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An integrated simulation method for interaction between manufacturing process and machine tool

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Abstract: The interaction between the machining process and the machine tool (IMPMT) plays an important role on high precision components manufacturing. In this paper a novel simplified method is proposed to realize the simulation of IMPMT by combining use the finite element method and state space method . In this method, the machine tool's transfer function is built as a small state space which obtained from the complicated finite element model of the machine tool. Furthermore, the control system of the machine tool is integrated with the transfer function of the machine tool to generate the cutting trajectory. Then, the tool tip response caused by cutting force is used for the machined surface prediction. Finally, a case study is carried out for a fly-cutting machining process, the IMPMT simulation is carried out on fly-cutting and the machining results have verified the effectiveness of this method.

Keywords: Dynamic analysis; state space; surface generation; machining interaction; precision machining

1. Introduction

In order to meet stringent tolerance and surface finish, a great deal of attention has been paid to the machining process and the machine tool (MPMT) itself [1-3]. Most of the improvement was made on the MPMT performance separately, while the interaction between them has been always overlooked. It has been recognized that the interaction of machining process and the machine tool (IMPMT) plays an important role in the machining performances, particularly for the precision machining, where IMPMT directly affects the material removal rate, and machined surface quality and dimensional as well as form accuracy [4-7]. It is necessary to research into the IMPMT to improve the machining accuracy, and optimize the machining process and the next generation machine tool design. Therefore, it is essential to carry out the investigation on IMPMT.

There are two main methods for the machine tool dynamic performance analysis, one is the numerical method based on lumped mass models. It is suitable for the single and simple component analysis and has been applied to the design of spindles [8-10] and slides [11]. Although this method is less time consuming, the simulation accuracy is limited due to the finite dimension approximation. Another method is the finite element (FE) method, which is widely employed as an effective approach to studying statics, dynamics, and thermal aspects of single components or whole complex machine tools [12-16] due to its high computational accuracy, convenience of result interpretation and capture of time/spatial details. However, a FE model of the machine tool usually has more than 10,000 degrees of freedom (DOFs), which causes

the transient response calculation of the machine tool under the cutting force very time consuming, and hence affects the computational efficiency and the design cycle of the machine tool development. For studying the IMPMT more flexible, a simple but accuracy simulation method is urgently needed, which can realize the IMPMT quickly and accurately. In this study, a novel method for machine tool dynamic response analysis under the cutting force is proposed by combining the state space model and the FE method. The proposed integrated method has the advantages both the lumped mass model and FE method. In this method, a dynamic model of the machine tool in state space is built based on the results of complex FE models representing the machine tool system, which reduces the amount of computational significantly, but still provides precise responses for the forcing function input and desired output points. The dynamic response of the machine tool is used to simulate the contour profile of the machined surface, and in order to realize the precise simulation in the whole size of the workpiece, the control system is also considered to generate the cutting trajectory in the whole machining process, the cutting force generated in the manufacturing process introduce the IMPMT. Thus, the influence of IMPMT on the machined results is achieved rapidly and precisely.

2. Integrated method for machine tool dynamic performance analysis

The traditional and the proposed integrated method for machine tool dynamic performance are outlined in Fig.1. The modelling procedure of this method can be described as follows. Firstly, the CAD model of a machine tool is converted into a FE model which contains all the details required to describe the dynamic performance of

the machine tool system, such as geometries of machine structures, moving components and bearings. Solutions are then obtained under the specified boundary and loading conditions. Secondly, different to the traditional method where time-consuming dynamic analysis on the full FE model must be performed, the modal analysis results of the FE model are used to establish a mathematical model, which can be solved by means of a computer program rapidly. Lastly, the machining trajectory generated by the control system is integrated with the mathematical model, and the machining process simulation and the machined surface generation are carried out in the state space model instead of running the time-consuming FE models. The machine tool dynamics and the IMPMT are still built into the model, however, the computational time is reduced significantly.

Fig.1 The framework of the integrated method.

2.1 Establishment of the state space model

The establishment of the state space model is the critical step in this method. It starts out with the modal analysis of the FE model to obtain eigenvalues and eigenvectors (resonant frequencies and mode shapes). The Block Lanczos method is used as eigenvalue extraction technique which can calculate all the eigenvalues and eigenvectors in a specific frequency range. The size of mass matrix M , stiffness matrix K and damping matrix C is $n \times n$, respectively, where n denotes the DOFs of the model. There are as many eigenvalues and eigenvectors as DOFs for the model, although they provide considerable insight into the system dynamics, the extraction of all these eigenvalues and eigenvectors would account for extremely high computational resource in the simulation, and it is typically too large to be used in a mathematical model. To reduce computational resource and to obtain the solutions in an efficient manner, only the DOFs of the nodes where forces applied and outputs of interests are extracted. Then the state space is built using these eigenvalue results obtained from the modal analysis. After then, the frequency and time domain analyses are carried out based on the state space model. This step realizes the transformation of the finite element model to the state space model, which can reduce the time cost in the following frequency and time domain analyses.

2.2 Theoretical basis:

In the second step, the aim is to obtain an efficient, 'small' model representing the mechanical and servo system of the machining system. It needs to reduce the size of the model and keep the accuracy of calculation, as well as maintaining the desired input-output relationship. The theoretical basis of establishing a 'small' state

space model using limited eigenvalues and eigenvector information is explained as follows (eigenvector entries for all modes only for input and output degrees of freedom). For a given structure, a FE model can be established with mass matrix M , damping matrix c and stiffness matrix K , the size of each matrix is $n \times n$. Therefore, the fundamental equation describing the dynamic behaviour of a structure discretized by FE can be written as:

$$M\ddot{x} + c\dot{x} + Kx = F \quad (1)$$

where F denotes a n -dimensional vector designating the force applying on each degree of freedom, x denotes the displacement vector, caused by the force. In order to uncouple the Eq. (1) the damping matrix can be replaced by using the Lord Rayleigh's hypothesis:

$$c = \alpha M + \beta K \quad (2)$$

where α is the constant mass matrix multiplier for alpha damping, and β is the constant stiffness matrix multiplier for beta damping. Thus Eq.(1) can be rewritten as:

$$M\ddot{x} + (\alpha M + \beta K)\dot{x} + Kx = F \quad (3)$$

After the coordinate decoupling transformation, Eq. (3) is changed as:

$$\ddot{x}_p + (\alpha I + \beta \Omega)\dot{x}_p + \Omega x_p = F_p \quad (4)$$

which can also be written as:

$$\ddot{x}_{pi} + 2\varepsilon_i \dot{x}_{pi} + \omega_i^2 x_{pi} = F_{pi} \quad (i=1, 2, \dots, n) \quad (5)$$

where, ω_i is the value of i_{th} natural frequency;

$\Omega = \text{diag}(\omega_i^2)$ is a diagonal matrix;

$\varepsilon_i = \frac{\alpha + \beta \omega_i^2}{2\omega_i}$ is the damping ratio for the i^{th} mode.

The state space method can be convenient to tackle the problem of frequency domain response and time domain response so the dynamic equation described by state space is,

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (6)$$

defining states:

$$\begin{cases} x_1 = x_{p1} \\ x_2 = \dot{x}_{p1} \\ x_3 = x_{p2} \\ x_4 = \dot{x}_{p2} \\ \dots\dots\dots \\ x_{2n-1} = x_{pn} \\ x_{2n} = \dot{x}_{pn} \end{cases} \quad (7)$$

differentiating Eq(7) gives

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega_1^2 x_1 - 2\varepsilon_1 \omega_1 x_2 + F_{p1} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\omega_2^2 x_3 - 2\varepsilon_2 \omega_2 x_4 + F_{p2} \\ \dots\dots\dots \\ \dot{x}_{2n-1} = x_{2n} \\ \dot{x}_{2n} = -\omega_n^2 x_{2n-1} - 2\varepsilon_n \omega_n x_{2n} + F_{pn} \end{cases} \quad (8)$$

Eq(8) can be written in the matrix form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dots \\ \dot{x}_{2n-1} \\ \dot{x}_{2n} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ -\omega_1^2 & -2\varepsilon_1 \omega_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & -\omega_2^2 & -2\varepsilon_2 \omega_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & -\omega_n^2 & -2\varepsilon_n \omega_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dots \\ x_{2n-1} \\ x_{2n} \end{pmatrix} + \begin{pmatrix} 0 \\ F_{p1} \\ 0 \\ F_{p2} \\ \dots \\ 0 \\ F_{pn} \end{pmatrix} \quad (9)$$

and its short form:

$$\dot{x} = A x + B \quad (10)$$

Where system matrix A is constituted by the natural frequency and the damping ratio, the ω_i and x_n are obtained from modal analysis.

The input matrix B is formed by F_p .

$$\begin{Bmatrix} F_{p,1} \\ F_{p,2} \\ \dots \\ F_{p,2n} \end{Bmatrix} = \begin{Bmatrix} x_{1,1} & x_{2,1} & \dots & x_{2n,1} \\ x_{1,2} & x_{2,2} & \dots & x_{2n,2} \\ \dots & \dots & \dots & \dots \\ x_{1,2n} & x_{2,2n} & \dots & x_{n,2n} \end{Bmatrix} \begin{Bmatrix} F_1 \\ 0 \\ \dots \\ 0 \end{Bmatrix} = \begin{Bmatrix} x_{1,1}F_1 \\ x_{1,2}F_1 \\ \dots \\ x_{1,2n}F_1 \end{Bmatrix} \quad (11)$$

For the space state Eq(9), applying F_l on the DOFs of the nodes where forces applied, the other DOFs are set to be zero, so when calculating F_p only the first column needed, as shown in Eq(11).

The x solved in Eq(9) are in principal coordinates, it should be transferred to physical coordinates, $y = x_n x$, Rewriting it in the matrix form:

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ \dots \\ y_{2n-1} \\ y_{2n} \end{Bmatrix} = \begin{Bmatrix} x_{n11} & 0 & x_{n12} & 0 & \dots & x_{n1n} & 0 \\ 0 & x_{n11} & 0 & x_{n12} & \dots & 0 & x_{n1n} \\ x_{n21} & 0 & x_{n22} & 0 & \dots & x_{n2n} & 0 \\ 0 & x_{n21} & 0 & x_{n22} & \dots & 0 & x_{n2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{nn1} & 0 & x_{nn2} & 0 & \dots & x_{nnn} & 0 \\ 0 & x_{nn1} & 0 & x_{nn2} & \dots & 0 & x_{nnn} \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dots \\ x_{2n-1} \\ x_{2n} \end{Bmatrix} \quad (12)$$

which is

$$y = Cx + D \quad (13)$$

In the response analysis, the DOFs where inputs applied and outputs of interests are assumed as the first m DOFs. For modal analysis, the modal vector of these m DOFs is then used to form the modal matrix $X_{m \times n}$.

For the state space Eq(12), only the displacement response of the first m DOF is

needed:

$$\text{First } m \text{ DOF: } \begin{Bmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{Bmatrix} = \begin{Bmatrix} x_{n11} & 0 & x_{n13} & 0 & \dots & x_{n1n} & 0 \\ x_{n21} & 0 & x_{n22} & 0 & \dots & x_{n2n} & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{nm1} & 0 & x_{nm2} & 0 & \dots & x_{nmn} & 0 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dots \\ x_{2n-1} \\ x_{2n} \end{Bmatrix} \quad (14)$$

Therefore, the size of the matrix is reduced significantly.

3. A case study – IMPMT of fly-cutting machining

3.1 The modeling process of the integrated method

The integrated method for IMPMT has been used to simulate the potassium dihydrogen phosphate (KDP) crystals fly-cutting machining process [17]. The dynamic performance has been found to have significant influence on the machining results for this types of machine tools i.e. periodicities ripples have been found on the machined KDP crystal surfaces. The periodicities ripples would lead different mechanisms and differing degrees of crystal laser damage threshold [18-20], which directly affect the component's function. To study the generation reason of the ripples, the attention should be paid on the IMPMT. The integrated method proposed in this study is used to obtain the dynamic response of this machine tool during the machining process and establish the relationship between the machine process and the machine tool. The modelling process of the method is illustrated in Fig. 2. The model is generated in commercial FE software, the elements use in the FE model of the machine tool are as follows: the solid element 186 is used to mesh the solid parts of the machine tool; the spring element combin14 is used to represent the aerostatic

bearing in the spindle and hydrostatic bearing in the slide, and the axial stiffness of the linear motors; the joint performance between the adjacent components are simulated by contact elements conta173 and targe170 elements; pre-load element Prets179 is used to model the bolt joint in the machine tool. There are 17000 elements and approximately 100,000 degrees of freedom in this model. Then, a state space model is created to generate low order models of complicated systems by defining the DOFs required for the desired frequency response. In this study, the cutting force only apply at the node locating at the tool tip in the x , y and z direction, and the output is the corresponding displacement of the tool tip cause by the cutting force. Therefore, only this DOFs of one node of the tool tip is required for the state space model. It makes the degree numbers required for the state space model reduced from 100,000 to 3, which is approximately 33000 times smaller than the original FE model.

Fig. 2 The state space built from the FE model.

Fig. 3 shows the results of tool tip response calculated by the integrated method and the finite element method, respectively. It can be found that the computational error is less than 2% between the two methods. The finite element method takes three hours for the transient response under cutting force, while the integrated method only takes one second, all of the simulation are carried out with the same computer configuration (i5-2300@2.8GB). It shows that the integrated method improves the computational efficiency significantly.

Fig.3 The tool tip response comparison between the FE method and the integration method.

Figure 4a) illustrates the structure diagram of the fly-cutting machine tool, the diamond cutter rotates with a air spindle to remove the material of the workpiece feeding by , and a hydrostatic slide.

Fig.4 The forming shape movements generated by the control system.

For calculating the cutting force response by using with the MATLAB function “*lsim*”, a time vector, “*t*” and input vector “*u*” are defined. The “*t*” and “*u*” are generated by the control system. The product of material-dependent cutting coefficient and instantaneous area of the uncut chip are used to model the cutting force. In precision fly-cutting, the tool radius R is significantly larger than the feedrate f , and depth of cut h , therefore, the instantaneous area of the uncut chip can be approximately as fh . The cutting, F , is obtained by:

$$F = kfh\delta \quad (15)$$

Where k is the specific cutting energy; f is feed speed; h is the depth of cut; δ is used to toggle the intermittent contact of the workpiece and tool and is defined as unity during contact and zero the remainder of the time.

The cutting path generated by the control system is shown in Fig.4 b), it can be found three regions named A', B', and C' on the workpiece. Fig.4 c) shows a cutting force over two revolutions in region B'. The duration time of the of the cutting force in each part is expressed by Eq(16).

$$t' = \begin{cases} 2R \arccos\left(\frac{R-ft}{R}\right) / 2\pi Rn' & , \quad t \leq \sqrt{R^2 - \frac{L^2}{4}} / f \\ 2R \arcsin \frac{L}{2R} / 2\pi Rn' & , \quad \sqrt{R^2 - \frac{L^2}{4}} / f < t < L/f \\ 2R \left(\arcsin \frac{L}{2R} - \arccos \frac{R-(ft-L)}{R} \right) / 2\pi Rn' & , \quad t \geq L/f \end{cases} \quad (16)$$

where t' is the cutting time on the workpiece of each rotate (s), L is the length of the workpiece (mm), n' is the rotation speed of the spindle (rev/s), R is the radius of the

cutting head (mm). Then, the whole process of the integrated method used for the ultra precision fly-cutting machine tool is shown in Fig.5. The tool tip responses under the cutting force obtained from the state space method are coupled with the cutting path to generate the machined surface. Fig. 6 a) shows the machined surface profile obtained by this simulation model. It can be seen that this surface simulation model can realize the waviness and the profile simulation.

Fig.5 The integrated method used for a fly-cutting machine tool.

3.2 Experimental verification

The fly-cutting machining tests are carried out using the following machining parameters: a spindle rotational speed of 300 r/min , a feed rate of 60 $\mu\text{m/s}$ and a depth of cut of 15 μm . The material of the workpiece is KDP crystal and the size of the workpiece is 415mm $\times$ 415mm as shown in Fig.6 a). The Wyko RST-plus (Veeco Metrology Group, Santa Barbara, CA, USA) is used to test the machined surface, and the tested result is shown in Fig. 6 b). Fig.6 c) and Fig.6 d) show the simulated and

tested surface profile of the workpiece, respectively. It can be found that there are seven waviness on the simulation and tested surface, the wavelength is 57 mm, the amplitude of the waviness are 50 nm and 60 nm, respectively. The simulation waviness and profile on the machined surface agree well with the experimental result. It indicates that the proposed simulation method is viable on simulation large size workpiece machining. Further, the testing results also validated that the integrated method proposed in this study could assure the surface generation simulation in precision machining.

Fig.6 The surface profile. a) Machine surface; b) Tested surface; c) Simulated profile b)

Tested profile.

4. Conclusion:

This paper presents an integrated method which can realize machine tool dynamic performance analysis rapidly without sacrificing computational accuracy.

The main conclusions drawn are as follows

- (1) A simple machine tool dynamic analysis model is established by integrated the state space and finite element method. Based on this method the dynamic response analysis of the machine tool is achieved rapidly, and hence the computational efficiency is improved significantly.
- (2) The contour profile simulation method of the machined surface was proposed by integrating the whole machining path generated by the control system and the dynamic responses of the machine tool at each machining path. Using this method can achieve the surface simulation with short time and high accuracy.
- (3) This method was used for an ultra-precision fly-cutting machine tool dynamic analysis, the result shows that it can improve the computational time more than 10800 times with less than 2% error compared with traditional method. The experimental test evaluate and validate the effectiveness of this method.

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Abbreviations

α — The constant mass matrix multiplier for alpha damping;

β — The constant stiffness matrix multiplier for beta damping;

δ — is used to toggle the intermittent contact of the workpiece and tool and is defined as unity during contact and zero the remainder of the time;

ω_i — The value of i_{th} natural frequency;

Ω — A diagonal matrix;

ε_i — The percentage of critical damping for the i^{th} mode;

A — System matrix;

B — Input matrix;

c — Damping matrix;

C — Output matrix;

D — Direct transmission matrix;

f — Feed speed;

F — Force applied on each degree of freedom;

F_p — The forcing function;

h — The depth of cut;

k — The specific cutting energy;

K — Stiffness matrix;

L — The length of the workpiece;

M — Mass matrix;

n' — The spindle rotate speed per second;

R — The radius of the cutting head;

t' — The cutting time on the workpiece each rotate;

u — The input scalar;

x_{pi} — Displacement of mode i ;

$\dot{x}_{pi}$ — Derivative of displacement of mode i .