Adamson R, Hobbs M, Silcock A, Willis MJ. [Steady-state optimisation of a multiple cryogenic air separation unit and compressor plant](#). *Applied Energy* 2017, 189, 221-232.

Copyright:

© 2017. This manuscript version is made available under the [CC-BY-NC-ND 4.0 license](#)

DOI link to article:

<http://dx.doi.org/10.1016/j.apenergy.2016.12.061>

Date deposited:

04/01/2017

Embargo release date:

23 December 2017

This work is licensed under a [Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International licence](#)

Steady-state optimisation of a multiple cryogenic air separation unit and compressor plant

Richard Adamson^a, Martin Hobbs^b, Andy Silcock^b and Mark J. Willis^a

^aSchool of Chemical Engineering and Advanced Materials, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom

^bBOC Gases Ltd., Bawtry Road, Brinsworth, Rotherham, S60 5NT, United Kingdom
richard.adamson1@newcastle.ac.uk, mark.willis@newcastle.ac.uk

Abstract

The development and on-line application of a steady-state optimisation strategy for a multiple cryogenic air separation unit and compressor plant is discussed. Implemented using mixed integer linear programming (MILP), it is demonstrated that the optimiser improves site efficiency at steady state by reduction of power consumption by up to 5% (a significant saving for such an energy intensive process) while meeting customer demand specifications. This is achieved through determination of the production distribution of the air separation units and optimal load distribution of the compression network, while simultaneously ensuring network material balance and network component operating constraints are met. In addition, the work demonstrates achievable benefits of demand side load management during peak power pricing periods, using liquid oxygen as an effective energy storage device. A key constituent of the optimisation strategy is linear modelling to predict individual unit power consumption. Piece-wise linear data-based models of compressor and air separation unit power are shown to provide accurate models which improve existing on-site power prediction by up to 80% for compressors and 60% for the air separation units.

Keywords: Air separation, compressors, MILP, optimisation, piece-wise linear modelling.

Highlights

- Material and power consumption modelling of a multi-unit gas separation network.
- Piece-wise linear data-based modelling of compressor and ASU power consumption.
- Development and implementation of a MILP approach to network optimisation.
- Demonstrates achievable optimal demand side load management scheduling benefits.

Nomenclature

Abbreviations

ASU – Air Separation Unit
HP – High Pressure
LN – Liquid Nitrogen
LP – Low Pressure
MP – Medium Pressure

GO – Gaseous Oxygen
IC – Internally Compressed
LO – Liquid Oxygen
ME – model Mean Error (kW)
TLO – Total Liquid Oxygen

Parameters

$\hat{b}$ – model coefficient
 C_{LO} – liquid make cost (£/HCM)
 δ – Boolean coefficient
 F – flow rate (HCMS/hr)
 P_D – discharge pressure (bar)
 y – flow/Boolean auxiliary variable

C_{kw} – spot power cost (£/MWh)
 C_s – liquid use cost (£/HCM)
 ε – machine efficiency (kW/HCM)
 J – cost function (£/hr)
 W – power consumption (kW)

Subscripts

c	– compressor	j	– compressor or ASU number
k	– pseudo-machine number	m	– model number
s	– liquid back-up supply	u	– unit (ASU)
v	– valve		

1.0 Introduction

Cryogenic air separation plants are energy intensive processes consuming significant amounts of power (electricity) as a result of air separation into oxygen, nitrogen etc. and the subsequent compression or liquefaction of the gas products. Key peculiarities of industrial air separation are well documented; air is the only raw material (it is free and supply is unlimited), the cost of power (electricity) is the primary operational cost, >90% (Yan 2010), which varies throughout the day and the networks simultaneously produce gas and liquid that may be used to satisfy customer demand (the stored liquid being used to satisfy demand at times when electricity is at its most expensive) e.g. see Manenti and Rovaglio (2013). The purpose of this paper is to report the development and on-line application of a steady-state optimisation strategy that aims to minimise power consumption of a network of air separation and gas compression units. This is achieved by optimal production distribution of the air separation units and load distribution of the compression network.

Related to this work therefore, is the optimisation of industrial gas supply chains that comprise of networks of pipework, compressors and gas production units which are operated to meet customer demand and optimised for economic and environmental cost reductions, e.g. see Uster et al. (2014), Azadeh et al. (2015) and Gao and You (2015), who use a mixed integer nonlinear programming (MINLP) algorithm to optimise economic and environmental objectives. In Cortinovis (2016), a MINLP approach to the optimisation of a natural gas transmission system comprising of parallel compressors power consumption is also reported. This allows the monitoring of power regression curves against real time data to track compressor performance and identifying when maintenance is required. The optimal load sharing of a network of compressors has also been considered by Han et al. (2004) and Abbaspour et al. (2005) who developed off-line optimisation approaches while Paperella et al. (2013) and Xenos et al. (2015) considered the on-line optimisation of a network of air compression units. In Paperella et al. (2013) the optimal load distribution of a number of parallel compressors in a natural gas pipe-line were considered. While Xenos et al. (2015) optimised an industrial compressor station that served compressed air to air separation and chemical plants. Where compressor load sharing performance can be robustly modelled, Øvervåg (2013) uses a MINLP model predictive controller to optimally load share using efficiency curve data.

Development of predictive models of both compressor and air separation unit power consumption is a particular focus of this work. The operating principles of compressors (e.g. their characteristic performance and operation limits) are normally described in terms of compressor efficiency and detailed hybrid models of power consumption in industrial multistage compressors have been developed, e.g. see Han and Han (2003). Similarly, mechanistic models of air separation units exist within the literature which may be used as the basis for prediction of power consumption e.g. see Huang et al. (2009). However, the development of a set of robust and reliable models from fundamental principles would be an onerous task for a complex mix of industrial air separation units and compressors. Therefore, an empirical data-based modelling approach is used in this work. Related work includes Puranik et al. (2016) who used nonlinear regression models as the basis of a MINLP

approach to the optimisation of an oxygen and nitrogen customer network. Similarly, in Cao (2011), an MINLP approach was used to determine the optimal production rates required to meet customer nitrogen demands and assess dynamic compressor performance. In addition, Kopanos et al (2015) and Xenos et al. (2015) also describe power consumption modelling using nonlinear empirical models to capture the relationship between power consumption, flows, temperatures and pressures for use within an optimiser to improve the operational costs of a parallel network of air compressors and to optimally schedule maintenance.

In all previous work on air separation unit and compressor plant optimisation, site optimisation for gas network demand is developed using a MINLP approach with dynamic optimisation using demands predicted for the days and weeks ahead. A key operational aspect of the process considered in this work is that customer demands are unknown and unpredictable, with time horizons of hours, not days. The implementation of an MINLP approach using nonlinear empirical models, reported in Adamson et al (2015) generally yielded excessive solution times rendering the on-line application impractical. In this work the steady-state optimisation problem is formulated as an MILP in order to ensure robust and efficient on-line optimisation.

To develop a MILP optimisation model, the power consumption of network components (ASU and compressors) are determined using a piece-wise linear modelling approach. Lin et al. (2013) present a review of the use of piecewise linearisation techniques, finding methods can be used to efficiently discover approximated globally optimal results. There are many recent examples in literature regarding the use of piece-wise linearisation techniques to partially or fully formulate a MILP approach to gas network optimisation. For example, Martin (2006) formulates a large MILP problem to solve the optimisation of natural gas networks using piece-wise linear approximations of nonlinear constraints at steady state. While Kolb et al. (2007) describe how gas network optimisation can be better achieved by developing piece-wise linear approximations of network components. In Camponogara et al. (2011) and Aguiar et al. (2014) gas-lifted oil field production costs are optimised by combining convex nonlinear regression curves with piece-wise linear approximations of these curves at given pressure and routing constraints. While Domschke et al. (2011) solve a complex natural gas network optimisation problem by integrating piece-wise linear approximations to partially linearise a nonlinear cost function. Furthermore, in Correa-Posadaa et al. (2014) the piecewise linearisation of general gas flow equations and the development of linear models of other gas network components, such as machine power are discussed. Adopting a similar approach, in this work, it is demonstrated that piecewise linear models can accurately predict the power consumption of both air separation units and compressors with prediction accuracies comparable to the best nonlinear alternative.

Where power consumption can be accurately predicted at a given time of use, the operational network cost can be determined ahead of use, subject to the predicted price of power. Flexible power loads can be selectively purchased on spot power markets at variable prices to reduce overall commodity costs in conjunction with demand side load management, Merkert et al. (2014). We refer to the examples in Karagiannopoulos (2014) in which flexible loads are manipulated into an overall less energy efficient operation strategy whilst still delivering operational cost savings by reducing load at peak power price. Similarly, Zhang et al. (2015) use a MILP scheduling approach to liquefy air off-peak and generate power directly via turbine at peak on underutilised air separation plants. However, expansion processes deliver a lower round-trip efficiency than load shifting the power requirement of usable product production. In this work, we consider a similar approach to Xenos et al. (2016) in which liquid oxygen is used directly as an energy storage device for load management benefits by consuming liquid and temporarily stopping product compression, load shifting additional

power consumption to off-peak periods for liquid production. ASUs are considered fixed loads and external product compression is flexibly load managed as non-dispatchable for power price peak-shaving activities.

The key contribution of this paper is the combined minimisation of the production distribution of a network of air separation units and external compressors using a MILP technique which incorporates accurate piece-wise linear network component power models. In particular, we develop an optimisation strategy that must operate using unpredictable and frequent changes in customer demands (not day ahead predictions). Our on-line application results demonstrate that the optimiser improves site efficiency at steady state by reduction of power consumption by up to 5% (a significant saving for such an energy intensive process¹) while meeting customer demand specifications using fast, efficient and readily available commercial (and open source) software. Furthermore, we demonstrate the optimiser can be used as an advanced scheduling tool to more than recover the costs of demand side load management actions taken to reduce the network's exposure to variation in spot market power pricing.

1.1 The multiple cryogenic air separation unit (ASU) and compressor plant

The multiple cryogenic air separation unit (ASU) and compressor plant considered in this work is an oxygen gas supply network operated to meet customer demands of oxygen gas at two pressures. A schematic of the network (which is the Margam site in the UK) is shown in Figure 1. It comprises a number of centrifugal and reciprocating compressors with various combinations of series and parallel operation. Three air separation units (ASUs) provide production flows of gaseous and liquid oxygen. Gaseous oxygen (GO) is compressed and sent directly to the customer while liquid oxygen (LO) is stored in vessels on site for gas network back up during compressor trips, to meet customer demands at times of high electricity prices and to meet local bulk liquid customer demands by road tanker. Two ASUs produce low pressure (LP) and the third is an internally compressed (IC) ASU capable of producing medium pressure (MP) and high pressure (HP) GO directly, along with liquid nitrogen (LN). Four separate air compressors feed the three ASUs with high pressure compressed air.

Three centrifugal compressors raise LP GO to MP GO and three reciprocating compressors raise LP GO or MP GO to HP GO. A pressure control loop allows cross over between the HP GO to MP GO stream and the network is supported, when required, by an oxygen tank pump vaporiser system converting LO to GO when the HP pipeline pressure falls due to under-production or over-consumption of HP gas. When there is overproduction of LP gas for the current compression configuration, a pressure controlled LP GO gas spill valve will open. However, in normal operation the spill valve, v1, remains shut as ASUs should preferentially be ramped down to prevent spill as production of oxygen consumes power. In other words, an open spill valve often implies an incorrect process configuration.

All ASUs produce gaseous nitrogen streams and a secondary nitrogen network consisting of four compressors is operated in parallel to the oxygen network. As the site is operated in oxygen lead, nitrogen gas is almost always produced in excess and spilled to atmosphere after heat exchange with the compressed air feed. For this paper, the production and supply of nitrogen gas is therefore not considered as part of the plant-wide steady-state optimisation strategy.

¹ e.g. the Air Liquide Group has more than 400 air separation units worldwide. The group's total electricity consumption in 2010 corresponded to more than one thousandth of the world's total electricity consumption (Li, 2011)

Figure 1: Margam supply network and compression configuration for oxygen production (Table 1 provides a description of each network flow). The network comprises of three air separation units (ASUs) (u1, u2 and u3) fed by four air compressors, three centrifugal oxygen compressors (c1, c2 and c3), three reciprocating oxygen compressors (c4, c5 and c6), one spill valve (v1), one cross over valve (v2) and flows to liquid oxygen (F_{LO}) and nitrogen storage (F_{LN}) tanks.

Table 1: Margam oxygen gas supply and compression network components and descriptions.

Flow	Description
$F_{GO,u1}$	Production flow of gaseous LP oxygen from ASU u1
$F_{GO,u2}$	Production flow of gaseous LP oxygen from ASU u2
$F_{GO,u3i}$	Production flow of gaseous MP oxygen from ASU u3
$F_{GO,u3ii}$	Production flow of gaseous HP oxygen from ASU u3
$F_{GO,c1}$	LP to MP oxygen flow through centrifugal compressor c1
$F_{GO,c2}$	LP to MP oxygen flow through centrifugal compressor c2
$F_{GO,c3}$	LP to MP oxygen flow through centrifugal compressor c3
$F_{GO,c4}$	LP to HP oxygen flow through reciprocating compressor c4
$F_{GO,c5}$	MP to HP oxygen flow through reciprocating compressor c5
$F_{GO,c6}$	MP to HP oxygen flow through reciprocating compressor c6

$F_{GO,v1}$	Flow of oxygen spilled from the LP line through valve v1
$F_{GO,v2}$	Flow of oxygen expanded from HP to MP through valve v2
$F_{GO,s}$	Flow of oxygen vaporised from liquid oxygen storage
$F_{GO,MP}$	Flow of MP oxygen to the customer
$F_{GO,HP}$	Flow of HP oxygen to the customer
$F_{LO,u1}$	Production flow of liquid oxygen from ASU u1
$F_{LO,u2}$	Production flow of liquid oxygen from ASU u2
$F_{LO,u3}$	Production flow of liquid oxygen from ASU u3
F_{TLO}	Total production flow of liquid oxygen from ASUs to storage
$F_{LN,u3}$	Production flow of liquid nitrogen from ASU u3

1.2 Current operating policy

The network's manager, BOC Gases, receives communication of independent demands of MP and HP GO from the customer. ASU production rate set points, managed by a linear model predictive controller and the compression network configuration and loads are then manipulated from the current configuration to meet the customer order. To do this, the operators attempt to minimise overall power usage of the plant which is a difficult task as all network components vary in specifications, including capacities and efficiencies. Furthermore, the characteristics of all network components change over time and vary dependant on current operating conditions leading to inconsistency in decisions resulting in sub-optimal re-configuration of the plant.

Current re-configuration strategies include ramping units individually in order to meet changes in customer demand instead of load sharing between units and selecting compressor loads and the production distribution of the ASUs using knowledge of machine efficiencies. As key performance indicators of machine wear and fouling, machine efficiencies are calculated monthly as the average power consumption divided by the average flow through the machine. This method fails to accurately capture the relationship between the operating efficiency of a unit and the process variables that affect it (temperatures, pressures, flows etc.) at given throughputs over time.

2.0 Plant-wide modelling for steady-state optimisation

The concept of real-time optimisation scheme (e.g. see Love, 2007) can be tailored to the operation of the integrated plant of ASU's and compressors. In this application, the aim of the real-time (or steady-state) optimisation scheme is to determine the optimal flows of the oxygen gas throughout the network in order to minimise total operating costs. The outputs of the optimiser are the optimal ASU production rate set-points and the running requirement of each compressor within the network. For significant demand changes, this will also allow the prediction of the optimal network configuration, i.e. whether a particular ASU or compressor should be turned on or off. To implement the optimisation strategy a mathematical representation of the oxygen gas network must be constructed. The three components of this optimisation model are **a)** steady state material balance relationships which are used to ensure an optimal configuration meets customer demand **b)** a description of the power consumption of each of the ASUs and compressors within the network which relates operating cost to the individual flows throughout the network **c)** a description of the total operating cost of the gas supply network. The mathematical model is then used in conjunction with process operational data (operating pressures, flow-rates etc.) to optimise the plant configuration.

2.1 Steady-state material balances

In order to define the material balance relationships across the site it is assumed that there are no dynamic gas losses due to spill from machines or fouling. Furthermore, as the network is relatively local and pipelines between machines short, pipeline friction losses are not modelled. The material balance relationships can therefore be defined based upon the steady-state temperature and pressure corrected² flowrates through the various components of the network.

Low pressure oxygen

Referring to Figure 1, the flow, F , in hundred cubic meters per hour, HCMs/hr, of gaseous oxygen, GO, production from the air separation units u1 and u2 is equal to the flow of gaseous oxygen through compressors c1, c2, c3 and c4 plus the gaseous oxygen flow leaving the network via the low pressure spill valve, v1,

$$F_{GO,u1} + F_{GO,u2} = F_{GO,c1} + F_{GO,c2} + F_{GO,c3} + F_{GO,c4} + F_{GO,v1} \quad (1)$$

Medium pressure gas flow

The customer demand of medium pressure, MP, gaseous oxygen is met by the production of MP from ASU3, u3i, compression of LP gas from c1, c2 and c3, the cross over flow through the let-down valve from the HP line, v2 minus the suction inlet of MP required for the HP compressors, c5 and c6,

$$F_{GO,MP} = F_{GO,u3i} + F_{GO,c1} + F_{GO,c2} + F_{GO,c3} - F_{GO,c5} - F_{GO,c6} + F_{GO,v2} \quad (2)$$

High pressure gas flow

The customer demand of high pressure, HP, gaseous oxygen is met by the production of HP from ASU3, u3ii, compression of MP gas from c5 and c6, and the liquid back up supply flow, $F_{GO,s}$, minus the cross over flow through the let-down valve from the HP to the MP line, v2,

$$F_{GO,HP} = F_{GO,u3ii} + F_{GO,s} + F_{GO,c4} + F_{GO,c5} + F_{GO,c6} - F_{GO,v2} \quad (3)$$

Liquid oxygen production

Liquid oxygen is produced by air separation units u1, u2 and u3, and the total site production, F_{TLO} , is given by,

$$F_{TLO} = F_{LO,u1} + F_{LO,u2} + F_{LO,u3} \quad (4)$$

Each ASU requires a minimum LO production flow at all times to purge hydrocarbons from the column liquid sump. When additional LO production is required to build liquid stocks, a production target can be met by optimisation to select the most efficient available ASUs.

2.2 Power consumption of each unit within the network

The efficiency of a compressor is also known as its specific power. This represents the power, W (kW), required to compress 1 HCM of gaseous oxygen. Specific power, ϵ , (kW/HCM), is normally used to calculate the power required by the machine to compress the swept volume through the machine. Kurz et al. (2010) describe the differences between the power consumption (and hence efficiency) of centrifugal and reciprocating compressors at a constant discharge pressure and temperature. As gas flow increases through a centrifugal

² Site flow meters are calibrated regularly and twinned with pressure and temperature meter readings to provide a calculated standardised flow which assumes ideal gas behaviour.

compressor, the power consumption increases proportionally presenting a constant efficiency, however, machines exhibit nonlinear behaviour approaching the lower and upper limits of operation. Reciprocating compressors have higher fixed mechanical power requirements, simply moving the piston, resulting in an efficiency relationship which favours the maximum design gas flowrate. In this work it is assumed that specific power may be mathematically modelled as a function of flow through the machine and the discharge pressure, therefore the power consumption (W_{cj}) of compressor cj , is given by,

$$W_{cj} = F_{GO,cj} \cdot \varepsilon_{cj}(F_{GO,cj}, P_{Dj}) \quad (5)$$

Where the efficiency profile (functional relationship) depends on the type of product compressor, either centrifugal or reciprocating.

The power consumption of the two low pressure oxygen producing air separation units is defined using a modified version of equation 5. Air separation unit power may be calculated using the difference between the power of the air compressor/s feeding the separation column and the power generated from the turbine (which is used to balance ASU column temperatures). As ASU power is directly related to total oxygen production (gaseous oxygen plus liquid oxygen), the power consumption of units $u1$ and $u2$, $W_{uj} (j=1,2)$, is given by,

$$W_{uj} (j=1,2) = (F_{GO,uj} + F_{LO,uj}) \cdot \varepsilon_{uj}((F_{GO,uj} + F_{LO,uj}), P_{Dj}) \quad (6)$$

Where the machine efficiency is taken to be a function of total oxygen production rate (gaseous oxygen plus liquid oxygen production rates) and the air compressor discharge pressure, which for an ASU is the column pressure. Column pressure is known to be a function of total oxygen production rate as it increases with increasing compressed air supply flow rate. As ASU $u2$ has two modes of operation, being supplied by the main air compressor only, or both the main and booster air compressors in parallel feeding the unit, power models of each air compressor feed operation were developed.

The third ASU on the plant ($u3$) generates MP, HP and LO to storage simultaneously therefore the power consumption of oxygen production is,

$$W_{u3} = (F_{GO,u3i} + F_{GO,u3ii} + F_{LO,u3}) \cdot \varepsilon_{u3}((F_{GO,u3i} + F_{GO,u3ii} + F_{LO,u3}), P_{Dj}) - F_{LN,u3} \cdot \varepsilon_{LN} \quad (7)$$

However, ASU $u3$ also produces LN directly through internal compression. As the network is optimised to minimise the cost of gaseous oxygen production, the LN product flow, $F_{LN,u3}$, is multiplied by a fixed LN production efficiency term, ε_{LN} , to approximate the power used to produce LN. This is then subtracted from the overall ASU power consumption of the unit.

Kurz et al. (2010) describe how gas density, a function of temperature, is known to have an effect on the work done by a compressor. Whilst studying the empirical data sets, product compressor feed temperature was observed as roughly constant due to temperature control of the product streams exiting the ASU which negate the majority of temperature variation. In addition, compressed air streams into ASUs are cooled by direct coolers and chillers to cooler-than-ambient conditions. In this work, the effect of temperature on power consumption on the models is therefore smaller than the influence of discharge pressure and gas flow due to temperature control management. Auxiliaries such as cooling water and water chiller duty have not been included in the models of network power consumption as these are not a manipulated variable of the optimisation technique but a systematic cost of operating the site.

2.3 Total operating cost of the network

The total network operating cost, J (£/hr) is a function of the sum of compressor and air separation unit power multiplied by the cost of power, C_{kW} (£/kW) plus the cost of consuming liquid oxygen, C_S (£/HCM), when a backup flow, F_S , is required plus the cost of producing LO, C_{LO} (£/HCM),

$$J = C_{kW} \cdot \left(\sum_{j=1}^6 W_{cj} + \sum_{j=1}^3 W_{uj} \right) + C_S \cdot F_S + C_{LO} \cdot F_{TLO} \quad (8)$$

The cost of power, C_{kW} , is the current spot market price plus non-commodity costs such as charges and taxes, the price of liquid back up consumption is assumed to be the average cost of its generation and the cost of making LO is the average cost of production, which involves consuming liquid nitrogen.

3.0 Data-based modelling of machine efficiency / power

As discussed in section 2.2, machine efficiency (and hence power) is a nonlinear function of flow through a machine and the discharge pressure. With years of historical data available in the data historian, it is possible to produce efficiency curves as well as discover the flow limits for each machine (discussed in section 4 of the paper). 15 minute averaged data of oxygen gas flow, machine discharge pressure, power usage and other key normal operation indicators, such as recycle valve opening position etc., were compiled over 6 months for each machine in the compression arrangement. Data was pre-screened to remove missing data and data not recorded during normal operation, e.g. at machine start up. Both univariate and multivariate plots of the process data were ‘eye-balled’ in order to ascertain interdependencies between the variables before data-based modelling was undertaken. As an example, the nonlinear relationship between the power consumption (and hence efficiency) of a reciprocating compressor and the flow and discharge pressure may be observed in Figure 2. Where the compressor recycles due to anti-surge control at low flow rates and is overloaded at high flow rates, a reduction in machine efficiency is observed. As a result, machines often run more efficiently per unit of gas when fully loaded and the resulting non-linear relationships are convex. Linear regression would not capture these complexities.

In Adamson et al. (2015) and Xenos et al. (2015) polynomial nonlinear regression models of compressor or ASU power were used. Referring to equations 5, 6 and 7, efficiency is multiplied by flow to calculate power, where efficiency is a function of flow and pressure. This introduces combination and modelling non-linearity and can be simplified by building multivariate regression models of power directly. A typical non-linear model being of the form (where the $\hat{b}_{0,j}$, ..., $\hat{b}_{4,j}$ are model coefficients),

$$\hat{W}_{cj} = \hat{b}_{0,j} + \hat{b}_{1,j}F_{GO,cj} + \hat{b}_{2,j}P_{Dj} + \hat{b}_{3,j}F_{GO,cj}^2 + \hat{b}_{4,j}P_{Dj}^2 \quad (9)$$

However, a nonlinear model cannot be embedded within a MILP approach to network optimisation which requires all components of the optimisation model to be linear.

Figure 2: Power consumption as a function of flow (HCMs/hr) and discharge pressure (bar). The process data demonstrates a multivariate nonlinear relationship. The data shown is for reciprocating compressor (c4) with the centrifugal compressors also demonstrating nonlinear relationships between power, flow and discharge pressure.

3.1 Piecewise linear modelling

Piece-wise linear models are the combination of many linear models to provide a pseudo-nonlinear overall model. The typical structure of a univariate piecewise linear model is (where u_1 is a single input and y is a process output and N are the number of data records),

$$y_j (j=1,\dots,N) = \left\{ \begin{array}{ll} \hat{b}_{01} + \hat{b}_{11}u_{1j} & u_{10} \leq u_{1j} < u_{11} \\ \hat{b}_{02} + \hat{b}_{12}u_{1j} & u_{11} \leq u_{1j} < u_{12} \\ \vdots & \vdots \\ \hat{b}_{0k} + \hat{b}_{1k}u_{1j} & u_{1k-1} \leq u_{1j} < u_{1k} \end{array} \right\}$$

The constraints represent the regions in which each model is valid (between the minimum, u_{10} and maximum, u_{1k} value of u_1). A number of approaches have been developed to create linear piecewise models from large data sets e.g. see Yang (2016). The primary difficulty being to optimally determine the total number of models, their parameters ($\hat{b}_{01}, \dots, \hat{b}_{1k}$) and the breakpoints ($u_{11}, \dots, u_{1k}$) that define the regions of validity for each model. In this work a pragmatic approach to piece-wise linear modelling was adopted, simplifying the model development task by assuming a maximum of five linear models, $k_{max} = 5$, are required for accurate relationship capture. This gives the following cost function,

$$J_m = \sum_{k=1}^{k_{max}} \|y_j - \hat{b}_{0k} - \hat{b}_{1k}u_{1j}\|_1 \quad \forall y_j, u_{1j} \in (u_{1k-1} \leq u_{1j} < u_{1k}) \quad (10)$$

A nonlinear optimiser, similar to the method employed by Szűcs et al (2012), is then used to minimise (10) in order to determine each of the model parameters $\hat{b}_{0k}, \hat{b}_{1k}$ ($k = 1, \dots, k_{max}$) and iteratively determine the position of the model break-points $u_{1k}, \dots, u_{1k_{max}-1}$. In an outer-loop, the value of k_{max} is increased from an initial value of two to the maximum value of five. The resulting models are validated on a second data-set and the piecewise linear model that minimises the validation error chosen as the optimal model.

Multivariate piece-wise linear modelling of the compressors

Each piece-wise segment of the compressor data are regressed to the following model,

$$\hat{W}_{cj,k} = \hat{b}_{0,k} + \hat{b}_{1,k}F_{GO,cj} + \hat{b}_{2,k}P_{Dj} \quad (11)$$

For the compressors, the discharge pressure³ is used to define the model breakpoints, i.e. denote the points at which one model begins at and the previous one ends. To develop the models, plant data was pre-screened and erroneous data points indicating non-running, non-steady state or operation outside normal operating range were removed. Figure 3 shows a typical set of data for compressor c4. Here, four piece-wise linear models were found to provide the best fit with the lowest modelling error to generate estimated power consumption from the measured flow and pressure data.

Figure 3: Plot of power with respect to flow for compressor c4 between the limits shown in Figure 2. The separate colours denote flow data within the constraint boundary of each piece-wise linear model. To assist visualisation of the model predictions the discharge pressure is not shown, rather, the black lines present the prediction obtained using each piece-wise linear model at the discharge pressure midpoint of each region. Additional, straight-line relationships would be obtained given any discharge pressure within a particular constraint boundary.

The model equations and the optimal breakpoints are,

$$\hat{W}_{c4} = \begin{cases} -320.75 + 22.01P_{D4} + 10.26F_{GO,c4} & 26.33 \leq P_{D4} < 30.12 \\ -285.47 + 18.88P_{D4} + 11.06F_{GO,c4} & 30.12 \leq P_{D4} < 34.68 \\ -244.07 + 15.49P_{D4} + 12.23F_{GO,c4} & 34.68 \leq P_{D4} < 40.21 \\ -182.19 + 12.18P_{D4} + 13.39F_{GO,c4} & 40.21 \leq P_{D4} < 43.14 \end{cases} \quad (12)$$

In other words, the empirical model defines ‘pseudo-machine’ models, each with a lower and upper pressure bound, i.e. for the purpose of building the steady-state optimiser the model of

³ Models could have been developed using either flow or discharge pressure breakpoints, however the use of pressure breakpoints were found to produce models that gave more accurate prediction of machine power.

compressor c4 may be considered as four separate (linear) machines with the appropriate pseudo-machine being determined by the current discharge pressure.

Univariate piece-wise linear modelling of the ASUs

Ideally, a piece-wise linear model of an ASU would use identical methodology to compressors. However, as column pressure is directly related to the total oxygen production rate, unlike pipeline and discharge pressure of compressors, the current column pressure cannot be fed into the model to estimate the ASU power consumption at the optimised flow. Therefore, ASU piece-wise linear power models must be univariate, only considering production flow from the ASU, with flow breakpoints and are given by,

$$\hat{W}_{uj,k} = \hat{b}_{0,k,uj} + \hat{b}_{1,k,uj}(F_{GO,uj} + F_{LO,uj}) \quad (13)$$

ASU u2 can be fed by either the main air compressor only or both the main and booster air compressors simultaneously. Each compressed air supply configuration has a separate piece-wise linear modelling strategy. Using $k_{max} = 5$ as a starting point, the piece-wise linear optimiser determined that the optimal model comprised three piece-wise segments for each compressor feed operation. An overlap between models demonstrates network configurations where the operator should have considered turning off one air compressor to reduce the overall ASU power consumption. Figure 4 shows the six piece-wise linear models with production flow breakpoints for ASU u2 (the actual model coefficients are omitted due to industrial confidentiality).

Figure 4: Plot of power with respect to flow for ASU u2. The separate colours denote flow data within the constraint boundary of each piece-wise linear model. Black lines present the prediction obtained using each piece-wise linear model within the particular regions. Six piece-wise regions are shown, three with one air compressor feed and three with two air compressor feeds. Two piece-wise linear models, one from each air supply configuration, overlap with respect to total oxygen production flow rate.

Analysis of the difference in operation of ASUs in summer and winter shows a reduction in the maximum production rate range of operation of ASU u2. This may be due to temperature differences in the air increasing the difficulty of compressing warmer, less dense air in the

summer. To represent this, the pseudo machine with the highest production flow range is removed from the optimiser during the summer months.

4.0 Machine processing capacity

Realistic modelling of machines requires the determination of the minimum and maximum flow production or processing ability of the ASUs and compressors. For example, the lower and upper flow limits of a compressor could be assumed constant and given by,

$$F_{GO,cj}^{min} \leq F_{GO,cj} \leq F_{GO,cj}^{max} \quad (14)$$

However, process operation limits are known to be a function of machine size, discharge pressure and weather conditions. For the compressors, operational data shows the lower and upper limits of flow change with the discharge pressure of the machine, i.e.

$$F_{GO,cj}^{min}(P_{Dj}) \leq F_{GO,cj} \leq F_{GO,cj}^{max}(P_{Dj}) \quad (15)$$

Therefore, two polynomial regression fits using data on the edge of the operating regions are used to capture the relationships between flow and discharge pressure. The regression model can then be used to determine the lower and upper flow constraint, equation 15. As the steady state optimiser does not alter pressure, the bounds are fixed prior to solving and nonlinearity is not introduced into the cost function.

Figure 5 shows the upper and lower flow bounds of compressor c4, with respect to discharge pressure. The lower (and upper) bound was obtained using the following regression model to the data points indicated in Figure 5.

$$\hat{F}_{GO,cj}^{min}(P_{Dj}) = \hat{a}_{0,cj} + \hat{a}_{1,cj}P_{Dj} + \hat{a}_{2,cj}P_{Dj}^2 \quad (16)$$

Figure 5: Plot of flow and discharge pressure of compressor c4 at steady state between the limits shown in Figure 2. Nonlinear regression models (16) were regressed to selected points (in red) to determine the lower and upper operational bounds at a given pressure. The black lines are the predicted flow constraints (15) obtained from model (16). Data points outside the regressed lower and upper flow bounds were outside of ideal operating range, ie running with gas recycle or overloaded.

5.0 Total operating cost model linearisation

The total operating cost of the network, equation 8 may be rewritten to include the piece-wise power models,

$$J = C_{kW} \cdot \left(\sum_{j=1}^6 \sum_{k=1}^{mj} \hat{W}_{cj,k} \cdot \delta_{cj,k} + \sum_{j=1}^3 \sum_{k=1}^{mj} \hat{W}_{uj,k} \cdot \delta_{uj,k} \right) + C_s \cdot F_s + C_{LO} \cdot F_{TLO} \quad (17)$$

The network cost is therefore a sum of each machine power, which is a sum of all the pseudo-machine model powers ($k = 1, \dots, m$), multiplied by the cost of power and the additional liquid costs and mj are the number of piecewise linear models obtained for each unit after model validation. In addition, Boolean coefficients, $\delta_{cj,k} \in \{0,1\}$ and $\delta_{uj,k} \in \{0,1\}$ are introduced into the cost function in order to remove machines from the network when not required as well as ensuring that machines are operating between the minimum and maximum flow constraints of each pseudo-machine region. A zero indicates the machine is off (therefore removing the respective power term from the operating cost) and one multiplied by the power indicates it is on. To prevent the optimiser selecting multiple pseudo-machines simultaneously, mutually exclusive constraints are then added for the sum of Boolean coefficients for each machine. For example, for the ' mj ' Boolean coefficients of models for compressor ' j ',

$$\sum_{k=1}^{mj} \delta_{cj,k} \leq 1 \quad (18)$$

Prior minimisation of equation 17, the current plant discharge pressures are imported. With no further updates, the discharge pressure of each unit remains constant during optimisation therefore the only decision variables in the in the piece-wise models are flow and the associated Boolean coefficients. As the multiplication of the Boolean and flow variables introduces combinational nonlinearity, in order to use a MILP solver they must be removed. The combination of the estimated power model and the Boolean gives,

$$\hat{W}_{cj,k} = (\hat{b}_{0,k} + \hat{b}_{1,k} P_{Dj}) \cdot \delta_{cj,k} + \hat{b}_{2,k} F_{GO,cj,k} \cdot \delta_{cj,k} \quad (19)$$

The second term in equation 19, $\hat{b}_{2,k} F_{GO,cj,k} \cdot \delta_{cj,k}$, causes nonlinearity therefore an auxiliary variable $y_{cj,k}$ is introduced where,

$$y_{cj,k} = F_{GO,cj,k} \cdot \delta_{cj,k} \quad (20)$$

When directly optimising for $y_{cj,k}$, the variable's lower and upper limits are indirectly subject to changes in the Boolean variable value. For example for a pseudo-machine of a compressor,

$$\hat{F}_{GO,cj}^{min}(P_{Dj}) \cdot \delta_{cj,k} \leq F_{GO,cj} \leq \hat{F}_{GO,cj}^{max}(P_{Dj}) \cdot \delta_{cj,k} \quad (21)$$

Therefore, $F_{GO,cj} = 0$ if $\delta_{cj,k} = 0$ and $\hat{F}_{GO,cj}^{min}(P_{Dj}) \leq F_{GO,cj} \leq \hat{F}_{GO,cj}^{max}(P_{Dj})$ when $\delta_{cj,k} = 1$.

Therefore, the cost function used to minimise total network cost is,

$$J = C_{kW} \cdot \left(\sum_{j=1}^6 \sum_{k=1}^{mj} (\hat{b}_{0,k,cj} + \hat{b}_{1,k,cj} P_{Dj}) \cdot \delta_{cj,k} + \hat{b}_{2,k,cj} \cdot y_{cj,k} + \sum_{j=1}^3 \sum_{k=1}^{mj} \hat{b}_{0,k,uj} \cdot \delta_{uj,k} + \hat{b}_{1,k,uj} \cdot y_{uj,k} \right) + C_s \cdot F_s + C_{LO} \cdot F_{TLO} \quad (22)$$

This is minimised with respect to the binary variables $\delta_{cj,k} \in \{0,1\}$, $\delta_{uj,k} \in \{0,1\}$, the auxiliary variables, $y_{cj,k}$, $y_{uj,k}$ and the flows F_s and F_{TLO} . The minimisation is performed with respect to the material balance constraints, equations 1-4, and the flow range limits, equation 15, and the constraints that define the state of the Boolean variable, equations 21.

6.0 Implementation Aspects

A supply network mimic was produced in Microsoft Excel as an optimiser interface (see Figure 6). The user inputs the current customer demand, which is linked to the mass balance constraint cells, to optimise the compression arrangement. The reasons for using Excel were that **a)** the steady-state optimiser had to be compatible with all computers within the company allowing the optimisation results to be integrated with exiting data access methods using data historian Excel plug ins and implemented by the process operators, **b)** it had to be cheap to run and easily updateable once developed as machine specifications change, **c)** the optimiser interface had to be easy for process operators to understand.

The optimised flows through the machine, the upper and lower flow limits, and the estimated machine power are presented in a machine representation of the network.

Figure 6: Spreadsheet-based optimiser operator interface to present the optimal production flow rate set points and compression configuration by network mimic (all numerical values are false). The user inputs the customer demand combination in the top left corner ready for solving. The optimisation result is displayed within the relevant boxes representing the machines in the network. Boxes prior to the ASUs display the air compressors required to meet the oxygen production rate.

An automated spreadsheet running scheduler positioned on a 24 hour server accommodates the macro run spreadsheet solving method. On opening, the spreadsheet downloads the current demand, pressure, flow, temperature, power price and machine availability tag data required for power estimation and the material balance constraints. A visual basic program then automatically runs the MILP (solved using Excel's built in solver routine) to minimise overall steady state network cost, it then updates the machine requirement running tags (0 = off, 1= on) and ASU production flow set points and exports this information to the data historian. The scheduler runs the optimisation spreadsheet every 15 minutes and as the results are only relevant at steady state and the network must be reconfigured from the current position to the optimal configuration over time by the process operator.

6.1 Machine availability

Machines were often unavailable for periods of time during maintenance or after a machine trip. A selection of dropdown text boxes were positioned on the operator interface and if the machine was unavailable, the machine's Boolean coefficient was forced to be zero (using an equality constraint, e.g. $\delta_{cj,k} = 0$). Adding this flexibility allowed the optimiser to cater for any network configuration. Forcing the Boolean coefficient to zero for liquid back up supply also allowed liquid back up to be prevented for most situations as favoured by liquid schedulers who wish to preserve stocks.

7.0 On-line application results

This section assesses the performance of the piece-wise linear models, running estimations of machine power in parallel with actual power consumption. It also demonstrates the application of the steady state optimiser to the process plant, optimising steady state network production and load distributions.

7.1 Prediction of power consumption

Validation of piece-wise linear models was conducted on-line by accessing the actual unit power consumption and comparing this with each of the models prediction of power consumption (using measured flow and pressure data) to determine the prediction accuracy of the models. The piece-wise linear models are compared with **a)** the current site models, which use a fixed value specific power (which is the current average efficiency of the compressor or ASU) multiplied by the flow through the machine, and **b)** the best-fit nonlinear multivariate models using equation 9. As a representative example of the relative accuracies of the models of the compressors, Figure 7 shows the power consumption predictions for compressor c4 in the network.

The results shown cover a period of 85 hours of operation where the three models are used in parallel. Model accuracy was determined by calculation of mean error (ME) between the actual and estimated power consumption at 15 minute intervals. Over this period, the ME for the current on-site constant efficiency specific power model is 99.81kW, for the nonlinear multivariate power model the ME is 20.57kW and for the piece-wise linear power model the ME is 20.10kW. Similar results are obtained for the other compressors in the network with around an 80% improvement in estimation of site power consumption when compared to the existing method used (and a comparable accuracy between piecewise linear and the nonlinear multivariate models). The ME of the piece-wise linear compressor models produced is typically equivalent to around $\pm 2\%$ of total compressor power.

Figure 8 shows the power consumption predictions for ASU u2, with operation across the full range of piece-wise linear models. During this period, the second air compressor feeding u2 was turned off, resulting in a temporary power increase (observed on the fourth vertical grid line) this is because the first machine is ramped up prior to stopping of the second machine.

The results cover the same period of 85 hours of operation and the model accuracy was again determined by calculation of the ME at 15 minute intervals. Over this period, the ME for the current on-site specific power model is 832.95kW and for the piece-wise linear power model the ME is 322.77kW. The ME of the piece-wise linear ASU models produced is typically equivalent to around $\pm 3\%$ of total ASU power. Similar results were obtained for the two other ASUs demonstrating that the piece-wise linear models of power consumption are around 60% more accurate when compared to power consumption estimation using the current on-site method. The use of the nonlinear multivariate model would increase the accuracy by a further

10% however; the optimiser should not be fed with pressure data at the current steady state production flows as pressure is a function of the total oxygen production rate.

Figure 7: Models of power consumption of compressor c4 over 85 hours, where the black trend is the actual power, the red trend is the estimate obtained from the current site specific power model, the green trend line is the estimate obtained from the multivariate nonlinear model, equation 9, and the blue trend is the multivariate piece-wise linear model estimation of power, equation 12.

Figure 8: Models of power consumption of ASU u2 over 85 hours, where the black trend is the actual power, the red trend is the estimate obtained from the current site specific power model, the green trend is the estimate obtained using the multivariate nonlinear model, equation 9, and the blue trend shows the univariate piece-wise linear model estimation of power.

7.2 Power cost optimisation savings

To demonstrate the operating cost savings that are achieved using the steady-state optimiser, steady state demands were optimised using the MILP solver and compared actual network configurations and flows using process data. Figure 9 shows the customer demand of MP and HP over a period of one day and the actual customer flows through the site flow meters. Large, temporary increases in HP supply demonstrate that liquid back up was required to flow through the meters and boost supply pipeline pressure. Three steady state positions were used to estimate the potential network power savings.

Figure 9: Customer demand for MP (blue) and HP (red) oxygen gas and metered gas delivered over a period of 24 hours. Three numbers indicate the selected steady state points and real network gas flows are ramped up or down towards the steady state points over a period of time.

Table 2 shows a comparison between the actual network configuration and ASU production flows and the output of the optimiser at steady state points 1-3, assuming a power price of £55/MWh. Binary values represent whether the machine was on or off and the flow from the network component is presented as a percentage of the upper limit. The estimated power consumption of the actual network configuration and optimised solution is calculated and the percentage difference between the two displayed.

Table 2 demonstrates that compression changes are not always required to yield power reduction benefits as only steady state point 2 involves a compression change over. This is understandable as the power consumption of the ASUs is significantly larger when compared to individual product compressors. For the tested steady state demands, both the process operator and the optimiser correctly decided ASU u1 was not required. Overall, the steady-state optimiser demonstrates a percentage reduction in power consumption and ultimately network costs of up to 5% at steady state.

Table 2: Recorded and optimised oxygen supply network outputs for steady state points 1-3.

Cost Function Component	Steady State Point 1		Steady State Point 2		Steady State Point 3	
	Actual	Optimiser	Actual	Optimiser	Actual	Optimiser
$F_{GO,u1}$	0%	0%	0%	0%	0%	0%
$F_{GO,u2}$	56.7%	58.6%	64.3%	85.3%	80.2%	88.0%
$F_{GO,u3i}$	57.3%	43.9%	85.5%	43.9%	87.8%	79.4%
$F_{GO,u3ii}$	11%	32.6%	33.3%	70.4%	37.0%	68.9%
δ_{c1}	0	0	0	0	0	0
δ_{c2}	0	0	0	0	0	0
δ_{c3}	1	1	1	1	1	1
δ_{c4}	0	0	0	1	1	1
δ_{c5}	1	1	1	1	1	1
δ_{c6}	0	0	0	0	0	0
$F_{GO,v1}$	0%	0%	0%	0%	0%	0%
$F_{GO,v2}$	3%	50%	25%	84%	27%	0%
$F_{GO,s}$	0%	0%	0%	0%	0%	0%
$F_{LO,u1}$	0%	0%	0%	0%	0%	0%
$F_{LO,u2}$	17%	17%	17%	17%	17%	17%
$F_{LO,u3}$	36%	30%	0%	0%	0%	0%
$F_{LN,u3}$	57.4%	53.2%	92.1%	72.7%	95.8%	100%
Power Consumption Differential (%)	-4.89%		-4.87%		-3.25%	

7.3 Liquid oxygen as an energy storage device

In addition to the cost savings generated by reducing the overall gas network power consumption for a given customer demand combination by optimisation, the potential benefits of planned demand side load management were simulated using the steady state optimiser, subject to variation in power pricing. Optimal steady state point 2 is considered at three power price values, **a)** the average power price considered for typical steady state optimisation at £55/MWh, **b)** the price of power during a typical low surplus peak at £250/MWh and **c)** off-peak overnight pricing at £45/MWh.

For steady state point 2 under average power price conditions, the optimal compression configuration is of three product compressors running, compressors c3, c4 and c5, with the combined power consumption being the flexible load. Running the optimiser using the peak power pricing value automatically suggests turning off all running product compressors and consuming LO to meet the created shortfall in customer demand. In this case, the network is reconfigured to open the cross over valve, v2, for HP to MP GO expansion and the spill valve, v1, to expel LP GO to atmosphere. ASU production rates and power consumptions are assumed fixed as temporarily ramping ASUs during peak is not considered safe practise.

In order to offset the liquid consumed in one hour during the peak pricing simulation and determine the benefits of such a procedure, steady state point 2 is optimised to recover the liquid consumed over three hours at the off-peak power price. The total predicted LO consumed during peak is divided by three and used in addition to the required minimum liquid production target, i.e. above what was required for LO purge. Optimisation results in an alternative product compression configuration with an increased ASU u3 MP GO production flow and a higher ASU u2 LO production flow to replace the liquid consumed

during peak. The cost incurred generating product off peak in additional power consumption and LN injection is calculated to be around 43% of the cost reduction achieved during peak avoidance, a significant benefit. The optimiser output at each of the various power prices for the customer demand combination is presented in Table 3.

Table 3: Optimised oxygen supply network outputs for steady state point 2 at three different power price values.

Cost Function Component	Optimal Configuration	Power Price Event	Off-Peak Liquid Make
C_{kW}	£55/MWh	£250/MWh	£45/MWh
$F_{GO,u1}$	0%	0%	0%
$F_{GO,u2}$	85.3%	85.3%	61.5%
$F_{GO,u3i}$	43.9%	43.9%	67.7%
$F_{GO,u3ii}$	70.4%	70.4%	70.4%
δ_{c1}	0	0	0
δ_{c2}	0	0	0
δ_{c3}	1	0	1
δ_{c4}	1	0	0
δ_{c5}	1	0	1
δ_{c6}	0	0	0
$F_{GO,v1}$	0%	100%	0%
$F_{GO,v2}$	84%	100%	41.5%
$F_{GO,s}$	0%	65.5%	0%
$F_{LO,u1}$	0%	0%	0%
$F_{LO,u2}$	17%	17%	97.7%
$F_{LO,u3}$	0%	0%	0%
$F_{LN,u3}$	72.7%	72.7%	72.7%

8.0 Discussion and conclusions

It has been shown that piece-wise linear models of network component power allow for improved estimation of power consumption (when compared to the existing site models) and achieve a similar accuracy to nonlinear regression models. The benefit of generating piece-wise linear models is that they can be incorporated into a steady-state optimisation strategy solved using a MILP approach, a strategy not yet attempted by other authors. It has been demonstrated using plant data that the developed optimisation strategy can reduce site power consumption costs by up to 5% during steady state conditions.

The fidelity of the optimisation scheme is dependent on the accuracy of the power consumption models. Generally, real-time optimisation strategies up-date model parameters when new steady-state conditions are detected. This is not the strategy adopted here, rather as with the existing on-site model, the aim is to monitor model accuracy periodically updating the models, e.g. on a monthly basis. It is suspected that greater improvements in model accuracy can be achieved through integration of further variables known to affect efficiency into the multivariate power models, such as ambient conditions (Fu and Gundersen 2012).

In addition to providing energy savings, other benefits of the steady-state optimiser are that the automated system releases operator time for other tasks, such as improving overall site control and aiding network configuration manipulation from the current network position to the optimal steady state end point. Furthermore, it could be used to provide decisions as to whether it is cheaper to consume liquid and stop compressors during high power price

periods. Including forecasts of spot market power pricing, a schedule for liquid oxygen production can be developed to allow release of liquid inventory during periods of high power or non-commodity costs and production during low pricing periods. In some cases, spilling gas or consuming liquid can be used to deliver an overall network cost benefit by avoiding consuming power at peak pricing (non-dispatchable activities) or by generating revenues from frequency or demand side response activities (dispatchable activities) as suggested by Merkert et al. (2014). This may also be useful for oxygen requests where a larger, under-loaded and less efficient compressor is required instead of another. Steady state optimisation cannot, however, automatically and optimally schedule over periods of lower power pricing to recover those stocks by producing liquid at other times but this can be achieved either by introducing a liquid production target schedule or by developing a dynamic optimisation method (Manenti et al. 2013).

Dynamic optimisation could deliver considerable further benefits including reduction of power consumption during network transitions, minimisation of liquid back up consumption, reduction of LP GO gas spill and minimisation of machine wear due to starting and stopping compressors. Furthermore, it would ensure the customer order is adhered to during ramps and that all ASU ramping constraints are adhered to for safety reasons. Large ramps can take place over several hours with operators required to alter production set point targets periodically to meet customer demands at all times.

Acknowledgments

The authors gratefully acknowledge the financial support of EPSRC grant EP/G037620/1 and BOC Gases through the Biopharmaceutical Bioprocessing Technology Centre at Newcastle University, UK.

References

- Abbaspour, M., Chapman, K. S., Krishnaswami, P. (2005) "Nonisothermal compressor station optimisation." *Journal Energy Resource Technology*, Trans ASME 2005, 127:131–41.
- Adamson, R., Hobbs, M., Silcock, A., Montague, G. (2015). "Real Time Optimisation of Industrial Gas Supply Networks." *IFAC-PapersOnLine* 48(8): 355-360.
- Aguiar, M. A. S., Camponogara, E. and Silva, T. L.. (2014). "A Mixed-Integer convex formulation for production optimization of gas-lifted oil fields with routing and pressure constraints" *Brazilian Journal of Chemical Engineering*, 31(2), 439-455.
- Azadeh, A., A. Z. Anaraki, et al. (2015). "Optimisation of multi-stage supply chain systems by integrated simulation-variable neighbourhood search algorithm." *International Journal of Services and Operations Management* 21(1): 1-26.
- Camponogara, E., M. P. de Castro, Plucenio, A., Pagano, D. J. (2011). "Compressor scheduling in oil fields." *Optimization and Engineering* 12(1): 153-174.
- Cao, Y., Swartz C. L. E., Baldea M. (2011). Design for dynamic performance: Application to an air separation unit. *Proceedings of the 2011 American Control Conference*.
- Correa-Posadaa, C. M., and Sánchez-Martín, P. (2014). "Gas network optimization: A comparison of piecewise linear models." *Optimization Online*
- Cortinovis, A., Mercangöz, M., Zovadelli, M., Pareschi, D., De Marco, A., Bittanti, S. (2016). "Online performance tracking and load sharing optimization for parallel operation of gas compressors." *Computers & Chemical Engineering* 88: 145-156.

- Domschke, P., Geißler, B., Oliver, K., Jens, L., Alexander, M., Antonio, M. (2011). "Combination of Nonlinear and Linear Optimization of Transient Gas Networks." *INFORMS Journal on Computing* 23(4): 605-617.
- Fu, C. and Gundersen, T. (2012), "Using exergy analysis to reduce power consumption in air separation units for oxy-combustion processes", *Energy*, 44, 60-68
- Gao, J., and You, F. (2015) "Shale Gas Supply Chain Design and Operations toward Better Economic and Life Cycle Environmental Performance: MINLP Model and Global Optimization Algorithm" *ACS Sustainable Chemistry & Engineering* 3 (7), 1282-1291
- Han, I. S. and Han, C. (2003). "Modeling of multistage air-compression systems in chemical processes." *Industrial and Engineering Chemistry Research* 42(10): 2209-2218.
- Han, I. S., Han, C. and Chung, C. B. (2004). "Optimization of the air- and gas-supply network of a chemical plant" *Chemical Engineering Research and Design* 82(10): 1337-1343.
- Huang, R., Zavala, V. M. and Biegler L. T. (2009). "Advanced step nonlinear model predictive control for air separation units." *Journal of Process Control* 19(4): 678-685.
- Karagiannopoulos, S., Vrettos, E., Andersson, G., Zima, M. (2014) "Scheduling and real-time control of flexible loads and storage in electricity markets under uncertainty," 11th International Conference on the European Energy Market, Krakow, 1-5.
- Kolb, O., Lang, J. and Bales, P. (2007). "Adaptive linearized models for optimization of gas networks." *Proceedings in Applied Mathematics and Mechanics*, 7, 1061301–1061302
- Kopanos, G. M., Xenos, D. P., Cicciotti, M., Pistikopoulos, E. N., Thornhill, N. F. (2015). "Optimization of a network of compressors in parallel: Operational and maintenance planning – The air separation plant case." *Applied Energy* 146: 453-470.
- Kurz, R., Winkelmann B. and Mokhatab S. (2010). "Efficiency and Operating Characteristics Of Centrifugal And Reciprocating Compressors." *Pipeline and Gas Journal* 237(10).
- Li, T., Roba T., Bastid M. and Prabhu A.. (2011) "Real time optimization of air separation plants." *Proceedings of ISA Automation Week 2011*.
- Lin M.-H., Carlsson J. G., Ge D., Shi J. and Tsai J.-F. (2013), "A Review of Piecewise Linearization Methods," *Mathematical Problems in Engineering*, vol. 2013.
- Love, J. (2007). "Process Automation Handbook: A Guide to Theory and Practice", Springer Science & Business Media: 849-899.
- Manenti, F. and Rovaglio M. (2013). "Market-driven operational optimization of industrial gas supply chains." *Computers & Chemical Engineering* 56: 128-141.
- Martin, A., M. Möller and S., Moritz (2006). "Mixed integer models for the stationary case of gas network optimization." *Mathematical programming* 105(2-3): 563-582.
- Merkert, L., Harjunkski, I., Isaksson, A., Säynevirta, S., Saarela, A., Sand, G. (2014) "Scheduling and energy – Industrial challenges and opportunities" *Computers and Chemical Engineering* 72 183-198
- Øvervåg, T. F. (2013) "Centrifugal Compressor Load Sharing with the use of MPC." Master of Science in Engineering Cybernetics, Norwegian University of Science and Technology.
- Paparella F., Domínguez L., Cortinovis A., Mercangöz M., Pareschi D and Bittanti S. (2013) "Load sharing optimization of parallel compressors." In: European control conference (ECC), July 17–19, 2013, Zürich, Switzerland.

- Puranik, Y., M. Kiliç, Sahinidis, N. V., Li, T., Gopalakrishnan, A., Besancon, B., Roba, T. (2016). "Global optimization of an industrial gas network operation." *AIChE Journal* 62(9): 3215-3224.
- Szucs, A., Kvasnica M. and Fikar M. (2012). "Optimal piecewise affine approximations of nonlinear functions obtained from measurements." *IFAC Proceedings Volumes (IFAC-PapersOnline)*.
- Üster, H. and Dilaveroğlu Ş. (2014). "Optimization for design and operation of natural gas transmission networks." *Applied Energy* 133: 56-69.
- Xenos D. P., Cicciotti M., Kopanos G. M., Bouaswaig A. E. F., Kahrs O., Martinez-Botas R. and Thornhill N. F. (2015). "Optimization of a network of compressors in parallel: Real Time Optimization (RTO) of compressors in chemical plants – An industrial case study" *Applied Energy* 114 51-63
- Xenos, D. P., Noor, I. M., Matloubi, M., Cicciotti, M., Haugen, T., Thornhill, N. F., (2016) "Demand-side management and optimal operation of industrial electricity consumers: An example of an energy-intensive chemical plant." *Applied Energy* 182 418–433
- Yan, L., Yu Y., Li Y., Zhang Z. (2010). "Energy Saving Opportunities in an Air Separation Process." 12th International Symposium on Process Systems Engineering and 25th European Symposium on Computer Aided Process Engineering.
- Yang, L., Liua S., Tsokab S., Papageorgiou L. G. (2016). "Mathematical programming for piecewise linear regression analysis." *Expert Systems with Applications* 44: 156-167.
- Zhang, Q., Grossmann, I. E., Heuberger, C. F., Sundaramoorthy, A., Pinto, J. M. (2015) "Air separation with cryogenic energy storage: Optimal scheduling considering electric energy and reserve markets." *Process Systems Engineering* 61(5) 1547-1558.