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Parametric Identification of Nonlinear Vibration Systems via Polynomial Chirplet Transform

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Abstract: The dynamic response of a nonlinear system can be characterized by its instantaneous amplitude (IA) and instantaneous frequency (IF) features, which dependent on the physical properties of the system. Accordingly, the system properties can be inferred from the IA and IF features if they can be identified accurately. To fulfil such an idea, a nonlinear system parameter identification method is proposed in this paper with the aid of Polynomial Chirplet Transform (PCT), which has been proved a powerful tool for processing non-stationary signals. The procedure of the proposed system identification method is summarized as: firstly, the instantaneous characteristics, IA and IF, are extracted by using the PCT from the nonlinear responses of the system; secondly, calculate the instantaneous modal parameters and present them to backbone and damping curves, which characterize the inherent nonlinearities of the system; and finally, estimate the physical property parameters of the system through fitting the identified average nonlinear characteristic curves. The proposed nonlinear system identification method is experimentally validated in the paper. The experimental results have shown that the proposed method is superior to two existing Hilbert transform based methods, particularly on the robust performance against noise. In other words, the proposed method can work very well in system identification under any noise contamination condition, while the HT based methods can work only in the absence of noise.

Key words: nonlinear system identification, parameter estimation, parameterized time-frequency analysis, polynomial chirplet transform

1. Introduction

Due to the existence of a variety of nonlinearities in mechanical systems [1-11],

nonlinear identification has received considerable attention in recent years. The nonlinearities include geometric nonlinearity (e.g. large elastic deformations [2]), material nonlinearity (e.g. deformation-dependent elasticity [3,4,6]), structural nonlinearity (e.g. gaps or clearances between mounting brackets [7]) properties, and defect-caused nonlinearity [9-11]. Two basic goals of nonlinear system identification are to enhance the dynamic characteristics of the systems with wanted nonlinearities, and detect unwanted nonlinearities for structure monitoring purposes. Nonlinear identification involves two tasks, i.e., identification of nonlinear characteristics and estimation of system parameters. Although it has been fully studied, nonlinear system identification, as a typical reverse problem, is still a challenge due to the large diversity of nonlinear systems. Ref. [12] presented a comprehensive summary of popular methods that have been applied to nonlinear system identification, which include equivalent or statistic linearization methods [13-16], time-domain methods (taking the forms of types of time series analyses, e.g., NARMAX model [17,18]), frequency-domain methods (e.g., higher-order frequency response function [19-21]), and artificial neural networks [22,23]. These methods show advantages in some aspects of system identification, however none of them is universally effective to all nonlinear systems. For example, NARMAX model is limited by large storage requirement and poor stability in calculation [24]; Neural network is difficult to train and moreover often suffers convergence and efficiency issues [25], etc.

With the advance of signal processing theory, a variety of time-frequency analysis (TFA) methods have been applied to nonlinear system identification in the past decades. Among them, Hilbert-Huang transform (HHT) [26,27], Wigner-Ville distribution (WVD) [28], Gabor transform (GT) [29,30] and wavelet transform (WT) [31,32] are the most popular ones. In contrast to conventional time- and frequency-domain analyses, the TFA methods project the time series into a space, in which signal components are separable and signal filtering could be more easily performed [12]. Since 1990s, the TFA based methods have been widely used to deal with various structural dynamics issues. For example, Franco and Pauletti [29] used Gabor spectrogram to reveal nonlinear features of system dynamics. Bellizzi, Guillemain and Kronland-Martinet [30] extended GT to identify nonlinear modal parameters of MDOF nonlinear systems with light damping. Staszewski [32] proposed a nonlinear identification procedure for SDOF and MDOF systems using multi-scale ridges and skeletons of WT. Pai [33] used perturbation solutions and time-frequency decomposition in combination to accomplish nonlinear system identification. However, the system identification capability of these conventional signal-independent TFAs are still limited due to the inexplicit time-frequency distribution (TFD) resulted by them.

Recently, the potential of Hilbert transform (HT) in the application to nonlinear vibration system identification is attracting increasing interests [34-39]. A number of HT-based system identification algorithms, such as FREEVIB [34] and FORCEVIB [35], have been successfully developed. However, the FREEVIB and FORCEVIB only consider the primary component (i.e., the fundamental quasi-harmonic solution) of system response. Such a simplification limits the capability of both algorithms in characterizing the nonlinearities in a dynamic system. Given high-order ultra-

harmonics, Feldman [36] developed a signal decomposition technique for system identification, named Hilbert Vibration Decomposition (HVD) [37]. However, HT-based methods are noise-sensitive and difficult to determine congruent modal parameters [36].

In this paper, an innovative nonlinearities identification and parameter estimation method is proposed, which can be implemented by following 4 steps: (1) transforming a nonlinear system into a slow-varying system of primary solution; (2) using polynomial chirplet transform (PCT) to extract instantaneous characteristics of primary component that is heavily contaminated by noise; (3) reconstructing backbone and damping curves using the estimated average modal parameters and instantaneous characteristics; and (4) fitting the identified average nonlinear characteristic force to estimate coefficients in initial force terms. In the proposed method, the PCT was employed to perform parameter estimation as it has been proved effective in enhancing TFD [40, 41].

2. Nonlinear System Model and Identification

2.1 Model of Nonlinear system

A nonlinear system can be expressed as

$$m\ddot{x} + f(\dot{x}) + k(x) = z(t) , \quad (1)$$

where m denotes the mass of oscillator, $f(\dot{x})$ and $k(x)$ indicate the general forms of velocity-dependent nonlinear damping and displacement-dependent nonlinear elasticity, respectively, $z(t)$ is time-varying excitation applied to the nonlinear oscillator. Usually, memoryless anti-symmetric nonlinear damping force and nonlinear restoring force can be written as

$$f(\dot{x}) = 2 \sum_{i=0}^I \beta_i |\dot{x}|^i \text{sign}(\dot{x}) , \quad (2)$$

$$k(x) = \sum_{j=0}^J \alpha_j |x|^j \text{sign}(x) . \quad (3)$$

The system given by Eq. (1) can be transformed to be an equivalent fast-varying quasi-linear system with unit mass [35], i.e.

$$\ddot{x} + 2h(t)\dot{x} + \omega^2(t)x = z(t)/m , \quad (4)$$

where $h(t)$ and $\omega^2(t)$ are fast-varying damping coefficient and fast-varying instantaneous natural frequency, respectively. Both of them contain two parts in time domain, one is much slower term and another one is much faster term with respect to the fundamental frequency component [35], i.e.,

$$\begin{aligned} h(t) &= h_0(t) + h_{fast}(t) , \\ \omega^2(t) &= \omega_0^2(t) + \omega_{fast}^2(t) . \end{aligned} \quad (5)$$

If fundamental frequency component $u(t)$ dominates the energy of response signal, it is called as primary solution. Both of excitation $z(t)$ and primary solution $u(t)$ show slower time-varying characteristics in contrast to high-order ultra-harmonics of system response. Decomposing the terms in Eq. (4) into a fast-varying part and a slow-varying part, an equation that contains only slow-varying part can be obtained by using the methods proposed in [42], i.e.

$$\ddot{u} + 2h_0(t)\dot{u} + \omega_0^2(t)u = z(t)/m . \quad (6)$$

It is noted that the dynamic system described by Eq. (6) is a slow time-varying system of primary solution. Hereinafter, it acts as substitution for nonlinearity identification and parameter estimation.

2.2 Nonlinear identification and parameter estimation

Backbone and damping curves of dynamic system are known as two important nonlinear characteristics of a nonlinear system. For nonlinear elasticity and nonlinear damping considered in this paper, average natural frequency is used to map approximate backbone to real backbone. Assume operator " $\{\}$ " represents averaging procedure that is realized through low-pass filtering. If primary vibration is $u(t) = A_p(t)\cos\varphi_p(t)$, the approximate backbone [42] can be expressed as,

$$\{\omega_0^2(A_p)\} = (\int_0^{2\pi} k(A_p\cos\varphi_p)\cos\varphi_p d\varphi_p)/(\pi mA_p) , \quad (7)$$

Similarly, after mapping average damping coefficient versus IA of primary velocity solution $\dot{u}(t)$, one can obtain an approximate damping curve [42] as,

$$\{h_0(A_{\dot{u}})\} = -(\int_0^{2\pi} f(-A_{\dot{u}}\sin\varphi_p)\sin\varphi_p d\varphi_p)/(2\pi mA_{\dot{u}}) , \quad (8)$$

where $A_{\dot{u}}$ denotes amplitude of primary velocity solution.

It is worthy to note that backbone cannot be approximated by Eq. (7) for certain anti-symmetric elasticity. For example, the exact backbone of backlash [43] is given by

$$\omega_0(A) = \sqrt{\alpha}/(1 + 2/\pi/(A/\delta - 1)), A > \delta , \quad (9)$$

where α denotes the stiffness of a linear elastic spring outside of dead zone, δ is gap width and A is vibration amplitude. Another example is piecewise linear spring [43], which is given by

$$\omega_0(A) = \pi/(2\arcsin(\sqrt{\mu}/\sqrt{(A/\Delta - (1 - \mu))^2 + \mu - \mu^2})/\sqrt{k_1} + 2\arctan\sqrt{((A/\Delta - (1 - \mu))^2 - \mu^2)/\mu}/\sqrt{k_2}), A > \Delta . \quad (10)$$

where k_1 and k_2 indicate stiffness of two linear elasticities inside and outside of zone Δ , respectively, μ equals to the ratio of k_1/k_2 , where $k_1 < k_2$.

In terms of parameter estimation, both FREEVIB and FORCEVIB algorithms will result in inaccurate nonlinear characteristic forces if all high-order ultra-harmonics are ignored [44]. To address this issue, it was proposed to use identified average nonlinear characteristic force to re-characterize the original ones. Firstly, by selecting extrema of displacement (x_{max}) and velocity ($\dot{x}_{max}$), respectively, one can obtain the exact values of both original elastic static force and damping force [43],

$$k(x_{max}) = \omega_0^2(t_{x_{max}})A(t_{x_{max}}) , \quad (11)$$

$$f(\dot{x}_{max}) = 2h_0(t_{\dot{x}_{max}})A_{\dot{x}}(t_{\dot{x}_{max}}) , \quad (12)$$

where $t_{x_{max}}/t_{\dot{x}_{max}}$ indicates time when the maximum displacement/velocity is reached.

Then, integrate both sides of the equations over the full period of primary solution, the average elastic static force and the average friction force can be derived as [45],

$$\{k(x_{max})\} = \{\omega_0^2(t_{x_{max}})\}A_p(t_{x_{max}}) , \quad (13)$$

$$\{f(\dot{x}_{max})\} = 2\{h_0(t_{\dot{x}_{max}})\}A_{\dot{u}}(t_{\dot{x}_{max}}) , \quad (14)$$

Finally, identified discrete points are required to fit by the function of average characteristic force. The fitting curves can approximate the theoretical average characteristic forces respectively, which are defined as,

$$\overline{k(x)} = \left(\int_0^{2\pi} k(A\cos\varphi)\cos\varphi d\varphi \right) x / (\pi A) , \quad (15)$$

$$\overline{f(\dot{x})} = 2 \left(- \int_0^{2\pi} f(-A_{\dot{x}}\sin\varphi)\sin\varphi d\varphi \right) \dot{x} / (2\pi A_{\dot{x}}) . \quad (16)$$

With the identified average characteristic forces, one can estimate the coefficient of each term of nonlinear force by referring to the theoretical coefficients in Eqs. (15) and (16).

It is noticed that in the scenario of backlash and bilinear springs, theoretical average nonlinear elastic forces cannot be obtained according to Eq. (15), because integration parameter φ is a function of time instead of displacement.

Firstly, in the case of backlash spring, average elastic force is a continuously differentiable function of displacement, whose slope gradually changes from 0 to α . Starting-point of variation in displacement domain is inflection point of piecewise elastic force curve in displacement domain. Theoretically, there exists an asymptote line of real average nonlinear elastic force curve $F_a = kx + b$, in which the slope and the intercept are given by

$$k = \alpha; b = -4\delta\alpha/\pi . \quad (17)$$

Secondly, in the case of bilinear spring, corresponding asymptote can be obtained with the slope and the intercept as

$$k = k_2; b = -4(k_2 - k_1)\Delta/\pi . \quad (18)$$

According to Eqs. (17) and (18), one can also carry out parameter estimation for the above two nonlinearities.

3. Extraction of instantaneous characteristics of a nonlinear system

The instantaneous characteristics can be extracted by using a variety of signal processing methods. But TFA methods are more favored since they can facilitate signal filtering for multi-component signals. Traditional TFA methods that can be used for IF and IA estimation include the short-time Fourier transform (STFT), the continuous wavelet transform (CWT), the Wigner-Ville distribution (WVD) and the chirplet transform (CT), etc. These methods are inadequate for good analysis and representation of multi-component non-stationary responses of nonlinear systems (WHY? It is necessary to explain the reason). In this study, the PCT is used to estimate instantaneous characteristics, including IA and IF, from system response. In this section, a HT based instantaneous characteristics extraction method that is used in FREEVIB and

FORCEVIB algorithms [34, 35] will be introduced first. Then, the PCT-based estimation method will be developed and demonstrated.

3.1 Instantaneous characteristics extraction based on the HT

For a non-stationary signal $x(t) = A_1(t)\cos \int \omega_1(t)dt + A_2(t)\cos \int \omega_2(t)dt$, its two components can be characterized by slow-varying amplitude and IF. Supposing that the first component is primary component and at any time $t = t_i$, the IA (envelope) and IF of the two components will satisfy the following conditions $A_1(t_i) \gg A_2(t_i)$ and $\omega_1(t_i) < \omega_2(t_i)$. Then, $A(t)$ and $\omega(t)$ of the signal can be expressed as,

$$A(t) = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\omega_2 - \omega_1)t} , \quad (19)$$

$$\omega(t) = \omega_1 + \{(\omega_2 - \omega_1)[A_2^2 + A_1A_2 \cos(\omega_2 - \omega_1)t]\}/A^2(t) . \quad (20)$$

where both the envelope and IF also consist of two parts, i.e., a slow-varying part and a fast-varying part. In practice, averaging time-varying parameters is achieved by filtering out fast-varying part with the aid of a low-pass filter. From Eq. (19) and (20),

one can obtain the remaining averaged slow-varying part, $\overline{A(t)} = \sqrt{A_1^2 + A_2^2}$; $\overline{\omega(t)} = \omega_1$. Such a procedure is equally effective even when the signal contains more than two components. When the primary component dominates a signal, one can obtain the averaged slow-varying part, $\overline{A(t)} = \sqrt{\sum A_i^2} \approx A_1(t)$, $\overline{\omega(t)} = \omega_1(t)$.

It is necessary to note that the HT based instantaneous characteristics extraction method is only applicable to mono-component signals. For multicomponent signals, an approximation method was developed in relation to FREEVIB and FORCEVIB algorithms for primary component. Firstly, process the multicomponent signal by using the HT to obtain its instantaneous characteristics; Secondly, averaged instantaneous characteristics are obtained though performing low-pass filtering operation to approximate instantaneous characteristics of primary component. Within the bandwidth of primary solution with time-varying IF features, neither low-pass filter nor narrow-band filter can cancel noise, thus errors occur in the identified envelope and IF as a consequence.

3.2 Instantaneous characteristics extraction based on PTFA

Parametric time-frequency analysis (PTFA) of a signal $x(t) \in L^2(R)$ is defined as [40]

$$PTFA_x(t_0, \omega, \Psi; \sigma) = M(t_0) \int_{-\infty}^{+\infty} \bar{z}(t) w_\sigma(t - t_0) \exp(-j\omega t) dt , \quad (21)$$

where $z(t)$ denotes the analytic signal of $x(t)$, $\bar{z}(t)$ indicates mathematical processing of the analytic signal using particular operators, and t_0 is the center of window function. $M(t_0)$ is a complex number whose modulus is equal to 1.

$$z(t) = x(t) + j\tilde{x}(t) , \quad (22)$$

$$\bar{z}(t) = z(t)\Phi^R(t)\Phi^M(t, t_0) , \quad (23)$$

where $\tilde{x}(t)$ indicates the HT of $x(t)$. $w_\sigma(t)$ in Eq. (21) refers to symmetric non-negative real window function. In most cases, $w_\sigma(t)$ employs a Gaussian window, i.e.

$$w_\sigma(t) = \exp(-0.5(t/\sigma)^2)/(\sqrt{2\pi}\sigma) , \quad (24)$$

The details for choosing an appropriate value of σ is depicted in [40].

In the above equations, Ψ refers to a nonlinear kernel function. $\Phi^R(t)$ and $\Phi^M(t, t_0)$ denote two mathematical operators, namely frequency-rotating operator and frequency-shifting operator. Considering an operator that is in the form of a complex signal, its IF is required to approximate the IF trajectory of the signal of interest.

In this paper, the PCT is adopted and the two operators in the PCT are expressed as [40],

$$\Phi_{\alpha_1, \dots, \alpha_n}^R(t) = \exp(-j \sum_{k=2}^{n+1} \alpha_{k-1} t^k / k) , \quad (25)$$

$$\Phi_{\alpha_1, \dots, \alpha_n}^M(t, t_0) = \exp(j \sum_{k=2}^{n+1} \alpha_{k-1} t_0^{(k-1)} t) , \quad (26)$$

where $(\alpha_1, \dots, \alpha_n)$ denotes characteristic parameters of a polynomial kernel function.

Assume that the IF of a non-stationary signal with continuous phase can be approximated by polynomial of appropriate order on a closed and bounded interval. The PCT has been proved effective in achieving explicit TFD of the signals with time-varying frequencies. According to the Weierstrass approximation theorem, polynomial approximation can achieve any degree of accuracy, which largely depends on kernel characteristics parameters.

In practice, the exact characteristic parameters cannot be determined in one time of operation. To address this issue, Peng [40] proposed a ridge-detection based method to estimate characteristic parameters for the PCT through an iteration process. Characteristic parameters in each iteration are obtained by polynomial fitting the ridge extracted from TFD. In the end, these parameters are converged to some values, which are taken as the estimated values of the parameters. It has been proved that the PCT-based IF extraction method can estimate the parameters accurately even under noisy conditions [41].

3.2.1 IF estimation with the aid of the PCT

It is necessary to estimate IF before reconstructing IA since the estimation accuracy of IF can significantly influence the accuracy of IA estimation. However, the ridge detection method proposed in [40] may raise error in IF estimation if the signal being inspected has been seriously polluted by noise. In [46], it is assumed that peaks in TFD are directly associated with energy distribution of the noisy signal, which, however, is probably invalid or biased when the signal-to-noise ratio (SNR) is low.

Inevitably, error will be introduced into the estimation of polynomial kernel parameters $\Delta A_1 = \Delta(\alpha_1, \dots, \alpha_n)^1$ during polynomial-fitting. If performing PCT with inappropriate kernel parameters, the resultant TFD would be incorrect and thus lead to wrong IF estimation. For this reason, [47,48] applied continuous and smooth constraints to refine the IF trajectory derived from the PCT under heavy noise conditions.

The continuity-smoothness criterion is defined as: for the i th sampling point in time domain ($1 \leq i \leq N$, and N is signal length), an IF trajectory is assumed to be continuous and smooth transition between the i th point and its neighbors, i.e., the

$(i - 1)th$ point and the $(i + 1)th$ point on the trajectory. In other words, the continuity-smoothness indicator defined as Eq. (27) is not more than 1 for the i th point

$$\gamma_i = \max(|S_{i-1} - S_i|, |S_{i+1} - S_i|) . \quad (27)$$

in which S_i indicates the rank of the i th time domain sampling point in frequency domain sampling. Assume M denotes sample length in frequency domain, thus $1 \leq S_i \leq M$, and, generally, $M < N$.

The IF estimation algorithm is developed as follows,

- 1) Conduct the PCT (in the first iteration, all the kernel parameters are set to be zero), then detect the energy peak in the TFD and find out the point with the largest energy density $(t_p, f_p), 1 \leq p \leq N$;
- 2) Compute the continuity-smoothness indicators for all points on the initial peak ridge in two directions, respectively, i.e., from t_{p-1} to t_1 and from t_{p+1} to t_N along with time axis;
- 3) Skip the point whose continuity-smoothness indicator is less than or equal to 1, then continue the same process for next point. Otherwise, if indicator of a point is more than 1, the value of this point will be set to zero. Then, peak is detected at this point along frequency axis again;
- 4) Repeat Step 3) until all points of the trajectory satisfy the continuity-smoothness criterion and eventually obtain the IF estimation in this iteration;
- 5) Determine kernel parameters with the estimated IF and repeat Steps 1) ~ 4) until preset convergence condition of kernel parameters is met. The estimated IF in the last iteration is regarded final estimation of true IF.

3.2.2 Instantaneous amplitude estimation with the aid of the PCT

The definition of PTFA given by Eq. (21) can be rewritten as,

$$PTFA_x(t_0, \omega, \alpha_1, \dots, \alpha_n; \sigma) = \int_{-\infty}^{+\infty} z(t) \Theta_{(t_0, \alpha_1, \dots, \alpha_n, \sigma)}^*(t) \exp(-j\omega t) dt , \quad (28)$$

where $\Theta_{(t_0, \alpha_1, \dots, \alpha_n, \sigma)}^*(t)$ is a complex window given by

$$\Theta_{(t_0, \alpha_1, \dots, \alpha_n, \sigma)}^*(t) = w_\sigma(t - t_0) \exp(-j \sum_{k=1}^n \theta_k (t - t_0)^{k+1} / (k + 1)) , \quad (29)$$

in which θ_k is function of t_0 and kernel parameters $(\alpha_1, \dots, \alpha_n)$,

$$\theta_k = \alpha_k + \sum_{i=k+1}^n i \alpha_i t_0^{i-1} , \quad (30)$$

Thus, PTFA can be interpreted as the product of the STFT of the analytical signal and the complex window $\Theta_{(t_0, \alpha_1, \dots, \alpha_n, \sigma)}^*(t)$. Through comparing (21) and (28), the complex window can also be expressed in the terms of the two frequency operators depicted by Eqs. (25) and (26), i.e.

$$\Theta_{(t_0, \alpha_1, \dots, \alpha_n, \sigma)}^*(t) = M(t_0) w_\sigma(t - t_0) \Phi^R(t) \Phi^M(t, t_0) , \quad (31)$$

where

$$M(t_0) = \exp(-j \sum_{k=1}^n \theta_k (-t_0)^{k+1} / (k + 1)) . \quad (32)$$

Assume that the signal being analyzed is a mono-component signal, the corresponding analytical signal can be expressed as,

$$z(t) = A(t)e^{j\Omega t} . \quad (33)$$

The PTFA of this signal is,

$$PTFA_x(t_0, \omega) = \langle z(t), \Theta(t)exp(j\omega t) \rangle = \int_{-\infty}^{+\infty} A(t)\Theta^*(t)exp(-j(\omega - \Omega)t)dt , \quad (34)$$

Amplitude $A(t)$ can be derived using Taylor's formula around center point of complex window, i.e., at the point $t = t_0$. Assume envelope $A(t)$ is slowly varying with time. Neglect the high order items in Eq. (34), yields

$$PTFA_x(t_0, \omega) = \int_{-\infty}^{+\infty} \left(A(t_0) + o(A'(t_0)) \right) \Theta^*(t)exp(-j(\omega - \Omega)t)dt , \quad (35)$$

where the prime indicates derivative. $W_\sigma(\omega)$ is the Fourier transform of Gaussian window $w_\sigma(t)$, and $\Theta^*(\omega)$ is the Fourier transform of the complex window $\Theta^*(t)$. When the first order item (infinitesimal) is neglected, has

$$\begin{aligned} PTFA_x(t_0, \omega) &= A(t_0)\Theta^*(\omega - \Omega) \\ &= M(t_0)A(t_0)W_\sigma(\omega - \Omega)exp(-j\omega t) , \end{aligned} \quad (36)$$

Thus

$$|PTFA_x(t_0, \omega)| = A(t_0)|W_\sigma(\omega - \Omega)| , \quad (37)$$

On IF trajectory, the IA can be obtained by

$$|PTFA_x(t_0, \Omega)| = A(t_0)|W_\sigma(0)| . \quad (38)$$

in which $|W_\sigma(0)|$ indicates the modulus of complex value of Gaussian window at zero frequency. For a mono-component signal, Eq. (38) means that it is able to estimate the IA from the IF trajectory obtained by using the PCT.

3.3 Demonstration of instantaneous characteristics extraction

For the sake of simplicity, a mono-component signal with polynomial phase and decaying amplitude is designed, i.e.

$$x(t) = exp(-0.03t^2) sin(2\pi(10t + 2.5t^2 + 0.4t^3 - 0.03t^4))/(1 + 0.1t) . \quad (39)$$

where $(0 \leq t \leq 10)$.

The IF is

$$IF(t) = 10 + 5t + 1.2t^2 - 0.12t^3 (Hz) , \quad (40)$$

and the IA is

$$A(t) = exp(-0.03t^2)/(1 + 0.1t) .$$

Three methods, i.e. the PCT, the HT and the CWT were applied to extract the IF and IA of the signal. For the PCT, an unnormalized Gaussian function $w(t) = exp(log(0.005)t^2)$ was used as window function. For the CWT, Morlet wavelet function was employed. The corresponding IA and IF extraction results are shown in Fig.1.

Fig. 1 Extracted IF and the IA: (a) extracted IF, (b) extracted IA

From Fig.1, it can be seen that basically, all three methods achieve accurate estimation of IF and IA of the signal, except the HT based method shows errors in IF estimation at both boundaries of the signal.

To test the robustness of the three methods in instantaneous characteristics extraction under noise condition, zero-mean Gaussian white noise is added to the signal. The SNR of the noise contaminated signal is $2dB$. Fig. 2(a) shows the IF extracted by the HT with the assistance of a low-pass filtering strategy. Fig. 2(b) shows the fitting IF curve of the TFD ridge through the CWT. Fig. 2(c) gives the IF extracted by the PCT based method introduced in Section 3.

Fig. 2 IF extracted by using HT (a), CWT (b) and PCT (c) under $SNR = 2dB$

Fig. 3 IA extracted by using HT and PCT under $SNR = 2dB$

Since the low-pass filter cannot cancel the noise within the bandwidth of $40Hz \sim 70Hz$, significant error appears in high frequency zone (6s~10s) for the HT method. In Fig. 2(b), it can be seen that from 5s to 10s, the noise also causes significant error to the result of the CWT. On the contrary, the IF extracted by the PCT based method matches real IF trajectory very well. The extracted IA is shown in Fig. 3. Obviously, in contrast to the HT based method the PCT-based method has significantly improved the accuracy of IA extraction results, so that the extracted IA basically matches the variation tendency of the real IA curve.

4. Numerical simulations and verification experiments

To validate the proposed method, several anti-symmetric nonlinear oscillators in both free and forced vibrations are considered in this section. In numerical experiments, a low SNR ($SNR < 10dB$) is assumed to mimic heavy noise condition. In practical experiment, a SDOF ruler spring-mass system is used to fulfil verification. If without specific notation, displacement response signal is acquired and corresponding

velocity/acceleration signals are obtained via a differentiator with filtering measure.

4.1 Numerical experiemnts

4.1.1 Free vibration

Case 1. This case considers a nonlinear system with a high-order nonlinear elasticity.

$$\ddot{x} + 2h\dot{x} + \alpha_1 x + \alpha_5 x^5 = 0, \quad (42)$$

where $\alpha_1 = (15 \times 2\pi)^2$, $\alpha_5 = 8 \times 10^9$, $h = 2.5$. Impulse excitation is imposed to the system, and the initial conditions are,

$$x(0) = 0, \dot{x}(0) = 45.$$

For such a nonlinear system, the theoretical expressions of backbone and damping curves derived by Eq. (7) and (8) are

$$\{\omega_0(A)\} = (\alpha_1 + 5\alpha_5 A^4/8)^{1/2}, \quad (43)$$

$$\{h_0(A_{\dot{x}})\} = h. \quad (44)$$

System response is calculated using the 4th order Runge-Kutta method. Zero-mean Gaussian white noise is artificially added to the system response with $SNR = 2dB$. The noise-contaminated system response and its FFT spectrum are shown in Fig. 4(a) and 4(b), respectively.

Fig. 4, the noise-contaminated response (a) and its FFT spectrum (b) for Case 1

(c)

Fig. 5 The TFD (a), the estimated IF (b) and the IA (c) of the primary component for Case 1

In the spectrum, it can be seen that the primary bandwidth range of response is from 15Hz to 100Hz, within which the noise effect is significant and cannot be easily cancelled through filtering. The PCT based method is used to estimate the instantaneous characteristics of the system response. Fig. 5(a) shows the TFD of the response. The IF extracted from the TFD is shown in Fig. 5(b), in which solid line is ridge and dashed line is IF curve generated by fitting the ridge using polynomial. Fig. 5(c) shows the IA of primary component, in which dashed line is theoretical one and solid line is the estimated one generated by Eq. (38).

With the extracted IF and IA, the procedure given in Section 2 is used to identify the backbone and the damping curve. In Fig. 6, the identified backbone is given by dotted line and the theoretical one calculated by Eq. (43) is displayed by solid line. Accordingly, identified damping curve takes the form of vertical line approximately.

Fig. 6 The identified backbone and the damping curve for Case 1

The theoretical average nonlinear elastic force and average nonlinear damping force

are obtained by Eq. (11) and (15), Eq. (12) and (16), respectively. Final identified results are shown in Fig. 7. Solid lines indicate theoretical solutions. Fitting these discrete dots with the form of theoretical solution, one can estimate the nonlinear characteristic parameters accurately.

Fig. 7 The identified average nonlinear characteristic forces for Case 1

Table 1 shows the errors in percentage of estimated parameters by the proposed method and the FREEVIB, with the former on the left side of slash and the latter on the right side. Apart from the case of $SNR = 2dB$, other two noise conditions are considered. In the table, symbol “*” indicates invalid estimation.

Table 1 PCT-based/ FREEVIB-based results under different noise conditions. (Case 1)

SNR	100dB	20dB	2dB
Coefficients			
h	-0.64%/-0.56%	0.72%/*	-0.13%/*
α_1	-1.8%/-1.9%	-3.8%/*	0.62%/*
α_5	4.6%/13.8%	1.5%/*	-7.1%/*

From Table 1, it is found that the FREEVIB algorithm can only achieve accurate parameters estimation when $SNR = 100dB$ (i.e. noise-free). Once the signal is polluted by noise, e.g., $SNR = 20dB$, significant errors will occur and thus lead to the failure of estimation. By contrast, the proposed PCT-based method can achieve accurate estimation in all SNR scenarios.

Case 2. This case considers a nonlinear system that is characterized by turbulent damping, viscous damping and a preloaded elasticity spring.

$$\ddot{x} + 2c_1\dot{x} + 2c_2\dot{x}|\dot{x}| + kx + F_0\text{sign}(x) = 0, \quad (45)$$

where $c_1 = 1.0$, $c_2 = 45$, $k = (2\pi \times 20)^2$, $F_0 = 1.2 \times 10^{-5} \times (2\pi \times 20)^2$, and the initial conditions are $x(0) = 0.001$, $\dot{x}(0) = 0$.

The theoretical backbone and damping are

$$\{\omega_0(A)\} = (k + 4F_0/(\pi A))^{1/2} , \quad (46)$$

$$\{h_0(A_{\dot{x}})\} = c_1 + 8c_2 A_{\dot{x}}/(3\pi) . \quad (47)$$

Likewise, system responses were obtained under the condition of $SNR = 2dB$. The corresponding instantaneous feature extraction results are shown in Fig.8. Where, Fig. 8(a) shows the IF estimated from the TFD ridge by the PCT based method. In Fig. 8(b), the solid line is the estimated IA of the primary component of the response and the dashed line is the theoretical IA.

Fig. 8 The estimated IF (a) and the IA (b) of the primary component for Case 2

The backbone and the damping curves are shown in Fig. 9. Where, the estimated results are indicated by dotted lines and the theoretical results by solid lines.

Fig. 9 The identified backbone and the damping curve for Case 2

Then, the average elastic static and average friction forces are identified. The results

are indicated by the dots in Fig.10. Where, theoretical solutions for both kinds of forces are also illustrated for comparison, which are indicated by solid lines.

Fig. 10 The identified average nonlinear characteristic forces for Case 2

Subsequently, system parameters are estimated by using the same fitting procedure that has been adopted in Case 1. The parameter estimation results obtained by using the PCT based method and the FREEVIB under different noise conditions are listed in Table 2. Likewise, the symbol “*” indicates invalid estimation.

Table 2 PCT-based/ FREEVIB-based results under different noise conditions. (Case 2)

SNR	100dB	20dB	2dB
Coefficients			
c_1	-1.6%/-1.4%	-3.2%/*	9.5%/*
c_2	1.5%/0.51%	2.5%/*	2.3%/*
k	-0.76%/-0.087%	-0.38%/*	0.44%/*
F_0	5.0%/0.53%	0.11%/*	-5.4%/*

From Table 2, it is found that FREEVIB fails to work in presence of noise, while the proposed PCT-based method can work very well and accomplish an accurate estimation of the parameters regardless of the noise. Even in the worst case when $SNR = 2dB$, its maximum estimation error is still below 10%.

4.1.2 Forced vibration

Case 3. A nonlinear system that contains Van-der-Pol and Duffing nonlinearities is considered in this case, i.e.

$$\ddot{x} + h\dot{x} + \mu\dot{x}(\dot{x}^2 - 1) + kx + \alpha x^3 = z, \quad (48)$$

with $h = 0.02, \mu = 0.1, k = 1, \alpha = 0.4$.

The excitation z is a harmonic excitation, i.e.

$$z(t) = 0.005 \cos(2\pi \times 0.16t) .$$

Theoretical backbone and damping are

$$\{\omega_0(A)\} = (k + \frac{3}{4}\alpha A^2)^{1/2} , \quad (49)$$

$$\{h_0(A_{\dot{x}})\} = 0.5((h - \mu) + \frac{3}{4}\mu A_{\dot{x}}^2) , \quad (50)$$

The forced response was calculated by using the 4th order Runge-Kutta method under the same noise condition $SNR = 2dB$. The results are shown in Fig.11. Where, Fig. 11(a) and (b) show the TFD and the IF estimation of response during the period of 50s~250s, Fig. 11(c) shows the estimated IA of primary component.

Fig. 11 The TFD (a), the estimated IF (b) and the IA (c) of the primary component for Case 3

From the extracted IF and IA, the backbone and the damping curves are identified. The corresponding identification results are shown in Fig. 12.

Fig. 12 The identified backbone and the damping curve for Case 3

Then, both average elastic static force and average friction force are estimated. The estimation results are shown in Fig. 13.

Fig. 13 The identified average nonlinear characteristic forces for Case 3

To investigate the robust performance of the proposed method against noise, the parameter estimation errors resulted under different noise conditions are listed in Table 3. In which, the estimation errors arisen by the FORCEVIB algorithm are also listed for facilitating comparison. Likewise, “*” indicates invalid estimation.

Table 3 PCT-based/ FORCEVIB -based results under different noise conditions. (Case 3)

SNR	100dB	20dB	2dB
Coefficients			
$(h-\mu)$	1.4%/-0.65%	2.8%/*	0.73%/*
μ	-0.48%/2.4%	-6.3%/*	-4.1%/*
k	1.3%/-0.080%	0.7%/*	5.2%/*
α	7.4%/1.3%	10%/*	10%/*

From the results listed in Table 3, it can be clearly seen that same as FREEVIB does, the FORCEVIB algorithm can work very well only in the absence of noise. Once noise is present, the FORCEVIB algorithm will give invalid estimation to the parameters. By contrast, the proposed PCT-based method can implement accurate estimation regardless of noise. Thus, it can be concluded that the PCT-based method possesses strong robust performance against noise and thus is more suitable to be used to deal with those engineering issues involving heavy noise pollution.

Case 4. A nonlinear vibration system contains combined backlash and dry friction nonlinearities is considered in this case. It is expressed as

$$\ddot{x} + 2h\text{sign}(\dot{x}) + k(x) = z, \quad (51)$$

where

$$k(x) = \begin{cases} 0, & \text{if } |x| \leq \delta \\ \alpha(|x| - \delta)\text{sign}(x), & \text{if } |x| > \delta \end{cases},$$

and $h = 650, \alpha = (2\pi \times 30)^2, \delta = 1.5, z(t) = 13000\sin(2\pi(10 + 7t)t)$.

The theoretical backbone is Eq. (9) and the theoretical damping is

$$\{h_0(A_{\dot{x}})\} = 4h/(\pi A_{\dot{x}}). \quad (52)$$

Apply the same procedure adopted in Case 3 to the system, the IF and IA of the system response extracted by using the PCT-based method are shown in Fig. 14.

Fig. 14 The estimated IF (a) and the IA (b) of the primary component for Case 4

The corresponding backbone and damping are shown in Fig. 15, and the corresponding average elastic force and average friction force are shown in Fig.16. On the left side of Fig. 16 the dashed line indicates the original elastic force, while the discrete dots are the identification results calculated by Eq. (13). According to the parameter estimation algorithm for backlash nonlinearity in Section 2.3, the asymptote

line of the average elasticity curve is parallel to that of the original one, but these two paralleled lines exhibit different intercepts on the abscissa axis. By fitting the discrete dots with the form of the asymptote line, the system parameters can be approximately calculated by using Eq. (28).

Fig. 15 The identified backbone and the damping curve for Case 4

Fig. 16 The identified average nonlinear characteristic forces for Case 4

The errors arisen in the estimations respectively by the proposed PCT based method and the FORCEVIB algorithm under various noise conditions are listed in Table 4 for comparison. Symbol “*” indicates invalid estimation.

Table 4 PCT-based/Feldman's method-based results under different noise conditions. (Case 4)

SNR Coefficients	100dB	20dB	2dB
h	6.6%/-6.9%	6.9%/*	7.5%/*
α	3.3%/-0.34%	1.2%/*	1.78%/*
δ	-2%/-2.5%	-3.3%/*	0.27%/*

Apparently, the strong noise robust performance of the proposed PCT based method in estimating nonlinear system parameters is proved once again by the results listed in Table 4.

4.2 Practical experiment

To further verify the effectiveness of the proposed method in real life, a test rig consisting of rigid foundation, tension mechanism with an adjustable tightening slider, ruler springs and aluminum mass is developed as shown in Fig.17. Since the deflection of the thin steel rulers is nonlinear, the test rig is intrinsically an SDOF nonlinear vibration system.

For a structure made of metal material, its deformation will change nonlinearly against applied force after the deformation reaches a critical value. In the meantime, phase lag will occur between structure deformation and the applied force due to inner friction and energy dissipation of the material. This hysteretic nonlinearity in stiffness of steel ruler is characterized by the elastic restoring force, which contains nonlinear orders of displacement and nonlinear orders of velocity [49,50].

Fig. 17 The test rig: mass (1), tension mechanism (2), rigid foundation (3), and ruler springs (4)

The setting up of the experiments are shown in Fig. 18. Where, a laser displacement sensor was used to measure free response of the system in hammer striking tests and an LMS system was used to collect impact force and displacement response. An example of the impact force and displacement response that are measured in the experiment is shown in Fig.19. Where, the measured impact force is plotted in Fig.19a, and dynamic response in Fig.19b.

Fig. 18 Setting up of the experiment: (a) test rig and hammer; (b) data acquisition device

Fig. 19 Time-domain impulse force (a) and response signal (b)

From Fig.19, it is found that the noise contained in the measured signals is small and ignorable. To fulfil system parameter estimation, the first 16s of dynamic response signal shown in Fig.19b was analyzed by using both conventional spectral analysis and the PCT. The results are shown in Figs.20a and b. The corresponding IF and the IA curves extracted from primary solution are shown in Figs. 20c and d.

Fig. 20 The analysis of response signal: the spectrum (a), the time-frequency distribution (b), the instantaneous frequency (c), and the instantaneous amplitude (d)

Based on extracted IF and IA, one can estimate the instantaneous modal parameters and identify backbone and damping curves, which are shown in Figs. 21a and b, respectively. The results shown in Fig.21a suggests that polynomial dominates almost all system response except the preloaded nonlinearity dominates the response only when the response is small. The identified damping curve shown in Fig.21b indicates that damping could be expanded into polynomial. Considering the aforementioned hysteretic nonlinearity in stiffness and referring to the popular hysteretic nonlinear models in Ref. [51], a tractable model with hysteresis loop, $F = k_1x + k_2x^3 + c_1\dot{x} + c_2\dot{x}^3$, is selected to describe the nonlinear stiffness. It is shown in Fig. 21c.

Fig. 21 Identified backbone (a), the damping curve (b), and the hysteresis loop of cubic nonlinearity model (c)

Subsequently, equivalent average nonlinear elastic restoring force $\{k(x)\} = k_1x + \frac{3}{4}k_2x^3 + \frac{4}{\pi}F_0\text{sign}(x)$ with $F_0 = \alpha k_1$ and equivalent average nonlinear damping force $\{f(\dot{x})\} = c_1\dot{x} + \frac{3}{4}c_2\dot{x}^3$ are assumed. Their identification result and corresponding fitted lines are shown in Fig. 22. Where, discrete points indicate the identification results and dotted lines are the fitted results. Then, the specific parameters in each term can be estimated and following which, the estimated parameters are substituted into the identified model to obtain the dynamic equation of the system.

Fig. 22 Identified equivalent nonlinear characteristic forces

Finally, the mass-normalized motion differential equation can be represented as

$$\ddot{x} + c_1\dot{x} + c_2\dot{x}^3 + k_1x + k_2x^3 + \alpha k_1\text{sign}(x) = 0 \quad (53)$$

where $c_1 = 0.2167, c_2 = 6.1213, k_1 = (2\pi \times 37.8539)^2, k_2 = 6.9330 \times 10^9, \alpha = 8.8707 \times 10^{-8}$.

In order to confirm that the equation above can correctly describes the dynamic characteristics and responses of the nonlinear vibration system, the system response calculated by using the estimated system parameters and the actual system response measured from the test rig are compared, as shown in Fig.23a. To facilitate analysis, the estimation errors are also calculated. The results are shown Fig.23b.

Fig. 23 Comparison between the artificial response and the real response: the time-domain signal (a), the absolute error (b)

From Fig.23c, it is seen that the estimation errors are very small and ignorable. Thus, it can be concluded that the proposed method is indeed an effective method for system identification.

5. Concluding remarks

In view of the limitations of existing nonlinear system identification techniques, a new system identification method is studied in this paper based on the PCT, which has been proved a powerful tool in achieving explicit TFD of non-stationary signals. The proposed PCT based system identification method is implemented by 4 steps, i.e. 1) nonlinear system is transformed into a slow-varying system of primary solution; 2) instantaneous modal parameters are estimated with IF and IA of response; 3) mapping average instantaneous modal parameters versus instantaneous characteristics to obtain backbone and damping curve; 4) parameter estimation is achieved by fitting curves of the identified average nonlinear force. (This should not be part of research conclusion !!)

Both numerical and experimental verifications have proved that the proposed PCT based method is superior to the standard FREEVIB and FORCEVIB algorithms and show strong robust performance against noise. The experiments has shown that the PCT based method is still able to perform accurate nonlinear system identification even under serious noise contamination condition (e.g. SNR=20 dB), while the FREEVIB and FORCEVIB algorithms can work only in the absence of noise or when the noise is very small and ignorable (e.g. SNR=100 dB).

In the future, the work will be extended to deal with MDOF systems and the further verification of the capability of proposed PCT based method in processing multicomponent signals.

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Appendix A

Glossary & Abbreviation

Abbr.	Expansion	Abbr.	Expansion
CT	Chirplet transform	PTFA	Parametric time-frequency analysis
CWT	Continuous wavelet transform	PCT	Polynomial chirplet transform
FREEVIB	Free vibration identification method	SDOF	Single-degree-of-freedom
FORCEVIB	Forced vibration identification method	SNR	Signal noise ratio
GT	Gabor transform	STFT	Short time Fourier transform
HT	Hilbert transform	TFA	Time-frequency analysis
HHT	Hilbert-Huang transform	TFD	Time-frequency distribution
HVD	Hilbert Vibration Decomposition	TFPCT	time-frequency fusion technique based on PCT
IA	Instantaneous amplitude		
IF	Instantaneous frequency	WVD	Wigner-Ville distribution
MDOF	Multi-degrees-of-freedom	WT	Wavelet transform

Appendix B

There are two approaches, one is FREEVIB based on free vibration and another is FORCEVIB based on forced vibration, can be used to identify the modal parameters of nonlinear systems, of which either or both stiffness and damping can be nonlinear. However, they only take into account the primary solution of the system. While, all the other high-order frequency components are ignored by both algorithms. The FORCEVIB can identify the inertia modal parameter, while FREEVIB only allows the identification of the elastic and damping modal parameters. The procedure of implementing FREEVIB and FORCEVIB can be briefly described as follows:

- Perform the HT of the measured excitation and response signals and extract their envelopes and IFs;
- Calculate the instantaneous modal parameters, e.g. frequency, damping, and mass value;
- Filter the obtained modal parameters with the aid of an appropriate low-pass filter, calculate the scale factor functions around the selected extrema points of displacement and velocity, and **scale the smooth modal parameters** (Not sure what does this mean).

- Present the results to backbones and damping curves, FRF, and static characteristics of excitation force.

Then, the initial nonlinear vibration model can be identified based on the static characteristics of excitation force. (Already mentioned earlier before procedure description, so deleted)