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External spur gear root bending stress: A comparison of ISO 6336:2006, AGMA 2101-D04, ANSYS finite element analysis and strain gauge techniques

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Abstract

The International Organisation for Standardisation ISO 6336:2006 and the American Gear Manufacturers Association AGMA 2101-D04 are the two most predominant empirically based analytical gear stress analysis methods for establishing contact and root bending stress in involute spur and helical gears. Although based on the same fundamental principles, they have evolved to such an extent that, for various reasons, they are not necessarily in agreement. The use of commercial universal numerical finite element software has become an increasingly popular alternative for gear stress analysis and this research compares the root bending stresses in external spur gears established using ISO 6336:2006, AGMA 2101-D04 and numerical finite element analysis (ANSYS) with experimental strain gauge validation.

Keywords: External gear; Root bending stress; ANSYS finite element analysis; Strain gauges, ISO 6336:2006; AGMA 2101-D04.

1 Introduction

Although the historical development of gear bending stress analysis has been well documented throughout the 20th century, parts are discussed here in brief for convenience. Starting with the work of Lewis in 1893 [1], who suggested gear teeth could be analysed as parabolic beams in bending, highlighted the importance of accounting for the actual tooth thickness at the base of the beam. Albeit, he did not account for the root fillet radius and its resulting stress concentration, or the radial component of tooth loading. This preceded a pioneering age of gear stress analysis utilising novel photoelastic experiments, and in 1926, Timoshenko and Baud [2] applied the photoelastic method to gear tooth models, concluding that the maximum root bending stress was significantly higher than that achieved using the simple Lewis formula. As a consequence, they established a simple stress concentration factor to account for the influence of the root fillet radius and chordal length. Subsequently, Heywood [3], Jacobson [4], Kelley and Pederson [5] and Allison and Hearn [6] all conducted photoelastic experiments to further investigate the complexity of root fillet stresses. Perhaps most notably because of its historic impact on gear design, regardless of its accuracy, Dolan and Broghamer [7] improved on the work of Lewis by establishing a stress concentration factor based on photoelastic experiments, that was considered of such importance that it filtered down into the American AGMA 2101-D04 standard [8] and is still in use today. There now exists potentially more accurate gear stress analysis techniques such as numerical finite element analysis (FEA), though this does not detract from the quality of the early innovators. In 1973, Wilcox and Coleman [9] utilised the numerical finite element method to make a direct comparison with the work of Jacobson [4], Kelley and Pederson [5] and Dolan and Broghamer [7], stating that the observed differences may be attributable to the method of photoelasticity rather than FEA.

Indeed today, a potential draw back with regards to the application of universal FEA code such as ANSYS and ABAQUS is the amount of work undertaken without experimental engineering validation. Although FEA has the ability to be incredibly accurate, it requires no gear expertise and can produce wrong and misleading results often as a consequence of over simplified gear models, incorrect boundary conditions and poor mesh quality.

Both ISO 6336:2006 [10] (a revision of ISO 6336:1996) and AGMA 2101-D04 [8] (the metric edition of 2001-D04 which supersedes 2101-C95) are analytical methods which establish gear root bending stresses based on beam bending theory with additional factors introduced to account for internally and externally generated stress and load modifying factors. Although based on the same fundamental principles, ISO 6336:2006 and AGMA 2101-D04 can produce different results. In part, this is due to the manner in which many of the load and stress modifying factors are calculated and in part, due to the way in which the fundamental nominal stresses are calculated. Only the latter of which is the subject of this investigation.

For clarification throughout, 1) the bending stresses calculated without any load or stress modifying factors, such as K_A , K_v , $K_{F\beta}$, $K_{F\alpha}$ will be termed nominal stresses, 2) any further reference to the ISO 6336:2006 and AGMA 2101-D04 methods will be simply referred to as ISO and AGMA, 3) all 3D FEA gear models have been accurately produced using the generated transverse tooth profile based on hob cutter geometry using Dontyne Systems Gear Production Suite which adopts the methods of [11], and 4) all FEA was conducted using Design Unit's high performance computing facility operating ANSYS 12.1 with up to 32 cores and 160GB RAM.

2 ISO 6336:2006 and AGMA 2101-D04 root bending stress

The ISO and AGMA methods have been developed over many decades for establishing the root bending and contact stress in gears. These stresses are compared with allowable strength values which have been generated from actual gear testing, thus establishing safety factors. Although both methods are complex in their entirety, their basis for calculating nominal root bending stress, for which this research relates is simply based on beam bending theory. They differ somewhat in the techniques used to establish the gear geometry required for this calculation and their stress concentration factors.

In accordance with ISO and AGMA the nominal root bending stresses are,

$$\sigma_{F(ISO)} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \quad (1)$$

$$\sigma_{F(AGMA)} = \frac{F_t}{b} \cdot \frac{K_f}{Y} \quad (2)$$

Where F_t is the tangential load, b is the face width and m_n is the normal module. Y_F and Y are the ISO and AGMA form and geometry factors respectively which accounts for the beam bending height and second moment of area at the critical section i.e. the point of maximum root bending stress. Y_S and K_f are the ISO and AGMA stress concentration factors respectively. These account for the sudden increase in cantilever beam bending stress due to the root fillet radius and its influence as a stress concentration. The geometry required to establish these factors can be derived directly from gear metrology scans, if available, or via the ISO and

AGMA methods. In summary, the differences between the nominal root bending stresses established in accordance with ISO and AGMA are as follows:

- In accordance with ISO, the location of maximum root bending stress occurs at the 30 degree tangent point regardless of load or its point of application.
- In accordance with AGMA, the point of maximum root bending stress occurs at the tangency between the inscribed Lewis parabola [1] and the tooth root. Unlike ISO, this is a product of the loaded position.
- ISO disregards the compressive root stress caused by the radial component of tooth loading. In an actual gear, it is this component of force which causes the compressive root stress on the unloaded side of a gear tooth to be greater than the tensile stress on the loaded side.
- AGMA includes and superposes the compressive root stress caused by the radial component of load with the tensile root stress caused by the tangential component of load. The complete derivation is published in AGMA 908-B89 Appendix C [12].
- With regards to the stress concentration factor Y_s , ISO calculates the root fillet radius precisely at the 30 degree tangent point.
- With regards to the stress concentration factor K_f , based on the work of Dolan and Broghamer [7], AGMA establishes the overall minimum root fillet radius regardless of its position relative to the tangency of the inscribed Lewis parabola, as has previously been explained by Kelley and Pederson [5].

Numerical FEA was chosen to investigate its accuracy for gear stress analysis whilst simultaneously examining the discrepancies associated with the ISO and AGMA standards. Indeed, comparing FEA, ISO and AGMA is not novel and Kawalec et al [13] provide an excellent systematic review of their differences. However, here, to validate the accuracy of all three methods (ISO, AGMA and FEA) a fourth experimental technique was chosen based on strain gauge technology utilising a novel large tooth design to provide an accurate comparison. Before presenting the numerical and experimental results, a brief summary of the problems associated with the application of the finite element and strain gauge methods for gear stress analysis are discussed.

3 Finite element analysis

Assuming appropriate boundary conditions, element type and mesh refinement, FEA has the ability to be a more accurate gear stress analysis technique than ISO or AGMA. Unfortunately, due to the numerical nature of FEA it can require large computational resources to achieve accurate results. As with most finite element simulations, the efficiency of the analysis can be improved using simplifications such as adopting 2D elements, if appropriate. Furthermore, with regards to both internal and external gears a minimum of three teeth [13-19] or in some instances partial adjacent teeth [20] has also been shown to be a valid compromise. To further illustrate the errors associated with incomplete tooth models, an arbitrary spur gear pair ($m_n=4$, $z_1=32$, $z_2=48$ and $\varepsilon_a<2$) was analysed with single tooth contact at the pitch point, using predominantly 20 node quadratic hexahedral elements which were refined until root bending and contact stress had converged to within 1%. Contact stress was an average value of that experienced on both the pinion and wheel. Steel was the chosen gear material with a Young's modulus of 207GPa, along with frictionless flank contact utilising the pure penalty formulation and program controlled normal contact stiffness factor which was set, by default, to unity. Torque was applied to the pinion bore together with a cylindrical support to restrict radial and axial deformation, but permitting tangential displacement. The wheel bore had all degrees of

freedom constrained. Figs. 1 and 2 illustrate that a minimum of three whole teeth is required for gears with a contact ratio less than 2. When compared to the full gear model, this resulted in bending stress errors of approximately 1%.

Fig. 1 Single and multiple tooth bending stress.

Fig. 2 Single and multiple tooth contact stress.

With regards to bending stress, it has been shown [21, 22] that the accuracy of a single tooth FEA model can be improved if the bore and radial faces are fully constrained. This can be further illustrated using ANSYS, the results of which are presented in Fig. 3, whereby a single pinion gear, identical to that used to generate the results presented in Figs. 1 and 2, has been analysed with both bore and radial faces constraints, and an applied load over a small area at the tip, normal to the tooth flank. Note that the magnitude of tip loading applied to generate the bending stresses presented in Fig. 3 was chosen to produce a level similar to that generated in Fig. 1, however they are unrelated. The results presented in Figs. 1-3 adopt full tooth increments, however, it should be reiterated that a single tooth with partial adjacent teeth (one and a half teeth) has also been shown to provide accurate results [20]. This important concept is exploited in the experimental design stage, Section 5.

Fig. 3 Single and multiple tooth bending stress.

4 Strain gauge stress analysis

Strain gauging is an inherently accurate technique that has been widely adopted for gear stress analysis [23-33]. However, much of the literature fails to overcome the errors associated with the positional accuracy of the gauge. For example the FEA root bending stress profile for an arbitrary chosen 10mm module spur gear with an applied tip load is illustrated in Fig. 4. There exist two logical places for gauge placement 1) the point of maximum bending stress or 2) the central root position. Position one is attractive because the stress is a maximum and its gradient is zero, however without definite knowledge of its actual position (i.e. the 30 degree tangent point or Lewis parabola) the results would be questionable. Furthermore, on a meshing gear pair, tip interference may require the gauge to be placed in the centre of the root, position 2, the point at which the stresses are lower and the stress gradient may be high which is more problematic. Note that the bending stress continues to fall a significant distance up the adjacent tooth flank. Assuming a gauge positional accuracy of $\pm 0.5\text{mm}$, an amount the author has realistically achieved with manual gauge application, a strain gauge located in the central root position for the relatively large 10mm module spur gear illustrated in Fig. 4 would still establish

a maximum potential root bending stress error of approximately 20%. For smaller gears the error could be exacerbated: This is a limitation of strain gauging gears for accurate stress however it should be noted that positional accuracy is insignificant for establishing changes in stress (i.e. face load distribution) as the positional errors can be calibrated out of the system. The problem associated with strain gauging gears can be somewhat alleviated by utilising numerous gauges, such as the work conducted by [24] whereby multiple active gauges are applied, though there still exists gaps between each gauge. It should be noted that the actual stress profile, such as that illustrated in Fig. 4, is heavily dependent on the basic rack geometry.

Fig. 4 Root bending stress going from tip to root.

5 ISO, AGMA, FEA and strain gauge comparison

A comparison of root bending stresses established using ISO, AGMA, FEA and strain gauges was made based on a single tooth geometry. To reduce the errors associated with the positional accuracy of the strain gauges, a large 50mm module spur gear was designed (Table 1) and manufactured, based on an accurate DXF tooth profile, using electrical discharge machining, as illustrated in Fig. 5, with a maximum profile tolerance of ± 50 microns. The gear end faces were precision ground to ensure perpendicularity of the tooth profile, thus minimising non uniform load distribution.

Table 1 Gear geometry.

Face width (mm)	b	10
Normal module (mm)	m_n	50.00
Normal pressure angle (deg)	α_n	20.00
Reference diameter (mm)	d	1150.00
Tip diameter (mm)	d_a	1283.593
Root diameter (mm)	d_f	1000.365
Base diameter (mm)	d_b	1080.646

A jig was designed to hold the gear tooth as a cantilever beam which could be subjected to a base tangent force via a flat faced anvil mounted on an Instron loading machine, statically calibrated with UKAS traceability, as illustrated in Fig. 6. The anvil has been specifically designed to be flexible in the direction of the gear axis, such that it reduces non uniform load distribution across the facewidth of the gear. Using FEA, it was established that only one full tooth and a portion of the adjacent teeth were required without affecting the maximum root bending stress. It may be apparent that the gear should be subjected to a thin rim factor since the backup ratio is obviously below 1.3 [34] however the jig was designed to be stiff whilst fully constraining the gear bore, thus eliminating rim deflections. Using FEA the bore diameter was increased such that it did not have a significant influence on the maximum bending stress. Consequently, thin rim factors are ignored throughout.

Fig. 5 Wire spark eroded 50mm module spur gear tooth.

(a)

(b)

Fig. 6 INSTRON 1603 loading machine (a) and jig (b).

Finite element analysis of the 50mm module spur gear tooth as presented in Fig. 7 emphasised the point of maximum root bending stress does not coincide precisely with either the 30 degree tangent point or the inscribed Lewis parabola. As a consequence, three strain gauges were applied, one at each of the respective locations (ISO, AGMA and FEA) using uni-directional 0.79 mm active width constantan foil gauges aligned in the transverse plane. Each gauge was wired using a quarter bridge configuration connected to an RDP transducer indicator, and utilised three additional gauges acting as completion resistors thus eliminating errors due to temperature fluctuations. To calibrate the system, the shunt calibration method was used. Stress analysis techniques such as FEA, ISO or AGMA could not be used to calibrate the system as this would defeat the objective.

Fig. 7 FEA root bending stress profile going from the tip to root.

6 Results

To compare FEA stresses with those established from ISO/AGMA and the strain gauge experiments, two separate ANSYS simulations were required because no single FEA model would accurately replicate either scenario. The first FEA model utilised predominantly 20 node quadratic hexahedral elements and consisted of the gear tooth with a fully constrained bore and applied line load. This was used to compare the stresses established in accordance with ISO and AGMA based on the geometry illustrated in Fig. 8, using Eqs. (1) and (2) presented in Section 2.

Fig. 8 FEA, ISO and AGMA stress analysis geometry.

The second FEA model compared the stresses established from the strain gauge experiments illustrated in Fig. 9. This consisted of all the jig components with the load applied through the anvil via a frictional contact ($\mu=0.125$) using the augmented lagrange formulation with program controlled normal contact stiffness. Jig bolts and preload were omitted from the FEA analysis as it was assumed their proximity to root fillet was such that it would not influence the bending stress. All contacting jig faces utilised the ANSYS bonded constraint. Force was applied to the top of the anvil which was free to displace in the vertical direction only. The base of the jig had all degrees of freedom fixed. The difference between the FEA results established from the models illustrated in Figs. 8 and 9 was minimal, however this research strived to make an accurate comparison of all the stress analysis methods and it would be unfair and inaccurate to compare all the methods based on a single FEA model.

Fig. 9 Experimental (a) and FEA (b) models.

Both linear elastic FEA models assumed steel as the chosen gear material, the hardness of which was irrelevant since Young's modulus and therefore deflections were assumed unchanged with hardness, furthermore Young's modulus is not a factor in classic beam bending theory when establishing stress, although it was necessary for the experimental analysis when converting strain to stress. Based on steel at room temperature, a value of 207GPa was chosen in accordance with Callister [35]. Higher order hexahedral and tetrahedral elements were systematically refined until root bending stress had converged within 1.0%, then three loads of 5, 10 and 15kN were applied, the results of which are presented in Figs. 10 and 11 along with the ISO, AGMA and strain gauge results. Note that the mesh size shown in Figs. 8 and 9 are for illustration purposes only as the gear was refined further until stress had converged.

Fig. 10 FEA, ISO and AGMA root bending stress results.

Fig. 11 FEA and strain gauge root bending stress results.

7 Discussion and conclusions

Stress analysis methods such as ISO, AGMA, FEA and strain gauges have long been used for the analysis and comparison of root bending gear stresses. A fundamental drawback of the experimental strain gauge technique is the error associated with the positional accuracy of the gauge, when applied in areas of high stress gradient, such as those experienced in gear roots. Here, a large module gear tooth was designed and manufactured to reduce these errors to a suitably low level, such that it could be used to accurately validate the stresses associated with the ISO, AGMA and FEA methods.

The results presented in Figs. 10 and 11 confirm that the finite element method can be regarded the most accurate gear stress analysis technique for establishing nominal root bending stress, as all strain gauge results were on average, within 0.5% of those produced using ANSYS. With regards to the experimental results, a measurement uncertainty evaluation was conducted in accordance with [36] accounting for 1) strain gauge positional accuracy, 2) UKAS load cell uncertainty 3) accuracy of the point of loading, 4) measurement resolution error, 5) shunt calibration accuracy and 6) measurement repeatability. This produced an average expanded uncertainty (95% CI) of approximately 1.5%.

Neglecting the location of critical stress, and only comparing maximum values, ISO and AGMA over and underestimated the root bending stress by a maximum of 5.2% and 6.4% respectively. These differences can be attributed to the fact that 1) ISO establishes the critical section based on the 30 degree tangent point whilst AGMA adopts the Lewis parabola, 2) ISO ignores the radial component of load whilst AGMA does not, 3) the stress concentration factors have been established using different techniques and 4) both ISO and AGMA calculate the root fillet radius using an iterative procedure influenced by the hob tip radius, however ISO calculates the root fillet radius at the critical section whilst AGMA calculates the overall minimum root fillet, regardless of its position. The AGMA root fillet radius ($\rho_f=20.65\text{mm}$) was 31.8% lower than ISO ($\rho_f=30.28\text{mm}$), yet the AGMA stress concentration factor ($K_f=1.37$) was 9.3% lower than ISO ($Y_s=1.51$).

Here, only a single gear geometry was analysed providing no trend with regards to the errors associated with ISO or AGMA. Indeed, that was not the objective of this work, and these errors will change depending on the gear tooth geometry; high pressure angle gears are a good example of the increased errors associated with ignoring the radial component of tooth load. However if done correctly FEA should always give the most accurate result, regardless of the complexity of the gear type, yet it must be re-iterated that these are only nominal bending stresses; ANSYS is not a fully developed gear pair stress analysis tool because,

- In comparison to bending stress, achieving pure FEA contact stress, whilst not impossible, requires much greater levels of mesh refinement, far smaller than the width of the Hertzian contact patch. This problem is often overcome by utilising a semi analytical and numerical procedure whereby FEA only establishes load distribution and is combined with traditional Hertzian contact theory equations.
- Establishing the location and magnitude of maximum bending and contact stress requires a gear pair to be modelled and rotated through an angle such that the highest (bending) and lowest (contact) point of single tooth contact is analysed. This is achievable but requires large computational resources.
- Including transverse ($k_{H\alpha}$ $k_{F\alpha}$) and axial ($k_{H\beta}$ $k_{F\beta}$) stress increasing factors is difficult when using non gear specific, general purpose FEA software such as ANSYS.
- Dynamic loads (K_v) are not accounted for.

- Peak load factors (K_A) are not accounted for.
- ANSYS does not provide permissible gear fatigue data. ISO and AGMA, albeit with their differences, provide material data that has been established experimentally using their respective procedures; hence it is relevant. Based on the results of this research, it would be inaccurate to combine ANSYS stresses with ISO or AGMA material data to produce safety factors.
- Although FEA can be more accurate, the errors associated with the ISO and AGMA standards are insignificant when compared to the errors associated with knowing accurate in-service loads. This, in all likelihood, presents the gear designer with the biggest source of stress analysis error.

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