Shi C, Hu Z, Ringsberg J, Luo Y.

[Validation of a temperature-gradient-dependent elastic-plastic material
model of ice with finite element simulations.](#)

Cold Regions Science and Technology 2017, 133, 15-25.

Copyright:

© 2016. This manuscript version is made available under the [CC-BY-NC-ND 4.0 license](#)

DOI link to article:

<http://dx.doi.org/10.1016/j.coldregions.2016.10.005>

Date deposited:

03/11/2016

Embargo release date:

20 April 2018

This work is licensed under a

[Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International licence](#)

1 Validation of a temperature-gradient-dependent elastic-plastic material

2 model of ice with finite element simulations

3 Chu Shi¹, Zhiqiang Hu^{1,2*}, Jonas Ringsberg³, Yu Luo¹

4 1. State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, China

5 2. School of Marine Science and Technology, Newcastle University, Newcastle upon Tyne, UK

6 3. Department of Shipping and Marine Technology, Chalmers University of Technology,
7 Gothenburg, Sweden

8

9 Nomenclature

f	function of the yield surface (MPa ²)
σ	Cauchy stress (MPa)
p	hydrostatic pressure (MPa)
J_2	deviatoric stress tensor (MPa ²)
a_0	first material constant in the yield function (MPa ²)
a_1	second material constant in the yield function (MPa)
a_2	third material constant in the yield function
ε_{eq}^p	effective plastic strain
ε_f	failure strain
ε_0	initial failure strain
$p_{cut-off}$	cut-off pressure (MPa)
σ_v	von Mises stress (MPa)
σ_i	normal stress (MPa)
τ_{ij}	shear stress (MPa)

10

11 **Abstract**

12 A temperature-gradient-dependent elastic-plastic material model of ice is proposed for the
13 numerical study of the influence of temperature-gradient on impact force in ship-iceberg
14 collisions. The model is based on the 'Tsai-Wu'-type yield surface, and an empirical failure
15 criterion is adopted. A series of yield surfaces with different sizes but the same shape are
16 obtained from the linear interpolation of test results to represent the continuous temperature
17 range in an iceberg. Temperature dependence is defined as the change in ice properties due to
18 the temperature gradient as a function of depth of the iceberg. Based on field test data, three
19 types of iceberg temperature profiles are assumed. The ice model is implemented as a
20 user-defined subroutine in the commercial explicit finite element code LS-DYNA. Collisions
21 between a rigid plate and different geometric iceberg shapes are simulated to analyse the
22 influence of iceberg geometry and ice model temperature. The calculated contact area-pressure
23 curves are compared with design laws to further calibrate the proposed ice model. Both a sharp
24 temperature profile and low temperature range can increase the local contact pressure and
25 global contact force as the penetration increases. The simulation results show that the ice model
26 can capture and be used to demonstrate the influence of temperature-gradient on contact force
27 in ship-iceberg collisions.

28

29 Keywords: Ship-iceberg collision; Ice material model; Temperature-dependent; Yield surface;
30 Numerical simulation

31 **1. Introduction**

As the climate changes in the Arctic, ice coverage and thickness continues to decrease during the summer season. Regular transport through the northeastern and northwestern Arctic Sea has become possible. Moreover, it is estimated that approximately 25% of the world's total new oil and gas reserves may be located in the Arctic [1]. These changes will lead to a significant increase in marine and offshore activities in the Arctic region in the coming years. The probability of collisions between icebergs and ships or offshore structures might increase, and severe collisions may lead to oil leakage, causing environmental pollution. Therefore, from the viewpoints of environmental protection and economic demand, research on the crashworthiness and safety of marine and offshore structures under the scenario of iceberg impact must be conducted.

Sea ice is a complex material consisting of solid ice, brine, gas and, depending on the temperature, various types of solid salt [13]. The mechanical properties of sea ice, such as the failure criterion, compressive strength and flexible strength, depend on many factors, e.g., temperature, porosity, salinity, density, microstructure, loading rate and confinement ratio. Temperature, as a basic thermal-mechanical parameter in the growth of sea ice, significantly influences the physical properties of sea ice. For instance, as the temperature decreases, the density of ice crystals increases and the dislocation mobility decreases. The stiffness of ice increases by approximately 25% as the temperature decreases from near the melting point to zero Kelvin [27]. Therefore, temperature is an important factor in research on the mechanical properties of sea ice and ship-iceberg collisions.

According to the NORSOK N-004(2004) code [26], when a structure is designed according to the accidental limited state format, the collision between the iceberg and a rigid plate belongs to the strength design, which implies that the structure is capable of crushing the ice with moderate

structural deformation. In this strategy, the temperature gradient in the iceberg can be completely reflected in the collision process; therefore, temperature may significantly influence the total contact force in ship-iceberg collisions. In this paper, a temperature-gradient-dependent elastic-perfect-plastic ice model is proposed based on the 'Tsai-Wu' yield surfaces presented by Ahmed A. Derradji [15]. These surfaces were fitted from triaxial compressive experiments of icebergs conducted by Gagnon and Gammon [2]. The influence of temperature on the ice model is reflected by the different sizes of the Tsai-Wu yield surfaces. Linear interpolation is applied to obtain yield surfaces at different temperatures. Three iceberg temperature profiles are assumed. The compressive and tensile behaviour of ice are described separately. The effective plastic strain and pressure-driven failure criterion proposed by Gao Y. et al. [17] and Liu Z. et al. [12] are used to determine the element failure during the simulation. Then, the ice model is applied in the simulation cases to study the influence of the iceberg's temperature range and temperature profile on the collision process. Based on a spherical iceberg-rigid plate collision, the effects of temperature on the high-pressure zone of simulated area-pressure curves are discussed. In the simulation of collisions between different iceberg shapes and a rigid plate, the influence of temperature on the total contact force and the significance of temperature effects for different iceberg shapes are analysed. The simulation is conducted with the commercial code LS-DYNA 971. The ice model is realized by a user-defined subroutine.

The simulation of compressive ice with failure is under development for decades and many aspects are not understood in full to develop a consistent material model for design loads. In this study, we do not try to develop a consistent material model to simulate such complex behaviours of ice in ice-structure interaction. The temperature-gradient-dependent elastic-plastic material

model is limited for iceberg under constant range of strain rates. The focus is study the influence of temperature profiles on impact force. Therefore, the details of failure process are ignored. Iceberg properties given to ice elements are from the experimental results conducted at strain rate around $4 \times 10^{-3} \text{ s}^{-1}$, corresponding to ductile-to-brittle transition strain rate in this study [27]. At transition strain rate, ice has the strongest strength and is most dangerous to vessels [31]. At this relatively high strain rate and short impact time, the visco-plastic effect is considered not strong. The irrecoverable strain is approximately considered behave as plastic model. Once again, this model is trying to capture global response of sea ice instead of all details of ice behaviors. If the model is able to simulate are-pressure curve, it is considered to be sufficiently acceptable for the simulation of contact force. Nevertheless, more accurate model, such as viso-elastic, visco-plastic and crack, should be considered in the future. The temperature-gradient model can be incorporated with these models.

2. Temperature characteristics of iceberg

Temperature of icebergs is not constant and changes significantly from the surface to the core of iceberg [7]. In field tests of ship-iceberg or structure-iceberg interactions, there is a limited amount of measured data on the temperature profile of iceberg samples compared with their velocity and approximate mass. This lack of data may be because special equipment, such as a temperature probe, is required to measure the iceberg temperature in the field [8]. In 1995, impact tests between icebergs and an engineering structure were conducted on Grappling Island [9]. The temperature gradient was quite sharp, from $-4 \text{ }^{\circ}\text{C}$ at a depth of 0.05 m into the ice to $-12 \text{ }^{\circ}\text{C}$ at a depth of 0.5 m into the ice. Larger penetrations led to a larger contact force and

pressure with the same iceberg [10]. Iceberg impact tests with the icebreaker CCGS Terry Fox were conducted in 2001, and the temperature profile of some iceberg samples were measured [8][11].

Figure 1 Iceberg temperature data from Jones [7]

One of the most comprehensive reviews of field-testing results is that by Jones [7], in which 27 groups of iceberg temperature data are compared and analysed. Part of these data is shown in Figure 1, in which the temperature decreases with depth because these data were measured in summer or spring, when the air temperature was higher. However, for 6 groups, temperature increases with depth because it was measured in winter, when the atmospheric temperature was low. Temperature typically decreases rapidly from the surface to a depth of 8 m and remains constant at depths deeper than 8 m. Because low temperature leads to firm ice, which has been observed experimentally, the rapid decrease in temperature means that the strength of ice near the surface increases rapidly with increasing depth.

In ship-iceberg collisions, the contact force may increase rapidly as the penetration increases. This phenomenon was observed in a field test by Ralph et al. [10] and was estimated by Timco [9] based on their field test results. Nevertheless, there are few published references about the influence of using temperature-dependent ice material models in the analysis of contact forces in ship-iceberg interactions.

3. Presentation of a material model of ice

3.1 Description of the yield surface and its temperature dependence

In the simulation of ship-iceberg collisions, the ice model is a significant factor influencing the simulation results and typically depends on experimental results. The results of several triaxial compressive experiments with granular sea ice are compared to analyse the effects of temperature on yield surfaces. The advantages and disadvantages of the 'Tsai-Wu' yield surfaces adopted in this paper are discussed.

Tsai-Wu-type yield function (Riska and Frederking [4], Liu Z et al. [12] and Ahmed A. Derradji [14] [15]) and n-type yield function (Timco et al. [5] [6] [13]) are two types of functions have been used to describe the yield surface of ice. Based on the formula derivation in Appendix I, for isotropic ice, these two yield functions can be transited to each other. Therefore, yield surfaces fitted by these two functions can be compared, as shown in Figure 2, and the influence of temperature on yield surface can be studied in a wider experimental data.

Iceberg ice can be regarded as granular sea ice [4] [12]. Therefore, both types of yield functions can be used to represent the mathematical behaviour of icebergs. Few triaxial experiments on iceberg ice have been reported. Triaxial experiments on other types of granular sea ice (e.g.,

multi-year floe ice, laboratory-prepared granular ice) can be applied to verify the yield surface of an iceberg and study the influence of temperature on the iceberg's yield surface. The results from triaxial compressive experiments conducted at a strain rate of approximately 10^{-3} s^{-1} have been presented by several researchers. These results, shown in Figure 2, are described in p-J₂ space to more easily implement in the FE model. The experimental conditions are listed in Table 1.

Table 1 Experimental conditions of the triaxial compressive experiments

Reference	Ice location	Ice structure	Density (kg/m ³)	Salinity (‰)	Failure type	Strain rate (0.001 s ⁻¹)	Tempera- ture (°C)
Riska	Eastern Canadian	Variable grain structure	875~	0.1~0.3	brittle	2	-2/-10
1987	Arctic	(multi-year floes)	900				
Kierkegaard	-	-	-	-	-	-	-
1993							
Timco	Beaufort Sea	granular/discontinuous-colu	-	4.2	ductile	0.2	-12
1984		mnar sea ice					
(biaxial)		(large piece of ice)					
Barrette	Northeastern coast-	homogeneous	895	-	-	1.6-6.5	-6.2~-20
2003	line of Newfoundland	(iceberg)					
Sammonds	Buckingham Island	columnar ice with isotropic	903	1.5	ductile	1	-3~-40
1998		in horizontal plane					
		(multi-year floes)					
Y. Mizuno	Prepared in the	freezing an aggregation of	905	-	Brittle/	2/3	-11
1998	laboratory	snow particle water			ductile*		

R.E. Gagnon	Okak Bay, Labrador	homogeneous	0.7~	-	Brittle/	3~5	-1~-16
1995***		(iceberg)	3.5%**		ductile*		

*Changing with confining pressure; **Fractional porosity; ***Fitted by Ahmed A. Derradji [15]

Figure 2 Triaxial experiment results of granular sea ice

From the series of experiments by each researcher, the influence of temperature is notable and coincident. Based on the experiments of Sammonds [3], Y. Mizuno [28], Riska [4] and Ahmed A. Derradji [15], the second invariant of the deviatoric stress tensor increases with decreasing temperature at the same hydrostatic pressure. Because the yield point of ice is determined by the effective modulus, ice has a larger effective modulus with decreasing temperature. The effective modulus is different from the elastic modulus (E) because the former contains the influence of delayed elastic strain, which must be considered in elastic behaviour [13]. The results are scattered because the strength of ice is affected by many factors, such as the hydrostatic pressure, grain size and porosity, and these factors are not the same in the different experiments.

The yield surfaces from Riska [4] are considerably smaller than the other fitted yield surfaces and most points in the experimental results, possibly due to the low salinity and high porosity of

the ice resulting from brine drainage during transportation. The yield surface proposed by Kierkegaard [12] covered the majority points in the experimental results. However, it lacks experimental information, such as temperature and strain rate, and cannot be applied to study temperature effects. Ahmed A. Derradji [15] obtained a series of yield surfaces with different temperatures (1, -6, -11 and -16 °C, shown in Figure 3). The strength of the ice increases linearly with decreasing temperature in the range of -1 to -11 °C. In the temperature range of -11 to -16 °C, the yield surface increases sharply. The yield surface at -16 °C is larger than most experimental results but fits with part of the test results from Y. Mizuno [28] and Sammonds [3]. In general, all of the surfaces fitted by Ahmed A. Derradji [15] are located within a reasonable region compared with the other results and cover all points of the tests results, which will produce conservative results during simulation. The temperature range (-1 to -16 °C) corresponds well with the temperature gradient from the iceberg surface to most penetration depths in ship-iceberg collisions.

An increase in hydrostatic pressure causes a decrease in the melting temperature of ice [16]. For example, ice will melt at -4 °C when confined by approximately 50 MPa of pressure. This phenomenon is called pressure melting. Hydrostatic pressure also significantly affects the deformation mechanism during compressive experiments – cracking at low pressure and dynamic recrystallization at high pressure [2]. Therefore, the high-pressure (dotted lines in Figure 2) and low-pressure parts of the P - J_2 yield surface will not be symmetrical. The high-pressure part, which is extrapolated from experimental results at low pressure, is not accurate. The accurate high-pressure part of the P - J_2 yield surface must be further investigated. In our simulation below, the pressure of each element is approximately 20 MPa, which does not belong to the

high-pressure range.

3.2 Implementation of the Tsai-Wu yield surface model

The Tsai-Wu yield surfaces fitted by Ahmed A. Derradji [15] are used to study the influence of temperature on ice properties in ship-iceberg collisions due to the reasons discussed in section 3.1. Two ice samples from the same series will not behave equally as they are affected by many factors. Nevertheless, scatter from the same series experiments is not severe. Therefore risk of accuracy of material data from the same series experiments is acceptable. At different temperatures, yield surfaces with the same shape but different sizes are applied to ice. Gagnon and Gammon [2] conducted triaxial iceberg compressive experiments at different temperatures, strain rates and confining pressures. Based on these experiments, Ahmed A. Derradji [15] proposed a series of Tsai-Wu-type yield surfaces (Equation (2)) for different temperatures (-1, -6, -11 and -16 °C) and strain rates (from 4×10^{-5} to $2.7 \times 10^{-1} \text{ s}^{-1}$).

$$f = J_2 - (a_0 + a_1 p + a_2 p^2) = 0 \quad (2)$$

The sea ice strength is affected by many factors, such as strain/stress rate, temperature and porosity. In some cases, for example uniaxial compression strength, the dominated factor is strain/stress rate besides ice temperature. Nevertheless, in this study, the focus is the influence of temperature-gradient on sea ice behaviour. Therefore, a constant strain rate is assumed. As the compressive strength reaches a maximum at the ductile-to-brittle transition, strain rate corresponding to the transition is adopted in this study. In the scale of this study (around 3m), the transition strain rate is $10^{-4} \sim 10^{-2} \text{ s}^{-1}$ [27]. Therefore, triaxial compressive experiments of Gagnon and Gammon [2] conducted at strain rate around $4 \times 10^{-3} \text{ s}^{-1}$ are adopted. The

Tsai-Wu-type yield surfaces with different temperatures and a uniform strain rate, approximately $4 \times 10^{-3} \text{ s}^{-1}$, are adopted (see Figure 3). The parameters of these elliptical curves are listed in Table 2. The increase rate of parameters a_0 , a_1 and a_2 between -11 and -16 °C is considerably higher than that between -1 and -11 °C. Therefore, to obtain yield surfaces at temperatures between the test values (-1 , -6 , -11 and -16 °C), linear interpolation is applied between -1 and -11 °C and between -11 and -16 °C, respectively.

Table 2 Parameters of Tsai-Wu yield surfaces for different temperatures

Temperature (°C)	Strain rate (10^{-3} s^{-1})	a_0	a_1	a_2
-1	4.33	22.794	2.051	-0.02279
-6	3.75 to 5.57	31.736	2.856	-0.03174
-11	4.4	39.366	3.542	-0.03937
-16	3.75 to 5.57	65.921	5.932	-0.06592

Figure 3 Elliptical yield surfaces with different temperatures

211

212 The temperature of ice in an iceberg typically decreases with increasing depth of the iceberg.

213 From the field test results [7][8] (some of the results are shown in Figure 1), the temperature

214 profile is linear or exponential between depths of 0 and 2 m, which is the penetration range of

215 ship-iceberg collisions. Therefore, the temperature profiles applied in ship-iceberg collisions are

216 assumed to be linear curves and exponential curves, respectively, as illustrated in Figure 4. If

217 using the field data, interpolation between two data points has been done when the depth of ice

218 element locates between these two points. The implementation of interpretation in FE model is

219 tedious. Also it is not accurate and the obtained distribution of temperature is discontinuous. In

220 order to get accurate and continuous value of temperature of each ice element and to

221 conveniently calculate temperature value, the assumed temperature profiles are used. The

222 maximum depth is 1.3 m in this figure, slightly shallow compared with 2 m. This small maximum

223 depth is because the penetration depth in our iceberg model is approximately 1.3 m. To reflect

224 the entire temperature profile, the maximum depth is set at 1.3 m. A-exponential curve is

225 moderately steeper than the field test results because some rare temperature gradients in

226 icebergs were reported, such as [10]. B-exponential and linear types are coincident with most of

227 the field test results.

Figure 4 Assumed temperature profiles of icebergs and equations of these curves

The implementation of temperature effects in the FE model is illustrated in Figure 5. In the simulation program, the distances between each ice element and the surface of the iceberg are calculated before the collision calculation. Based on the distance, the precise temperature of each element is obtained from the temperature profiles (shown in Figure 4). Then, based on linear interpolation of values in Table 2, accurate yield surfaces, which will be applied in the following impact calculations, can be obtained for each ice element.

Figure 5 Flowchart of the implementation of temperature effects in the FE model

3.3 Failure criterion applied in the ice material model

The empirical failure criterion proposed by Gao Y. et al. [17] and Liu Z. et al. [12] is adopted in this paper. The failure criterion is based on the effective plastic strain and hydrostatic pressure:

$$\varepsilon_{eq}^p = \sqrt{\frac{2}{3} \varepsilon_{ij}^p \cdot \varepsilon_{ij}^p}$$

$$\varepsilon_f = \varepsilon_0 + \left(\frac{p}{10^8} - 0.6\right)^2 \quad (3)$$

$$p < p_{cut-off}$$

If the pressure is smaller than the cut-off pressure, $p_{cut-off}$, or the effective plastic strain is larger than the failure strain, ε_f , the ice element fails and is deleted from the calculation to simulate erosion.

Figure 6 Energy curves from the simulation of iceberg-rigid wall collision

Energy curves directly from iceberg-rigid wall collision simulation results are illustrated in Figure 6 to show energy balance during eroding process. Total energy and external work are almost the same means energy ratio (Energy ratio=total internal/(initial internal energy+external work)). Initial internal in this model is zero) is around 1. This fits well with energy balance requirement (energy ratio=1). Hourglass energy is much less than 10% of total internal energy means hourglass deformation is under good control. Total internal energy equals to the sum of eroded internal energy and existed internal energy. This shows that energy is balance in eroding process. Erosion of elements violates the global mass equilibrium. Nevertheless, when the ice element is going to be deleted, its apparent stiffness is decreasing significantly which means it can not sustain any load. Therefore, erosion of elements has minor effect on the simulation of mechanical process. The erosion of elements is capable of describing some ductile failures of ice, such as pressure melting. Also, it is considered as be capable of describing the small extrusions during interaction process. In case of brittle crushing, erosion can not capture the fracture and

265 brittle flaking ice. The method from fracture mechanics, for example, cohesive element can be
266 applied in describing this phenomenon. Though there are some shortcomings of erosion method,
267 up to now, erosion is a common way to simulate failure in ice especially when we focus on the
268 simulation of mechanical process. In the simulation of ship-iceberg collisions, once the element is
269 deleted, the neighbouring element behind the eroded element will partially unload over a short
270 penetration length (one element length). When the 'neighbouring element' impacts the ship, it
271 will be reloaded. This cyclic load behaviour is not described in the current ice model because no
272 cyclic loading experiments have been conducted on icebergs. Ahmed A. Derradji [18] proposed
273 an elastic-plastic constitutive model for freshwater, columnar-grained S-2 ice that considers the
274 cyclic loading condition.

275 In summary, a temperature-dependent isotropic, elastic-plastic ice model is formulated. The
276 influence of temperature is reflected by different sizes of 'Tsai-Wu' yield surfaces. A cut-off
277 pressure and effective strain failure criteria are applied in tensile and compressive stress states,
278 respectively. The implementation of temperature effects is illustrated in Figure 5. The elastic
279 response of the ice element is calculated with Hooke's law, and the cutting plane algorithm is
280 used to calculate the plastic consistency parameter and stress tensor in the plastic state. For
281 more details of the implementation of the elastic-plastic calculation in the FE model, the reader is
282 referred to Gao Y. et al. [17]. This material model is incorporated into the commercial software
283 LS-DYNA by a user-defined subroutine.

284 **4. Finite element simulations**

285 Loads from iceberg collisions belong to an Abnormal Level Ice Event (ALIE), which corresponds
286 to the accidental limit state in modern codes for offshore structures [12]. The contact

area-contact pressure relationship proposed by Masterson et al. [19] is recommended by the ISO code, ISO/CD 19906(2010) [30], for the ice loads in an ALIE. As ice failure process in ice-structure interaction is complex (for example, number and locations of high pressure zones keep changing, stochastic micro- and macro-fractures appear and propagate), measured area-pressure data are relatively extensive. The global pressure design curve in ISO 19906(2010) is in fact an regression line fitted to the average plus 3 times standard deviation and covers most of measured data. The probability of ice load can not be reflected in this model and the focus here is the simulation of design area-pressure curve.

In the first part of this section, the ice model is calibrated by comparison between simulated area-pressure curves and design curves, including the ISO curve. Because the local shape of an iceberg significantly influences ship-iceberg collisions, the temperature effects on different iceberg geometries are analysed in the second part.

4.1 Sphere-shaped Iceberg collisions with a rigid steel plate

As recommended by McKenna [29], a sphere can represent the mean iceberg model shape. Therefore, a spherical iceberg-rigid plate collision is simulated in this section. The area-pressure curves are compared with design curves to analyse temperature effects. The sphere-shaped iceberg with a radius of 1 m is fixed through the opposite part of the collision side, shown as white points in Figure 7. The mesh size of the iceberg is determined based on the convergence analysis conducted of Gao Y. et al. [17] and is set at 50 mm × 50 mm × 50 mm. The details of the iceberg material are shown in Table 3, which are the same as those in reference [17]. The AUTOMATIC_SURFACE_TO_SURFACE contact algorithm is applied for the iceberg-ship collision, and 'soft option 2' is used to obtain more accurate results. Considering the erosion of icebergs,

ERODING_SINGEL_SURFACE is applied to simulate contact between the new ice surfaces generated from erosion, and 'soft option 1' is used in this contact algorithm. The static and dynamic friction coefficients are both set to 0.15. The collision phenomena (e.g., high-pressure and low-pressure zones) and sensitivity analysis of the material parameters were studied by Gao Y. et al. [17]; here, we focus on the temperature effects. Three temperature profiles, shown in Figure 4, are applied to study the influence of different temperature profiles. Because the temperature range of an iceberg may change with seasons and locations, it is useful to study the influence of the temperature range on the area-pressure curve. The temperature ranges adopted here are -1~-16 °C (means from -1 to -16 °C), -8~-16 °C and a constant -1 °C.

Figure 7 Illustration of a spherical iceberg-rigid steel plate collision. White points are fixed, and a constant velocity of 1 m/s in the z-axis is applied to the rigid steel plate.

Table 3 Iceberg material details in reference [17]

Element type	Solid	Density (kg/m ³)	900
--------------	-------	------------------------------	-----

Number of element nodes	8	Poisson's ratio	0.3
Number of element integration point	1	Young's modulus (MPa)	9500
Element length (mm)	50	Cut-off pressure; tension strength (MPa)	2
Strain rate	$>10^{-3}$	Strain hardening function	None
Limit of the elastic strain	10^{-3}	Limit of elastic stress	9.5

Figure 8 Comparison of the contact area-contact pressure relationships between the simulation and design codes

Figure 9 Details of the contact area-contact pressure relationship (logarithmic expression) within a small contact area

Figure 8 compares pressure-area relationships between the simulation results and design curves. The simulated area-pressure exhibits fluctuations, particularly at the beginning of contact. This may be because the mesh size (50 mm) of the iceberg is relatively large when the analysis pressure is in a small contact area. The deletion of one element may cause a large fluctuation in contact force. When the contact areas are large, the pressures stabilize. For a contact area larger than 1.5 m², the design curves achieve consensus, and the simulated curves fit well with all of the design curves except for that proposed by Timco [20], which is considerably lower than the others. In a small contact area (smaller than 0.75 m²), significant differences exist among the different design curves. The average line of the simulated area-pressure curves corresponds well with the Molikpaq design curve [20]. In an extremely small contact area (smaller than 0.1 m²), the simulated results increase rapidly. The maximum pressure is considerably higher than that of the Molikpaq design curve and is close to the API/CSA [21] and ISO [19] curves. The typical character of the area-pressure curve, rapid pressure increase as the contact area become smaller, is observed in the simulated results, and the simulated curves are located in a reasonable range compared with the different design curves. All of the simulated area-pressure curves with different temperatures exhibit the same trend, which means that temperature only affects the size of the pressure-area curve. According to Figure 8, temperature mainly affects the high-pressure zone, which determines the maximum local pressures sustained by the ship and is significant in the design process.

To study the temperature effects in detail, the high-pressure zones of the area-pressure curves are illustrated in logarithmic form in Figure 9. First, in same temperature range of $-1\sim-16\text{ }^{\circ}\text{C}$, the results from different temperature profiles are compared. The curve for exponential-b is slightly larger than that of the linear type, and the exponential-a type is considerably larger than both of the former curves in the majority of the contact area. This trend occurs because the exponential-a profile is steeper than the other two profiles, indicating that the iceberg has many more low-temperature elements around its surface. Ice at low temperatures has a 'larger' yield surface (see Figure 3) and requires a higher deviatoric stress to be damaged than ice at high temperatures. Therefore, low-temperature ice leads to a high contact force. This detail is discussed in section 4.2. The exponential-b profile is moderate and similar to the linear type, especially for the portion at low temperatures, which is why these two results are similar. Second, three temperature ranges with the same linear profile, -1 , $-1\sim-16$ and $-8\sim-16\text{ }^{\circ}\text{C}$, are adopted. The $-8\sim-16\text{ }^{\circ}\text{C}$ curve is larger than the other two curves, and the $-1\sim-16\text{ }^{\circ}\text{C}$ curve is slightly larger than the $-1\text{ }^{\circ}\text{C}$ curve. This is coincident with the fact that low-temperature elements lead to a high contact force. The linear type at a temperature range of $-8\sim-16\text{ }^{\circ}\text{C}$ is similar to the exponential-a type at a temperature range of $-1\sim-16\text{ }^{\circ}\text{C}$ because both the temperature profile and temperature range directly affect the distribution of ice at low temperatures around the surface of the iceberg. Therefore, at some time, they have the same effects on the contact force. From Figure 9, the order of the curve sizes does not remain constant at all times, particularly for the linear curve at $-1\text{ }^{\circ}\text{C}$.

The high-pressure zone of the area-pressure curve is important because it significantly influences the local design process. Therefore, based on the analysis of simulated area-pressure

curves, the temperature profile and temperature range should be considered in the design process, especially for local structure design. In fact, temperature can also influence the global response of the ship in ship-iceberg collisions, which will be discussed in section 4.2.

4.2 Combined effects of the geometric shape and temperature-gradient-dependent material of ice

The local shape of icebergs can affect their behaviour in ship-iceberg collisions. Using the FEM, Storheim et al. [22] and Gao Y. et al. [23] analysed the sensitivity of iceberg shape in ship-iceberg interactions with different material models of icebergs. They found that blunt-shaped icebergs act rigidly and that sharp-shaped icebergs crush easily. Iceberg material models have a greater influence on sharp icebergs than on blunt ones. Frederking and Timco [24] conducted laboratory impact tests with large-scale ice floes that had different local shapes. They found that the maximum load was a function of the shape of the impact interface.

A combination of the geometric shape and temperature-gradient-dependent material properties of ice is assessed. Four different types of icebergs, prism, cone, sphere and tube, are applied in the simulation of an iceberg-rigid plate collision. The ice material is the same as that in section 4.1. The geometry and mesh of the icebergs are illustrated in Figure 10. The typical mesh size is 50 mm × 50 mm × 50 mm. Because the time step of the computation depends on the smallest element size, the central part of the cone, which has smaller elements than the outer part and does not collide with the rigid plate, is ignored to acquire a larger time step. To supply an even force distribution on the iceberg and avoid local ice element deletion near the load application region, a layer of rigid elements is attached to the back of the iceberg. A constant

velocity of 2 m/s is applied on the rigid layer. The iceberg temperature gradients from the surface to the inside are assumed as follows: -1~-16 °C, -8~-16 °C and constant -1 °C. The temperature profiles are assumed to be linear and exponential-a. The computational time is sufficiently long to reflect the temperature gradient.

Figure 10 a) Cube length=3 m; b) cone height=1.75 m, top radius=3.3 m, down radius=2.3m; c) prism height length=3 m; d) sphere radius=2 m

Figure 11 Force-penetration relationships of conical, spherical and prismatic icebergs.

Figure 12 Force-penetration relationships of a cubic iceberg at different temperature ranges.

The influence of temperature on the global contact force is analysed for spherical, cubic and conical icebergs. Figure 11 and 12 show the contact force of different iceberg shapes with different temperature ranges and temperature profiles. As penetration increases, the amount of contact elements increase, which leads to a larger contact force. The temperature gradient is reflected, and more ice elements with low temperatures contact the rigid plate. Therefore, the differences between the different temperature ranges become larger. The maximum contact force decreases as the temperature range increases. The temperature profile has the same effect on the contact force. From the comparison between the linear and exponential temperature profiles of a conic iceberg, a sharper temperature profile yields a larger contact force. From Figures 10 and 11, a sharp shape leads to a lower contact force than does a blunt shape. This corresponds with the simulation results of references [19] and [20] and the trends of the experimental results from [24].

The detailed reasoning behind low temperatures leading to a high contact force in the

simulation is discussed here. At low temperatures, a 'large' size of the yield surface is adopted; therefore, at the same hydrostatic pressure (p), the yield of the ice element requires a high second invariant of the deviatoric stress tensor (J_2). J_2 has a linear relationship ($\sigma_v = \sqrt{3J_2}$) with the von Mises stress, which is used to measure the strength of the material under multiaxial loading conditions. This means that the ice is stiff at high J_2 and requires a high amount of force to fail. Therefore, high J_2 leads to a higher contact force when the ice element has failed. For instance, in conic iceberg collision, Figure 13 shows the J_2 at an exponential temperature profile of -1~-16 °C and at a constant temperature of -1 °C, respectively. In this figure, the amount of elements with higher J_2 at an exponential profile is considerably larger than that at a constant temperature. Therefore, the contact force of the exponential profile is high.

Figure 13 J_2 at a constant temperature gradient of -1 °C (left); J_2 at an exponential temperature gradient of -1~-16 °C (right)

The significance of temperature effects on the contact force follows different trends for different iceberg shapes. Figures 10 and 11 illustrate that temperature has the most significant effect on cubic icebergs and has a moderate effect on spherical and conic icebergs. For prismatic icebergs, there is only a slight difference between the different temperatures, possibly because the stress concentration is severe in prism-shaped icebergs. Only a small contact force yields high stress in ice elements. At a temperature range of -1~-16 °C, a small contact force is sufficient to

make the ice element yield; therefore, there is nearly no difference between the different temperatures cases. Furthermore, the confinement of elements is also weak in prism-shaped icebergs. Hydrostatic pressure (p), which can reflect the confinement condition of elements, is shown in Figure 14. At the moment shown in the figure, the contact areas of the prism, cone and sphere are the same (3.2 m^2). The contact area of the cube remains constant in the collision process, and it is larger than that of the other three shapes. Based on the 'Tsai-Wu' yield surface at a pressure range of 0 to 45 MPa, the difference in J_2 between different temperatures increases with the growth of hydrostatic pressure. Therefore, the influence of temperature is more significant with heavier confinement. The percentage of the area with high confinement elements is treated as the criterion of the confinement station. From Figure 14, the confinement of elements in a cubic iceberg is the strongest, and the prismatic iceberg has the weakness confinement. This may be another reason for the lack of influence of temperature in the prismatic iceberg in collision with a rigid plate. Several separate high confinement areas and low confinement areas can be observed in the icebergs in Figure 14. Though this does not look like pressure distributions from real ice, it reflects, to some extent, the characteristic of separated high-pressure and low-pressure zones and the characteristic of birth and death cycle of high pressure zones [25].

Figure 14 Hydrostatic pressure of different iceberg shapes. At this moment, the contact areas are

the same except for the cube shape. a-cube; b-prism; c-sphere; d-cone

5. Conclusions

Under accidental limited state design conditions, a temperature-gradient-dependent, elastic-perfect plastic ice model is proposed for the simulation of ship-iceberg collisions. Temperature effects are calibrated using several design and experimental area-pressure curves, including the ISO rule. Based on the analysis and comparison of existing triaxial experiments on granular sea ice, the results of Gagnon and Gammon [2] are adopted for the 'Tsai-Wu'-type yield surfaces in this ice model. The influence of temperature on the ice model is reflected by a series of yield surfaces, which are fitted from experimental results at different temperatures. Linear interpolation is adopted to obtain yield surfaces between the test temperatures. Based on field test data, three iceberg temperature profiles are assumed. The cut-off pressure and effective plastic strain based on Gao Y. et al. [17] and Liu Z. et al. [12] are applied as failure criteria for the tensile stress state and compressive stress state, respectively. The advantages and shortcomings of the yield surface applied in this model are discussed. The n-type yield function for granular sea ice is proven to be equivalent to the 'Tsai-Wu'-type yield surface by formula derivation and comparison of experimental results.

Based on the simulation of a spherical iceberg-rigid plate collision, the influence of the

temperature range ($-1\text{ }^{\circ}\text{C}$, -1 to $-16\text{ }^{\circ}\text{C}$ and -8 to $-16\text{ }^{\circ}\text{C}$) and temperature profiles (exponential-a, exponential-b and linear) of spherical ice on the local contact pressure are analysed. Using four types of icebergs—cube, cone, sphere and prism—a combination of geometrical-shape- and temperature-gradient-dependent material properties of ice is studied. The analysis focuses on the global contact force generated by different types of icebergs. Reasons for the different influence of temperature on different iceberg shapes are proposed. The most important results are as follows:

(1) A temperature-gradient-dependent, elastic-perfect plastic ice material model is proposed for the numerical study of the influence of temperature-gradient on contact force in ship-iceberg collisions in the accidental limited state design condition. Temperature effects are reflected by a series of ‘Tsai-Wu’ yield surfaces. The model is calibrated by comparison between simulated area-pressure curves and design lows.

(2) The n-type yield function for isotropic granular sea ice is proven to be equivalent to the ‘Tsai-Wu’ yield surface of sea ice.

(3) The high hydrostatic pressure part of the ‘Tsai-Wu’ yield surface should be lower than the low-pressure part, instead of being symmetrical with it, due to the pressure melting of ice.

(4) In the simulation of iceberg-rigid plate collisions, a low-temperature range and sharp temperature profile leads to high contact pressure in local contact conditions and high contact force in global contact conditions. Temperature has a greater effect on highly confined iceberg shapes (e.g., cubic iceberg) than on less confined iceberg shapes (e.g., prismatic iceberg).

The collision between an iceberg and rigid plate belongs to the strength design in accidental

limited state format. The influence of temperature on local contact pressure and global contact force can benefit this design process. In the future, the ice model can also be applied in the simulation of contact force in ship structure-iceberg collisions, which belong to the shared-energy design. Relatively random distribution of ice load is one of the main characters in iceberg structure interaction. The current model can not reflect probability of ice load mainly due to the absence of description of stochastic fracture behavior. In the future, crack simulation method, for example cohesive elements, can be incorporated with this model to capture probability of ice load.

Acknowledgements

The work contained in this paper is part of a joint-research project between the State Key Laboratory of Ocean Engineering at Shanghai Jiao Tong University and the Department of Shipping and Marine Technology at Chalmers University of Technology. The Natural Science Fund of China (Grant No. 51239007) also supported this research. This support is greatly appreciated by the authors. Moreover, the authors would like to thank Dr. Yan Gao at Shanghai Jiao Tong University for her help on the programming work of the user subroutine code in LS-DYNA.

References

- [1]Bergan, P.G., Cammaert, G., Skeie, G., Tharigopula, V., 2010. On the potential of computational methods and numerical simulation in ice mechanics. IOP Conference Series: Materials Science and Engineering 10.
- [2] GAGNON R. E, GAMMON P. H. Triaxial experiments on iceberg and glacier ice[J]. Journal of

521 Glaciology, 1995.

522 [3] Sammonds P R, Murrell S A F, Rist M A. Fracture of multiyear sea ice[J]. Journal of Geophysical
523 Research Oceans, 1998, 103(C10):21795–21815.

524 [4]K. Riska, R. Frederking, 1987. Ice load penetration modelling. Proc. of the 9th Port and Ocean
525 Eng. under Arctic Conditions Conf. Fairbanks, Alaska, USA, pp.317-327.

526 [5] Timco G W, Frederking R M W. Confined compression tests: Outlining the failure envelope of
527 columnar sea ice[J]. Cold Regions Science & Technology, 1986, 12(86):13–28.

528 [6] Timco G W, Frederking R M W. An investigation of the failure envelope of
529 granular/discontinuous-columnar sea ice[J]. Cold Regions Science & Technology, 1984, 9(84):17–
530 27.

531 [7] Jones S J. A review of the strength of iceberg and other freshwater ice and the effect of
532 temperature[J]. Cold Regions Science & Technology, 2007, 47(3):256–262.

533 [8] Ralph F, Mckenna R, Gagnon R. Iceberg characterization for the bergy bit impact study[J]. Cold
534 Regions Science & Technology, 2008, 52(1):7–28.

535 [9] Timco G W. Isolated ice floe impacts [J]. Cold Regions Science & Technology, 2011, 68(1):35–
536 48.

537 [10]Ralph, F., Mckenna, R., Crocker, G., Croasdale, K., Pressure/area measurement from the
538 Gappling Island iceberg impact experiment. Proceedings IAHR Symposium on Ice, 2, pp. 171-178,
539 St. Petersburg, Russia.

540 [11]Johnston M, Ritch R, Gagnon R, et al. Comparison of impact forces measured by different
541 instrumentation systems on the CCGS Terry Fox during the Bergy Bit Trials[J]. Cold Regions
542 Science & Technology, 2008, 52(1):83-97.

543 [12] Liu Z, Amdahl J, Løset S. Plasticity based material modelling of ice and its application to ship–
544 iceberg impacts[J]. Cold regions science and technology, 2011, 65(3): 326-334.

545 [13] Timco G W, Weeks W F. A review of the engineering properties of sea ice[J]. Cold Regions
546 Science & Technology, 2010, 60(2):107–129.

547 [14] Derradji-Aouat A. Multi-surface failure criterion for saline ice in the brittle regime[J]. Cold
548 Regions Science & Technology, 2003, 36(2):47–70.

549 [15] Derradji-Aouat A. A unified failure envelope for isotropic fresh water ice and iceberg ice. Proc.
550 of the ETCE/OMAE-2000 Joint Conference, Energy for the New Millennium, New Orleans, USA,
551 OMAE2000/P&A-1002.

552 [16] Nordell B. Measurement of P-T coexistence curve for ice-water mixture[J]. Cold regions
553 science and technology, 1990, 19(1): 83-88.

554 [17] Gao Y, Hu Z, Ringsberg J W, et al. An elastic–plastic ice material model for ship-iceberg
555 collision simulations[J]. Ocean Engineering, 2015, 102: 27-39.

556 [18] Derradji-Aouat A, Sinha N K, Evgin E. Mathematical modelling of monotonic and cyclic
557 behaviour of fresh water columnar grained S-2 ice[J]. Cold regions science and technology, 2000,
558 31(1): 59-81.

559 [19] Masterson D M, Frederking R M W, Wright B, et al. A revised ice pressure-area
560 curve[C]//Proceedings of the International Conference on Port and Ocean Engineering Under
561 Arctic Conditions. 2007.

562 [20] Timco G W, Sudom D. Revisiting the Sanderson pressure–area curve: Defining parameters
563 that influence ice pressure[J]. Cold regions science and technology, 2013, 95: 53-66.

564 [21]API. American Petroleum Institute Recommended Practice for Planning, Designing and

565 Constructing Structures and Pipelines for Arctic Conditions. 1220L Street N.W., Washington, DC
566 200005:1995.

567 [22] Storheim M, Kim E, Amdahl J, et al. Iceberg shape sensitivity in ship impact assessment in
568 view of existing material models[C]//ASME 2012 31st International Conference on Ocean,
569 Offshore and Arctic Engineering. American Society of Mechanical Engineers, 2012: 507-517.

570 [23] Gao Y, Hu Z, Wang J. Sensitivity Analysis for Iceberg Geometry Shape in Ship-Iceberg Collision
571 in View of Different Material Models[J]. Mathematical Problems in Engineering, 2014, 2014.

572 [24] Frederking R, Timco G. Sea Ice Floe Impacts-Large Scale Basin Experiments[C] //Proceedings
573 of the Tenth (2000) International Offshore and Polar Engineering Conference ISOPE'00. 2000, 1:
574 640-645.

575 [25] Gagnon R E. A numerical model of ice crushing using a foam analogue[J]. Cold regions
576 science and technology, 2011, 65(3): 335-350.

577 [26] NORSOK N-004, 2004. Design of Steel Structures, Standards Norway, Oslo, Rev 2.

578 [27] Schulson E M, Duval P. Creep and fracture of ice[M]. Cambridge: Cambridge University Press,
579 2009.

580 [28] Mizuno Y. Effect of hydrostatic confining pressure on the failure mode and compressive
581 strength of polycrystalline ice[J]. The Journal of Physical Chemistry B, 1998, 102(2): 376-381.

582 [29] McKenna R. Iceberg shape characterization[C]//Proceedings 18th International Conference
583 on Port and Ocean Engineering under Arctic Conditions. 2005, 2: 555-564.

584 [30] ISO/CD 19906, 2010. Petroleum and Natural Gas Industries-Arctic Offshore Structures, ISO
585 TC 67/SC 7/WG 8, Final Draft International Standard. International Standardisation Organization,
586 Geneva, Switzerland (2010), p. 434.

[31] Michel B, Toussaint N. Mechanisms and theory of indentation of ice plates [J]. Journal of Glaciology, 1978, 19:285-300.

APPENDIX I

Ralston [5][6] proposed an n-type yield function to describe granular sea ice, as shown in formula (1)

$$f(\sigma_i, \tau_{ij}) = a_1(\sigma_x^2 + \sigma_y^2) + a_2(\sigma_x - \sigma_y)^2 + a_3(\sigma_x + \sigma_y) - 1 = 0 \quad (1)$$

It can be rewritten as

$$\begin{aligned} f(\sigma_i, \tau_{ij}) &= a_1((\sigma_x + \sigma_y)^2 - 2\sigma_x\sigma_y) + a_2((\sigma_x + \sigma_y)^2 - 4\sigma_x\sigma_y) + a_3(\sigma_x + \sigma_y) - 1 \\ &= (a_1 + a_2)(\sigma_x + \sigma_y)^2 + a_3(\sigma_x + \sigma_y) - (2a_1 + 4a_2)\sigma_x\sigma_y - 1 \\ &= (a_1 + a_2)A^2 + a_3A - (2a_1 + 4a_2)B - 1 = 0 \end{aligned} \quad (2)$$

where $A = \sigma_x + \sigma_y$ and $B = \sigma_x\sigma_y$.

Formula (3) is the 'Tsai-Wu' yield surface function.

$$f = J_2 - (b_0 + b_1p + b_2p^2) = 0 \quad (3)$$

where J_2 is the deviatoric stress tensor and p is the hydrostatic pressure. Their expression is shown in Equations (4) and (5), respectively.

$$J_2 = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2] \quad (4)$$

$$p = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \quad (5)$$

For the x-y plane condition, $\sigma_z = 0$. Then, Equations (4) and (5) can be rewritten as (6) and (7), respectively.

$$J_2 = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_x)^2 + (\sigma_y)^2] = \frac{1}{6}[2(\sigma_x + \sigma_y)^2 - 6\sigma_x\sigma_y] = \frac{1}{3}A^2 - B \quad (6)$$

$$p = \frac{\sigma_x + \sigma_y}{3} = \frac{A}{3} \quad (7)$$

609 From (6) and (7), we can obtain $A = 3p$ and $B = 3p^2 - J_2$. Substituting these two formulas

610 into (2), we can obtain

611
$$f(\sigma_{ij}) = J_2 + \left(\frac{3a_1 - 3a_2}{2a_1 + 4a_2} \right) p^2 + \frac{3a_3}{2a_1 + 4a_2} p - \frac{1}{2a_1 + 4a_2} \quad (8)$$

612 Equation (8) can be equivalently transferred to Equation (3). Therefore, the n-type yield function

613 and 'Tsai-Wu' yield function are actually the same for granular sea ice.

614