertical ultimate strength under different combinations of longitudinal curvature. Each point on the interaction diagram describes the horizontal and vertical ultimate strength under a prescribed combination of bending moments. In each case the ratio of vertical to horizontal bending moment is kept constant and the magnitude is increased until the peak ultimate strength is reached. This approach is relatively simple to apply in the Smith method by ensuring the formulation is used with increments of bending moment. However, in the FEM simulations, pure bending is applied through increments of curvature, with the resulting reaction bending moment measured. This ensures the simulation continues into post collapse. This means that the ratio between horizontal and vertical

bending moment can change somewhat. However, in the present study, the ratio was found to remain fairly constant throughout the simulations and therefore the use of curvature ratios was considered suitable.

Fig. 16. Simplified progressive collapse results

Because the box is vertically symmetrical, the interaction plot would expect to also show a symmetrical relationship about the vertical axis. This means that, for the simplified analyses, only results for positive horizontal bending moment are required. However, the FEM results show slight asymmetry, which is probably due to the influence of the geometric imperfections imposed on the plates and stiffeners. These create a slightly asymmetrical section because the imperfection pattern alternates between adjacent elements. For example, the width wise plate imperfection shape alternates between distorting one plate with imperfection lobes pointing into the box centre and the adjacent plate imperfection lobes running opposite and out from the box centre.

4.8. Intact Analysis Summary

The comparative results for the intact IST box girders demonstrate a number of key features of the FEM and simplified methodologies. The theoretical results compare well to the experimental data when the boundary conditions in the physical tests are properly taken into account. The FEM study

has shown that a prismatic section can be constrained adequately to impart pure bending moment across the section. The analyses have also demonstrated that static and dynamic FEM solvers can produce very similar results when used to model a progressive collapse failure of a box structure. Finally, the results correlate closely to the simplified Smith progressive collapse method. It is important to note that the theoretical results are significantly different in terms of ultimate strength prediction compared to the original experiment data. This is an important finding because it provides a case in point that physical test results must be critiqued as intensely as equivalent numerical results to ensure that the boundary conditions are not influencing the result unduly.

5. Damage Simulation

The latter part of this paper presents progressive collapse analysis of the F200 IST box girder with damage first sustained from an artificial indenter. The purpose of the analysis is to compare the predictions of damaged box girder ultimate strength with and without residual stress using dynamic explicit, implicit and static analysis. This section discusses the simulation of indentation damage of the box girder using dynamic explicit analysis. The results focus on the load of the indenter that punches into the box girder with constant velocity and to a depth of 0.3m. The following section presents and discusses the results from subsequent progressive collapse analyses of the damaged structures.

5.1. Simulation Approach

The numerical simulations are undertaken with 3 load steps:

- Step 1: penetration of the indenter into the box girder at 3m/s and to a depth of 0.3m,
- Step 2: retraction of the indenter at 3m/s,
- Step 3: apply incremental bending moment up to and beyond the ultimate capacity

Where Step 3 is completed using the dynamic solver a step time of 1 second was used to increment the applied curvature from zero to the post collapse region.

The indenter is defined as a rigid body cylinder with a hemisphere tip. The indenter has a size of 0.75m height and 0.35m diameter. The FEM analysis used the dynamic explicit analysis capabilities of ABAQUS. Three different indentation scenarios were completed, damaging the bottom, the side and the top flanges of the box. In each case the indenter was targeted at the exact centre of the flange (Fig. 17). The penetration was sufficient to severely rupture the flange around the targeted area. The box boundaries are constrained in all six degrees of freedom for the first two steps. All other initial settings for the FEM analysis were the same as for the intact analyses, with superimposed average geometric imperfections.

The indentation of the box girder at top, bottom and side are completed in a separate simulation file before the subsequent bending moment analyses, using the restart capabilities of ABAQUS. This reduced the time cost of the various analyses considerably as multiple bending moment simulations could be completed using a single indentation simulation.

Fig. 17. Indentation Scenarios (a) Top Indentation, (b) Side Indentation and (c) Bottom Indentation

5.2. Indentation Results

5.2.1. Top Panel Damage

Fig. 18 shows the simulation stress plot of the ruptured panel and the displacement force graph for the top indentation of the box girder. The stress plot shows the stress state once the indenter has been removed. The maximum lateral penetration resistance force and displacement of the indentation are $F_y = -579.5\text{kN}$ and 83.9mm respectively. The start of the rupture of the plate occurred at 86.93mm penetration after which the resistance force starts to decline. The rupture zone extends to the outermost stiffeners on the top flange. The side flanges are highly stressed but do not exhibit significant out of plane deflection.

The stress plot shows the high residual stresses in the region around the ruptured hole. The box returns to an equilibrium state once the indenter is removed, which means the stresses are effectively locked in to the structure subsequent to the impact. The residual stress is predominantly tensile in the area around the rupture with equilibrating compressive stresses of much lower magnitude occurring in structure away from the hole. The residual stress reduces in magnitude across the side flanges and is very small in the bottom flange.

Fig. 18. Force displacement of box girder for top indentation

5.2.2. Bottom Panel Damage

Fig. 19 shows the simulation stress plot of the ruptured panel and the displacement force graph for the bottom indentation of the box girder. The graph of bottom damage shows the resistive force is proportional to the displacement until it reaches the maximum indentation force of $F_y=622.34\text{N}$ and displacement of 89.9mm . The rupture occurs almost immediately after the indentation force reaches its maximum value as shown in Fig. 19. The rupture takes place at about 3mm of further penetration of the indenter after the maximum force has been reached and the force then sharply declines.

The stress plot shows similar characteristics to the top damage model, with high tensile residual stresses in the area close to the rupture and equilibrating compressive stresses elsewhere. The rupture extends further than for the top panel, primarily because there are less stiffeners on the panel to resist the impact load. Significant distortion occurs across the whole bottom flange and into the side panels.

Fig. 19. Force displacement of box girder for Bottom indentation

5.2.3. Side Panel Damage

Fig. 20 shows the simulation stress plot of the ruptured panel and the displacement force graph for the side indentation of the box girder. When placed under side damage, the transverse frame trips at 54mm after penetration of indenter and load $F_x = -387.586\text{kN}$. This leads to a very sudden displacement with a corresponding drop in penetration force. The maximum indentation force and displacement for side damage peaks at $F_x = -674.6\text{kN}$ and 111mm respectively. The rupture initiates at 114.0mm and lower load is then required until the penetrator reaches the maximum indentation.

With stress similar to the other damage models, the ruptured zone extends over most of the side flange and significant distortion at the corners with the top and bottom panels are also exhibited.

Fig. 20. Force displacement of box girder for side indentation

5.2.4. Comparison of lateral force of box girder to all the side shells

The graph below shows the comparison of lateral force for the three different damage scenarios.

The top and bottom damage produces a very similar path of load displacement force. The side damage slightly deviates from the other lines at 54-70mm indentation. The side damage also produces a higher penetration force than the other cases. This happens because the indenter is targeted between the two longitudinal stiffeners and is thus capable of absorbing more energy and

elongating more before rupture. In comparison, the top and bottom indentations are targeted directly at the central longitudinal. The top damage scenario produces a lower maximum load compared to side and bottom penetration. This occurs due to the additional rigidity of the top structure, meaning it is less capable of absorbing higher energy during indentation. This leads to higher stress concentration along the corner section and earlier rupture early compared to the other cases.

Fig. 21. Force displacement of box girder for top, bottom and side indentation

6. Progressive Collapse Analysis of Damaged Girders

6.1. Representation of Damage in FEM Progressive Collapse Method

On completion of the indenter analysis steps, the ruptured box girder FEM models are further subjected to incremental bending moments to assess their progressive collapse and ultimate strength characteristics. The FEM progressive collapse analyses of the damaged box girders are carried out using three approaches, which are now summarised.

The setup is generally as detailed previously, with the boundary conditions modified to the constraints shown in Fig. 4. A full range of curvature combinations are simulated to produce complete interaction curves for comparison with the equivalent intact results. The analysis also allows comparison of several different FEM solver approaches. Three distinct setups were followed:

an explicit analysis including the residual stresses due to damage; an explicit analysis excluding the residual stresses due to damage; and an implicit analysis also excluding the residual stresses due to damage.

6.1.1. Dynamic FEM Solver

The easiest method in terms of setting up the FEM solver is to continue from the indenter simulation (steps 1 and 2) with a subsequent explicit load step (step 3). This is invoked by restarting the indenter simulation using the *RESTART capabilities in ABAQUS. Because the bending moment analysis is continued directly from the indentation simulation, the residual stresses created around the ruptured area are also sustained into the bending moment load step. The continued use of the explicit analysis solver means that the simulation time for each analysis is relatively long compared to an equivalent implicit analysis.

6.1.2. Static FEM Solver

For comparative purposes an implicit solver approach was also followed, which is more efficient in terms of computation time but requires a more complex setup procedure. However, ABAQUS is unable to directly restart an explicit analysis with an additional implicit load step. Instead, using the implicit solver requires a “new” input file to be created, whilst retaining the ruptured geometry from the damage analysis. This was achieved by exporting the final node coordinates from the indenter output file and re-building the damaged girder in a new input file with an implicit load step. The resulting FEM model has the same ruptured geometry, but the residual stresses due to damage are removed and therefore any influence they have in the bending moment characteristics of the girder are neglected.

6.1.3. Dynamic FEM Solver with Zero Residual Stress

A third solver approach was also followed using the zero residual stress model and with an explicit load step. This enables a further comparison of implicit and explicit approaches which otherwise start from an identical FEM model setup.

6.2. Representation of Damage in the Simplified Progressive Collapse

Method

The progressive collapse method has been applied to the analysis of ship structures in various damaged conditions [18,41–43]. In all these cases, the residual strength of the damaged hull girder is assessed by modifying element load shortening curves in the damaged area and/or removing elements in the ruptured zone. This usually has a negative effect on the overall bending capacity of the girder. Although this relatively simple approach to represent damage can be effective, the limitations of the Smith method in dealing with a structure which is no longer in its “as designed” condition must be appreciated. The principal assumptions of interframe buckling and independence between elements inherent in the Smith method remain, which means that the undamaged structure adjacent to a damaged zone is assumed to be unaffected by the damage. In particular, the method cannot account for a change in buckling mode due to the change in boundary conditions caused by rupture or the effect of damage over several frame spaces, which may reduce the effectiveness of the transverse frames to ensure interframe collapse. These phenomena may have a significant detrimental effect on the strength of the area around a damaged zone, which would then have a negative influence on the global strength as compared to the equivalent interframe damage assumed within the standard Smith approach.

The damaged cross sections used in the simplified progressive collapse analyses are shown in Fig. 22. The extent of the rupture zone was sized by inspection of the equivalent FEM models and refined by comparison to the FEM results. The top damage scenario (Fig. 18) appears to cause a smaller rupture

zone than the side (Fig. 19) and bottom (Fig. 20) damage scenarios, which is likely to be due to the more extensive stiffening in the top flange. Plate and stiffener elements adjacent to the rupture zone were assumed to be unaffected by the damage and thus were assigned load shortening curves consistent with the intact case (see Fig. 15).

Fig. 22. Simplified Progressive Collapse Method Damage Models (a) Top Damage, (b) Side Damage and (c) Bottom Damage

All three damage scenarios showed a high degree of sensitivity to the damage extent. As an example, a comparison of the vertical sag bending moment-curvature relationships for the top damaged girder with different sized rupture zones (with length d as shown in Fig. 22) are shown in Fig. 23. This highlights some of the difficulties of accurately modelling damage using the simplified approach. An increase in the damage length of 15cm from 0.6m to 0.75m shows a corresponding 30% reduction in the ultimate strength. The ruptured zone lengths of all three models were therefore fine-tuned to the dimensions as shown in Fig. 22 so as to correspond with the equivalent finite element results in vertical sag.

Fig. 23. Simplified progressive collapse analysis of top damage box girder – comparison of the ruptured zone extents

6.3. Results

Complete interaction curves are produced from the FEM analyses of all three box girder damage scenarios under combinations of horizontal and vertical bending moment. The plots allow clear comparison between the different solver methods and with the intact box girder solution.

6.3.1. Top Damage

The interaction plots (Fig. 24) show the significant reduction in ultimate capacity of the top damaged box compared to the intact strength in almost all cases. The reduction is most pronounced in the upper part of the interaction plot, where the box is under sagging bending moment. In this circumstance the damaged region is placed under compressive in plane load. This is predominantly taken by the upper parts of the box sides. Compared to the intact box, the compressive load portion of the cross section is also increased for the same curvature because the neutral axis is lower. These effects combine to cause much earlier buckling in the upper parts of the box, which corresponds to a much lower ultimate strength. The reduction in ultimate strength is less significant in the lower (hog) part of the interaction plot.

Fig. 24. Interaction Diagram for Top Damage Case

The interaction plot also shows considerable differences between the FEM analyses where the residual stress due to damage is maintained (dynamic FEM with residual stress) and the equivalent analyses undertaken with no residual stresses included (static FEM). The simplified progressive collapse results show close correlation to the static FEM. Remarkably, under a predominant hogging bending moment the dynamic FEM ultimate strength results are greater than for the intact case, although the capacity is still much reduced in the upper quadrants of the interaction plot.

These results suggest that the residual stresses in the structure, which are particularly high in the region adjacent to the ruptured zone, have a significant effect on the ultimate strength of the girder, in this case by increasing the capacity for all combinations of applied curvature. Both the static FEM and simplified progressive collapse results do not account for the influence of the residual stresses.

The top damage box girder results from the static approach and the dynamic – zero residual stress approach are compared in Fig. 25. The results reiterate the findings from the intact analyses showing that the two solvers produce almost identical results so long as the initial conditions in the mesh are identical. This also shows conclusively that the differences between the static and dynamic results presented above are due to the residual stresses in the mesh resulting from the impact.

Fig. 25. Interaction Diagram for Top Damage Case. Comparison of implicit and explicit (zero residual stress) solver methods

6.3.2. Bottom Damage

Interaction plots for the bottom damage scenario are presented in Fig. 26. In comparison to the differences shown for the top damage case, the plot demonstrates much closer correlation between the static FEM, dynamic FEM and simplified progressive collapse results. However, the dynamic FEM with residual stress results still shows higher ultimate strength than the equivalent zero stress static FEM when the box is predominantly under a hogging bending moment, which corresponds to the damage region being placed under compressive load.

The closer correlation between results is likely to be because the top flange, which in this scenario is left intact, is the dominant load bearing region of the structure. Therefore, the influence of the ruptured zone and the associated tensile residual stress field in the bottom flange has less influence on the overall strength of the box under longitudinal bending.

Fig. 26. Interaction Diagram for Bottom Damage Case

6.3.1. Side Damage

Interaction plots for the side damage scenario are presented in Fig. 27. The plot shows similar correlation between results as seen in the side damage scenario, which again reflects the dominance of the top flange in determining the overall strength of the girder.

Fig. 27. Interaction Diagram for Side Damage Case

7. Conclusions

The strength of three small scale box girder structures have been analysed using several numerical methods under combinations of longitudinal bending moment. The girders have been tested whilst intact and also with three damage scenarios, whereby the girder had been previously subject to an impact simulation which ruptured a large hole in the top, side or bottom flange.

The results have provided numerical verification of the IST box girder experiments [3], and have further elaborated on the reasons why numerical results do not correlate closely with the original experiments. A hypothesis is proposed and validated showing that the friction between the load box and the test section may give rise to an artificial increase in the strength of the box girder. Although this hypothesis is only based on the information of the physical test as published, it provides a reasonable explanation for the discrepancy between numerical and experimental results.

The comparison between static and dynamic FEM solvers for the intact box girder has shown that either approach is valid for the purposes of progressive collapse analysis. The assumptions inherent in the static method, which neglects the influence of time dependent mass and damping effects, are acceptable for progressive collapse even when the buckling is essentially a dynamic phenomenon. The results have also demonstrated that the simplified progressive collapse method provides very good results in comparison to the more computationally expensive FEM.

However, the damage simulations and the subsequent progressive collapse analyses have shown that the residual stress caused by the impact from a rigid body has a significant influence on the ultimate strength characteristics of a ruptured box girder. Residual stresses are not accounted for in the simplified progressive collapse method or in the static FEM analyses as conducted in this study. In the dynamic FEM analyses the residual stress, in the form of large tensile zones close to the ruptured zone and lesser compressive stresses elsewhere, have the effect of increasing the box girder strength by up to 10%.

The results presented in this paper have important implications in the development of approximations of rupture and damage in both FEM and simplified methods. If rupture is modelled by simply removing structural elements the influence of the residual stresses caused during the initiation of the damage is not accounted for and thus its influence on the overall strength of the girder is neglected. A more rigorous approach to representing the structure adjacent to a ruptured zone may be needed so as to properly account for the significant influence of the stresses within the structure.

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