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Contradiction between Conservation Property and Mass Conservation in Adaptive Grid Based Shallow Flow Models: Cause and Solution

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SUMMARY

When performing shallow flow simulations on adaptive grids, the conservation property (C-property) and the mass conservation may not be simultaneously preserved, i.e. either C-property or mass conservation is likely to be violated following grid refining or coarsening. The cause of such a contradiction is analysed in detail in this work, which essentially links to the reconstruction of bed and flow information in those newly created cells during grid adaptation. An effective approach is subsequently proposed to resolve the contradiction by locally modifying the bed elevation in certain problematic cells when reconstructing flow variables and bed elevation by means of linear interpolation as part of the grid adaptation procedure. Copyright © 0000 John Wiley & Sons, Ltd.

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KEY WORDS: mass conservation; C-property; adaptive grids; shallow water flow models

1. INTRODUCTION

Adaptive mesh refinement (AMR) has been broadly used to achieve high-resolution simulations only in those regions of interest, for example in [1, 2, 3, 4], in order to achieve a satisfactory level of numerical accuracy without compromising the overall computational efficiency. AMR may be commonly classified as block adaption and hierarchical grid adaption [5, 4]. These types of adaptive grids have been widely applied to solve the shallow water equations (SWE) for different applications, for example in [6, 7, 8, 9, 3]. In addition, [4] developed a new adaptive grid system that involves simple allocation of subdivision levels on given coarse background cells, removing the necessity of any complex data structure to store grid information.

Despite the improved computational efficiency compared with the uniformly refined mesh, an adaptive grid based SWE model may not simultaneously preserve the water surface conservation and the mass conservation during a simulation, as reported in [9, 5]. The preservation of water surface is closely related to the conservation property (i.e. C-property) [10] of a SWE model. C-property essentially requires a model to numerically preserve a quiescent steady flow and is usually employed to reflect the well-balancedness of a numerical scheme, e.g. in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. The well-balancedness refers to the ability of a numerical scheme solving the SWE to balance the flux gradients and the source terms [28, 29, 30, 31]. In other words, the C-property affects the accuracy and numerical stability of a SWE model and thereby is

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an important feature such a model should possess. On the other hand, mass conservation is another key criterion reflecting the accuracy and reliability of numerical solutions and should be reinforced during a simulation. However, as previously mentioned, they may not coincide during a simulation involving AMR.

The contradiction between the C-property and mass conservation means that only one of them will be satisfied during AMR simulations, which may lead to numerical instability or loss of solution accuracy. For instance, considering a flow at rest over an uneven bed profile is simulated by an adaptive grid based SWE model that involves generation of refined mesh at wet-dry fronts to precisely capture the shoreline (Figure 1). While the mass conservation is reinforced on the newly refined cells, spurious waves are likely to be generated at the wet-dry fronts and propagate towards the wet domain, as demonstrated in Figure 2 where the simulation is done by the SWE model presented in [32] on the grid generated through applying the adaptive grid system in [4]. The generation of the unphysical waves will inevitably affect the accuracy of the simulation results.

Therefore, simultaneous preservation of mass conservation and C-property remains to be a research issue yet to be resolved. This work therefore aims to investigate more in-depth the cause of the contradiction between the C-property and the mass conservation during AMR simulations and subsequently propose an effective way to resolve the problem. The rest of the paper is arranged as follows: the cause of problem is analysed in detail in Section 2; a solution to the issue is proposed in Section 3; then Section 4 provides the mathematic proof and numerical test is carried out in Section 5; finally brief conclusions are drawn in Section 6.

2. THE CAUSE OF THE ISSUE

In the context of SWE models, the well-balanced condition generally refers to the ability of a numerical scheme to properly balance the flux gradients and source terms [28, 29, 30, 31]. In practice, a scheme may be considered to be well-balanced if it satisfies the conservation property (C-property) as defined in [10], i.e. when the quiescent steady state is preserved, at the computational level [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. The surface water level must maintain to be constant when the flow velocity is zero in a quiescent flow. If a numerical scheme fails to satisfy the C-property, spurious momentum fluxes will be created and subsequently lead to inaccurate results or numerical instability.

On static grids, the C-property can be preserved by carefully designing techniques to balance the slope source terms and flux terms. These techniques include upwind discretization of the bed slope terms [33, 34], surface gradient methods (SGM) [35], flux correction method [36], hydrostatic reconstruction [37], mathematical balancing [18], slope flux method of [27], among others. However, the C-property is likely to be violated on dynamically adaptive grids when the mass conservation is also required, even when a proper balancing technique is implemented to deal with slope source terms [9, 5]. This can be clearly demonstrated by the numerical results obtained for a flow at rest test as illustrated in Figure 2 (b). On the other hand, if the C-property is maintained during a simulation involving adaptive grids, the mass conservation is subjected to be violated.

In this work, water surface level, water depth and bed elevation, denoted respectively by η , h and z_b , are used in the model and defined in Figure 3. The aforementioned issue of contradiction is likely to rise during the mesh refining or coarsening procedure that involves specifying flow variables and bed elevation in the new cells. Figure 4 (a) and (b) respectively plots the coarse parent cells and their refined sub-cells with relatively high water surface level, i.e. the water surface levels in the refined cells are higher than the bed elevations. In this case, as long as the water surface level and bed elevation are both linearly reconstructed in the refined cells and parent cell, the C-property and the mass conservation can be simultaneously satisfied. It should be noted that the linear reconstruction herein for bed elevation may be in the different forms according to the data availability. For example in Figure 4, if fine data are available, the value at a coarsened cell is averaged from all sub-cells

Figure 1. Refined meshes near to wet-dry fronts for the simulation as shown in Figure 2.

in grid coarsening, while the values in refined cells are directly determined from the available fine data in grid refining. Otherwise, if we only have coarse data, the values at refined cells are linearly interpolated from the available information in neighboring coarse cells, while the values at coarsened cells are the right coarse data in grid refining. The feature of preserving simultaneously the C-property and the mass conservation on the adaptive grids is proven in the cell i as shown in Figure 4. The quiescent flow has a constant water surface level and so the water surface levels in the sub-cells i_{-1} and i_{-2} are the same as that in the corresponding coarse cell i , i.e.

$$\eta_i = \eta_{i_{-1}} = \eta_{i_{-2}}. \quad (1)$$

According to the linear relation, the bed elevation can be expressed by

$$z_{bi} = \frac{1}{2} (z_{bi_{-1}} + z_{bi_{-2}}). \quad (2)$$

Assuming a 1D problem, the volume of water in the parent cell i is calculated by

$$m_i = h_i \Delta x, \quad (3)$$

Figure 2. Predicted surface water level for flow at rest test at $t = 200$ s using: (a) uniform grid, (b) adaptive grid refined near to wet-dry fronts (Figure 1).

where Δx represents the cell size. The total volume of water in the two sub-cells i_{-1} and i_{-2} is

$$m_{i_{-1}} + m_{i_{-2}} = h_{i_{-1}} \frac{\Delta x}{2} + h_{i_{-2}} \frac{\Delta x}{2}. \quad (4)$$

From relations in equations (1) and (2) and $\eta = h + z_b$, we have $h_i = 0.5 (h_{i_{-1}} + h_{i_{-2}})$ and in turn

$$m_i = m_{i_{-1}} + m_{i_{-2}}. \quad (5)$$

Equation (5) indicates that the mass conservation is satisfied; meanwhile the water surface level is preserved to ensure the C-property during grid adaption. However, this is not always the case as the reconstructed water surface level in a sub-cell may be lower than the bed elevation, for example the refined cell i_{-2} in Figure 5 (b). Such a case may occur near wet-dry interface over uneven bed where the gradient of water depth is likely to be lower than that of bed elevation. Subsequently, if the water surface level is maintained in all cells to preserve the C-property, the reconstructed bed elevation in a sub-cell may rise above the water surface level, for instance, the bed level as shown

Figure 3. Notations of bed and flow variables for shallow water flows.

in cell i_{-2} in Figure 5 (b). The water pattern is obviously different from the previous case as a new dry sub-cell emerges. The total water volume in the two sub-cells is evaluated by

$$m_{i_{-1}} + m_{i_{-2}} = h_{i_{-1}} \frac{\Delta x}{2}. \quad (6)$$

Since the volume of water in the parent cell i is given by $m_i = h_i \Delta x$, the mass conservation is only preserved when $h_{i_{-1}} = 2h_i$. As shown in Figure 5 (a) and (b), $h_i = \eta_i - z_{bi}$ and $h_{i_{-1}} = \eta_{i_{-1}} - z_{bi_{-1}}$. If $h_{i_{-1}} = 2h_i$ is assumed, we have

$$\eta_{i_{-1}} - z_{bi_{-1}} = 2(\eta_i - z_{bi}). \quad (7)$$

As $\eta_{i_{-1}} = \eta_i$, the above equation can be rearranged to become

$$z_{bi} = \frac{1}{2}(\eta_i + z_{bi_{-1}}). \quad (8)$$

According to equations (8) and (2), $\eta_i = z_{bi_{-2}}$ is derived. This is apparently incongruous to the fact that the reconstructed bed elevation is higher than the corresponding water surface level in sub-cell i_{-2} . Therefore, the assumption of $h_{i_{-1}} = 2h_i$ is invalid and the mass conservation is therefore violated. In contrast, if the mass conservation is preserved, the water surface level will no longer keep uniform as shown in Figure 5 (c), because the volume of water in sub-cell i_{-1} is equal to that in cell i and thus can not guarantee the same water surface level as that in other wet cells. The contradiction can also be found in the grid coarsening procedure and the cause is similar. The above analysis clearly demonstrates the reason of why the C-property and the mass conservation cannot be satisfied at the same time on adaptive grids when the bed elevation in the sub-cells is lower than the reconstructed water surface level.

3. PRESERVATION OF C-PROPERTY AND MASS CONSERVATION ON ADAPTIVE GRIDS

It is known from last Section that the contradiction between the C-property and mass conservation is actually caused by coarsening or refining the cells with relatively small water depth over abrupt bed, e.g. the wet-dry fronts. As static grids will not give rise to this problem, a straightforward approach to handle the problem is to avoid grid adaption in the problematic regions. However, this approach

Figure 4. Reconstruction of flow information during grid refinement for case 1: (a) flow information in parent cells; (b) reconstructed flow information in sub-cells that satisfies the C-property and mass conservation.

of enforcing static grids inevitably compromises the merit of the adaptive grids that allow flexible change of grid resolution according to the needs. Herein, a novel approach is devised to resolve the contradiction between the C-property and mass conservation on adaptive grids.

Last section reveals that the problem will not occur if the parent and sub-cells are all wet and the water depth/level and bed elevation are both reconstructed in a linear way which must be carried out in both refining and coarsening procedures. In engineering applications, however, the available coarse and refined bathymetric or topographic datasets may not have the linear relationship as required. For instance, if the given bed elevation in wet cell $i-1$ in Figure 4 is not equal to the averaged value of those in the two wet sub-cells $i-1_1$ and $i-1_2$, the bed elevation in the coarse wet cell should be modified to the averaged value from the corresponding fine cells, rather than use the available coarse ones, i.e.

$$z_{bi-1} = \frac{1}{2} (z_{bi-1_1} + z_{bi-1_2}). \quad (9)$$

In the case of one or more sub-cells with reconstructed bed elevation higher than the corresponding water surface level, the contradiction may be resolved by modifying the bed elevation locally in the relevant cells during adaption. The modification for bed elevation is demonstrated herein in 1D cells in grid coarsening and refining to a higher and lower levels, respectively.

Figure 5. Reconstruction of flow information during grid refinement for case 2: (a) flow information in parent cells; (b) reconstructed flow information in sub-cells that satisfies the C-property, (c) reconstructed flow information in sub-cells that satisfies the mass conservation.

3.1. Local modification of bed elevation during cell refining

Figure 6 shows how to modify the bed elevation in the two newly refined cells. When a cell is refined to a higher adaptation level, the flow variables and bed elevation must be reconstructed in the centres of two refined sub-cells, normally by means of linear construction. For example, the bed elevation and water surface level in sub-cell i_1 can be respectively computed from

$$z_{bi_1} = z_{bi} - \frac{\Delta x}{4} \nabla z_{bi}, \quad (10)$$

$$\eta_{i_1} = \eta_i - \frac{\Delta x}{4} \nabla \eta_i, \quad (11)$$

where Δx is the cell dimension of the parent cell; ∇z_{bi} and $\nabla \eta_i$ denote the slope of the bed elevation and the water surface level, respectively, in the parent cell i . Other flow variables can be evaluated in the same way. ∇z_{bi} is termed as the original bed slope in Figure 6(b), which will be modified to a new bed slope $\bar{\nabla} z_{bi}$. In the same figure, the linearly reconstructed bed elevations of the two sub-cells are denoted as z_{bi_1} and z_{bi_2} . This case will be problematic as the reconstructed bed elevation is higher than the corresponding water surface level in sub-cell i_2 (Figure 6(b)). This will retain the C-property but violate the mass conservation as $h_{i_1} \neq 2h_i$. In order to overcome the issue and ensure mass conservation, the bed elevation in the wet sub-cell i_1 may be locally modified by altering the bed slope ∇z_{bi} to be

$$\bar{\nabla} z_{bi} = \frac{\min(z_{bi_2}, \eta_{i_2}) - z_{bi}}{0.25\Delta x}. \quad (12)$$

where the bed elevations and water surface level on the right hand side are obtained from linear reconstruction using the original slopes. Then the bed elevation in wet sub-cell i_1 is calculated based on $\bar{\nabla} z_{bi}$ as

$$\bar{z}_{bi_1} = z_{bi} - \frac{\Delta x}{4} \bar{\nabla} z_{bi}. \quad (13)$$

For the dry sub-cell i_2 whose bed elevation is higher than the water surface level, it is not necessary to modify its bed elevation as it neither affects the C-property nor mass conservation, as shown in Figure 6(c).

3.2. Local modification of bed elevation during cell coarsening

During grid coarsening, local bed modification must also be applied to resolve the contradiction between the C-property and mass conservation. Figure 7 illustrates the case when the two sub-cells i_1 and i_2 are coarsened to create a new (parent) cell i . If a dry sub-cell has a bed elevation higher than the linearly extrapolated water surface level, e.g. cell i_2 in Figure 7(b), simple linear reconstruction is likely to cause incorrect mass, i.e. $z_{bi} = 0.5(z_{bi_1} + z_{bi_2})$ leading to $h_i \neq 0.5h_{i_1}$, as shown in Figure 7(b), although the uniform water surface level is preserved. Therefore it is necessary to modify the bed elevation in the coarsened cell i to conserve mass by the following equation

$$\bar{z}_{bi} = \frac{1}{2} [\min(z_{bi_2}, \eta_{i_2}) + \min(z_{bi_1}, \eta_{i_1})], \quad (14)$$

The relationship between $\bar{z}_{bi}$ and z_{bi} may be found in Figure 7(c).

3.3. 2D extension

When refining a cell for 2D problems, the cell is normally divided into four sub-cells and we may use the available fine topographic data to accurately reflect the bed elevations in refined cells, rather than extrapolate them linearly from the coarse cell center like in equation (10). If this is the case, there may be no unique linear 2D slope for bed elevation in a coarse cell and thus we can not directly employ the approach proposed for 1D problem to modify the bed elevations for 2D problems.

Figure 6. Local modification of bed elevation during cell refining: (a) original state of the flow in the parent cell; (b) linearly reconstructed flow information and bed elevation in the two sub-cells; (c) bed elevation after local bed modification.

To preserve the C-property and mass conservation for 2D problems through using the proposed approach, the bed elevation is modified in a 1D cross manner, e.g. the bed elevation in sub-cell i_{-1}

Figure 7. Local modification of bed elevation during cell coarsening: (a) original state of the flow in the two sub-cells; (b) linearly reconstructed flow information and bed elevation in the coarsened cell; (c) bed elevation after local bed modification.

Figure 8. A 2D parent cell and sub-cells.

is modified from the information in sub-cells i_1 and i_3 , see Figure 8. It is not the case when coarsening 2D cells and we can directly extend the equation 14 to

$$\bar{z}_{bi} = \frac{1}{4} [\min(z_{bi_1}, \eta_{i_1}) + \min(z_{bi_2}, \eta_{i_2}) + \min(z_{bi_3}, \eta_{i_3}) + \min(z_{bi_4}, \eta_{i_4})]. \quad (15)$$

4. PROOF OF THE C-PROPERTY AND THE MASS CONSERVATION

The well-balanced cell-centered finite volume scheme presented in [32] is adopted in this work to solve the SWE. The C-property is preserved as long as the water surface level does not change after grid adaption for quiescent flow. As shown in Figures 6 and 7, the water surface levels in wet cells remain to be the same after grid adaption and consequently the C-property is satisfied when linear reconstruction is used to obtain the values of the bed elevation and flow variables in the newly created cells. The key point here is to prove whether the approach proposed in last section is capable of ensuring mass conservation.

When refining a cell, the volume of water in wet sub-cell i_1 can be evaluated by

$$m_{i_1} = \frac{1}{2} h_{i_1} \Delta x = \frac{1}{2} (\eta_{i_1} - \bar{z}_{bi_1}) \Delta x. \quad (16)$$

From equations (12) and (13),

$$\begin{aligned} \bar{z}_{bi_1} &= z_{bi} - \frac{\Delta x}{4} \bar{\nabla} z_{bi} \\ &= z_{bi} - \frac{\Delta x}{4} \left[\frac{4(\eta_{i_2} - z_{bi})}{\Delta x} \right] \\ &= 2z_{bi} - \eta_{i_2}. \end{aligned} \quad (17)$$

Substitution of $\bar{z}_{bi_1}$ into equation (16) gives

$$m_{i-1} = \frac{1}{2} (\eta_{i-1} + \eta_{i-2} - 2z_{bi}) \Delta x. \quad (18)$$

As $\nabla \eta = 0.0$, η_{i-1} and η_{i-2} can be computed from equation (11) and subsequently $\eta_{i-1} = \eta_{i-2} = \eta_i$. Therefore, equation (16) can be rewritten as

$$m_{i-1} = \frac{1}{2} (2\eta_i - 2z_{bi}) \Delta x = h_i \Delta x. \quad (19)$$

The water volume in cell i is equal to $h_i \Delta x$ and that in the dry sub-cell $i-2$ is zero, i.e. $m_i = m_{i-1} + m_{i-2}$, indicating that the new approach preserves mass conservation as well as C-property during grid refining.

As shown in Figure 7, the mass before coarsening the two sub-cells is expressed as

$$m_{i-1} + m_{i-2} = h_{i-1} \frac{\Delta x}{2} = \frac{1}{2} (\eta_{i-1} - z_{bi-1}) \Delta x. \quad (20)$$

After modifying locally the bed elevation using equation (7), the new mass in cell i created following grid coarsening is

$$m_i = h_i \Delta x = (\eta_i - \bar{z}_{bi}) \Delta x. \quad (21)$$

From equation (14), we have $\bar{z}_{bi} = 0.5 (\eta_{i-1} + z_{bi-1})$ and equation (21) is rearranged to be

$$m_i = \frac{1}{2} (\eta_{i-1} - z_{bi-1}) \Delta x. \quad (22)$$

Therefore we have $m_i = m_{i-1} + m_{i-2}$, which proves that the new approach preserves mass conservation in grid coarsening.

5. NUMERICAL TEST

Through using the new approach in the quiescent flow simulation shown in Figure 2, the computed spurious waves on the adaptive grid completely disappear and no artificial motion is detected throughout the simulation, meanwhile, the total mass keeps constant during the simulation. To test the performance of the new approach which is designed to simultaneously preserve the C-property and mass conservation on adaptive grid in a dynamic case, the theoretical test developed in [38] is applied, as it describes a shallow water flow with wetting and drying over uneven bed. In this test case, the adaptive grid system proposed in [4] is under consideration but the adaption criterion is changed to use the water depth, i.e. the grid is refined if the water depth is smaller than a certain value h_c and it is coarsened when the water depth is higher than $2h_c$ ($h_c = 0.035\text{m}$ for this test case). The purpose of such a criterion is to undertake the adaption in the domain part involving wetting and drying which is likely to bring about the contradiction between the C-property and the mass conservation.

In this test case, the 2D frictionless parabolic bed topography is defined as

$$z_b(x, y) = -h_0 \left[1 - \frac{(x - x_0)^2 + (y - y_0)^2}{a^2} \right], \quad (23)$$

where $[x_0, y_0]$ represents the center of the parabolic bowl; h_0 denotes the water depth at the domain center; a is the distance from the center to the shoreline of zero elevation. The exact solution of this test case is given by

$$\begin{aligned} \eta(x, y, t) = & \frac{\sigma h_0}{a^2} [2(x - x_0) \cos(\omega t) \\ & + 2(y - y_0) \sin(\omega t) - \sigma], \end{aligned} \quad (24)$$

Figure 9. Computed water level (a) through using the new approach on the corresponding adaptive grid (b) at $t = 4.49s$.

$$u(t) = -\omega\sigma \sin(\omega t), \quad v(t) = \omega\sigma \cos(\omega t), \quad (25)$$

in which σ is a constant and $\omega = \sqrt{2gh_0}/a$ is the frequency of the rotation.

In this work, the parameters are set to be $h_0 = 0.1$ m, $a = 1.0$ m and $\sigma = 0.5$ m. A $4 \text{ m} \times 4 \text{ m}$ computational domain with the center of (2 m, 2 m) and four solid boundaries is chosen and the cell size varies between 0.08m and 0.02m according to the water depth as shown Figure 9. The SWE model presented in [32] is employed and the Courant number is 0.5.

Figure 10 plots the computed water level by using the new approach in the cross section of $y = 2.0$ m at $t = 10.17s$ and $t = 12.37s$. A fairly good agreement with the exact solution is detected, indicating the new approach performs well in simulating wetting and drying over uneven bed on adaptive grid. If the conventional approach which only preserves the C-property is employed to compute the shallow water flow, less accurate result is produced comparing to the new approach, because the mass is not preserved in the grid adaption, see Figure 12. If the conventional approach preserving just the mass conservation in grid adaption, the result is even worsen as demonstrated in Figure 13, as the violation of the C-property is liable to cause spurious momentum which in turn leads to unrealistic flow pattern. In a word, the new approach is capable of handling such numerical problems and thus can improve the accuracy of shallow water flow models on adaptive grids.

6. CONCLUSIONS

This work intends to investigate the cause of contradiction between the C-property and mass conservation when numerically solving the SWEs on adaptive grids and to propose an effective approach to resolve the issue. It is found that the problem lies in the way by which the flow and bed information is reconstructed in the newly created cells. If all of refined sub-cells are wet, both the C-property and the mass conservation can be satisfied by using linear reconstruction. If the reconstructed water surface level is lower than the bed elevation in a sub-cell but higher than that of another sub-cell that is wet, the grid refining or coarsening will violate either the C-property or the mass conservation. Based on the detailed analysis of the cause of the problem, an effective approach is proposed to resolve this contradiction by means of local bed modification. The capability of the new approach in simultaneously preserving the C-property and mass conservation on adaptive grids

Figure 10. Computed water level through using the new approach in the cross section of $y = 2.0$ m at (a) $t = 10.17$ s, (b) $t = 12.37$ s.

is proven mathematically. Besides, a dynamic test case demonstrates that the new approach performs better in terms of accuracy than the conventional ones which preserve either the C-property or the mass conservation.

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(a)

(b)

Figure 11. Computed L_1 errors through using the new approach and the conventional one only preserving the C-property: (a) $L_1(h)$, (b) $L_1(q)$ and $q = \sqrt{q_x^2 + q_y^2}$.

Figure 12. Computed total mass in terms of water volume through using the new approach and the conventional one only preserving the C-property.

Figure 13. Computed water level through using the conventional one only preserving the mass conservation.

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