Yigit S, Chakraborty N.

[Rayleigh-Bénard power-law fluid convection in rectangular enclosures.](#)

***Journal of Thermophysics and Heat Transfer* 2017, 31(4), 805-816.**

Copyright:

Copyright © 2016 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. All requests for copying and permission to reprint should be submitted to CCC at www.copyright.com; employ the ISSN 0887-8722 (print) or 1533-6808 (online) to initiate your request. See also AIAA Rights and Permissions www.aiaa.org/randp.

DOI link to article:

<http://dx.doi.org/10.2514/1.T5108>

Date deposited:

30/12/2016

Rayleigh-Bénard power-law fluid convection in rectangular enclosures

Sahin Yigit* and Nilanjan Chakraborty†

Newcastle University, Newcastle-Upon-Tyne, NE1 7RU, UK

Influences of aspect ratio (ratio of height to length) on laminar Rayleigh-Bénard convection of power-law fluids in rectangular enclosures have been numerically investigated for constant wall heat flux boundary condition for horizontal walls. The steady state simulations have been conducted for the range of aspect ratio 0.25 to 4, nominal Rayleigh number range 10^3 to 10^5 , power-law index 0.6 to 1.8 for a representative single value of nominal Prandtl number (10^3). It has been found that convective transport weakens with increasing aspect ratio and thermal conduction dominates thermal transport for tall enclosures. Moreover, the critical Rayleigh number for the onset of convection increases with increasing values of power-law index and aspect ratio. Thermal convection irrespective of the value of aspect ratio has been found to augment with increasing (decreasing) Rayleigh number (power-law index) due to strengthening of buoyancy force in comparison to viscous resistance with increasing Rayleigh number (shear-thinning behaviour with decreasing power-law index). The simulations reveal that flow patterns and mean Nusselt number are dependent on the initial condition, and it is possible to obtain different steady-state solutions for different initial conditions. The numerical findings have been explained with the help of scaling arguments and in turn have been utilised to propose a correlation for the mean Nusselt number.

* PhD student, School of Mechanical and Systems Engineering, Newcastle University Newcastle Upon Tyne NE1 7RU, UK / s.yigit1@ncl.ac.uk

† Professor of Fluid Dynamics, School of Mechanical and Systems Engineering, Newcastle University Newcastle Upon Tyne NE1 7RU, UK / nilanjan.chakraborty@ncl.ac.uk

Nomenclature

AR	Aspect ratio [-]	α	Thermal diffusivity [m ² /s]
a_0, a_1	Correlation parameter [-]	β	Thermal expansion coefficient [1/K]
b	Correlation parameter [-]	δ	Velocity boundary layer thickness [m]
c_p	Specific heat at constant pressure [J/kgK]	δ_{th}	Thermal boundary layer thickness [m]
e	Relative error [-]	θ	Dimensionless temperature [-]
e_{ij}	Rate of strain tensor [s ⁻¹]	μ	Dynamic viscosity [Ns/m ²]
g	gravitational acceleration [m/s ²]	ν	Kinematic viscosity [m ² /s]
Gr	Grashof number [-]	ρ	Density [kg/m ³]
H	Height of the enclosure [m]	$\tau_{ij}(\tau)$	Stress tensor (stress) [N/m ²]
h	Heat transfer coefficient [W/m ² K]	ψ	Stream fuction [m ² /s]
K	Consistency [N.s ⁿ /m ²]	Ψ	Dimensionless stream fuction ($\Psi = \psi/\alpha$) [-]
k	Thermal conductivity coefficient [W/mK]	Subscripts	
L	Length of the enclosure [m]	a	apparent
n	Power-law index [-]	C	cold wall
Nu	Nusselt number [-]	eff	effective
$\overline{Nu}$	Mean Nusselt number [-]	H	Hot wall
P	Pressure [Pa]	Special characters	
Pr	Prandtl number [-]	ΔT	Temperature difference hot and cold wall
q	Heat flux [W/m ²]	$\Delta_{min, cell}$	Minimum cell distance
r_e	Grid expansion ratio [-]		
Ra	Rayleigh number [-]		
T	Temperature [K]		
u_i	i th component of velocity [m/s]		
U	Dimensionless horizontal velocity ($U = u_1 L/\alpha$) [-]		
V	Dimensionless vertical velocity ($U = u_2 L/\alpha$) [-]		
x_i, x_j	Coordinate in i th and j th directions [m]		

I. Introduction

Natural convection of Newtonian fluids in enclosed spaces has been studied extensively (e.g. Refs. [1-4]) but limited attention directed to non-Newtonian fluids despite their potential applications in cooling of electronics, food and chemical processing, solar and nuclear power systems. Many common man-made and biological fluids exhibit shear-thinning (e.g. ketchup, blood) and shear-thickening (e.g. mixtures of corn starch and water, so-called “bullet-proof” custard) behaviours. For example, aqueous solutions of polymers (e.g. carboxymethyl cellulose, polyethylene oxide, xanthan gum) show a decrease in viscosity with increasing shear rate (i.e. shear-thinning behaviour). Recently, it has been reported in Ref. [5] that 0.1-0.2 % aqueous solutions of xanthan gum exhibit an augmentation of convective heat transfer rate in comparison to water for laminar Rayleigh-Bénard convection in an enclosure. This is in agreement with the findings of several previous studies on laminar Rayleigh-Bénard convection of shear-thinning fluids in enclosures [6-9]. Power-law model of viscosity was used by the pioneering analysis Ozoe and Churchill [10] to model the shear rate dependence of viscosity for natural convection of both shear-thinning and shear-thickening fluids. Existing literature on natural convection of power-law fluids can be divided in two categories. The first category concentrates on the critical conditions for the onset of convection of power-law fluids [11-13]. The second category focuses on the effects of Rayleigh and Prandtl numbers on the heat transfer characteristics beyond the critical conditions for the onset of fluid motion [14-27]. The specific contributions of the aforementioned references have been summarised in detail in Table 1. Several investigations belonging to the second category concentrated on the effects of aspect ratio AR (i.e. height to length ratio) and boundary conditions (i.e. constant wall temperature and constant wall heat flux) on natural convection of power-law fluids in rectangular enclosures with differentially heated vertical walls [14]. It has been found that the wall boundary condition significantly influences the aspect ratio AR dependence of the mean Nusselt number in the case of natural convection of power-law fluids in rectangular enclosures with differentially heated vertical walls [14]. Recently, Ref. [9] analysed effects of aspect ratio on natural convection of power-law fluids in rectangular enclosures with differentially heated horizontal walls with heated bottom wall (i.e. Rayleigh-Bénard configuration) for constant wall temperature boundary condition. It has been found that convection weakens with increasing aspect ratio and heat transfer takes place purely due to thermal conduction for tall enclosures. Previous analyses [14-17] indicated that the Nusselt number behaviour for the constant wall heat flux (CWHF) boundary condition can be significantly different from the corresponding results for the constant wall temperature (CWT) boundary condition. Thus, this analysis concentrates on the influences of aspect ratio on Rayleigh-Bénard convection of power-law fluids in rectangular enclosures for CWHF boundary condition for horizontal walls. In

this investigation, simulations have been carried out for a range of values of aspect ratio, nominal Rayleigh number Ra and power-law exponent n ($0.25 \leq AR \leq 4$, $10^3 \leq Ra \leq 10^5$, $0.6 \leq n \leq 1.8$) for a single value of nominal Prandtl number (i.e. $Pr = 10^3$). Practical power-law fluids such as aqueous solutions of polymers including carboxymethyl cellulose, polyethylene oxide, polyacrylamide and xanthan gum exhibit Prandtl numbers of the order of 10^3 (or greater than this value). Thus, the current analysis is conducted for single value of $Pr = 10^3$ because previous analyses [16,20] indicated that the mean Nusselt number of power-law fluids remain independent of Prandtl number for $Pr \gg 1$. In this respect, the specific objectives of present analysis are:

- (i) To indicate the effects of aspect ratio, nominal Rayleigh number and power-law index on Rayleigh-Bénard convection of power-law fluids in rectangular enclosures with differentially heated horizontal walls subjected to constant heat flux.
- (ii) To indicate the effects of initial conditions on streamline and isotherm patterns and mean Nusselt number in the aforementioned configuration.

The remainder of the paper will be organised as follows. The information pertaining to mathematical background and numerical implementation are provided in the next two sections, followed by the presentation of results and subsequent discussion. The main findings will be summarised and conclusions will be drawn in the final section of this paper.

II. Mathematical Background

For the power-law model the viscous stress tensor τ_{ij} is given as follows:

$$\tau_{ij} = \mu_a e_{ij} = K(e_{kl}e_{kl}/2)^{(n-1)/2} e_{ij}, \quad (1)$$

where $e_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i)$ is the rate of strain tensor, K is the consistency and n is the power-law index and $\mu_a = K(e_{kl}e_{kl}/2)^{(n-1)/2}$ is the apparent viscosity. The apparent viscosity μ_a decreases (increases) with increasing shear rate for $n < 1$ ($n > 1$) and thus fluids with $n < 1$ ($n > 1$) are referred to as shear-thinning (shear-thickening) fluids, whereas $n = 1$ represents Newtonian fluids. The present analysis is conducted in non-dimensional form for the purpose of generalization where the spatial co-ordinates, velocity components, pressure, shear stress and temperature are non-dimensionalised in the following manner:

$$x_i^+ = \frac{x_i}{H}; u_i^+ = \frac{u_i}{U_{ref}}; P^+ = \frac{P}{\rho U_{ref}^2}; \tau_{ij}^+ = \frac{\tau_{ij} H}{\rho \alpha U_{ref}}; \Theta = \frac{T - T_{ref}}{\Delta T_{ref}}, \quad (2)$$

where U_{ref} is the reference velocity scale defined as α/H , T_{ref} is the reference temperature and ΔT_{ref} is a reference temperature difference, which is taken to be $\Delta T_{ref} = qH/k$. The temperature at the geometrical centre

of the annular enclosure is also taken as reference temperature (i.e. $T_{ref} = T_{cen}$). Several previous analyses [12,21] indicate that the temperature dependence of K does not significantly affect the qualitative nature of the temperature and velocity distributions in natural convection of power-law fluids and the variation of $\overline{Nu}$ in response to Ra and n . Therefore, the thermo-physical properties (e.g. k , c_p , n and K) are taken to be temperature independent in the current analysis for the sake of simplicity following several previous analyses [6,7,9]. This yields the following non-dimensional forms of conservation equations for power-law fluids with temperature-independent thermo-physical properties:

Non-dimensional mass conservation equation

$$\frac{\partial u_i^+}{\partial x_i^+} = 0; \quad (3)$$

Non-dimensional momentum conservation equations

$$u_j^+ \frac{\partial u_i^+}{\partial x_j^+} = -\frac{\partial P^+}{\partial x_i^+} + \delta_{i2} Ra Pr \Theta + \frac{\partial \tau_{ij}^+}{\partial x_j^+}, \quad (4)$$

Non-dimensional energy conservation equation

$$u_j^+ \frac{\partial \Theta}{\partial x_j^+} = \frac{\partial^2 \Theta}{\partial x_j^+ \partial x_j^+}. \quad (5)$$

In Eq. 4 Kronecker's delta δ_{i2} ensures that the buoyancy term is operational only in the x_2 direction (i.e. vertical direction) and this term appears due to the Boussinesq assumption [7,9,14,21]. In Eq. 4, Ra and Pr are the nominal Rayleigh and Prandtl numbers, which are defined using the reference temperature difference ($\Delta T_{ref} = qH/k$) in the following manner:

$$Ra = g\beta q H^{2n+2} / \alpha^n k (K/\rho); \quad Pr = (K/\rho) \alpha^{n-2} H^{2-2n}. \quad (6)$$

It is possible to use same constitutive equations (i.e. Eqs. 3-5) and model τ_{ij} with different model of viscosity (i.e. Herschel-Bulkley) to study other generalised non-Newtonian fluids (e.g. yield stress fluids [27]). For the present analyses local heat transfer coefficient h is defined as:

$$h = \left| -k(\partial T / \partial x_2)_{wf} \times 1 / (T_{x_2=0} - T_{x_2=H}) \right|. \quad (7)$$

Accordingly, mean heat transfer coefficient $\bar{h}$ and the mean Nusselt number $\overline{Nu}_{cy}$ are evaluated as:

$$\bar{h} = \frac{1}{L} \int_0^L h(x_1) dx_1, \quad \overline{Nu}_{cy} = \bar{h} H / k. \quad (8)$$

Using Buckingham's pi theorem, it is possible to show that the Nusselt number for natural convection of power-law fluids in rectangular enclosures can be expressed as: $Nu = f(Ra, Pr, n, AR)$. The simulation domain is shown schematically in Fig. 1 where the two horizontal walls of a rectangular enclosure are subjected to constant heat fluxes, and the vertical walls are considered to be adiabatic in nature. Equations 3-5 are solved in conjunction with

the following boundary conditions. The two vertical walls are kept under adiabatic conditions (i.e. $\partial\theta/\partial x_1^+ = 0$ at $x_1^+ = 0$ and $x_1^+ = 1/AR$) and both velocity components (i.e. u_1^+ and u_2^+) are identically zero on each boundary because of the no-slip condition and impenetrability of rigid walls. The heat fluxes for horizontal hot and cold walls are specified (i.e. $-\partial\theta/\partial x_2^+ = 1$ at $x_2^+ = 0.0$ and $x_2^+ = 1.0$ respectively).

III. Numerical Implementation

The steady-state conservation equations (i.e. Eqs. 3-5) are solved iteratively in conjunction with boundary conditions in the context of the finite-volume methodology by using the commercial package ANSYS-FLUENT. This commercial package has been used successfully in simulating natural convection of power-law fluids in previous studies [14-20,27]. A second-order up-wind scheme is used for the discretisation of the convective terms, whereas a second-order central differencing scheme is used for discretisation of the diffusive terms. The well-known SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm [28] is used for coupling of the pressure and velocity. The convergence criteria were set to 10^{-6} for all the scaled residuals. It has been checked that a smaller value of convergence criterion does not alter any of the results presented here. Grid independence of the results has been established based on a careful analysis of a number of different non-uniform Cartesian meshes. For the $AR = 0.25$ case, non-uniform meshes of 180×220 and 200×240 have been used for establishing grid independence. The maximum difference in $\overline{Nu}$ for Newtonian fluids (i.e. $n = 1.0$) has been found to be smaller than 0.3% for the range of Ra (i.e. $10^3 \leq Ra \leq 10^5$). Hence the non-uniform mesh of 180×220 has been used for all $AR = 0.25$ simulations. The simulations for $AR = 0.5$ have been carried out for non-uniform meshes of 100×100 and 200×240 and the maximum uncertainty in $\overline{Nu}$ or Newtonian fluids was found to be smaller than 0.15%. Nevertheless, the non-uniform mesh of 200×240 has been used for the $AR = 0.5$ case for the sake of sensitivity of numerical results. For $AR = 1.0$ ($AR = 2.0$) non-uniform meshes of 100×100 and 240×240 (100×150 and 160×320) have been used to assess grid-dependency and the maximum uncertainty in $\overline{Nu}$ or Newtonian fluids was found to be smaller than 0.1%. Accordingly, the non-uniform meshes of 240×240 and 160×320 have been used for $AR = 1.0$ and $AR = 2.0$ cases respectively. Finally, a non-uniform mesh of 160×480 has been used for the $AR = 4.0$ case. The non-dimensional minimum grid spacing ($\Delta_{min,cell}/L$) grid expansion ratio (r_g) values for the meshes used here are 6.651×10^{-4} , 1.015 for $AR = 0.25$ (180×220), 5.779×10^{-4} , 1.015 for $AR = 0.5$ (200×240), 6.808×10^{-4} , 1.01 for $AR = 1.0$ (240×240), 4.904×10^{-4} , 1.015 for $AR = 2.0$ (160×320), 1.337×10^{-4} , 1.015 for $AR = 4.0$ (160×480) respectively.

Simulations have also been performed to estimate the grid-convergence for power-law fluids and the maximum numerical uncertainty in $\overline{Nu}$ for power-law fluids with $n = 0.6$ and $n = 1.8$ is found to be marginally greater than that in the corresponding Newtonian (i.e. $n = 1.0$) fluid case. Although it has been found that numerical uncertainty increases with decreasing power law index, the maximum numerical uncertainty in $\overline{Nu}$ for all values of n in power-law fluids is still found to be less than 1% for $0.25 \leq AR \leq 4$ and $10^3 \leq Ra \leq 10^5$ at $Pr = 10^3$.

The CPU time increases with decreasing values of AR and n (i.e. demand of CPU time is 10 times greater for $n = 0.6$ than one for $n = 1.0$ in the $AR = 0.25$). It will be shown later in Sections V and VI that the effective Rayleigh number increases significantly in comparison to the nominal Rayleigh number for shear-thinning (i.e. $n < 1$) fluids for $AR < 1$. Thus, the simulations with small values of AR and n take more number of iterations to converge and degree of stiffness of the momentum equations due to variation of viscosity increases with decreasing n . By contrast, the convective transport weakens with increasing n and thus only the energy transport plays a key role for large values of n for shear-thickening fluids, which takes smaller number of iterations and CPU time to converge than the corresponding Newtonian fluid case with same set of values of aspect ratio, nominal Rayleigh and Prandtl numbers.

IV. Validation

The mean Nusselt numbers $\overline{Nu}$ for laminar Rayleigh- Bénard convection of Newtonian fluids in square enclosures for $10^3 \leq Ra \leq 10^6$ and $Pr = 0.71$ have been compared to the benchmark data [29] in Table 2. It is evident from Table 2 that an excellent agreement has been achieved between the present results and the benchmark data [29]. Moreover, the variation of $\overline{Nu}/\overline{Nu}_{n=1}$ with n for natural convection of power-law fluids in square enclosures with differentially heated vertical side walls subjected to CWT boundary condition has also been compared to the corresponding results presented by Ref. [22] and an excellent agreement was also obtained (see Fig. 2 of Ref. [23]).

V. Scaling Analysis

Scaling analysis is conducted to elucidate the effects of Ra, Pr, n, AR on laminar Rayleigh-Bénard convection of power-law fluids. The wall heat flux q can be scaled as: $q = h\Delta T \sim k\Delta T/\delta_{th}$ which leads to $\overline{Nu} \sim qH/\Delta T \sim H/\delta_{th}$ where $f(Ra, Pr, AR, n)$ represents the ratio of hydro-dynamical and thermal boundary layer thicknesses (i.e.

δ/δ_{th}). Accordingly, equating order of magnitudes of the inertial and buoyancy forces in vertical direction (i.e.

$u_2 \sim \sqrt{g\beta q \delta_{th} H/k}$) leads to:

$$u_2^2 \sim g\beta q \delta_{th} H/k \sim (\alpha/H) Ra^{(n+1)/(n+4)} Pr^{(n+2)/(n+4)} f^{-(n+1)/(n+4)}. \quad (9)$$

Based on continuity equation (i.e. $u_1/L \sim u_2/H$) one obtains:

$$u_1 \sim u_2 AR^{-1} \sim (\alpha/H) AR^{-1} Ra^{(n+1)/(n+4)} Pr^{(n+2)/(n+4)} f^{-(n+1)/(n+4)}. \quad (10)$$

Similarly, equating the order of magnitudes of inertial and viscous resistance ($\rho u_2^2/H \sim K(u_2/\delta)^n(1/\delta)$ and $\rho u_1^2/L \sim K(u_1/\delta_1)^n(1/\delta_1)$ where δ and δ_1 are the hydro-dynamic boundary layer thicknesses on vertical and horizontal walls respectively) one obtains:

$$\delta \sim H Ra^{(n-2)/(n+4)} Pr^{n/(n+4)} f^{(2-n)/(n+4)}, \quad (11i)$$

$$\delta_1 = H Ra^{(n-2)/(n+4)} Pr^{n/(n+4)} AR^{(1-n)/(n+1)} f^{(2-n)/(n+4)}. \quad (11ii)$$

Using Eq. 11, the effective viscosity can be estimated in the vertical (horizontal) boundary layers as: $\mu_{eff}^V = K(u_2/\delta)^{n-1}$ ($\mu_{eff}^H = K(u_1/\delta_1)^{n-1}$). Accordingly, effective Rayleigh numbers (Ra_{eff}) can be evaluated in the vertical and horizontal boundary layers in the following manner:

$$Ra_{eff}^V = \frac{\rho^2 c_p g \beta q H^4}{\mu_{eff}^V k^2} \sim Ra^{(7-2n)/(n+4)} Pr^{(2-2n)/(n+4)} f^{(3n-3)/(n+4)}, \quad (12i)$$

$$Ra_{eff}^H = \frac{\rho^2 c_p g \beta q H^4}{\mu_{eff}^H k^2} \sim Ra^{(7-2n)/(n+4)} Pr^{(2-2n)/(n+4)} AR^{(2n-2)/(n+1)} f^{(3n-3)/(n+4)}. \quad (12ii)$$

Eq. 12 indicates that Ra_{eff}^H increases with decreasing values of AR and n for shear-thinning (i.e. $n < 1$) fluids, whereas Ra_{eff}^H decreases with decreasing values of AR for increasing values of n for shear-thickening (i.e. $n > 1$) fluids. Furthermore, Ra_{eff}^H assumes much greater values than Ra_{eff}^V and nominal Rayleigh number (i.e. Ra) for shear-thinning (i.e. $n < 1$) fluids for $AR < 1$. It is worth noting that the scaling estimates shown above should be considered in an order of magnitude sense for explaining the trends observed from numerical simulations.

VI. Results and Discussion

A. Effects of nominal Rayleigh number and power-law index

The variations of non-dimensional temperature ($\theta = (T - T_{cen})k/qL$) along the vertical mid-plane (x_2/H) and non-dimensional vertical velocity component ($V = u_2 L/\alpha$) along the horizontal mid-plane (x_1/L) are shown in Fig. 2 for different values of Ra and n at $AR = 0.25$ and $Pr = 10^3$. Figure 2 shows that the magnitude of V increases with increasing (decreasing) values of Ra (n). This indicates that thermal convection strengthens with increasing (decreasing) values of Ra (n) due to diminished viscous resistance with shear-thinning behaviour. This

also can be confirmed from the scaling relation given by Eq. 12 , which indicates that Ra_{eff} increases in both vertical (Ra_{eff}^V) and horizontal (Ra_{eff}^H) directions with increasing (decreasing) values of $Ra(n)$ for a given set of values of Pr and AR . It can be also seen from Fig. 2 that the θ decreases with increasing (decreasing) values of $Ra(n)$, which can be explained as follows:

$$\Delta T \sim \frac{q\delta_{th}}{k} \text{ and } \theta \sim \frac{q\delta_{th}k}{qLk} \sim O\left(\frac{\delta_{th}}{L}\right). \quad (13)$$

According to Eq. 13 θ decreases with decreasing δ_{th} . As δ_{th} (i.e. δ/f) is expected to decrease with increasing (decreasing) values of $Ra(n)$ (see Eq. 11), the maximum magnitude of θ decreases with increasing (decreasing) $Ra(n)$. Furthermore, the extent of non-linearity of θ variation with x_2/H can be considered as an indicator of the strength of convective transport, whereas the linearity indicates purely thermal conduction. The non-linearity of θ distribution with x_2/H rises with decreasing n and this distribution becomes linear for $n \gg 1$ because heat transfer takes place due to pure condition. The contours of non-dimensional stream functions ($\Psi = \psi/\alpha$) and the non-dimensional isotherms ($\theta = (T - T_{cen})k/qL$) are shown in Fig. 3 for different values of Ra and n at $AR = 0.25$. It is also evident from Fig.3 that the curvature of θ and magnitude of Ψ increase with increasing (decreasing) $Ra(n)$. This is found to be consistent with previous findings of Ref. [16] for square enclosures (i.e. $AR = 1$).

B. Effects of aspect ratio

The variation of $\overline{Nu}$ with AR for different values of Ra and n is shown in Fig. 4 for representative shear-thinning ($n = 0.8$), Newtonian ($n = 1.0$) and shear-thickening (i.e. $n = 1.4$) fluids. It can be seen from Fig. 4 that $\overline{Nu}$ decreases with increasing AR and approaches unity for $AR \gg 1$. This indicates that convective strength weakens for increasing AR and thermal transfer is dominated by conduction for tall enclosures (i.e. $\overline{Nu} = 1.0$ for $AR > 2$) for shear-thinning (i.e. $n < 1$), Newtonian (i.e. $n = 1$) and shear-thickening (i.e. $n > 1$) fluids. The effects of diffusive (convective) transport strengthen (weaken) with increasing AR , which can be explained using the energy flux integral at the vertical mid-plane. The energy flux integral can be shown as follows:

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{cond} = \int_0^H \rho c_p u_1 dx_2 - \int_0^H k(\partial T / \partial x_1) dx_2, \quad (14)$$

where the first term on the right hand side represents the effects of convective transport, whereas the second term on the right hand side accounts for thermal conduction. Here, $\dot{Q}_{conv}$ and $\dot{Q}_{cond}$ can be scaled in the following manner by using Eqs. 9-11:

$$\dot{Q}_{conv} \sim \frac{\rho c_p q \delta_{th} \delta_1 u_1}{k} \sim q L R a^{(3n-3)/(n+4)} P r^{(3n+2)/(n+4)} A R^{(2-2n)/(n+1)} f^{-(4n+1)/(n+4)}, \quad (15i)$$

$$\dot{Q}_{cond} \sim \frac{k q \delta_{th} H}{k L} \sim q A R \delta_{th} \sim q L R a^{(n-2)/(n+4)} P r^{n/(n+4)} A R^{(n+3)/(n+1)} f^{-2(n+1)/(n+4)}. \quad (15ii)$$

Eq. 15 suggests relative weakening (strengthening) of the contribution of $\dot{Q}_{conv}$ ($\dot{Q}_{cond}$) to the overall thermal transport with increasing AR for both shear-thinning (i.e. $n < 1$) and shear-thickening (i.e. $n > 1$) fluids. This is because the exponent of AR in $\dot{Q}_{cond}$ is greater than $\dot{Q}_{conv}$ (i.e. $(n+3)/(n+1) > (2-2n)/(n+1)$) for the same set values of Ra and Pr for both shear-thinning (i.e. $n < 1$) and shear-thickening (i.e. $n > 1$) fluids. The contours of Ψ and θ are shown in Fig. 5 for different values of AR for $Ra = 10^5$ and $n = 0.8$. It is worth noting that the dimensions of the configurations are scaled in such manner that the widths of the enclosures look the same merely for different heights of enclosures for the ease of plotting in Fig. 5. It can be seen from Fig. 5 that the curvature of the θ diminishes with increasing AR and for tall enclosures (e.g. $AR = 4$), the distribution of θ becomes totally parallel to the hot and cold walls. This is qualitatively similar to the previous findings of Ref. [9] in the case of CWT boundary condition. The magnitude of stream functions in the $AR = 4$ case at $Ra = 10^5$ is vanishing small so the streamline pattern has no significant importance (it is just numerical noise for the vanishing small magnitudes of Ψ) for this particular case because fluid flow is practically non-existent in this case and heat transfer takes place purely due to conduction which is reflected in the distribution of isotherms parallel to the horizontal walls. Although $\overline{Nu}$ mostly decreases with increasing AR , the mean Nusselt number $\overline{Nu}$ is found to be slightly greater in $AR = 0.5$ than the value for $AR = 0.25$ for $Ra \geq 5 \times 10^4$ for shear-thinning (i.e. $n < 1$) and $Ra = 10^5$ for Newtonian (i.e. $n = 1.0$) fluids. Both $AR^{(2-2n)/(n+1)}$ and $AR^{(n+3)/(n+1)}$ in Eqs. 15i and 15ii respectively assume higher values for $AR = 0.5$ than in $AR = 0.25$ for $n \leq 1$ and thus the modification of the relative strengths of convection and conduction with the variation in aspect ratio is responsible for the observed non-monotonic AR dependence of $\overline{Nu}$ between $AR = 0.25$ and 0.5 for $Ra \geq 5 \times 10^4$ for shear-thinning (i.e. $n < 1$) and $Ra = 10^5$ for Newtonian (i.e. $n = 1.0$) fluids.

C. Effects of initial conditions on mean Nusselt number and flow patterns

It has been demonstrated earlier [9,30,31] that the initial conditions significantly affect the steady two-dimensional solutions for Rayleigh-Bernard convection of Newtonian (i.e. $n = 1$) and shear-thinning (i.e. $n < 1$) fluids in rectangular enclosures. Two types of initial conditions have been considered in the current analysis. These initial conditions referred as the “quiescent flow (Q.F.)” and “established flow (E.F.)” initial conditions respectively. The Q.F. condition considers zero velocity initial condition, whereas the steady state simulation results obtained for a smaller value of Ra is used for the initial condition for the cases with $Ra \geq 1 \times 10^3$ in the E.F for Newtonian fluid simulations. In the context of Q.F. condition, the perturbation for initiating the flow is supplied by the numerical errors (e.g. truncation error) as in the case of several previous analyses [14-20]. The steady-state solution obtained

for Newtonian (i.e. $n = 1.0$) fluids based on E.F. initial condition for a given set of values of Ra and Pr has in turn been used as an initial condition for one set of simulations involving non-Newtonian power-law fluids (i.e. E.F. initial condition for power-law fluids), whereas the quiescent condition is used as an initial condition (i.e. Q.F. initial condition for power-law fluids) for the other set of power-law fluid simulations. In Fig. 6, the variation of $\overline{Nu}$ with n is shown for different values of Ra and AR at $Pr = 10^3$ for both E.F. and Q.F. conditions. It can be seen from Fig. 6 shows that the variation of $\overline{Nu}$ is sensitive to the initial conditions especially for $Ra \geq 5 \times 10^4$ in the case of shear-thinning fluids (i.e. $n < 1$). Additionally non-monotonic trend of $\overline{Nu}$ with n is observed for both initial conditions for different AR in the case of shear-thinning fluids (i.e. $n < 1$). The mean Nusselt number $\overline{Nu}$ can be scaled as:

$$\overline{Nu} \sim qH/\Delta Tk \sim H/\delta_{th} \sim Ra^{(2-n)/(n+4)} Pr^{-n/(n+4)} AR^{(n-1)/(n+1)} f^{6/(n+4)}. \quad (16)$$

Equation 16 shows that $\overline{Nu}$ is expected to increase monotonically with increasing (decreasing) Ra (n) for given values of Pr and AR . Thus, this non-monotonic trend of $\overline{Nu}$ with increasing (decreasing) Ra (n) can be explained by the change in flow patterns (i.e. number of cells). The distance between the isotherms close to the hot and cold walls gets affected by the modification of the flow pattern, and this modification is responsible for the non-monotonic variation of $\overline{Nu}$ with increasing (decreasing) Ra (n). In order to demonstrate the aforementioned changes in the flow patterns, the representative contours of non-dimensional stream function Ψ and the non-dimensional temperature θ are shown in Fig. 7 for both E.F. and Q.F. initial conditions. A number of simulations have been also conducted for convergence criteria set 10^{-7} for all the scaled residuals to check stability of the flow patterns are shown in Fig7. It has been found that flow patterns and the mean Nusselt numbers remain insensitive for convergence criteria set 10^{-6} and 10^{-7} for all the scaled residuals regardless of initial conditions. . It is also worth noting that qualitatively similar flow patterns were reported earlier (see Ref. [9]) even for constant wall boundary conditions for horizontal walls. The non-uniqueness of the steady state solution in this configuration suggests that some of the solutions may not be easily observed in reality under an experimental condition. Figure 7 indicates that flow pattern (i.e. number of cells) changes with increasing (decreasing) Ra (n) for shear-thinning fluids (i.e. $n < 1$), which was also previously reported in [9, 31]. It is also worth noting that the Ra_{eff} may assume much larger magnitude than its nominal value (i.e. Ra) for shear-thinning (i.e. $n < 1$) fluids since exponent of Ra (i.e. $(7 - 2n)/(n + 4)$) increases with decreasing n . This contributes to the change in the flow patterns (i.e. number of cells) as shown in Fig. 7 especially for shear-thinning (i.e. $n < 1$) fluids.

A correlation was proposed in Ref. [16] for mean Nusselt number for laminar Rayleigh-Bénard convection of power-law fluids in square enclosures (i.e. $AR = 1$) for $10 \leq Pr \leq 10^4$ based on scaling relations. This correlation was improved in Ref. [17] for shear-thickening (i.e. $n > 1$) fluids, which is given as follows:

$$\overline{Nu} = \overline{Nu}_1 \text{ when } \overline{Nu}_1 = 0.289Ra^{0.014} \frac{Pr^{0.217}}{(1+Pr)^{0.017}} \left(\frac{Ra^{1-n/2}}{Pr^{n/2}} \right)^{1/(n/2+2)} A > 1; \quad (17i)$$

$$A = \exp[(n-1)c_1 Ra^{c_2} Pr^{c_3}] \text{ for } n \leq 1; \quad (17ii)$$

$$c_1 = 0.345, c_2 = 0.129 \text{ and } c_3 = 0.103 \text{ for } 5 \times 10^3 \leq Ra \leq 1 \times 10^5; \quad (17iii)$$

$$A = n^d \text{ for } n > 1; \quad (17iv)$$

$$d = [0.136Pr^{0.04} \ln Ra]^{1.5} \text{ for } 5 \times 10^3 \leq Ra \leq 1 \times 10^4; \quad (17v)$$

$$d = [0.145Pr^{0.04} \ln Ra]^{1.5} \text{ for } 1 \times 10^4 < Ra \leq 1 \times 10^5; \quad (17vi)$$

$$\overline{Nu} = 1 \text{ when } \overline{Nu}_1 \leq 1. \quad (17vii)$$

It is worth noting that Eq. 17 is valid for where convection takes place as one cell flow pattern. Equation 17 has been modified here to include AR dependence of $\overline{Nu}$ in the following manner for one cell flow pattern for $0.25 \leq AR \leq 4$, $5 \times 10^3 \leq Ra \leq 1 \times 10^5$ and $10 \leq Pr \leq 10^4$:

$$\overline{Nu}(AR) = \overline{Nu}(AR = 1) + [(1.987 - 0.157 \ln Ra)(1 - AR)] \text{ for } n \leq 1; \quad (18i)$$

$$\overline{Nu}(AR) = \overline{Nu}(AR = 1) + \left[\left(\frac{1 - 0.124 \ln Ra}{1.452 - 0.183 \ln Ra} \right) (1 - AR) \right] \text{ for } n \geq 1. \quad (18ii)$$

Figure 8 shows the variation of $\overline{Nu}$ with n for different values of Ra and AR at $Pr = 10^3$ along with the predictions of Eq. 18. It can be seen from Fig. 8 that Eq. 18 satisfactorily predicts the variation of $\overline{Nu}$ with n for different values of Ra and AR (i.e. typical deviation is smaller than 4%) apart from over estimation of $AR = 2$ (i.e. 10%) for shear-thickening (i.e. $n > 1$) fluids.

D. Onset of convection of power-law fluids

It is well-known that a critical Rayleigh number Ra_{crit} needs to be exceeded to initiate the fluid motion in the case of Rayleigh-Bénard convection. The critical Rayleigh number Ra_{crit} is estimated by linear regression of $(\overline{Nu} - 1)$ in terms of Ra for Newtonian (i.e. $n = 1$) fluids where the instability is super-critical (not shown here but refer to Fig. 8a in Ref. [9]). In the case of shear-thinning (i.e. $n < 1$) fluids, where the instability is sub-critical, the critical Rayleigh number is estimated by reducing nominal Rayleigh number Ra in steps of 50, 250, 1000 for $AR \leq 1$, $AR = 2$ and $AR = 4$ respectively from an established flow condition where $\overline{Nu} \gg 1$. The quantity $(\overline{Nu} - 1)$ jumps abruptly from zero value with an increase in Ra for shear-thinning fluids (i.e. $n < 1$) according to the expectation

of sub-critical instability (not shown here but refer to Fig. 8a in Ref. [9]), this is because viscosity remains very large at low shear rate values for shear-thinning fluids (i.e. $n < 1$).

Table 3 lists the critical Rayleigh number Ra_{crit} for the onset of convection for shear-thinning and Newtonian fluids. Table 3 shows that Ra_{crit} decreases with decreasing values of AR and n , which is consistent with previous findings of Ref. [9] for the CWT boundary condition. The critical Rayleigh number Ra_{crit} values for shear-thinning (i.e. $n < 1$) and Newtonian (i.e. $n = 1$) fluids for square (i.e. $AR = 1$) enclosures are found to be consistent with the previous findings by Turan *et al.* [16,20]. The values of Ra_{crit} for shear-thinning (i.e. $n < 1$) and Newtonian (i.e. $n = 1$) fluids have been parameterized by Ref. [16] in the following manner for laminar Rayleigh-Bénard convection of power-law fluids in square enclosures (i.e. $AR = 1$) for $10 \leq Pr \leq 10^4$:

$$Ra_{crit}(AR = 1) = 1700n^2 \quad n \leq 1. \quad (19)$$

Equation 19 has been modified here to include AR dependence of Ra_{crit} in the following manner for $0.25 \leq AR \leq 4$ and $10 \leq Pr \leq 10^4$:

$$Ra_{crit}(AR) = 1700n^2 + [(AR - 1)(a_0n^b)]; \quad (20i)$$

$$a_0 = 65860/(1 + \exp((3.387 - AR)/0.789)) \text{ and } b = 2.38 + 0.27AR. \quad (20ii)$$

For shear-thickening (i.e. $n > 1$) fluids, $(\overline{Nu} - 1)$ cannot be used for a sufficiently accurate measure for the onset of fluid motion in the case of shear-thickening (i.e. $n > 1$) fluids since $(\overline{Nu} - 1)$ remains close to 0.0 for a large range of values of Ra . It has been found that fluid flow initiates for any non-zero value of nominal Rayleigh number by examining the variation of the maximum values of stream function ψ_{max} with Rayleigh number (not shown here but refer to Fig. 8b in Ref. [9]). Thus, the critical Rayleigh number for shear-thickening (i.e. $n > 1$) fluids can be considered as $Ra_{crit} = 0$ since viscosity remains very small at low shear rate values for shear-thickening fluids (i.e. $n > 1$). Therefore, shear-thickening fluids do not experience any resistance to fluid motion once the nominal Rayleigh approaches zero. This is found to be consistent with previous findings [9,13]. The values of nominal Rayleigh number $Ra_{\overline{Nu}=1}$ for which $\overline{Nu} = 1.001$ for shear-thickening (i.e. $n > 1$) fluids are listed in Table 4. The values of $Ra_{\overline{Nu}=1}$ presented in Table 4 for shear-thickening fluids have been parameterized in the following manner for $0.25 \leq AR \leq 4$ and $1 < n \leq 1.8$:

$$Ra_{\overline{Nu}=1} = 720 + \exp(3.85AR) [13.79/(1 - 0.352n)] \text{ for } 0.25 \leq AR \leq 1; \quad (21i)$$

$$Ra_{\overline{Nu}=1} = [1370 \exp(a_1AR)] - 4000 \text{ for } 1 < AR \leq 4; \quad (21ii)$$

$$a_1 = 1.281 + 0.462/(1 + \exp((1.462 - n)/0.09)). \quad (21iii)$$

Figure 9 shows the variations of $Ra_{crit}(AR)$ and $Ra_{\overline{Nu}=1}$ with AR along with the predictions of Eq. 20 and Eq. 21 respectively. It can be seen from Fig. 9 that Eq. 20 and 21 satisfactorily predict the variation of $Ra_{crit}(AR)$ and $Ra_{\overline{Nu}=1}$ with AR for $10 \leq Pr \leq 10^4$.

VII. Conclusions

The effects of Ra , n , AR on Rayleigh-Bénard convection of power-law fluids in rectangular enclosures have been numerically investigated for constant wall heat flux boundary condition for the active walls for a range of different values of aspect ratio, nominal Rayleigh number and power-law index (i.e. $0.25 \leq AR \leq 4$, $10^3 \leq Ra \leq 10^5$, $0.6 \leq n \leq 1.8$) for a single representative value of nominal Prandtl number ($Pr = 10^3$). It has been found that thermal convection strengthens (weakens) with decreasing (increasing) n as a result of shear-thinning (shear-thickening) behaviour. The convective thermal transport strengthens with increasing Ra due to the augmentation of buoyancy force irrespective of the value of n . However, convective transport weakens and the influences of thermal conduction strengthen with increasing AR . Thus, $\overline{Nu}$ increases with increasing (decreasing) $Ra(n)$ whereas the $\overline{Nu}$ decreases with increasing AR . However, the flow pattern (i.e. number of cells) varies depending on Ra, AR, n and initial condition, and non-monotonic variation of $\overline{Nu}$ with increasing (decreasing) $Ra(n)$ has been observed due to change in the flow patterns for different initial conditions. Finally, Ra_{crit} for onset of convection (i.e. $\overline{Nu} > 1.0$ for $Ra > Ra_{crit}$) decreases with decreasing values of AR and n . The aforementioned observations have been explained with the help of scaling arguments. The numerical findings have been utilised to propose a correlation for the mean Nusselt number in the case of one-cell flow structure for laminar Rayleigh-Bénard convection in rectangular enclosures with constant wall heat flux boundary condition for active walls.

References

- [1] Hussein A.K., Rout S. K., Fathinia F., Chand R., Mohammed H.A., "Natural convection in a triangular top wall enclosure with a solid strip," *Journal of Eng. and Sci.*, Vol. 10, 2015, pp. 1326-1341.
- [2] Hussain S.H., Hussein A.K., Mahdi M.M., "Natural convection in a square inclined enclosure with vee-corrugated sidewalls subjected to constant flux heating from below," *Nonlinear Analysis: Modelling and Control*, Vol. 16, 2011, pp. 152-169.
- [3] Hussain S.H., Hussein A.K. and Kadim K.A., "Numerical simulation of natural convection in a parallelogrammic enclosure containing volumetric heat source with non-uniformly heated left sidewall," *Heat Transfer- Asian Research*, Vol. 43, 2014, pp. 542-560.
- [4] Hussain S.H., Hussein A.K., "Numerical investigation of natural convection phenomena in a uniformly heated circular cylinder immersed in square enclosure filled with air at different vertical locations," *Int. Com. in Heat Mass Transfer*, Vol. 37, 2010, pp. 1115-1126.
doi: 10.1016/j.icheatmasstransfer.2010.05.016
- [5] Darbouli M., Métivier C., Leclerc S., Nouar C., Bouteera M., Stemmelen D., "Natural convection in shear-thinning fluids: Experimental investigations by MRI," *Int. J. Heat Mass Transfer*, Vol. 95, 2016, pp. 742-754.
doi: 10.1016/j.ijheatmasstransfer.2015.12.056
- [6] Ohta M., Akiyoshi M. and Obata E., "A numerical study on natural convective heat transfer of pseudo-plastic fluids in a square cavity," *Num. Heat Transfer Part A*, Vol. 4, 2002, pp. 357-372.
doi: 10.1080/104077802317261218
- [7] Lamsaadi M., Naïmi M., Hasnaoui M., "Natural convection of non-Newtonian power law fluids in a shallow horizontal rectangular cavity uniformly heated from below," *Heat Mass Transfer*, Vol. 41, 2005, pp. 239-249.
doi: 10.1007/s00231-004-0530-8
- [8] Inaba H., Daib C. and Horibe A., "Natural convection heat transfer of micro-emulsion phase-change material slurry in rectangular cavities heated from below and cooled from above," *Int. J. Heat Mass Transfer*, Vol. 46, 2003, pp. 4427-4438.
doi: 10.1016/S0017-9310(03)00289-8
- [9] Yigit S., Poole R.J., Chakraborty N., "Effects of aspect ratio on laminar Rayleigh-Bénard convection of power-law fluids in rectangular enclosures: A numerical investigation," *Int. J. Heat Mass Transfer*, Vol. 91, 2015, pp. 1292-1307.
doi: 10.1016/j.ijheatmasstransfer.2015.08.032
- [10] Ozoe H. and Churchill S.W. "Hydrodynamic stability and natural convection in Ostwald-De Waele and Ellis fluid: the development of a numerical solution," *AIChE J.*, Vol. 18, 1972, pp. 1196-1207.
doi: 10.1002/aic.690180617
- [11] Tien C., Tsuei H.S. and Sun Z.S., "Thermal convection of a horizontal layer of non-Newtonian fluid heated from below," *Int. J. Heat Mass Transfer*, Vol. 12, 1969, pp. 1173-1178.
- [12] Solomatov V., Barr A.C., "Onset of convection in fluids with strongly temperature-dependent power-law viscosity," *Phys. Earth & Planetary Interior*, Vol. 155, 2006, pp. 140-155.

doi: 10.1016/j.pepi.2005.11.001

- [13] Alloui Z., Ben Khelifa N., Beji H., Vasseur P., Guizani A., “The onset of convection of power-law fluids in a shallow cavity heated from below by a constant heat flux,” *J. Non-Newt. Fluid Mech.*, Vol. 196, 2013, pp. 70-82.

doi: 10.1016/j.jnnfm.2013.01.008

- [14] Turan O., Sachdeva A., Poole R.J., Chakraborty N., “Aspect ratio and boundary conditions effects on laminar natural convection of power-law fluids in a rectangular enclosure with differentially heated side walls,” *Int. J. Heat and Mass Trans.*, Vol. 60, 2013, pp. 722-738.

doi: 10.1016/j.ijheatmasstransfer.2013.01.017

- [15] Turan O., Sachdeva A., Poole R.J., Chakraborty N., “Laminar natural convection of power-law fluids in a square enclosure with differentially heated side walls subjected to constant wall heat flux,” *J. Heat Transfer Transactions of ASME*, Vol.134, 2012, pp. 122504.

doi:10.1115/1.4007123

- [16] Turan O., Fotso-Choupe F., Lai J., Poole R.J. and Chakraborty N., “Boundary condition effects on laminar natural convection of power-law fluids in a square enclosure heated from below with differentially heated horizontal walls,” *Ind. Eng. Chem. Res.*, Vol. 53, 2014, pp. 456-473.

doi: 10.1021/ie4023447

- [17] Yigit S., McRoberts C. and Chakraborty N., “Numerical investigation of laminar Rayleigh-Bénard convection of power-law fluids in square cross-sectional cylindrical annular enclosures,” *Int. Com. in Heat Mass Transfer*, Vol. 78, 2016, pp. 112-120.

doi: 10.1016/j.icheatmasstransfer.2016.08.025

- [18] Yigit S., Graham T., Poole R.J. and Chakraborty N., “Numerical investigation of steady-state laminar natural convection of power-law fluids in square cross-sectioned cylindrical annular cavity with differentially-heated vertical walls,” *Int. J. Num. Methods for Heat and Fluid Flow*, Vol. 26, 2016, pp. 85-107.

doi: 10.1108/HFF-01-2015-0030

- [19] Yigit S., Ford J., Poole R.J. and Chakraborty N., “Numerical investigation of boundary condition effects on laminar natural convection of power-law fluids in square cross-sectional cylindrical annular space with differentially heated vertical walls,” *Comp. Thermal Sciences*, Vol. 7, 2015, pp. 261-282.

doi: 10.1615/ComputThermalScien.2015014174

- [20] Turan O., Lai J., Poole R.J., Chakraborty N., “Laminar natural convection of power-law fluids in a square enclosure submitted from below to a uniform heat flux density,” *J. Non-Newt. Fluid Mech.*, Vol. 199, 2013, pp. 80-95.

doi: 10.1016/j.jnnfm.2013.06.002

- [21] Kaddiri M., Naïmi M., Raji A., Hasnaoui M., “Rayleigh-Bénard convection of non-Newtonian power-law fluids temperature-dependent viscosity,” *ISRN Thermodynamics*, Vol. 2012, 2012, pp. 614712.

doi: 10.5402/2012/614712

- [22] Kim G. B., Hyun J.M., Kwak H. S., “Transient buoyant convection of a power law non-Newtonian fluid in an enclosure,” *Int. J. Heat Mass Transfer*, Vol. 46, 2003. pp. 3605-3617.
doi: 10.1016/S0017-9310(03)00149-2
- [23] Turan O., Sachdeva A., Chakraborty N., Poole R.J., “Laminar natural convection of power-law fluids in a square enclosure with differentially heated side walls subjected to constant wall temperatures,” *J. Non-Newt. Fluid Mech.*, Vol. 166, 2011, pp. 1049-1063.
doi: 10.1115/1.4007123
- [24] Lamsaadi M., Naïmi M., Hasnaoui M., Mamou M., “Natural convection in a vertical rectangular cavity filled with a non-Newtonian power law fluid and subjected to a horizontal temperature gradient,” *Num. Heat Trans. Part A*, Vol. 49, 2006, pp. 969-990.
doi: 10.1080/10407780500324988
- [25] Lamsaadi M., Naïmi M., Hasnaoui M., “Natural convection heat transfer in shallow horizontal rectangular enclosures uniformly heated from the side and filled with non-Newtonian power law fluids,” *Energy Conversion and Management*, Vol. 47, 2006, pp. 2535 -2551.
doi: 10.1016/j.enconman.2005.10.028
- [26] Khezzar L., Signiner D., Vinogradov I., “Natural convection of power law fluids in inclined cavities,” *Int. J. Thermal Sci.*, Vol. 53, 2011, pp. 8-17.
doi: 10.1016/j.ijthermalsci.2011.10.020
- [27] Hassan M. A., Pathak M., Khan M.K., “Rayleigh-Bénard convection in Herschel-Bulkley fluid”, *J. Non-Newt. Fluid Mech.* Vol. 226, 2015, pp. 32-45.
doi: 10.1016/j.jnnfm.2015.10.003
- [28] Patankar S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, Washington, D.C., 1980.
- [29] Quertatani N., Cheikh N. B., Beya B. B., Lili T., “Numerical simulation of two dimensional Rayleigh- Bénard convection in an enclosure,” *C. R. Mecanique*, Vol. 336, 2008, pp. 464-470.
doi:10.1016/j.crme.2008.02.004
- [30] Venturi D., Wan X., Karniadakis G., “Stochastic bifurcation analysis of Rayleigh-Bénard convection,” *J. Fluid Mech.*, Vol. 650, 2010, pp. 391-413.
doi: 10.1017/S0022112009993685
- [31] Albaalaki B., Khayat R.E., “Pattern selection in the thermal convection of non-Newtonian fluids,” *J. Fluid Mech.*, Vol. 668, 2011, pp. 500-550.
doi: 10.1017/S0022112010004775

Tables

Table 1: Summary of the findings of existing analyses on natural convection in enclosures involving non-Newtonian fluids obeying power-law model of viscosity.

Ref.	Type	Enclosure	Configuration & Boundary conditions	$AR = H/L$	Model & power-law exponent	Ra, Pr	Correlation
[7]	A, N	Rectangular	Diff. heated horizontal wall (CWHF)	1/12	Power-law $0.6 \leq n \leq 1.4$	$10^3 \leq Ra \leq 5 \times 10^3$ $Pr = 1 - 10 - 100$	-
[9]	N	Rectangular	Diff. heated horizontal wall (CWT)	$1/4 \leq AR \leq 4$	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $Pr = 10^3$	$\overline{Nu} = f(Ra, Pr, AR, n)$
[10]	N	Square	Diff. heated horizontal wall (CWT)	1	1. Power-law $0.4 \leq n \leq 1$ 2. Ellis	$Ra = 6 \times 10^3$ $Pr = 0.1 - 1 - 10$	-
[11]	E, N	-	Diff. heated horizontal layers (CWT)	-	Power-law $0.4 \leq n \leq 1$	Ra_{crit} for $\overline{Nu} > 1$	-
[12]	N	-	Diff. heated horizontal layers (CWT)	-	Power-law $1 \leq n \leq 4$	Ra_{crit} for $\overline{Nu} > 1$ $\mu_\infty = f(T)$	-
[13]	A, N	Rectangular	Diff. heated horizontal wall (CWHF)	1/6	Power-law $0.8 \leq n \leq 1.2$	Ra_{crit} for $\overline{Nu} > 1$	-
[14]	N	Rectangular	Diff. heated vertical wall comparison (CWT-CWHF)	$1/8 \leq AR \leq 8$	Power-law $0.6 \leq n \leq 1.8$	$10^4 \leq Ra \leq 10^6$ $Pr = 10^3$	$\overline{Nu} = f(Ra, Pr, AR, n)$
[15]	N	Square	Diff. heated vertical wall comparison (CWT-CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^6$ $10 \leq Pr \leq 10^5$	$\overline{Nu} = f(Ra, Pr, n)$
[16]	N	Square	Diff. heated horizontal wall comparison (CWT-CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^5$	$\overline{Nu} = f(Ra, Pr, n)$

[17]	N	Cylindrical Annular $0 \leq r_i/L \leq 24$	Diff. heated horizontal wall comparison (CWT- CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, r_i/L, n)$
[18]	N	Cylindrical Annular $1/8 \leq r_i/L \leq 16$	Diff. heated vertical wall (CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^6$ $10^2 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, r_i/L, n)$
[19]	N	Cylindrical Annular $1/8 \leq r_i/L \leq 16$	Diff. heated vertical wall comparison (CWT- CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^6$ $10^2 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, r_i/L, n)$
[20]	N	Square	Diff. heated horizontal wall (CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^6$	$\overline{Nu} = f(Ra, Pr, n)$
[21]	N	Square	Diff. heated horizontal wall (CWHF)	1	Power-law $0.6 \leq n \leq 1.4$	$0 \leq Ra \leq 10^5$ $\mu_\infty = f(T)$	-
[22]	N	Square	Diff. heated vertical wall (CWT)	1	Power-law $0.6 \leq n \leq 1$	$10^5 \leq Ra \leq 10^7$ $10^2 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, n)$
[23]	N	Square	Diff. heated vertical wall (CWT)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^6$ $10 \leq Pr \leq 10^5$	$\overline{Nu} = f(Ra, Pr, n)$
[24]	A,N	Rectangular	Diff. heated vertical wall (CWHF)	1-12	Power-law $0.6 \leq n \leq 1.4$	$0 \leq Ra \leq 10^6$ $Pr = 1 - 10 - 100$	$\overline{Nu} = f(Ra, Pr, AR, n)$
[25]	A,N	Rectangular	Diff. heated vertical wall (CWHF)	$0.1 \leq AR \leq 1$	Power-law $0.6 \leq n \leq 1.4$	$0 \leq Ra \leq 10^6$ $Pr = 1 - 10 - 100$	$\overline{Nu} = f(Ra, Pr, AR, n)$
[26]	N	Rectangular $0^\circ \leq \phi \leq 90^\circ$	Diff. heated inclined horizontal wall (CWT)	$1/8 \leq AR \leq 1$	Power-law $0.6 \leq n \leq 1.4$	$10^4 \leq Ra \leq 10^5$ $10^2 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, AR, \phi, n)$

A: analytical; E: experimental; N: numerical

Table 2 Comparison of the mean Nusselt number $\overline{Nu}$ for Newtonian fluid with the benchmark data [22] for square enclosure ($AR = 1$) at $Pr = 0.71$.

Ra	Present Study	Ref. [22]
103	1.000	1.000
104	2.154	2.158
105	3.907	3.910
106	6.363	6.309

Table 3 Values of Ra_{crit} is estimated by linear regression for Newtonian fluids and reducing Ra in steps of 50, 250, 1000 for $AR \leq 1$, $AR = 2$ and $AR = 4$ respectively for shear-thinning (i.e. $n < 1$) fluids from an established flow condition where $\overline{Nu} \gg 1$.

n	$AR = 0.25$	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 4$
0.6	350	400	650	3000	22500
0.7	450	500	900	4250	37500
0.8	550	650	1150	6250	60000
0.9	700	750	1450	8750	95000
1.0	743	841	1676	11107	138850

Table 4 Values of Rayleigh number $Ra_{\overline{Nu}=1}$ for which $\overline{Nu}$ deviates from unity in the third decimal place (i.e. $\overline{Nu} \approx 1.001$) for shear-thickening fluids.

n	$AR = 0.25$	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 4$
1.2	775	875	1850	15000	250000
1.4	800	900	2000	20000	425000
1.6	825	925	2200	30000	1050000
1.8	850	950	2500	40000	1400000

Figure Captions

Fig. 1 Schematic diagram of the simulation domain.

Fig. 2 Variations of $\theta(V)$ along the vertical (horizontal) mid-plane for $AR = 0.25$ and $Pr = 10^3$.

Fig. 3 Contours of Ψ and θ for different values of Ra and n at $AR = 0.25$.

Fig. 4 Variations of $\overline{Nu}$ with AR for different values of Ra and n .

Fig. 5 Contours of Ψ and θ for different values of AR at $Ra = 10^5$ and $n = 0.8$.

Fig. 6 Variation of $\overline{Nu}$ with n for both established and quiescent flow initial conditions.

Fig. 7 Contours of Ψ and θ for both established and quiescent flow initial conditions.

Fig. 8 Variation of $\overline{Nu}$ with n along with the prediction of Eq. 18.

Fig 9 a) Ra_{crit} and b) $Ra_{\overline{Nu}=1}$ with AR with the predictions of Eq. 20 and Eq. 21.

Fig. 1 Schematic diagram of the simulation domain.

Fig. 2 Variations of $\theta(V)$ along the vertical (horizontal) mid-plane for $AR = 0.25$ and $Pr = 10^3$.

Fig. 3 Contours of Ψ and θ for different values of Ra and n at $AR = 0.25$.

Fig. 4 Variations of $\overline{Nu}$ with AR for different values of Ra and n .

Fig. 5 Contours of Ψ and θ for different values of AR at $Ra = 10^5$ and $n = 0.8$.

Fig. 6 Variation of $\overline{Nu}$ with n for both established and quiescent flow initial conditions.

$AR = 0.25$	Ψ	θ	$AR = 1.0$	Ψ	θ
$a_1(a_1^*)$			$a_3(a_3^*)$
b_1^*
c_1^*
d_1^*			b_3^*
$AR = 0.5$	Ψ	θ			
$a_2(a_2^*)$
b_2^*			$AR = 2.0$
c_2^*
d_2^*
e_2^*			b_4^*
f_2^*

Fig. 7 Contours of Ψ and θ for both established and quiescent flow initial conditions.

Fig. 8 Variation of $\overline{Nu}$ with n along with the prediction of Eq. 18.

Fig 9 a) Ra_{crit} and b) $Ra_{\overline{Nu}=1}$ with AR with the predictions of Eq. 20 and Eq. 21.