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MODELLING AND ANALYSIS OF ELECTROMAGNETIC TORQUE IN DIRECT DRIVE PM MACHINES WITH ROTOR ECCENTRICITY

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Abstract

The influence of rotor eccentricity on torque oscillations as well as average output torque of permanent magnet (PM) machines with integral number of slots per pole per phase has been studied using 1-dimensional (1D) analytical modelling and 2-dimensional (2D) finite element (FE) method. It is shown that cogging and ripple torque are amplified by rotor eccentricity due to the contribution of new flux density components created by eccentricity, while this amplification follows a quadrature function against the eccentricity degree. Furthermore, it is shown that the average electromagnetic torque can be increased by rotor eccentricity provided that magnetic saturation is negligible. The analytical predictions have been validated by FE analysis of a direct drive PM machine.

1 Introduction

Large machines such as wind power direct drive PM generators are prone to rotor eccentricity due to the manufacturing tolerances and aerodynamic force on the rotor. Many papers have investigated the rotor eccentricity influence on vibration and noise, covering a vast research on cogging torque, unbalanced magnetic pull (UMP), condition monitoring, 3-dimensional types of eccentricity and mitigation techniques, [1-19].

In case of vibrations, most of the research focus has been on Unbalanced Magnetic Pull (UMP). Specifically in symmetrical machines with number of slots per pole and phase equal to integer, only few studies investigate the eccentricity and torque correlation. None of existing research has carried out on the influence of eccentricity on the average torque.

In [13] and [18], Kim and Wang show that the cogging of symmetrical machines with 6-slot/ 8-pole, 9-slot/12-pole and 12-slot/ 8-pole, is unchanged by eccentricities as large as 50%, while the influences of SE and DE are identical.

Furthermore, they show that for asymmetric designs such as those with 9-slot/8-pole and 9-slot/10-pole, static and dynamic eccentricities create new cogging frequencies, viz. $8x$ and $10x$ for static and $8x \pm x$ and $10x \pm x$ for dynamic eccentricity, where x is the rotor mechanical frequency.

Similarly, in [9], it is shown that cogging for 9-slot/12-pole machine is unchanged by eccentricity, whereas in asymmetric designs such as that with 9-slot/8-pole, the amplitude of cogging waveform will significantly increase.

In [19], Yoon shows that the cogging of 6-slot/ 8-pole PM design will have extra frequency components due to static and dynamic eccentricities (SE and DE), viz. $8x$ and $6x$, respectively. This finding is in contrast with the one in [13] and needs further investigation.

Dorrell in [4] shows that the torque ripples change due to 50% eccentricity is negligible for a 4 pole-18 slot PM machine, while in [7] he suggests that the SE might reduce the average torque due to deterioration of flux linkage.

In the present paper the influence of eccentricity on cogging, torque ripple and average torque of symmetrical machines with integral number of slots per pole per phase is investigated using a simple 1D permeance model, while the predictions are assessed by FE modelling of a direct drive PM machine. To achieve a more physical understanding of the eccentricity influence and numerical results, the torque is viewed as a superposition of flux density interactions associated with different airgap lengths around the machine circumference.

2 Analytical modelling

2.1 Torque oscillations

The electromagnetic torque in a PM machine can be viewed as the resultant action of tangential stress, τ , on the stator/rotor surface, (1), while τ can be calculated using Maxwell stress tensor, (2). In (1) and (2), r_0 is the radius of the airgap, l_z is the active length of stator, α is the angular

location in the airgap in stator reference frame and in mechanical radians, finally indices r and t refer to radial and tangential directions.

$$T = r_0 I_z \int_{\alpha_1}^{\alpha_1+2\pi} \tau d\alpha \quad (1)$$

$$\tau = B_r(r, \alpha) B_t(r, \alpha) / \mu_0 \quad (2)$$

To have non-zero time-varying torque component, the space and time orders of the contributing flux density components must fulfil the conditions set in (3) and (4).

$$|n| = |n'| \quad (3)$$

$$\text{If } n + n' = 0, \text{ then } m + m' \neq 0 \quad (4)$$

$$\text{If } n - n' = 0, \text{ then } m - m' \neq 0$$

Based on (1-4), the oscillating torque for a concentric machine, T_{conc} , can be expressed by (5), in which the contributing flux density components fulfil the conditions (3) and (4) and for cogging they can be shown to be one due to magnet waves and the other due to slotting components.

$$T_{conc}(\omega_r t) = \sum_{i,j} B_{ri} \cos(n\alpha + m\omega_r t) B_{tj} \cos(n'\alpha + m'\omega_r t) \quad (5)$$

Including the additional flux density components for an eccentric machine, the oscillating torque can be expressed as (6), in which the upper most term is the non-perturbed component, viz. T_{conc} , and the rest of the terms are due to the interaction of new flux density components due to eccentricity. Furthermore, δ_0 is the effective airgap length at concentricity, and $\hat{\delta}_{ecc}$ is the change of airgap due to eccentricity.

$$\begin{aligned} T_{SE}(\omega_r t) = & \sum_{i,j} B_{ri} \cos(n\alpha + m\omega_r t) B_{tj} \cos(n'\alpha + m'\omega_r t) \\ & + \sum_{i,j} \left[\frac{1}{2} \left(\frac{\hat{\delta}_{ecc}}{\delta_0} \right) B_{ri} \cos((n \pm 1)\alpha + m\omega_r t) \right. \\ & \left. * \frac{1}{2} \left(\frac{\hat{\delta}_{ecc}}{\delta_0} \right) B_{tj} \cos((n' \mp 1)\alpha + m'\omega_r t) \right] \\ & + \sum_{i,j} \left[\frac{1}{4} \left(\frac{\hat{\delta}_{ecc}}{\delta_0} \right)^2 B_{ri} \cos((n \pm 2)\alpha + m\omega_r t) \right. \\ & \left. * \frac{1}{4} \left(\frac{\hat{\delta}_{ecc}}{\delta_0} \right)^2 B_{tj} \cos((n' \mp 2)\alpha + m'\omega_r t) \right] \end{aligned} \quad (6)$$

Based on (6), the following comments and conclusions can be made.

- a) It can be seen that the additional flux density terms in (6) are the eccentricity-borne children of the same mother waves which contribute to T_{conc} . As a result, the extra

oscillating torque due to eccentricity is strongly coupled to slotting, thereby minimizing the slotting effect would significantly reduce the former.

- b) The cogging and ripple torque are amplified due to the new flux density components created by rotor eccentricity. Note that for ripple torque, armature reaction adds to the airgap flux density components, but for a symmetric machine it can be shown that the new flux density components will have similar time and space orders as for no load conditions.
- c) The cogging and torque ripple are polynomial functions of the rotor eccentricity whose order, depending on the behaviour of tangential flux density components, may vary between 2 to 4. In (6), it is assumed that the modulation of tangential components is similar to radial parts.
- d) The oscillating torque at DE conditions can be derived in a similar procedure, where it can be shown that the torque components would have the same frequencies as in case of SE and concentricity.

2.2 Average torque

A. 1D permeance model

To obtain non-zero time-invariant torque in a PM machine, the airgap flux components must fulfil the condition (3), as well as (7). As a result, the average torque for a concentric machine, T_{DC} , can be approximated by (8), while the modified average torque at eccentricity conditions can be expressed by (9). In (8) and (9), the indices “arm” and “1,1” refer to armature reaction and fundamental flux density components.

$$\text{If } n + n' = 0, \text{ then } m + m' = 0 \quad (7)$$

$$\text{If } n - n' = 0, \text{ then } m - m' = 0$$

$$T_{DC,conc} \propto \hat{B}_{r,PM,1,1} \cdot \hat{B}_{t,arm,1,1} \quad (8)$$

$$T_{DC,SE} \propto \left(1 + \frac{1}{2} \left(\frac{\hat{\delta}_{ecc}}{\delta_0} \right)^2 \right) \hat{B}_{r,PM,1,1} \cdot \hat{B}_{t,arm,1,1} \quad (9)$$

Based on (8) and (9) the following comments and predictions can be made.

- a) The amplification factor of the fundamental radial flux density component is due to eccentricity and has been validated in [14].
- b) Rotor eccentricity may increase the average torque, depending on the amplitude modulation of eccentricity in

tangential direction, which is outside the scope of the 1D model.

B. 1D lumped-circuit model

To have a more physical understanding of the eccentricity impact on radial airgap flux density and average torque, the latter can be considered as a superposition of torque produced by each half slice of the machine, viz. the half with airgaps smaller than nominal airgap and the half with airgaps larger than the nominal airgap. In this case, the radial airgap flux on no load, B_{gap} , can be calculated for different airgap lengths, according to (10), where l_m is the radial thickness of the magnet, μ_r is the relative permeability of the magnet, B_r is the remanence flux density of the magnets, and $\Delta\delta$ is the extra virtual airgap accounting for magnetic saturation of the stator core.

$$B_{gap} = \frac{B_r}{1 + \frac{\mu_r (\delta + \Delta\delta(B_{gap}))}{l_m}} \quad (10)$$

Fig.1 shows the variation of airgap flux against different airgap lengths, for both NdFeB and Ferrite magnets. Based on Fig.1, the following comments and conclusions can be made.

- The increase and decrease of the radial flux density against different airgap lengths can be visually judged based on the slope of the corresponding curve shown in the figure.
- In case the saturation can be neglected, e.g. in the case of Ferrite magnets, the increase of radial flux in the slice with smaller airgaps is larger than the decrease of the same in the slice with larger airgaps.

(a)

(b)

Fig.1. Impact of airgap length on no load radial airgap flux density with SE60%. (a) Ferrite magnets. (b) NdFeB magnets.

- In case of strong magnets, viz. NdFeB magnets, the increase of the radial flux density in the slice with smaller airgaps is limited by extra reluctance due to saturated stator core. As a result, the increase of the radial flux in one slice of the airgap is almost equal to the decrease of the same in the other slice.
- As a result of (b) and (c), and due to the fact that average torque is proportional to the no load radial flux, it can be concluded that the eccentricity may increase the resultant average torque while this increase will be limited by saturation.

3 Finite element validation and analysis

3.1 Torque oscillations

As the presented 1D analytical model does not predict the behaviour of the flux density in tangential direction, FE analysis has been provided to both validate and extend the analytical findings. The same 3kW PM prototype as in [14] is chosen for FE modelling.

(a)

Fig.2. Impact of rotor eccentricity on time-variant torque. (a) No load. (b) Full load. (c) Cogging vs. eccentricity degree, for NdFeB. (d) Cogging vs. eccentricity degree, for Ferrite.

Fig.3. Impact of eccentricity on average torque of design with Ferrite.

The time-variant torque on both no load and full load conditions have been studied for different degrees of SE and DE and shown in Fig.2. To investigate the saturation impact, similar practices have been made for both NdFeB and Ferrite magnets, with remanence flux density of about 1.2 T and 0.4 T, respectively. Based on Fig.2, the following comments and conclusions can be made.

- SE and DE have same impact on cogging and ripple torque.
- It validates the analytical prediction that eccentricity may increase the cogging and ripple torque significantly. As this impact is valid for both strong and weak magnets, it is independent from magnetic saturation, Figs.2 (c) and (d).
- It validates the analytical prediction that cogging amplitude vs. eccentricity degree follows a polynomial function of order 2.

3.2 Average torque

The average torque of the available prototype has been studied by FE, employing both NdFeB and Ferrite magnets, Fig.2 (b) and Fig.3. As can be noticed, the average torque is increased in case of weak magnet, but almost unchanged in case of strong magnets. The following comments and conclusions can be made.

- The amplification of average torque is in consistence with the analytical prediction. It implies that the modulation of fundamental flux density component in tangential direction is less significant than in radial direction. This impact has been earlier realized in the flux density analysis in [14], where on no load, eccentricity was shown to cause no amplitude modulation of the fundamental flux in tangential direction.
- It confirms the 1D lumped-circuit model prediction that the increase of the average torque only occurs for designs where no magnetic saturation is present.

4 Conclusion

This paper shows that the cogging and ripple torque of a symmetric PM machine with integral number of slots per pole and phase may be significantly increased due to eccentricity, whereas the amplification may have a quadrature trend against the eccentricity degree. It is shown that the cogging and ripple amplification is independent from the magnetic saturation. Furthermore, it was shown that the average torque may be increased by eccentricity provided that the saturation can be neglected. FE analysis confirms the analytical predictions and supports them in the case of tangential flux density assessments.

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