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Predict Pitch and Heave Motion of Ship via Inverse Triangle Transform MGM(1,n) Model

Jihong Shen, Yan Li and Xiangyin Meng

Abstract—The prediction of the ship motion has a significant practical meaning to the safety of the ship controlling. The pitch and heave motions are the main two features in the vertical movement system of the ship. Considering the coupling relationship between the influence factors in the ship motion system including the wave, the Inverse Triangle Transform MGM(1,n) model is proposed to predict the pitch and heave motion. The Range Transform is applied to normalize the different quantities of the model inputs. The modified Inverse Triangle Transform is introduced to solve the problem that the data are periodic oscillating. After these two transforms, the data fit the grey model well. The MGM(1,n) model is used to predict. In the analysis of the predicted result, within the relative error of 20%, the predicted data can get 70 groups, which is about 4-5 seconds. The right oscillating shape of the pitch and the heave motions last more than one period. The result shows that the model can predict the ship motion in required precision, and the model is effective and practicable.

I. INTRODUCTION

Effected by the influencing factors of the sea and many other factors, the ship motion becomes a complex system consists of six degrees of freedom, in which the pitch motion and the heave motion are the main cases in vertical movement. To control the motion is the key point to the safety of the ship. Knowing the ship motion in upcoming seconds is necessary for the manipulation of the ship. So, the prediction has a significant practical meaning. Short time prediction of the ship motion is defined to predict the ship motion in the coming seconds via the certain method [1]. There are already many schoolers working on this topic, such as the time serial method, convolution method, Kalman filter, the period graph method [2], [3]. In the years around, with the development of the practical requirement, there comes the methods based on the NNet, the Wavelet theory and decoupling the motion equations and so on [4], [5].

Most of the methods mentioned above are based on the Strip Theory and seldom of them consider the coupling relationship among the influence factors in the whole ship motion system. To solve this problem, the Grey System Theory is introduced into the ship motion prediction. Some schoolers have studied on this field. To the property of

periodic oscillating in pitch motion, Jihong Shen applies the Triangle Transform to change the pitch data into the monotonic data series, and predict via the GM(1,1) model. The predicted time can be 12 seconds [6], [7], [8]. Du modifies the Triangle Transform and prolongs the predicted time in the same precision of Shen [9]. Jihong Shen and Jide Li apply the MGM(1,n) model and the Updating MGM(1,n) model to the system composed of pitch and heave motion, and get an acceptable result [10], [11], [12]. In this paper, the Grey System Theory and the real time predicted theory are combined, and the Inverse Triangle Transform and MGM(1,n) model are applied to predict the ship motion in grey system composed of pitch motion, heave motion and wave.

II. INVERSE TRIANGLE TRANSFORM MGM(1,N) MODEL

A. Grey System Theory

Grey System Theory is proposed by the Chinese scholar Deng Julong in 1982 [13]. It is a new step of the Control Theory. The Grey System Theory studies the objects with “partial known information and partial unknown information”. And it describes the level of having the information with “black”, “white” and “grey”. The all known information is defined as “white”, and the opposite is “black”. The partial known information is defined as the “grey” information. The Grey System Theory draws the useful information by handling the original data, and fits the transformed data with the thought of Least Square Criterion, then predicts. It studies the small sample and the system with lack information. With the grey relational analysis and grey generating, the Grey System Theory transfers the disordering series into the order one and draws the useful information from the original data and foresees the changing tendency of the system [14], [15]. Because the Grey System Theory can depict the system with few data and has no limitations of the original data [16]. It has been widely used in the past twenty years and its theory has a big development [17], [18], [19]. The development of Grey Theory can be divided into two main parts. One part is the application into a new engineering field. For the application requirement, the GM model is modified to fit the new problem, such as it is introduced into the ship motion prediction. The other part is to improve the precision of the GM model [20], such as the academic research on the solving of the recognized parameters and widening the requirement of the original data. Both researches are necessary for the development of the Grey System Theory. The first one may have a tough execution and the second one may put too

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many focus on the precision but not for the practical. So, to merge both of them and build a model can be applied into the practical engineering field with an accepted or maybe good precision is what we are working on in this paper.

B. MGM(1,n) Model

The MGM(1,n) model is one of the important model in Grey System Theory. It connotes to build an approximate differential equation mode, taking base on the series of main factors in the system. The MGM(1,n) model depicts the coupling relationships among the main factors in the system, and the influence to one factor effected by others. In this model, the n denotes the number of the variables. Where each variable x_i^0 has m data $x_i^0(k)$, in which $k = 1, 2, \dots, m$.

1) *Accumulated Generating Operation*: To each group of data, the Accumulated Generating Operation(AGO) is applied to get a new series $x_i^1(k)$ according to (1).

$$x_i^1(k) = \sum_{j=1}^k x_i^0(j) \quad (1)$$

where $i = 1, 2, \dots, n$, and $k = 1, 2, \dots, m$. The AGO transfers the original data with disorder into the monotone increasing one.

2) *Inverse Accumulated Generating Operation*: This transform is the inverse process of the AGO, and it subtracts the adjoining series in succession. It will be applied into the reverting of the predicted differential equation data. The calculating is according to (2)

$$x_i^0(k) = x_i^1(k+1) - x_i^1(k) \quad (2)$$

where $i = 1, 2, \dots, n$, and $k = 1, 2, \dots, m-1$.

3) *Differential Equation*: The MGM(1,n) model is built on the series generated via a group of constant differential equation of n variables, which is (3)

$$\begin{cases} dx_1^{(1)}/dt = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \dots + a_{1n}x_n^{(1)} + b_1 \\ dx_2^{(1)}/dt = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \dots + a_{2n}x_n^{(1)} + b_2 \\ \vdots \\ dx_n^{(1)}/dt = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \dots + a_{nn}x_n^{(1)} + b_n \end{cases} \quad (3)$$

We set $X^{(0)}(k) = [x_1^{(0)}(k), x_2^{(0)}(k), \dots, x_n^{(0)}(k)]^T$ and $X^{(1)}(k) = [x_1^{(1)}(k), x_2^{(1)}(k), \dots, x_n^{(1)}(k)]^T$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n} \quad (4)$$

$$B = [b_1, b_2, \dots, b_n]^T \quad (5)$$

We present A and B in the matrix form, and change the (3) into (6)

$$dx^{(1)}/dt = Ax^{(1)} + B \quad (6)$$

where $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)})$.

A and B are recognized parameters. The a_{ij} denotes the influence degree from the j th factor to the i th factor. The b_i denotes the influence from the factors besides the system-constituted factors to the change rate of the i th factor.

4) *Solve Differential Equation*: We discrete the differential equations [12]. And according to the trapezoid formula and the rectangle formula, we get (7).

$$x_i^{(0)}(k) = \sum_{j=1}^n \frac{a_{ij}}{2} (x_i^{(1)}(k) + x_i^{(1)}(k-1)) + b_i \quad (7)$$

where $i = 1, 2, \dots, n$, and $k = 1, 2, \dots, m$.

Set $a_i = [a_{i1} \ a_{i2} \ \dots \ a_{in} \ b_i]^T$, where $i = 1, 2, \dots, n$.

We get the recognized parameters $\hat{a}_i$ of a_i by the Least Square Criterion.

$$\hat{a}_i = \begin{bmatrix} \hat{a}_{i1} \\ \hat{a}_{i2} \\ \vdots \\ \hat{a}_{in} \\ \hat{b}_i \end{bmatrix} = (L^T L)^{-1} L^T Y_i \quad (8)$$

where $i = 1, 2, \dots, n$.

Set $\bar{x}_i^{(1)}(k) = 0.5(x_i^{(1)}(k) + x_i^{(1)}(k-1))$, and $k = 1, 2, \dots, m$.

In (8),

$$L = \begin{bmatrix} \bar{x}_1^{(1)}(2) & \dots & \bar{x}_n^{(1)}(2) & 1 \\ \bar{x}_1^{(1)}(3) & \dots & \bar{x}_n^{(1)}(3) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \bar{x}_1^{(1)}(m) & \dots & \bar{x}_n^{(1)}(m) & 1 \end{bmatrix}_{(m-1) \times (n-1)} \quad (9)$$

The $\hat{A}$ and $\hat{B}$ of A and B can be got

$$\hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \dots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \dots & \hat{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \dots & \hat{a}_{nn} \end{bmatrix}_{n \times n} \quad (10)$$

$$\hat{B} = [\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n]^T \quad (11)$$

The predicted data are got by (12):

$$\bar{x}_i^{(1)}(k) = e^{\hat{A}(k-1)}(x_1^{(1)}(1) + \hat{A}^{-1}\hat{B}) - \hat{A}^{-1}\hat{B} \quad (12)$$

where $i = 1, 2, \dots$, and $k = 1, 2, \dots$.

To revert the predicted model by (13)

$$\begin{cases} \bar{x}_i^{(0)}(1) = \bar{x}_i^{(1)}(1) \\ \bar{x}_i^{(0)}(k) = \bar{x}_i^{(1)}(k) - \bar{x}_i^{(1)}(k-1) \end{cases} \quad (13)$$

where $i = 1, 2, \dots$, and $k = 1, 2, \dots$.

C. Data Preparation

1) *The Ship Motion and Wave Data:* In the six degrees of freedom, the pitch and heave motion are the main two features in vertical movement of the ship. And in the vertical movement plane, the wave is the key influence factor. There are also other factors which effect the vertical movement. Their influences are depicted by B in MGM(1,n) model. The three series of model data are shown in Fig. 1.

Fig. 1. Ship motion and wave data

The x axis is the serial number of the data. it is relevant to the time. The y axis is the swing of the data in different measurement units. The measurement unit of pitch motion is degree. The measurement unit of heave motion is cm. The measurement unit of wave is cm.

2) *Range Transform:* The grey relation analysis (GRA) is pioneered by Deng Julong in 1984 [19]. It depicts the relationship among the elements in the system with the time going on. It analyzes the geometric shape of the time series. Take the ship motion system for example, the three main elements in the ship motion system have difference measurement units. And there is big difference in each order of magnitude, which makes it difficult to show the influences of every elements and the influences among the elements in the ship motion system. The influence from the elements in small magnitude will be omitted by the influence from the elements in big magnitude. To solve this problem, the Range Transform is introduced into the ship motion system to normalize measurement standard of the three elements. Suppose the original serials are $X_j = (X_j(1), X_j(2), \dots, X_j(m))$, where $j = 1, 2, \dots, n$, and $\bar{X}_j = (\bar{X}_j(1), \bar{X}_j(2), \dots, \bar{X}_j(m))$ are the serials after the Range Transform according to (14).

$$\bar{X}_j = \frac{X_j - X_{jmin}}{(X_{jmax} - X_{jmin})} \quad (14)$$

X_{jmax} and X_{jmin} denote the maximum and minimum elements of the serials X_j , where $j = 1, 2, \dots, n$. The Range Transform transfers the breadth of the pitch, heave motion and wave data into the interval of $[0, 1]$. With the same magnitude, the influence among the factors in the system is clear.

3) *Inverse Triangle Transform:* With the AGO, the grey model will have a good result based on the e^x form data [7]. But the periodic oscillating data can not being changed into this form after the AGO. Even if the whole series are moved above the 0, the AGO data still have the property of oscillating. The sin data after $\sin(x) + 1$ are applied by the AGO, which is shown in Fig. 2. This will lead the simulation and the prediction exceed over the precision requirement and fail to reflect the changing tendency of the whole system. So the Inverse Triangle Transform, which is a better contention between the periodic oscillating data and sectional monotone data than other transform, is introduced to solve this kind of problem.

Fig. 2. The sin data and its AGO

We illustrate the Inverse Triangle Transform by the data from the $\sin(x)$.

(15) is a preparation for the inverse Triangle Transform.

$$\bar{X}_j = \frac{X_j - (X_{jmax} + X_{jmin})}{0.5 \times (X_{jmax} - X_{jmin})} \quad (15)$$

Because the traditional Inverse Triangle Transform is not a one-one map, we change its definition with concerning the period element to solve this problem. The modified Inverse Triangle Transform is proposed according to (16).

$$\tilde{X} = \arcsin(\bar{X}) + M(\bar{X}) \cdot 2\pi \quad (16)$$

where, $M(\bar{X})$ is the period number of $\bar{X}$. Then the modified Inverse Triangle Transform becomes a one-one map.

As shown in Fig. 3, after the Inverse Triangle Transform, the periodic oscillating sin data is changed into the monotone increasing series. And its AGO fits the e^x form well, which provides the possibility of good prediction with the grey model.

D. Error Judgement

Because the oscillating property, the traditional error judging can not reflect how good or bad the predicted results are. To this point, we will judge the predicted result in two aspects of the swing and phase. Both of them are the key

Fig. 3. The Inverse Triangle Transform sin data and its AGO

properties of a oscillating series. Generally considering them can give a fair judgement to the predicted series.

In the aspect of swing, the traditional relative error is defined as (17).

$$err(k) = (X_{pre}(k) - X(k))/X(k) \quad (17)$$

When the $X(k) \rightarrow 0$, even if the $X_{pre}(k) - X(k)$ is small, the err is still big, which is shown in Fig. 4. Because the err can not illustrate the predicted result accurately, so we change the definition of relative error by (18).

Fig. 4. Small predicted error of swing

$$Err(k) = (X_{pre}(k) - X(k))/\max(|X(k)|) \quad (18)$$

The new definition of relative error depicts the relative difference between the prediction and real data comparing to the biggest changing of the system.

In the aspect of phase, the laggard phase of the prediction may lead the fetal fail of the ship motion control. But the advanced phase is acceptable.

E. Inverse Triangle Transform MGM(1,n) model

1) *Flowchart*: According to the calculating steps mentioned above, the Inverse Triangle Transform MGM(1,n) model (ITT-MGM(1,n)) is proposed. And its flowchart is shown in Fig.5.

Fig. 5. The flowchart of the ITT-MGM(1,n) model

In ITT-MGM(1,n) model, the pitch, heave motion and the wave data are changed by the Range Transform and the Inverse Triangle Transform separately. Then the three series of the modified data are used as the original inputs of the MGM(1,n). After the two inverse processes of the data preparation, the predicted data, including the predicted pitch and predicted heave motion are analyzed in the range of relative error.

III. NUMERICAL EXPERIMENT

A. The Predicted Time

The data in the numerical experiment are got from the simulation in the water pool. It has the resolution of 100 samples per second. And with the ratio between the pool simulation and the real sea, it is about 5-7 seconds with different sea conditions [10].

B. Data used

344 groups of the data are applied as the original data to predict. The largest amplitude of the pitch motion is 4.804. The pitch motion contains 3 periods. The largest amplitude of the heave motion is 69.355. The heave motion contains 3 periods. They are shown in Fig. 6. It should be motioned that not the more data used, the better predicted result are. With the different sea conditions, the number of the data group

original should be changed. It is an empirical parameter. Most of the time, around two or three periods of oscillating is a good choice. It provides the enough system information and does not cost too much calculating time for the real-time predicting system.

Fig. 6. The original pitch, heave motion and wave data

C. The Analysis of the Result

Within the relative error of 20%, the predicted pitch motion lasts about 70 groups. It is about 4-5 seconds and an oscillating period. Then, with the increasing of the phase of the pitch motion, the error begins to exceed the acceptable range. The period of the predicted heave motion occurs in front of the real heave motion about 10 groups, which is a phase of $(10/90) \times 2\pi$. It is in the acceptable range and also a period.

Fig. 7. The prediction of the ship motion

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

To consider the couple relationship between different factors in the ship motion system, the MGM(1,n) model is introduced. The pitch motion, the heave motion and the wave are the three main factors to influence the system and are the inputs of the model. The Range Transform and the Inverse

Fig. 8. Relative error of the predicted ship motion

Triangle Transform are introduced to make the data fit the requirement of grey model more. The pitch and heave motion are predicted in the relative error of 20%. And the predicted groups can be around 70, which is about 4 – 5 seconds. The model in this paper is practical.

B. Future Works

Though the ITT-MGM(1,n) model is effective, there is still some works need to be finished in future. In the high sea condition, the pitch and heave motion will change randomly and the model should be modified, such as taking the random model into account to fit this case. Some improvements of the grey model with the data transform process should be applied to increasing the AGO data fit the e^x form. They are the subjects for the scholars who are interested in Grey System Theory to research.

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