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Analytical structural reliability analysis of a suspended cable

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Abstract: Suspended cables, including transmission lines, suspension bridge cables, and edge cables of roof structures, feature in high profile and large span projects for architectural reasons, for their functional efficiency, and for ease of construction, particularly over large spans. A suspended cable predominantly reacts external loads by means of axial tension, and is, therefore, able to make full use of the material strength. Because of the slenderness of the suspended cable, the structural response is nonlinear, even if the material property is within the elastic range. From a mechanics perspective, therefore, these types of structure exhibit high levels of geometric non-linearity. For this reason, the nonlinear relationships between tension force, normal displacement, and the external loads need to be considered. In aiming to determine the structural safety of a suspended cable, and to understand which uncertainty features have the greatest influence, these relationships are written within a probabilistic framework.

This article briefly sets the analysis of suspended cables within the context of geometrically nonlinear elastic structures and corresponding finite element analysis methodologies. Analytical solutions to the tension and normal displacement of a suspended cable subjected to external loading are presented. Nonlinear performance functions, in the form of either the cable tension or normal displacement are stated. Analytical expressions for the required gradients of the performance function of a suspended cable with respect to the basic variables under static loads are developed. The structural reliability of a suspended cable is studied using a first-order reliability method (FOSM) and verified by comparison with Monte Carlo simulation (MCS) and Monte Carlo simulation based optimization principles (MCOP) for a number of examples. Load cases including, wind, snow, and temperature variation are included.

Key words: suspended cable, analytical solution, reliability analysis, FOSM, Monte Carlo simulation.

1. Introduction

Suspended cables, including transmission lines, suspension bridges and roofs, are widely used as long-span engineering solutions. Unlike conventional structures, a suspended cable is a flexible structural system, the shape of which cannot be prescribed, but must take a ‘form-found’ configuration determined by equilibrium and geometric constraints on the basis of a predefined initial stress state^[1-3]. Because of the slenderness and flexibility of a suspended cable, the structural responses are nonlinear even if the material property is within the elastic range. For this reason, geometric nonlinearity should be considered in the analysis of a suspended cable. The deterministic geometrically nonlinear analysis of a suspended cable is explained in references^[4-6] and others. In reality, the variables affecting the safety of structure are random because these parameters contain uncertainties introduced in the form of epistemic uncertainties in the design process, and aleatoric uncertainties in the form of material characteristics, construction tolerances, and service conditions

including loading. An accurate prediction of the performance of an analytically described suspended cable in the presence of uncertainty is presented in this paper.

Previously published work on the reliability analysis of cable structures has mainly focused on evaluating the reliability of geometrically nonlinear structures through the use of the finite element method. For instance, P.-L. Liu, A.D. Kiureghian^[7] introduced the finite element-based reliability method, formulated using FOSM and SORM principles, for geometrically nonlinear elastic structures, and created a general purpose reliability analysis code to evaluate structural reliabilities. Xinpei Zhang [8], proposed an algorithm to calculate the safety index with limit states based on element strength and in service performance of a cable structure using a nodal displacement algorithm and the checking point (JC) method of reliability. K.Imai, D.M.Frangopol^[9-10] and D. M. Frangopol, K. Imai^[11] considered the nonlinear relationships between strains and displacements and investigated the system reliability evaluation of suspension bridges by a probabilistic finite element analysis approach. The above methods, combining probabilistic theory with the traditional finite element analysis, are one of the more practically effective methods to analyse the reliability of complex structures in the broadest sense. However, little work has been done on the reliability analysis of a suspended cable based on classical analytical solutions at present, which, once available, negates the need to consider further numerical modelling, and offers both computational efficiency and clear definitions of assumptions that may contribute to otherwise unknown or unclear epistemic uncertainty.

The aim of this paper is to formulate and quantify the reliability of a suspended cable on the basis of a classical analytical structural mechanics solution. In the following section, the theory of suspended cable is described and the analytical solution for the tension of a suspended cable is provided. In section 3, the limit state function of suspended cable is established and the gradients of the limit state function with respect to the basic random variables are deduced. In section 4, the computational accuracy of FOSM used in suspended cable is demonstrated by comparison with Monte Carlo simulation (MCS) and Monte Carlo simulation based optimization principle (MCOP) by using an example of transmission line (suspended cable). Conclusions from the present study are drawn in the last section.

2. Analytic solution of a suspended cable

2.1 Selection of cable equation

Following the theoretical basis of Irvine^[1], we consider here a profile adopted by a uniform cable suspended between two rigid supports that are at the same level and subjected to a uniform distributed load mg along cable length, as shown in fig.1. It is assumed that the cable: (1) is perfectly flexible and devoid of flexural rigidity; (2) can sustain only tensile forces; (2) is composed of a homogeneous material which is linearly elastic.

Fig.1 a cable under a distributed load along the cable length and equilibrium of an element of cable

Considering the sketches in fig.1, the vertical and horizontal equilibrium of the isolated element of the cable located at (x, z) require that,

$$dV + mgds = 0 \quad (1)$$

$$dH = 0, \quad (2)$$

where, H is the horizontal component of cable tension. H is constant everywhere if no longitudinal loads act on the cable, or Equation (2) may be directly integrated; V is the vertical component of cable tension and can be written as

$$V = H \tan \theta = H \frac{dz}{dx}. \quad (3)$$

Differentiating equation (3) with respect to x and substituting it into equation (1), and noting that the following geometric constraint that must be satisfied,

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = 1, \quad (4)$$

the governing differential equation of the cable is obtained as,

$$H \frac{d^2z}{dx^2} + mg \sqrt{1 + \left(\frac{dz}{dx}\right)^2} = 0. \quad (5)$$

Solving the differential equation (5), the profile function of the cable that satisfies the boundary conditions in Fig.1 can be obtained as,

$$z = \frac{H}{q} \left[\cosh \frac{mgx}{2H} - \cosh \left(\frac{mgx}{H} - \frac{mgx}{2H} \right) \right] \quad (6)$$

This is a catenary equation of a suspended cable, fully determined by the coordinate x at any point, (for example, the sag f at mid-span). If the sag at mid-span of cable is f , namely, $x = l/2$, $z = f$, the horizontal component of cable tension H can be calculated from equation (6), as in,

$$f = \frac{H}{mg} \left(\cosh \frac{mgx}{2H} - 1 \right). \quad (7)$$

Because the catenary equation of the suspended cable and the equations derived from it all involve hyperbolic functions and, therefore, transcendental equations as functions of the problem defining variables, the solution of the system of differential equations is overly complicated. It is, however, possible to derive some relatively simple solutions for specific loading and boundary conditions, such as for a profile under a distributed load mg along the cable span, for example. In the context of a transmission line idealisation, we may consider the profile of a uniform cable spanning between two supports at the same level, generated by a uniformly distributed vertical load

mg , as shown in fig.2. The horizontal equilibrium of the isolated element of cable is the same as defined in equation (2). The vertical equilibrium of an element requires that,

$$dV + mgds = 0 \quad (8)$$

Fig.2 a cable under a distributed load and equilibrium of an element of the cable.

Differentiating equation (3) with respect to x and substituting it into equation (8), then,

$$H \frac{d^2z}{dx^2} = -mg \quad (9)$$

If the profile is relatively flat, so that the ratio of sag span is 1:8 or less, the differential equation governing the vertical equilibrium of an element is accurately specified by equation (9) ^[1]. Integrating equation (9) twice and applying the boundary conditions identified in Fig.2, the solution for z as a function of the system characteristic values and the longitudinal co-ordinate, x , is,

$$z = \frac{mg}{2H} x(l-x). \quad (10)$$

Given that the solution to the sag at mid-span of the cable is f , that is, at $x = l/2$, $z = f$, and substituting this condition into equation (10), the horizontal component of cable tension H can be obtained as,

$$H = \frac{mgl^2}{8f}. \quad (11)$$

Substituting equation (11) into equation (10), we can have the profile function of cable as,

$$z = \frac{4fx(l-x)}{l^2}. \quad (12)$$

Equation 12 describes a parabola that is fully determined by the sag f at the mid-span of cable. Comparing the deformed geometry of the catenary computed using iterative method using (6) and (7) with the parabola in equation (12), when the sag at the mid-span of two profile functions of the cable are equal, the maximum differences of the deformed geometry are summarised in Table 1 and illustrated in Fig.3 (for a sag to span ratio of approximately 0.15). The differences in the predicted

horizontal component of the cable force for the catenary and parabola, $\Delta H = H_c - H_p$ (H_c and H_p are the deformed geometries of the catenary and parabola solutions, respectively) as a percentage of the applied normal load, are listed in Table 1.

Fig. 3 Illustrative (exaggerated) difference ΔH of the deformed geometries of the catenary (equations (6) & (7)) and the parabola (equation (12))

Table 1 the maximum difference of the deformation and H of catenary and parabolic

f/l	0.05	0.1	0.15	0.2	0.25	0.30
δ/f	0.08%	0.32%	0.70%	1.19%	1.75%	2.37%
$\Delta H/mgl$	0.83%	1.63%	2.39%	3.09%	3.72%	4.28%

It is clear that the differences in the predicted deformed geometries and the horizontal components of cable tension, are very small, and as the sag tends to zero, the two solutions tend to converge. With the calculation of the catenary equation overly complex and involving an iterative solution approach, we may note that when mgl/H is small such that the cable length is only fractionally longer than the span, the substitution of a power series approximation for a hyperbolic function yields the properties associated with a parabola, which is then the limiting form of the catenary as the profile flattens. In a practical engineering context, given that the sag to span ratios of suspended cables are frequently relatively small, and that if the actual load distribution can be approximated as a uniformly distributed, then the errors arising from the parabolic approximation are acceptable^[1,2], and therefore forms the theoretical basis for the remainder of the paper.

2.2 Parabolic profile and response to a uniformly distributed load

It is an essential requirement of cable structural theory that the effects of geometric nonlinearity should be included in the analysis of suspended cable structures. In comparison with the initial sag of a suspended cable, the vertical deformation generated under a load increment supplementary to the self-weight of the cable may be substantial, especially in cases where the initial sag is small. The equilibrium equation of a suspended cable is not set by its initial state, but rather, it is developed by considering the change of the cable profile produced with the change of external loads and the preceding deformed state.

We consider a simply supported cable, with the two supports at the same level, subjected to a uniformly distributed load of intensity $mg + q$ per unit span on l . The profile of the initial state of the cable under the action of its self-weight mg is defined by equation (12) and the horizontal component of cable tension H is calculated using equation (11). w denotes the additional vertical deflection of the cable and h is the increment in the horizontal component of cable tension, arising from the action of the supplementary (to the self-weight mg) uniformly distributed load q . The

deformed profile of the cable under the combined loading $mg + q$ is obtained by augmenting equation (10), i.e.,

$$(H + h)(z + w) = \frac{(mg + q)x}{2}(l - x) \quad (13)$$

Expanding the left-hand side of equation (10) and making use of the configuration of the cable under self-weight (e.g. equation (10)), we obtain,

$$w = \frac{1}{2(1 + h)}(1 - h/q)x(1 - x) \quad (14)$$

where $w = w/(ql^2/H)$, $h = h/H$, $x = x/l$ and $q = q/mg$.

To complete the solution, h must now be calculated. The equilibrium equation for the cable (equation (14)) simply provides the relationship between the external load q , the vertical deflection w , and the horizontal component h of cable tension under the current configuration. The unknown increments in horizontal component of cable tension, and additional sag or deflection, h and w , are not independent. The current configuration represents an intermediate state in defining the final equilibrated form that cannot be defined by the single equilibrium equation (14). Given the applied load parameter q *a priori*, equation (14) is insufficient to mathematically solve for the unknown parameters w and h . The deformation of the suspended cable needs to be considered during the transition process from the initial state to the final state so as to both establish the equation of equilibrium and ensure compatibility and continuity along the cable length.

As constitutive modelling for cables is not within the scope of this paper, Hooke's law is used here to relate the changes in cable tension to changes in the cable geometry when the cable is displaced from its original self-weight equilibrium profile. Therefore, a change in length of a component length of the cable may be related to a corresponding change in axial tension according to,

$$\frac{t}{EA} = \frac{ds' - ds}{ds} \quad (15)$$

where t is the increment in tension exerted on the element, $t = hds/dx$; E is Young's modulus; A is the cross-section area of the cable (assumed uniform); ds is the original length of the element; and ds' is its final length, $ds' = \sqrt{(dx + du)^2 + (dz + dw)^2}$; and u is the longitudinal components of the displacement.

ds' may be expanded as a Taylor series, remaining sufficient to the second order of small quantities for a suspended cable with a shallow sag (i.e. sag to span ratio of 1:8 or less). Meanwhile, considering the increment of horizontal tension, h , to be constant given the absence of longitudinal loads are acting on the cable, the following equation may be derived if u and w are both considered to vanish at the supports;

$$\frac{hL_e}{EA} = \int_0^l \frac{dzdw}{dx^2} dx + \frac{1}{2} \int_0^l \left(\frac{dw}{dx} \right)^2 dx, \quad (16)$$

where L_e is a calculating quantity only a little greater than the span itself, $L_e \cong l[1 + 8(f/l)^2]$.

Substituting the definition for w from equation (14) into (16), and using the convenient dimensionless forms of the variables to complete the integration, we obtain a cubic function of h of the form,

$$h^3 + \left(2 + \frac{\lambda^2}{24}\right)h^2 + \left(1 + \frac{\lambda^2}{12}\right)h - \frac{\lambda^2}{12}q\left(1 + \frac{1}{2}q\right) = 0 \quad (17)$$

$$\lambda^2 = \frac{(mgl/H)^2 l}{HL_e/EA} \quad (18)$$

The independent parameter λ^2 , accounting for geometric and elastic effects, is of fundamental importance in the static response of suspended cables. Equation (17) characterises the inherent geometric non-linearity of a suspended cable.

With the definitions,

$$\begin{cases} a = 2 + \lambda^2/24 \\ b = 1 + \lambda^2/12 \\ c = -\lambda^2 q(1 + q/2)/12 \end{cases} \quad (19)$$

equation (17) may be reduced to the form,

$$h^3 + ah^2 + bh - c = 0 \quad (21)$$

where the coefficients a , b and c are real constants. The positive real root of equation (17) is obtained as;

defining,

$$\begin{cases} B = b - a^2/3 \\ D = 2a^3/27 - ab/3 + c \end{cases} \quad (22)$$

and $\Delta = (D/2)^2 + (B/3)^3 \geq 0$, then,

$$h = -a/3 + \left(-D/2 + \sqrt{(D/2)^2 + (B/3)^3}\right)^{1/3} + \left(-D/2 - \sqrt{(D/2)^2 + (B/3)^3}\right)^{1/3} \quad (23)$$

or $\Delta = (D/2)^2 + (B/3)^3 < 0$,

$$h = -a/3 + 2(-B/3)^{1/2}\cos(\theta/3), \text{ and } \theta = \arccos\left[\frac{-D/2}{(-B/3)^{3/2}}\right], \quad (24)$$

such that the method applicable to the calculation h depends on the parameter Δ , i.e. the relative magnitudes of B and D , which are determined by the parameter λ^2 and the loads q applied on cable. This completes the fundamental analytical approach in defining the solution (e.g. cable tension and displacement) to the problem of a suspended cable subjected to self-weight (to define the original configuration) and a uniformly distributed load.

Variations in the ambient temperature can change the length of a suspended cable, with correspondingly non-linear impacts on the cable sag and tension. Equation (17) may then be extended to;

$$h^3 + \left(2 + \frac{\lambda^2}{24} + \frac{EA}{HL_e} \alpha \Delta t L_t\right) h^2 + \left(1 + \frac{\lambda^2}{12} + \frac{2EA}{HL_e} \alpha \Delta t L_t\right) h - \frac{\lambda^2}{12} q \left(1 + \frac{1}{2} q\right) + \frac{EA}{HL_e} \alpha \Delta t L_t = 0 \quad (25)$$

where α is the coefficient of expansion, Δt describes a uniform temperature change, and L_t is defined as $L_t = l[1 + 16(f/l)^2/3]$.

Writing,

$$\begin{cases} a = 2 + \lambda^2/24 + EA\alpha\Delta t L_t/HL_e \\ b = 1 + \lambda^2/12 + 2EA\alpha\Delta t L_t/HL_e \\ c = -\lambda^2 q(1 + q/2)/12 + EA\alpha\Delta t L_t/HL_e \end{cases}, \quad (26)$$

the required solution for h (equation (25)) may be calculated from equation (23) or (24) according to the magnitude of Δ .

In practice, a cable is subjected to the action of vertical loads such as self-weight mg , ice accumulation, or other, similarly vertical, live loads, q_i ; similarly, a suspended cable may also be subjected to the action of horizontal live loads, q_w , arising from wind pressure, for example, and acting normal to the projected surface of the cable. Other secondary structural loads from other attached cables or structural membranes, for example, where the actions may be applied to the cable at arbitrary directions, may be resolved into vertical, q_i , and horizontal, q_w , components. The total imposed load on the cable may be defined by the vector sum of vertical and horizontal loads. The total load acting on the cable per unit length is,

$$q = \sqrt{(mg + q_i)^2 + q_w^2}. \quad (27)$$

Under the combined forces q_i and q_w , the cable rotates to an equilibrated plane at an angle β_0 , known as the windage yaw, to the vertical, where,

$$\beta_0 = \text{actan}\left(\frac{q_w}{mg + q_i}\right) \quad (28)$$

The components of self-weight mg , ice load q_i and wind load q_w in the local co-ordinate direction defined by the windage yaw is,

$$\begin{cases} mg' = mg \cos \beta_0 \\ q'_i = q_i \cos \beta_0 \\ q'_w = q_w \sin \beta_0 \end{cases} \quad (29)$$

and $q = mg' + q' = mg' + q'_i + q'_w$, and the relationships between q and the basic loads mg , q_i and q_w is showed as in Fig.4 (b).

Fig.4 Total load acting on a suspended cable under vertical and horizontal loads and the relationships between and the basic loads and mg .

3. Reliability calculation of a suspended cable

3.1 Structural reliability analysis approach

The first-order (FOSM) and second-order (SORM) reliability methods perform analytical probability integration to provide nominal safety indices or probabilities of failure ^[10, 12-14]. FOSM is considered to be one of the most reliable computational methods for expanding the limit state function as a first-order Taylor series expansion about the checking point, approximating the limit state function by a tangent (hyper-) plane. Similarly, SORM expands the limit state function as a second-order Taylor series expansion, approximating the limit state function by a paraboloid. The reliability index in FOSM or SORM, representing the shortest distance from the origin in standardized normal space to the hyper-plane or paraboloid, can be calculated by solving an optimization problem ^[13]. If the limit state function is nonlinear near the checking point, SORM may provide more accurate results, but it is an approach that is inherently more complex. However, if the limit state function is nearly linear in the vicinity of the checking point, the reliability indices provided by both methods may be closely equivalent ^[10]. From equation (23), (24) and (30), (31), it is clear that in principle the limit state function the analytically derived description of a suspended cable is non-linear.

Liu and Der. Kiureghian ^[7,11] investigated the displacement reliability of a two-dimensional geometrically nonlinear elastic structure in the form of a stochastic plate under random static loads, and concluded that, in spite of strong nonlinearity of the structural response, the results provided by FOSM and SORM were similar, with FOSM providing a good measure of the structural reliability. Initially based on these results, FOSM has been selected to estimate the reliability of a suspended cable described in the preceding section. Concurrently, in order to assess the validity of adopting FOSM, Monte Carlo simulation is also used to estimate the limit state probabilities.

3.2 Defining the limit state function

The tension, T , at any point, x , along the length of a cable with supports that are at the same level, is,

$$T = [(H + h)^2 + (mg + q)(l/2 - x)]^{1/2}, \quad (30)$$

and at the maximum sag point, $x = l/2$, the cable tension is $T = H + h$.

The limit state function, Z , at any point along the cable may then be written as,

$$Z = R - S = f_y A - T \quad (31)$$

where, R is the resistance of cable in tension; f_y is yield strength of material used for cable; A is area of crossed section of cable; S is load effect under different load; H , calculated by equation (11), and h , calculated by equation (23) or (24), respectively, are the horizontal component of cable tension under self-weight mg and external load q .

3.3 Analytical gradients – temperature independent case

The basic random variables affecting the reliability of a suspended cable, without considering the effects of a change of temperature, include material properties of the cable (yield (or working) strength of the material f_y , Young's modulus E), loads or actions (self-weight mg , horizontal live load q_w , vertical live load q_i), and geometric parameters (cross-sectional area A , cable sag f , and span length l). The dimensionless horizontal component of tension force h of the cable in the limit state function Z is calculated using equation (23) if $\Delta \geq 0$, and (24) if $\Delta < 0$.

The relationships between the limit state function and the basic random variables, without considering the temperature change, is shown in Fig. 5 and the relationship between q , in the windage yaw plane arising from combined horizontal and vertical loads, and mg , q_i and q_w is shown in Fig.4(b).

Fig. 5 Relationships between the limit state function and basic variables excluding temperature effects.

The gradients of the limit state function Z of a geometrically nonlinear elastic cable with respect to the basic random variables are derived from the complex relationships shown schematically in Fig.5 and Fig.4(b), using the chain rule of differentiation for the cases that $\Delta \geq 0$ and $\Delta < 0$.

If $\Delta \geq 0$, the parameter h in the limit state function Z is expressed by equation (23) and the logical relationships between Z and basic variables are shown in Fig.5 (a). The gradients are computed by taking the derivative of Z with respect to the random variables as;

$$\begin{aligned} \frac{\partial Z}{\partial (mg)} = & \left[-\eta_1 \frac{EA(8f/l)^3}{(mg \cos \beta_0)^2 l_e} - \frac{l^2}{8f} (1 + h) \right] (\sin^2 \beta_0 \cos \beta_0 + \cos \beta_0) \\ & - \frac{H \lambda^2 \xi_1}{72 mg} (1 + q) \left(\tan^2 \beta_0 + \frac{q_w}{mg} \tan \beta_0 + \frac{q_i}{mg} \right) \end{aligned} \quad (32a)$$

$$\frac{\partial Z}{\partial l} = -\eta_1 \frac{4EA(8f/l)^3[1 + 4(f/l)^2]}{mgl_e^2} - \frac{mgl}{4f}(1 + h) \quad (32b)$$

$$\frac{\partial Z}{\partial f} = \eta_1 \frac{8EA(8f/l)^2[3 + 8(f/l)^2]}{mgl_e^2} + \frac{mgl^2}{8f^2}(1 + h) \quad (32c)$$

$$\frac{\partial Z}{\partial q_w} = \frac{\lambda^2 H \xi_1}{36}(1 + q) \frac{\tan \beta_0}{mg} \quad (32d)$$

$$\frac{\partial Z}{\partial q_i} = \frac{\lambda^2 H \xi_1}{72}(1 + q) \frac{(1 - \tan^2 \beta_0)}{mg} \quad (32e)$$

$$\frac{\partial Z}{\partial E} = \eta_1 \frac{A(8f/l)^3}{mgl_e} \quad (32f)$$

$$\frac{\partial Z}{\partial A} = f_y + \eta_1 \frac{E(8f/l)^3}{mgl_e} \quad (32g)$$

$$\frac{\partial Z}{\partial f_y} = A \quad (32h)$$

where,

$$\eta_1 = \frac{H}{72} - \frac{H}{72} \frac{1}{9} \frac{B^2}{\sqrt{\Delta}} \left[\left(-\frac{D}{2} + \sqrt{\Delta} \right)^{-\frac{2}{3}} - \left(-\frac{D}{2} - \sqrt{\Delta} \right)^{-\frac{2}{3}} \right] \left(1 - \frac{a}{3} \right) - \frac{H \xi_1}{72} \left[\frac{a^2}{9} - \frac{b}{6} - \frac{a}{3} - q \left(1 + \frac{q}{2} \right) \right] \quad (32j)$$

$$\xi_1 = \left(-\frac{D}{2} + \sqrt{\Delta} \right)^{-\frac{2}{3}} \left(-1 + \frac{D}{2\sqrt{\Delta}} \right) - \left(-\frac{D}{2} - \sqrt{\Delta} \right)^{-\frac{2}{3}} \left(1 + \frac{D}{2\sqrt{\Delta}} \right) \quad (32k)$$

If $\Delta < 0$, the parameter h in the limit state function Z is expressed by equation (24) and the logical relationships between the limit state and the basic variables are shown in Fig.5 (b). Considering the limit state functions in the cases of $\Delta \geq 0$ and $\Delta < 0$, the only difference is in the expression for h (e.g., c.f. equation (23) with (24)). The difference in the gradients between the two cases is in the partial derivatives of h with respect to a , B and D , which can be calculated from equation (24). The remaining gradients are unchanged from the case $\Delta \geq 0$. Following the same analytical computational procedure, the gradients in the case of $\Delta < 0$, with the exception of the gradients of the limit state function with respect to the loads mg , q_w and q_i , are quite different from equation (32a), (32d) and (32e). In the case $\Delta < 0$, the gradient $\partial Z / \partial f_y$ is identical to equation (32h) where $\Delta \geq 0$, whilst the remaining gradients of the limit state function with respect to the parameters l , f , E and A (illustrated in equation (33a) – (33c)), require replacing η_1 in the corresponding equations with η_2 as defined in equation (33d).

$$\begin{aligned} \frac{\partial Z}{\partial(mg)} = & \left[-\eta_2 \frac{EA(8f/l)^3}{(mg)^2 l_e} - \frac{l^2}{8f} (1 + \mathbf{h}) \right] (\sin^2 \beta_0 \cos \beta_0 + \cos \beta_0) \\ & + \frac{\lambda^2 H \xi_2}{18 mg} (1 + q) \left(\tan^2 \beta_0 + \frac{q_w}{mg} \tan \beta_0 + \frac{q_i}{mg} \right) \end{aligned} \quad (33a)$$

$$\frac{\partial Z}{\partial q_w} = \frac{\lambda^2 H \xi_2}{9} (1 + q) \frac{\tan \beta_0}{mg} \quad (33b)$$

$$\frac{\partial Z}{\partial q_i} = \frac{\lambda^2 H \xi_2}{18} (1 + q) \frac{(1 - \tan^2 \beta_0)}{mg} \quad (33c)$$

$$\eta_2 = \frac{H}{72} + \frac{H}{36} \left[\left(-\frac{B}{3} \right)^{-1/2} \cos \left(\frac{\theta}{3} \right) - \frac{BD \xi_2}{3} \right] \left(1 - \frac{a}{3} \right) + \frac{H \xi_2}{18} \left[\frac{a^2}{9} - \frac{b}{6} - \frac{a}{3} - q \left(1 + \frac{q}{2} \right) \right] \quad (33d)$$

$$\xi_2 = (-B/3)^{1/2} \sin(\theta/3) \frac{1}{2\sqrt{(-B/3)^3 - (D/2)^2}} \quad (33e)$$

3.4 Analytical gradients – temperature dependent case

If the effects of a uniform temperature rise Δt need to be incorporated, a coefficient of thermal expansion α_t for the cable material and effects of a temperature change Δt are introduced in addition to the eight basic variables included in the case of without considering temperature change. The expressions of the limit state function and all parameters among the calculation proceeding are the same as that without considering temperature, with the exception of the expressions of a , b , c , in which a new item is added due to a temperature rise (see equation (26)). To obtain the partial derivatives it is convenient to introduce an intermediate variable, h_t , taking into account the effect of temperature in (26), is defined as

$$h_t = EA\alpha_t \Delta t L_t / H L_e \quad (34)$$

The relationships between the limit state function and the basic random variables, considering temperature effect, are shown in Fig.6, and the relationships between q in the windage yaw plane and mg , q_i and q_w are the same as without considering the temperature change, as shown in Fig.4(b).

In comparison with the temperature independent case, it is required to add the gradients of the limit state function with respect to α_t and Δt , along with the partial derivatives of parameters a , b , c with respect to the basic random variables where there is dependency on temperature variation.

Fig. 6 Relationships between the limit state function and basic variables including temperature effects.

The gradients considering the effect of temperature are obtained as;

For $\Delta \geq 0$

$$\frac{\partial Z}{\partial \alpha_t} = \eta_{t1} \frac{8fEA\Delta t l_t}{mgl^2 l_e} \quad (35a)$$

$$\frac{\partial Z}{\partial \Delta t} = \eta_{t1} \frac{8fEA\alpha_t l_t}{mgl^2 l_e} \quad (35b)$$

$$\begin{aligned} \frac{\partial Z}{\partial (mg)} = & \left[-\eta_1 \frac{EA(8f/l)^3}{(mg \cos \beta_0)^2 l_e} - \frac{l^2}{8f} (1+h) - \eta_{t1} \frac{8fEA\alpha_t \Delta t l_t}{(mg \cos \beta_0)^2 l^2 l_e} \right] (\sin^2 \beta_0 \cos \beta_0 + \cos \beta_0) \\ & - \frac{H\lambda^2 \xi_1}{72mg} (1+q) \left(\tan^2 \beta_0 + \frac{q_w}{mg} \tan \beta_0 + \frac{q_i}{mg} \right) \end{aligned} \quad (35c)$$

$$\frac{\partial Z}{\partial l} = -\eta_1 \frac{4EA(8f/l)^3 [1 + 4(f/l)^2]}{mgl_e^2} - \frac{mgl}{4f} (1+h) - \eta_{t1} EA\alpha_t \Delta t \frac{8f(4f/l)^4/3 + (8f/l)^2/3 + 2}{mgl_e^2} \quad (35d)$$

$$\frac{\partial Z}{\partial f} = \eta_1 \frac{8EA(8f/l)^2 [3 + 8(f/l)^2]}{mgl_e^2} + \frac{mgl^2}{8f^2} (1+h) + \eta_{t1} EA\alpha_t \Delta t \frac{4(4f/l)^4/3 + (8f/l)^2 + 8}{mgl_e^2} \quad (35e)$$

$$\frac{\partial Z}{\partial E} = \eta_1 \frac{A(8f/l)^3}{mgl_e} + \eta_{t1} \frac{8fA\alpha_t \Delta t l_t}{mgl^2 l_e} \quad (35f)$$

$$\frac{\partial Z}{\partial A} = f_y + \eta_1 \frac{E(8f/l)^3}{mgl_e} + \eta_{t1} \frac{8fE\alpha_t \Delta t l_t}{mgl^2 l_e} \quad (35g)$$

$$\eta_{t1} = \frac{H1}{3} - \frac{H}{27\sqrt{\Delta}} \left[\left(-\frac{D}{2} + \sqrt{\Delta} \right)^{-\frac{2}{3}} - \left(-\frac{D}{2} - \sqrt{\Delta} \right)^{-\frac{2}{3}} \right] \left(1 - \frac{a}{3} \right) + \frac{\xi_1}{6} \left(\frac{a^2}{9} - \frac{b}{6} - \frac{a}{3} + \frac{1}{2} \right) \quad (35h)$$

Comparing the corresponding case in Fig.5 and in Fig.6, the relationships between the limit state function Z and the live loads q_w and q_i , are independent of whether the effect of temperature is considered or not. Hence, the gradients of the limit state function Z with respect to the live loads q_w and q_i are same as equation (32d) and (32e), in which the effect of temperature is not considered and $\Delta \geq 0$, respectively. The gradient of the limit state function Z with respect to f_y is the same as equation (32h) in the case of $\Delta \geq 0$ without considering the effect of temperature.

If $\Delta < 0$ and considering the effect of temperature, the dimensionless horizontal component of the cable tension force h in the limit state function Z is expressed by equation (24) and the relationships between the limit state and basic variables are shown in fig.6 (b). The gradients of the limit state function with respect to the loads mg remain unchanged with the inclusion of temperature effects. For $\Delta < 0$, equation (35c) is replaced by equation (36a). The gradient $\partial Z / \partial f_y$ remains as equation (32h) in the case of $\Delta \geq 0$ without considering the effect of temperature. The outstanding gradients of the limit state function with respect to the parameters α_t , Δt , l , f , E and A are similar with the corresponding equations in the case of $\Delta \geq 0$, requiring replacement of η_1 and η_{t1} with η_2 expressed in (33d) and η_{t2} in (33d), respectively. The gradients of the limit state function Z with respect to the live loads q_w and q_i are same as equation (33b) and (33c), in which the effect of temperature is not considered and $\Delta < 0$, respectively.

$$\frac{\partial Z}{\partial (mg)} = \left[-\eta_2 \frac{EA(8f/l)^3}{(mg \cos \beta_0)^2 l_e} - \frac{l^2}{8f} (1+h) - \eta_{t2} \frac{8fEA\alpha_t \Delta t l_t}{(mg \cos \beta_0)^2 l^2 l_e} \right] (\sin^2 \beta_0 \cos \beta_0 + \cos \beta_0) + \frac{\lambda^2 H \xi_2}{18mg} (1+q) \left(\tan^2 \beta_0 + \frac{q_w}{mg} \tan \beta_0 + \frac{q_i}{mg} \right) \quad (36a)$$

$$\eta_{t2} = \frac{H}{3} + \frac{2H}{3} \left[(-B/3)^{-\frac{1}{2}} \cos(\theta/3) - \xi_2 BD/3 \right] \left(1 - \frac{a}{3} \right) + \frac{4H\xi_2}{3} \left(\frac{a^2}{9} - \frac{b}{6} - \frac{a}{3} + \frac{1}{2} \right) \quad (36b)$$

Comparing the corresponding gradients of the limit state function with respect to the basic random variables in section 3.3 and section 3.4, the effect of including temperature changes are that new items, denoted η_{t1} or η_{t2} , are generated in all gradients except for $\partial Z / \partial f_y$, along with two new gradients that are concerned with the temperature change and the coefficients of thermal expansion.

4. Cable reliability analysis: examples and discussion

Eight basic random variables are defined in the absence of temperature effects, which are increased to ten when considering changes due to temperature, all of which affect the reliability of the cable to varying degrees. More generally, there may be many basic random variables X_i ($i = 1, 2, \dots, n$) describing the structural reliability problem at the limit state function $G(\mathbf{X}) = 0$. The fundamental FOSM algorithm described by Melchers is adopted in this work ^[25].

A conductor or wire, widely used in overhead transmission lines in electrical engineering, is typical a suspended cable. An overhead transmission line is selected as an example to demonstrate the implementation of the preceding formulation. The structural safety and sensitivities of suspension cables are calculated.

An overhead transmission line may be subjected to external uncertainties including environmental conditions (wind and ice), secondary environmental factors, such as temperature

variations, for example, and intrinsic manufacturing uncertainties including axial stiffness of the cable, and self-weight. In design practice, these uncertainties are normally accounted for by the use of a factor (frequently described as a factor of safety) to give a permissible stress value (the ultimate tensile strength divided by a minimum factor safety) or the maximum permissible tension when scaled by a measure of the cross-sectional area. Any lack of confidence in the results of the analysis, in the form of epistemic uncertainty, may also be reflected in this single safety factor.

Current practice in the design of the transmission line conductors adopts the catenary equation (6) ^[18] or the parabolic equation (12) ^[19] and applies all loads to the conductor as a single load case to calculate the sag f , while the tension at the lowest point (i.e. the horizontal tension) is set in advance to be equal to the maximum permissible tension. In this case, however, it is not convenient to establish the limit state function and compute the reliability of the conductor in tension. Hence, the limit state function (31), considering the geometrically non-linear behaviour of suspension cable, is used to compute the reliability of the conductor in tension, and the sag f is assumed to be known priori in terms of engineering experience. The minimum factor of safety in respect of conductor tension is 2.5 at the lowest point on line and 2.25 at the suspension point ^[18].

Example 1: A 330kV overhead transmission line is suspended between two equal height supports. The transmission line is a round wire concentric layup overhead electrical stranded conductor, comprising aluminium clad steel wires (designation JL/G1A-240/30). It has a mass per unit length $m=0.9207\text{kg/m}$. The equivalent diameter of the conductor line, d , is 21.6mm, and the cross-sectional area, A , is 275.96mm^2 ^[15]. The span of the transmission line is 300m and the height of wire hanging on the structure is $h_s = 45\text{m}$, the calculating height for the wind load of wire is $h_c = 40\text{m}$ computed by the equation in reference [17], as shown in Fig.7.

Fig.7 The diagrammatic drawing of the suspended conductor

A single basic load case combination is considered: extreme radial ice thickness $\delta_{\max} = 20\text{mm}$ with a concurrent wind speed $v = 10\text{m/s}$ which is perpendicular to the line and a temperature variation of $\Delta t = -5^\circ\text{C}$. The wind loads and ice cover load acting on the line in this case of loads combination can be calculated according to the corresponding rules as follows ^[16, 17]:

If the density of ice is taken as 900kg/m^3 , the ice cover load on line per meter is:

$$q_i = 0.9 \times \pi \delta (\delta + d) g \times 10^{-3} \text{ N/m} \quad (37)$$

In general, the density of air is taken as 1.25kg/m^3 at a temperature of 15°C and an atmospheric pressure of 101.3 kPa at sea level, and the wind load on line per meter is:

$$q_w = 0.625 v^2 (d + 2\delta) \alpha_f \mu_z \mu_{sc} \times 10^{-3} \text{ N/m} \quad (38)$$

where, g represents the acceleration of gravity, $g = 9.80665\text{N/mm}^2$; α_f represents the wind pressure coefficient of non-uniformity, shown in Table 2; μ_{sc} is the shape coefficient of conductor line, shown in Table 3; δ is the icing thick on lines, and $\delta=0$ without ice cover; and μ_z represents the wind pressure exposure coefficients, varying with the height of wires and shown in Table 4.

Table 2 Wind pressure coefficient α_f ^[18]

wind speeds $v(\text{m/s})$	$v < 20$	$20 \leq v < 27$	$27 \leq v < 31.5$	$v \geq 31.5$
α_f	1.0	0.85	0.75	0.70

Table 3 Conductor shape coefficient μ_{sc} ^[18]

conductor surface condition	without ice cover		with ice cover
conductor diameter $d(\text{mm})$	$d < 17$	$d \geq 17$	
μ_{sc}	1.2	1.1	1.2

Table 4 Wind pressure exposure coefficients (excerpts) ^[18]

height of the conductor above ground or sea levels (m)	type of the ground roughness			
	A	B	C	D
20	1.63	1.25	0.84	0.62
30	1.80	1.42	1.00	0.62
40	1.92	1.56	1.13	0.73
50	2.03	1.67	1.25	0.84
60	2.12	1.77	1.35	0.93
Notes: for the type of ground roughness, Type A represents the places near the sea, islands, seashores, lakeshores or desert areas; Type B represents the fields, countryside, hilly areas, or towns and city suburbs where the houses is sparse; Type C represents urban districts with the crowded buildings; Type D represents urban districts with the crowded and high buildings.				

Supposing that the transmission line is located within an environment of ground roughness B, the following load case combination is considered: the extreme ice cover with the concurrent wind and the temperature, $mg=9.03\text{N/m}$; $q_w=7.21\text{N/m}$; $q_i=23.07\text{N/m}$; $\Delta t=-5^\circ\text{C}$.

For this example, all random variables are assumed to be normally distributed, and the coefficients of variation of variables mg , q_w and q_i are, respectively, 0.07, 0.193 and 0.181 for a 50 year reference period ^[20]; that of the tensile strength f_y and the Young's modulus E of conductor are assumed 0.10 and 0.05, respectively, with reference to [11, 21]; that of the geometrical variables f is assumed 0.1 because the minor change of cable length may generate a remarkable variation of f ^[1,2]; and that of the other parameters are assumed as in Table 5. The statistical characteristics of the basic variables are listed in Table 5.

Table 5 Statistical characteristics of the basic variables

Basic Variable	Statistical distribution	mean value	standard deviation	coefficient of variation
Self-weight mg (N/m)	Normal	9.03	0.632	0.07
Length of span l (m)	Assumed Normal	300	15.0	0.05
Sag f (m)	Assumed normal	7.50	0.90	0.10
Wind load q_w (N/m)	Assumed Normal	7.21	1.391	0.193
Icing load q_i (N/m)	Assumed Normal	23.07	4.176	0.181

Young's modulus E (N/mm ²)	Normal	73000	3650	0.05
Section areas A (mm ²)	Assumed Normal	275.96	13.798	0.05
Tensile strength f_y (N/mm ²)	Normal	274.03	27.403	0.10
Coefficient of thermal expansion α_t (1/°C)	Assumed Normal	1.96×10^{-5}	9.80×10^{-7}	0.05
temperature change Δt °C	Assumed Normal	-5	0.5	0.10

Based on the statistical characteristics of the basic random variables listed in Table 5, the preceding analytical FOSM-based algorithm is used to calculate the reliability of the conductors. The resulting values of β and p_f for the different cases are listed in table 6.

To demonstrate the validity and rationality of the results computed by the FOSM-based algorithm, the direct Monte-Carlo simulation (MCS) and the Monte-Carlo simulation based optimization (MCOP) methods are also used to evaluate the failure probability and/or the corresponding reliability index of the same example.

The well known direct Monte-Carlo simulation method determines the probability of failure by means of a large number of simple repeated sampling; it is adaptable to a general class of problems, including non-linear limit state functions. Convergence is related to the failure probability or the reliability index and is independent of the dimensionality of the problem, and may be slow and computationally expensive if not prohibitive. If a confidence value is selected as 95% to ensure the sampling error and the relative error is 0.2, the sampling numbers required is of the order of $100/p_f$ to obtain reliable estimates for p_f or the corresponding reliability index^[14]. In the light of the results of failure probability p_f calculated by FOSM in Table 6, the required sampling number for direct Monte-Carlo simulation is at least need 10^7 to obtain reliable p_f estimates in this example. To reduce the sampling number and assure simulation precision, MCOP is introduced for the small failure probability problem.

The principle of the Monte-Carlo simulation based optimization method is to define the random variables with known distributions by means of sampling on the limit state function and calculating the distance from the sampling points to the origin. The shortest distance is then taken to be the reliability index β . The specific calculation methodology initially involves transforming the constrained optimization problem into a non-constrained problem by combining the limit state function with n stochastic variables; that is, one of variables y_i in the limit state function $Z = g(y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_n)$ is expressed by the others, i.e., $y_i = f(y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_n)$, for example, f_y in equation (31); secondly, in order to define a combination of sampling points that are located on the limit state surface, $n - 1$ variables are sampled with the exception of y_i that is then calculated from $y_i = f(y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_n)$; finally, compute the distance from the sampling points to the origin and find the reliability index β ^[22].

Calculated values of β using the MCS and MCOP approaches as a function of the number of simulations are also listed in Table 6.

Table 6 Calculated values of reliability index β and failure probability p_f

Calculating method	FOSM	MCS	MCOP			
Numbers of simulation or iteration	6	10^7	10^4	10^5	10^6	10^7
Probability of failure p_f ($\times 10^{-5}$)	2.735	2.840	1.928	2.065	2.084	3.081
Reliability indices β	4.035	4.026	4.116	4.100	4.098	4.073

Difference $D(\%)$	-	-0.22	2.02	1.62	1.57	0.95
Sensitivity coefficient α_i calculated using FOSM	$\alpha_L=0.308; \alpha_f=-0.247; \alpha_A=-0.226; \alpha_{q_w}=0.050; \alpha_{mg}=0.100; \alpha_{q_i}=0.372;$ $\alpha_{f_y}=-0.800; \alpha_E=0.055; \alpha_{\alpha_t}=3.334 \times 10^{-3}; \alpha_{\Delta t}=-0.007$					

The reliability indices for example 1, listed in Table 6, and computed using the FOSM, is used as the basis for estimating the relative differences between values obtained from the MCS and MCOP algorithms, shown in line 5 in Table 6, and validating the FOSM approach. The following observations are made:

(1) The failure probability and the relevant reliability index of the conductor in this example computed using FOSM is very close to the results computed from MCS, with the difference in the reliability index being -0.22%.

Estimates based on the MCOP algorithm are also listed in Table 6 for comparison with the FOSM. Again, the differences in values are small, in the range 2.02%~0.95%, and reduce with increasing numbers of MCOP simulations.

The results validate the analytical reliability analysis of a suspended cable using the FOSM formulation presented in this paper.

(2) Analysing the reliability estimates computed by the FOSM and that by MCS or MCOP, two main reasons may be identified for their differences: firstly, the predictions from the MCS and MCOP are dependent on the number of simulations, although the reliability gradually converges with increased numbers of simulations but with randomness remaining; the second reason is that the limit state function is complex for such a strongly geometrical non-linear problem, such that employing only first order terms at the expansion of the checking point in the FOSM will introduce either an overestimate or an underestimate of the failure content. It is for this reason that the reliability analysis of a suspended cable using SORM may be beneficial, although it is noted that the FOSM predictions are sufficient for design.

(3) As expected, it is demonstrated that not all of the random variables have the same level of effect on the reliability for a specific case. The influence of the random variables on the reliability prediction can be, of course, identified by the magnitudes of the sensitivity coefficients α_i computed as part of the FOSM.

In example 1, material strength f_y have high sensitivity coefficients, meaning that this random variable play significant roles in the reliability of the conductor. The icing load q_i and the geometrical variables such as span length L , sag f and cross-section area A of conductor have intermediate levels of sensitivity. The reliability is not sensitive to the other variables including self-weight mg , elastic modulus E and wind load q_w , especially to temperature change Δt and coefficient of thermal expansion α_t . If the temperature change Δt and the coefficient of thermal expansion α_t is assumed to be deterministic, the reliability index of the conductor in Example 1 is $\beta = 4.035$ and the corresponding failure probability $p_f = 2.734 \times 10^{-5}$. Comparing with the results of dealing variables Δt and α_t with random variables listed in Table 6, the difference is so small that they can be assumed to be deterministic.

Example 2: The overhead transmission line is same as that in Example 1 with the exception that two basic load case combinations are considered: (1) a maximum wind speed $v_{\max} = 40$ m/s perpendicular to the line and a coincident temperature variation $\Delta t = 10^\circ\text{C}$ with respect to the reference temperature at installation of the conductor. The line is taken to be not subjected to ice accretion, such that the radial ice thickness is $\delta = 0$ mm; (2) extreme radial ice thickness $\delta_{\max} =$

15mm with a concurrent wind speed $v = 10\text{m/s}$ perpendicular to the line and a temperature variation of $\Delta t = -5^\circ\text{C}$. In this example, the FOSM is just used to compute the reliability of the conductor.

Similarly, supposing that the transmission line is located within an environment of ground roughness B, the wind loads and ice cover load acting on the line in two cases of loads combination can be calculated by equations (37) and (38) as follows:

In case 1: the extreme wind speed and the concurrent temperature, $q_w = 25.95\text{ N/m}$; $\Delta t = 10^\circ\text{C}$. The standard deviation of wind load is 5.008 calculated in line with the coefficient of variation in Table 5.

In case 2: the extreme ice cover with the concurrent wind and the temperature, $q_w = 6.04\text{ N/m}$; $q_i = 15.22\text{ N/m}$; $\Delta t = -5^\circ\text{C}$. The standard deviations of wind load and icing load are 1.116 and 2.275, respectively, calculated in line with the coefficient of variation in Table 5.

The probability distribution types of these random variables are assumed to be normally distributed. The statistic characteristics of the other variables are same as Example 1, as shown in table 5, and the variables Δt and α_t are assumed to be deterministic. Based on the statistical characteristics of the basic random variables shown above and in Table 5, the resulting values of β and p_f at the lowest point are calculated by FOSM for the different cases are listed in Table 7(a), meanwhile, the factor of safety for this wire are calculated as $K = f_y A / (H + h)$ and listed in Table 7(a).

Table 7(a) Reliability index β and failure probability p_f calculated using FOSM

Cases of loads combination	Case 1: extreme wind speed and concurrent temperature	Case 2: extreme ice cover with concurrent wind and the temperature
Sag f (m)	7.50	7.50
Factor of safety K	2.518	2.448
Reliability indices β	5.264	5.061
Probability of failure p_f	7.065×10^{-8}	2.089×10^{-7}
Sensitivity coefficient α_i	$\alpha_L = 0.212$; $\alpha_f = -0.127$; $\alpha_A = -0.173$; $\alpha_{q_w} = 0.698$; $\alpha_{mg} = 0.019$; $\alpha_{f_y} = -0.647$; $\alpha_E = 0.048$;	$\alpha_L = 0.292$; $\alpha_f = -0.269$; $\alpha_A = -0.208$; $\alpha_{q_w} = 0.056$; $\alpha_{mg} = 0.117$; $\alpha_{q_i} = 0.282$; $\alpha_{f_y} = -0.838$; $\alpha_E = 0.044$

Table 7(b) Reliability index β and failure probability p_f calculated using FOSM

Cases of loads combination	Case 1: extreme wind speed and concurrent temperature	Case 2: extreme ice cover with concurrent wind and the temperature
Sag f (m)	7.76	7.76
Factor of safety K	2.559	2.501
Reliability indices β	5.315	5.140
Probability of failure p_f	5.342×10^{-8}	1.376×10^{-7}
Sensitivity coefficient α_i	$\alpha_L = 0.214$; $\alpha_f = -0.132$; $\alpha_A = -0.173$; $\alpha_{q_w} = 0.697$; $\alpha_{mg} = 0.018$; $\alpha_{f_y} = -0.647$; $\alpha_E = 0.047$;	$\alpha_L = 0.290$; $\alpha_f = -0.271$; $\alpha_A = -0.206$; $\alpha_{q_w} = 0.056$; $\alpha_{mg} = 0.114$; $\alpha_{q_i} = 0.282$; $\alpha_{f_y} = -0.838$; $\alpha_E = 0.043$

The “Eurocode - Basis of structural design (EN 1990:2002) [23]” divides the Reliability Consequence (RC) into RC1, RC2 and RC3 associated with the three consequences classes. The recommended minimum reliability index for ultimate limit states associated with different RC is 3.3,

3.8 and 4.3, respectively, for a 50 year reference period. The “Unified standard for reliability design of engineering structures (GB 50153-2008) [24]” in China also divides the safety level of structural members into level 1, 2 and 3 according to the importance of structure. The target reliability index of a structural member subject to a ductile failure is specified for each level as 3.7, 3.2 and 2.7, respectively.

The results of example 2, listed in Table 7, and computed using the FOSM, show that the reliability indices in both cases are achieved above 5.0, far exceeding the recommended level of RC3 specified in EN 1990:2002 and level 1 specified in GB 50153-2008. The factor of safety K in case 2, however, is not satisfied with the minimum value at the lowest point on line specified by reference [18]. For the geometric nonlinearity of the cable, the tension force of cable will decrease with the increment of sag f if all the other parameters remain the same, and the factor of safety of cable K will increase. For example, the sag of the conductor is increased to $f=7.76\text{m}$ to let the factor of safety K just meet the demands of reference [18], the reliability index β is 5.140, as listed in Table 7(b).

The influence of the random variables on the reliability prediction in different case of loads combination is very different.

In case 1 of the extreme wind with the concurrent temperature, material strength f_y and wind load q_w have high sensitivity coefficients, meaning that these random variables play significant roles in the reliability of the conductor. The geometrical variables such as span length L , sag f and cross-section area A of conductor have intermediate levels of sensitivity. The reliability is not sensitive to the other variables including self-weight mg , and elastic modulus E .

In case 2 comprising extreme ice and the concurrent wind and temperature effects, the material strength f_y remains a significant influence on the reliability of conductor. The sensitivity of the safety of the suspended cable to the wind load q_w is reduced, consistent with the reduction in magnitude of this applied load. The sensitivity of ice load q_i is just above that of the geometrical variables, with the influence of the cable span becoming more prominent.

5. Conclusions

(1) Based on the classical parabolic analytical solution of a suspension cable, the reliability of suspended cable is explored using the FOSM. The necessary inclusion of geometric nonlinearity creates a limit state function that is complex. The calculation of the structural reliability of a suspended cable is a complicated problem with a large number of basic random variables. Nevertheless, it has been demonstrated that it is feasible to compute the reliability of a suspension cable using the FOSM.

(2) The first-order derivatives of the limit state function with respect to the basic random variables can be derived by means of the successive application of the chain rule of differentiation. This methodology provides a solid foundation for computing the reliability of suspension cable using FOSM.

(3) To verify the rationality and correctness of the results of FOSM, the Monte Carlo Simulation method (MCS) and the Monte Carlo Simulation Based Optimization principle (MCOP) have been implemented. The reliability or the failure probability evaluated using MCS and MCOP are very close to that by obtained from the FOSM. These outcomes imply that the structural reliability solutions for a suspended cable estimated by the implemented analytical FOSM are rational and correct.

(4) The stress-based structural reliability analysis of a suspended cable, with an assumed parabolic profile, with the same height supports, and subjected to horizontal and vertical loads, using the FOSM has been demonstrated in this paper. The structural reliability of a suspended cable with fewer restrictions on geometric form and boundary conditions and including deformation limit state functions will be studied further in a separate manuscript.

(5) If the safety factor K of the conductor is satisfied with a minimum value of 2.5 at the lowest points of suspended cable, the reliability indices β in different cases are far higher than the level of RC3 in EN 1990:2002 and that in GB 50153-2008. A safety index in excess of 5.1 would appear to be required to achieve an equivalent safety factor of 2.5.

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