Koulin G, Zhang J, Frazer RC, Shaw BA, Sewell I. [A new profile roughness measurement approach for involute helical gears](#). *Measurement Science and Technology* 2017

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DOI link to article:

<https://doi.org/10.1088/1361-6501/aa5d96>

Date deposited:

07/02/2017

Embargo release date:

01 February 2018

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A new profile roughness measurement approach for involute helical gears

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Abstract. With increasing quality requirements of involute gear contact surfaces resulting from grinding, superfinishing, polishing and coatings of various types, surface characterization at the roughness scale is becoming more important. Typical gear roughness measurements are made in an arbitrary coordinate system using roughness measuring instruments which are difficult to relate to gear meshing action and therefore gear functionality. A method has been developed which allows well defined, repeatable measurements to be taken using conventional roughness measuring instruments and then transform the results into the functional gear meshing coordinate system. As part of the method a mathematical model describing the gear geometry and alignment method was developed. The method has been validated by standard gear profile measurement.

PACS: 06.30.F- ; 06.60.Sx

Keywords: profile gear measurement, roughness gear measurement, involute gear geometry model

1 Introduction

Production of power dense, reliable, geared transmissions requires manufacture of high precision gears using metrology systems to provide information feedback to control the process. The measurements are used to both adjust the production machinery mid operation and also to finally quantify the quality of the manufactured gears. The most widely used are the gear measurement instruments (GMI) [1]. These instruments have been popular even before numerically controlled machines were available, since their operation allows deviations from a perfect involute to be measured accurately and with data point spacing uniform along the gear path of contact or line of action. Typical GMI profile measurements are acceptable for characterising form and waviness features of the surface.

General, stylus type surface roughness measuring instruments are commonly used by gear manufacturers for surface roughness characterisation [2]. A workpiece holding fixture is normally used to roughly align the gear so that the stylus is approximately perpendicular to the tooth surface to be

measured. Although there are guidelines set out by various geometric product specifications (GPS) [3-5] and code of inspection practices [2] on how to measure gear surface texture, this kind of measurements is still not very well defined.

Recent developments in the GMI functionality have allowed the measurement of roughness deviations using suitable roughness measurement attachments. These roughness attachments also measure deviations in profile using gear line of contact as a datum axis [6]. The roughness attachments use a diamond tip stylus with a radius of either 2 or 5 μm measuring normal to the gear surface. A skid is used to minimize gear geometry and form deviation effects.

The GMI based roughness measurement requires a state of the art GMI and roughness attachment. The issue of traceability of the GMI roughness measurement using the gear path of contact requires addressing.

The measurement method which is outlined in this paper, makes use of a general stylus measurement instrument (SMI). The capabilities of such SMIs are well developed and characterized. Since such machines are widely utilized throughout industry, adapting these for a 'hybrid' roughness and profile gear measurement will expand metrological capabilities, allowing gear profile roughness measurements to be evaluated with respect to the functional path of contact axis and provide a potential traceability route for GMI roughness measurement attachments.

There are other areas which would benefit from this. Consider characterisation of superfinished and coated gears. The effectiveness of such surface treatments can be analysed by failure development monitoring on test rigs, where the gear surface is measured at intervals throughout its operation. Development of the failure of superfinished or coated gears is at the order of the roughness scale, 1-2 μm [7]. Some coatings themselves are only 2-3 μm thick [8]. A measurement performed on a general GMI is not able to capture the high frequency content of the roughness characteristic.

Profile roughness measurement on gears usually have no relation with the exact position on the flank being measured and therefore it is difficult to relate the measured features to key meshing points. The method presented in this paper provides a datum for surface texture features which can be referenced to meshing position on the gear flank path of contact or length of roll. This can be used for lubricant condition simulations e.g. elastohydrodynamic lubrication (EHL) models [9].

This paper presents the methodology, description of the measurement system and shows the theoretical basis for the measurement data analysis based on well-known involute helicoid geometry [10].

The method is then applied to a test gear example to take repeated measurements on a single helical gear tooth flank. Standard GMI measurement of the same gear flank, sets a bench mark for the form profile measurement on the gear. Measurement made on a SMI is compared with the bench mark measurement. From this conclusions about the validity of the outlined method are drawn.

One of the benefits of the outlined measurement methodology is that similar position on a gear flank can be measured repeatedly. To investigate this claim, a number of repeated measurements are made. After each measurement taken, the measurement system was reassembled in order to test the ability of the method to produce repeatable results. The measurements are analysed and compared from which conclusions about the reproducibility are drawn.

2 Methodology

This measurement approach utilizes a standard SMI Form Talysurf Intra 50. A long series probe was used to increase the amplitude range with a $2\mu\text{m}$ radius 90° conical diamond tip. This was necessary to span part of the gear root, highest point on the flank and the tip of the gear. The amplitude resolution of the system is $0.032\mu\text{m}$, measurements were taken at the stylus speed of 0.5 mm/s with lateral point sampling of $0.5\mu\text{m}$. However, the outlined method can also be used with any SMI of similar operation.

The following methodology was used:

- An alignment fixture was designed to locate the gear repeatedly relative to the SMI's coordinate system. In order to optimize the fixture geometry, full computer aided design (CAD) model of the helical gear was used.
- For optimal gear location with respect to SMI's coordinate system, the following requirements have to be satisfied. To allow for most accurate measurement, the amplitude measurement axis of the stylus must be as normal to the measured surface as possible. Another requirement is to be able to take measurements at the same position repeatedly. More details about how the alignment fixture satisfies these requirements are provided in section 3.
- Once the gear is properly positioned, measurement along the helical surface can be taken. Care is needed to ensure the measurement remains within the vertical range of the instrument stylus. Longer stylus may be necessary to be able to measure the surface features of the root, which may fall outside of the vertical range of the standard length stylus. The measurement span of the measurement length should include the root region to provide a reference feature if this method is to be applied to monitoring of failure development. The root is a good reference for this purpose since it is a non-contacting region and will not change as the gear flank wears or fatigues.
- Roughness analysis requires the removal of the involute form of the surface usually with a polynomial. This method can result in excessive residual form deviations influencing the roughness analysis. A more accurate fitting model representing the involute form and additional design micro geometry corrections can be used. In the presented method the form is represented by a theoretical trace. A mathematical model is used to calculate the theoretical trace along the helicoid of the gear. The measured and theoretical traces are aligned, and the deviations from a theoretical trace are calculated. Since involute roll length for every point on the theoretical trace is known, roll length for each measured point can therefore be extrapolated.
- The final step of the method, constitutes a coordinate system transformation. The transformation is made from length versus amplitude coordinate system of the SMI, to roll length versus deviations coordinate system of the involute. The transformation is made into the coordinate system inherent to the gear, that same coordinate system in which the measurements are made by GMIs. Therefore the high resolution surface texture features measured by SMI can be mapped onto involute gear base tangent plane. This approach is examined for validity against GMI bench mark measurement and for reproducibility with repeated measurements along the same flank of a single involute helical gear.

The GMI Klingelnberg P65 was used as a reference for form measurement. This is a standard 4 axis gear measurement machine which uses involute generation measurement method to measure gear profile form deviations. This was selected as a datum for comparison purposes as it is the UK's primary gear measuring machine in the National Gear Metrology Laboratory and has traceable calibrated to

Physikalisch Technische Bundesanstalt, Germany for involute profile measurement. The GMI provides a reference for involute profile form measurement but is not used for assessing roughness measurement capability. Uncertainty (U_{95}) or CMC values are $\pm 1.2 \mu\text{m}$ for total deviation F_a , and profile form deviation f_{fa} and $\pm 1.0 \mu\text{m}$ for profile form deviation slope f_{Ha} . The amplitude resolution of the GMI is $0.0039 \mu\text{m}$, measurements were taken at the stylus speed of $\sim 2 \text{ mm/s}$ with lateral point sampling of 0.0575 mm . A stylus with a 2 mm diameter ball ruby tip was used.

3 Measurement system

The definition of the measurement system presented here includes measurement instrument, stylus, a moving stage that moves relative to the measurement instrument, gear alignment fixture and the gear being measured. In this section the description of the measurement system is made.

3.1 Gear geometry

The design of the fixture is specific to the gear geometry that is to be measured. For the purpose of examining and validating of the method under discussion, one flank of a single helical gear was considered. The main geometry parameters of the gear are summarized in table 1.

Table 1: Nominal involute helical gear geometry parameters.

Parameter	Value
Number of teeth	24
Normal module	6 mm
Reference pressure angle	20°
Reference helix angle	28.1°
Base helix angle	26.27°
Hand of helix	(+ve) right
Tip diameter or end of active profile (EAP)	175.103 mm
Reference diameter	163.242 mm
Base diameter	150.901 mm
Root diameter	146.444 mm
Start of active profile (SAP) diameter	155.083 mm
Start of tip relief (STR) diameter	168.634 mm
Tip relief	Linear $50 \mu\text{m}$

The gear has been manufactured from low carbon alloy gear steel 18CrNiMo7-6 Gear blank was turned from forged bar and nominal gear geometry was hobbled. The gear then undergone case carburising heat treatment process. The bore of the gear was precision turned to reinstate the datum which was used in finish form grinding process using aluminium oxide vitreous grinding wheel. The gear is manufactured according to tolerance class 5 according to ISO 1328-1:2013 standard [11], and thus the total form

tolerance (F_a) of the flank is 11 μm and profile deviation slope tolerance (f_{Ha}) is 6.5 μm . All of the measurements were performed on as ground gear prior to any running.

3.2 Gear alignment fixture

For the purpose of surface texture measurement a number of ISO GPS standards [3-5] and codes of inspection practice [2] are available. A fixture is typically used to roughly align the gear to ensure the stylus tip is approximately perpendicular to the tooth surface being measured. However, based on previous experience, this kind of arrangement fails to define the datum axes properly because

- If the fixture fails to align the gear accurately, the orientation of the measurement traces won't be consistent relative to the direction of the lay of the surface. This will result in inconsistency in the outcome of measurements.
- The length of the measurement trace may not cover the whole profile from start to end of active profile (SAP to EAP), with the starting and ending positions undefined. This will result in further inconsistency in the outcome of measurement, as gear surface texture may vary from dedendum to addendum of the gear.

The fixture design proposed in this paper, provides rigid, well defined mounting position of the gear, as shown in figure 1. The fixture enables a gear surface texture measurement to be carried out repeatedly along a similar trace, covering the whole tooth profile from SAP to EAP, yielding consistent and complete information about a gear's surface texture.

Figure 1: Gear alignment fixture CAD model.

The fixture provides rigid mounting and special orientation of the helical gear with respect to the SMI's coordinate system. The reference for the measurement coordinate system is provided by the crest of a hardened, precision sphere. The whole fixture is positioned on a moving stage which is able to translate in the y_m direction of the measurement coordinate system, see figure 2. Location between the fixture and the moving stage is provided by precision ground pins. The gear sits flush against an inclined plane and the bore of the gear is supported by two hardened spherical supports. For completeness, gear coordinate system is also illustrated in figure 1. The three dimensional gear body has six degrees of freedom in three dimensional space. The inclined plane constrains three of the six degrees of freedom,

translation in z_g direction and rotation about x_g and y_g axes. The hardened spherical supports constrain two additional degrees of freedom, translation in x_g and y_g directions. The last degree of freedom, rotation about the z_g axis is constrained by a setting block of a specific height. Accuracy of the height dimension of the setting block is of typical precision grinding tolerance $\pm 10 \mu\text{m}$ for the height of about 33 mm. A point on the gear tooth tip edge rests on the setting block, rigidly fixing the flank to be measured with respect to the measurement coordinate system. Tooth angular misalignment caused by tooth thickness and cumulative pitch errors are compensated through alignment to the theoretical trace. This may have effect on the gear form parameters due to the gear manufacturing errors but will not affect surface texture roughness and waviness parameters.

The first prototype of the gear alignment fixture has been designed for the specific gear geometry described in section 3.2. The supports are rigidly mounted and the inclination of the fixture accommodates gears of one particular base helix angle. The fixture can accept gears of equal base helix angle, either left or right handed, of similar bore and tip diameter. However for each new tip diameter a new setting block is needed to constrain rotation about z_g axis correctly. The fixture allows every tooth to be measured by manually indexing each tooth, the gear can also be flipped over to allow both flanks to be measured.

Figure 2: Measurement system: 1) measurement instrument; 2) stylus; 3) moving stage.

The stylus of the measurement instrument travels in the x_m direction and takes measurement of the vertical displacement in z_m direction. Therefore all of the measurements taken lie on x_m - z_m plane. One requirement of the fixture is to orient the amplitude measurement axis of the stylus, z_m axis, as perpendicular to the measured surface as possible. The measurement trace must also pass through SAP and EAP points that lie on the transverse plane of the gear, x_g - y_g plane, in order to be comparative to profile measurement on a GMI, which makes measurements in the transverse plane. It is hypothesized that these requirements are achieved by two conditions. These conditions are outlined below, as for the evidence of the validity of these conditions, it is presented in the subsequent sections.

- The inclination of the helical gear by the base helix angle β_b with respect to the amplitude measurement axis of the stylus, is needed.
- The points SAP and EAP which lie on the transverse plane of the gear must also lie on the measurement plane of the instrument. The intersection between the transverse plane of the gear and the measurement plane of the instrument is a line, which is parallel to x_m axis. Therefore the rotation of the gear must be such that the line passing through points SAP and EAP on the gear transverse plane, is parallel to x_m axis. SAP and EAP can be generally defined at any face width along the flank.

For the purpose of this paper, SAP and EAP on the mid-face are chosen, this is convenient since the primary profile measurements, using a GMI, are typically made along the mid face. The second condition is achieved by optimising setting block height (which defines rotation of the gear relative to its datum bore) with the aid of the fixture assembly CAD model.

4 Theoretical helicoid model

Analysis of roughness measurement firstly requires form removal, usually performed by filtering and/or polynomial form removal. Small scale roughness features are dominated by the larger scale form of the surface. Once the form of the surface is removed, roughness and waviness features are revealed.

There are various ways that the form of the surface can be represented. Where applicable, the form of a surface can be simply approximated with a polynomial. Alternatively if the measured surface can be represented by a basic geometrical entity, e.g. a circle, such an entity can be fitted to the data. The problem will constitute finding the size and the position of a chosen entity type that will fit the data with the least error [12].

For complex surfaces such as helical gears, a simplified approach of polynomial fitting is often applied. However more information about the measured surface can be obtained, if more sophisticated fitting methods are used [13, 14].

The approach presented in this paper, uses a theoretical description of the nominal form of the trace to be measured. The measured data is fitted to the theoretical trace. For this, a point to point absolute orientation least squares fitting method is used. Two dimensional adaptation of the three dimensional method described in the reference [15], was applied. Once the measured data is fitted, the form of the theoretical trace can be removed, revealing the roughness features. A benefit of this method is that the length of roll of each measurement point can be extrapolated from the theoretical trace. This allows the measurement to be presented in the coordinate system of the involute shape, consistent with GMI measurement formats and the functional operation of the gear.

The theoretical trace is the virtual measurement that would be measured on a perfectly smooth, ideal, design gear, by a perfect measurement system. The purpose of theoretical trace is to be an ideal reference profile for measured trace. In order to generate the theoretical trace, a mathematical model of the involute helicoid is used. The measurement system which was described in the previous section, makes planar measurements. Each measured trace is described by a series of length and amplitude data sets. Therefore the measurement trace is the intersection between the involute helicoid of the gear surface, and a plane in which the measurements are made.

4.1 Helicoid - plane intersection

First we define the helical gear geometry. The equation of the vector $\mathbf{r}_{\text{inv}}$ defines the points of the involute helicoid surface is [10]:

$$\mathbf{r}_{\text{inv}}(\theta, u) = (r_b \cos \theta + u \cos \lambda_b \sin \theta) \mathbf{i} + (r_b \sin \theta - u \cos \lambda_b \cos \theta) \mathbf{j} + (p\theta - u \sin \lambda_b) \mathbf{k} \quad (1)$$

where r_b is the involute base radius, p is the helix screw parameter, u and θ are curvilinear coordinates of the involute helicoid surface. Additionally $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the direction of axes x_g , y_g and z_g , respectively. Where the sum of helix base angle β_b and helix lead angle λ_b is a right angle:

$$\beta_b + \lambda_b = \frac{\pi}{2} \quad (2)$$

It is also useful to consider the involute helicoid surface unit normal $\mathbf{n}_{\text{inv}}$ [10]:

$$\mathbf{n}_{\text{inv}}(\theta) = \sin \lambda_b \sin \theta \mathbf{i} - \sin \lambda_b \cos \theta \mathbf{j} + \cos \lambda_b \mathbf{k} \quad (3)$$

The construction of involute helicoid geometry is illustrated in figure 3.

Figure 3: Construction of involute helicoid geometry.

Looking closely at the components of the unit normal $\mathbf{n}_{inv}$, it can be seen that the $\mathbf{k}$ component is independent of curvilinear coordinates θ or u and is only dependent on helix lead angle λ_b . Recalling right angle relationship in equation (2), $\cos\lambda_b$ can be replaced with $\sin\beta_b$. It can be summarized that the unit normal $\mathbf{n}_{inv}$ is inclined by base helix angle β_b with respect to gear x - y plane. Therefore this serves as a theoretical argument for the fixture inclination of the helical gear by base helix angle β_b with respect to the stylus measurement axis (refer to section 3.2), which will result in stylus measurement axis orientation as close to normal of the gear surface as possible, along the full measurement trace.

We now define a general plane which will intersect the involute helicoid. The general Cartesian equation of a plane [16]:

$$\mathbf{n}_p \cdot \mathbf{x} + d = 0 \quad (4)$$

where d is the negative displacement of the plane to the origin, $\mathbf{n}_p$ is the plane unit normal and $\mathbf{x}$ is the Cartesian coordinate vector:

$$\mathbf{n}_p = [n_{px} \quad n_{py} \quad n_{pz}] \quad (5)$$

$$\mathbf{x} = [x \quad y \quad z]^T \quad (6)$$

The intersection of the involute helicoid by a general plane is found by substituting equation (1) into (4) and obtaining this result:

$$n_{px}(r_b \cos \theta + u \cos \lambda_b \sin \theta) + n_{py}(r_b \sin \theta - u \cos \lambda_b \cos \theta) + n_{pz}(p\theta - u \sin \lambda_b) + d = 0 \quad (7)$$

Rearranging for curvilinear coordinate u we obtain a function of θ :

$$u_{\text{int}}(\theta) = \frac{n_{px}r_b \cos \theta + n_{py}r_b \sin \theta + n_{pz}p\theta + d}{-n_{px} \cos \lambda_b \sin \theta + n_{py} \cos \lambda_b \cos \theta + n_{pz} \sin \lambda_b} \quad (8)$$

Substituting equation (8) into (1) we obtain an expression for points of the intersection $\mathbf{r}_{\text{int}}$ between a plane and an involute helicoid:

$$\mathbf{r}_{\text{int}}(\theta) = \mathbf{r}_{\text{inv}}(\theta, u_{\text{int}}(\theta)) = (r_b \cos \theta + u_{\text{int}}(\theta) \cos \lambda_b \sin \theta)\mathbf{i} + (r_b \sin \theta - u_{\text{int}}(\theta) \cos \lambda_b \cos \theta)\mathbf{j} + (p\theta - u_{\text{int}}(\theta) \sin \lambda_b)\mathbf{k} \quad (9)$$

4.2 Tip relief

It is very common in gear design to apply some micro geometry modifications such as tip relief and crowning to improve gear meshing during operation by correcting for load dependent tooth deflection. The gear being analysed in this paper is modified by linear profile tip relief, refer to section 3.1. Equations presented here describe linear tip relief geometry, however these can be adapted for other types of modifications such as parabolic relief or profile crowning.

It is simpler to define tip relief on a section of the involute helicoid in the transverse plane perpendicular to the base cylinder axis. The linear tip relief is defined by start of tip relief (STR) roll angle, end of tip relief roll angle (this is usually the same as EAP) and the magnitude of correction, as illustrated in figure 4. In order to obtain an equation for linear tip relief deviation as a function of roll angle a system of two simulations equations must be solved. Matrix representation of this system of equations is the following:

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \phi_{\text{STR}} & 1 \\ \phi_{\text{EAP}} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \delta \end{bmatrix} \quad (10)$$

where ϕ is the involute section roll angle with subscripts *STR* and *EAP* relating to start of tip relief and end of active profile, respectively. Where c_1 and c_2 are coefficients of a polynomial, in the case of linear tip relief a straight line or first order polynomial, δ is the magnitude of tip relief.

Figure 4: Involute tip relief.

Equation for linear tip relief deviation as a function of roll angle is:

$$f_{TR}(\phi) = c_1\phi + c_2 \quad \text{for } \{\phi \in \mathbb{R} \mid \phi_{STR} \leq \phi \leq \phi_{EAP}\} \quad (11)$$

where f_{TR} is the tip relief deviation applied perpendicular to the involute line.

We can extend this definition of tip relief for the full surface geometry of the involute helicoid. It is worth mentioning that the involute helicoid roll angle θ is not the same as the involute section roll angle ϕ . Each point on the involute helicoid is defined by two curvilinear coordinates, see equation (1). Therefore we must be able to express the involute section roll angle ϕ in terms of the curvilinear coordinates. One can deduce that projection of curvilinear coordinate u onto x - y plane in the direction of z -axis, labelled $u_i \cos \lambda_b$ in figure 5, is commonly known as the roll length of the involute section.

Figure 5: Involute helicoid section roll angle. Geometric relation of curvilinear coordinates of the helicoid surface and its section by transverse x-y plane.

Hence the equation for the involute helicoid section roll angle ϕ in terms of u is:

$$\phi(u) = \frac{u \cos \lambda_b}{r_b} \quad (12)$$

Substituting equation (12) into (11) we obtain an expression for tip relief deviation f_{TR} as function of u :

$$f_{TR}(u) = c_1 \frac{u \cos \lambda_b}{r_b} + c_2 \quad \text{for } \{u \in \mathbb{R} \mid u_{STR} \leq u \leq u_{EAP}\} \quad (13)$$

Where the limiting values of u_{STR} and u_{EAP} can be found from values of ϕ_{STR} and ϕ_{EAP} , respectively, by means of equation (12). We now have an expression for the magnitude of tip relief, next let's consider the direction vector of the tip relief. We defined tip relief deviation f_{TR} such that it is applied normal to the section of involute line. Analogously this will be equivalent to component of the involute helicoid surface normal $\mathbf{n}_{inv}$, from equation (3), which lies on the x-y plane:

$$\mathbf{n}_{\text{invxy}} = \frac{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{n}_{\text{inv}}}{\left| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{n}_{\text{inv}} \right|} = \sin\theta \mathbf{i} - \cos\theta \mathbf{j} \quad (14)$$

Therefore applying tip relief to the involute helicoid geometry, from equation (1), we obtain the expression for points of the modified helicoid surface $\mathbf{r}_{\text{invTR}}$ in vector form:

$$\mathbf{r}_{\text{invTR}} = \mathbf{r}_{\text{inv}} + f_{\text{TR}} \mathbf{n}_{\text{invxy}} \quad (15)$$

Or in the expanded component form:

$$\mathbf{r}_{\text{invTR}}(\theta, u) = (r_b \cos\theta + uc_3 \cos\lambda_b \sin\theta + c_2 \sin\theta) \mathbf{i} + (r_b \sin\theta - uc_3 \cos\lambda_b \cos\theta - c_2 \cos\theta) \mathbf{j} + (p\theta - u \sin\lambda_b) \mathbf{k} \quad (16)$$

where

$$c_3 = \left(1 + \frac{c_1}{r_b} \right) \quad (17)$$

In similar fashion as for unmodified involute helicoid, we are able to find intersection of the modified involute helicoid with a general plane. Substituting equation (16) into (4) and rearranging for u we obtain function of θ :

$$u_{\text{intTR}}(\theta) = \frac{n_{\text{Px}}(r_b \cos\theta + c_2 \sin\theta) + n_{\text{Py}}(r_b \sin\theta - c_2 \cos\theta) + n_{\text{Pz}}p\theta + d}{-n_{\text{Px}}c_3 \cos\lambda_b \sin\theta + n_{\text{Py}}c_3 \cos\lambda_b \cos\theta + n_{\text{Pz}} \sin\lambda_b} \quad (18)$$

The equation for points of the intersection between a plane and a modified involute helicoid $\mathbf{r}_{\text{intTR}}$ is:

$$\mathbf{r}_{\text{intTR}}(\theta) = \begin{cases} \mathbf{r}_{\text{inv}}(\theta, u_{\text{int}}(\theta)) & \text{for } \{u \in \mathbb{R} \mid 0 \leq u_{\text{int}}(\theta) \leq u_{\text{STR}}\} \\ \mathbf{r}_{\text{inv}}(\theta, u_{\text{intTR}}(\theta)) & \text{for } \{u \in \mathbb{R} \mid u_{\text{STR}} < u_{\text{int}}(\theta) \leq u_{\text{EAP}}\} \end{cases} \quad (19)$$

4.3 Measurement plane orientation

We have defined an expression for intersection between a general plane and the involute helicoid. This general intersection can be adapted for a special case of intersection with a plane in which the measurements are made. The orientation of the measurement plane with respect to the gear was described in section 3.2. We will now define such a plane mathematically. The intersections of the involute helicoid are illustrated in figure 6.

Figure 6: Intersections of the involute helicoid. The colour of the intersection line is respective to the plane intersecting the helicoid geometry.

We discussed the requirement for the measurement to pass through two points on the transverse plane, points $\mathbf{r}_{SAP}$ and $\mathbf{r}_{EAP}$. Therefore a line passing through both of these points must lie on the measurement plane. Let us define unit vector $\mathbf{v}_L$ in the direction of that line:

$$\mathbf{v}_L = \frac{\mathbf{r}_{EAP} - \mathbf{r}_{SAP}}{|\mathbf{r}_{EAP} - \mathbf{r}_{SAP}|} \quad (20)$$

Another requirement discussed is that the measurement plane is tilted by base helix angle β_b with respect to the base cylinder axis. These requirements provide two constraints for the orientation of the measurement plane.

The orientation of the measurement plane is defined by its unit normal $\mathbf{n}_p$. Therefore the unit normal $\mathbf{n}_p$ is tilted by β_b with respect to the z-axis to satisfy the first requirement. The second requirement is satisfied if unit normal $\mathbf{n}_p$ is perpendicular to unit vector $\mathbf{v}_L$. These constraints provide two equations. The third equation is provided by the fact that $\mathbf{n}_p$ is a unit vector. Three equations let us solve for the three unknown components of the measurement plane unit normal $\mathbf{n}_p$. The equations to be solved are:

$$\begin{aligned}
\mathbf{v}_L \cdot \mathbf{n}_P &= 0 \\
\mathbf{n}_P \cdot \mathbf{k} &= \cos \beta_b \\
|\mathbf{n}_P| &= 1
\end{aligned} \tag{21}$$

Since both vectors $\mathbf{r}_{EAP}$ and $\mathbf{r}_{SAP}$ lie on the transverse plane, z component of vector $\mathbf{v}_L$ turns out to be zero. Using this simplification to solve equations (21) for components of $\mathbf{n}_P$ yields:

$$\begin{aligned}
n_{Px} &= \left[1 + \left(\frac{v_{Lx}}{v_{Ly}} \right)^2 + \frac{1 + \left(\frac{v_{Lx}}{v_{Ly}} \right)^2}{\sec^2 \beta_b - 1} \right]^{-1/2} \\
n_{Py} &= \frac{-v_{Lx} n_{Px}}{v_{Ly}} \\
n_{Pz} &= \pm \sqrt{\frac{n_{Px}^2 + n_{Py}^2}{\sec^2 \beta_b - 1}}
\end{aligned} \tag{22}$$

The solution for the z component of the measurement plane unit normal n_{Pz} can either be positive or negative. Negative root provides the correct plane orientation if the magnitude of β_b is positive, for a right hand helix. Alternatively, positive root for a left hand helix.

Once the intersection plane unit normal is found, the negative displacement of the plane to the origin d can be found from equation (4).

4.4 Tip relief magnitude in the measurement plane

So far we have defined unmodified and tip modified involute geometry and the planes that intersect these geometries. The magnitude of the tip relief is different in the measurement plane compared to the transverse plane. The view of the gear tip in the direction perpendicular to the base cylinder axis z is shown in figure 7. The magnitude of tip relief would appear smaller in the measurement plane compared to the transverse plane by cosine of the incline angle. Therefore in order to compare the tip relief magnitude measured in the measurement plane of the SMI with the measurements taken in the transverse plane with the GMI, a scale factor of cosine of the incline angle must be applied.

Figure 7: Perpendicular view of the gear tip.

5 Results & discussions

5.1 SMI measurement analysis

With the use of the measurement system defined, the described methodology was applied to repeated measurements on a single flank of the helical gear defined in section 3.1. It was mentioned earlier that a requirement of an SMI, is that the measurement plane of the machine is as normal to the measured surface as possible. Recall that in order to satisfy this condition, the alignment fixture inclines the gear by the base helix angle. This was confirmed by theoretical description of the involute helicoid normal vector, see section 4.1 and equation (3). However an involute helicoid normal forms a curved surface which cannot intersect with a flat plane for all points. Therefore there will be an angle error between the involute helicoid surface normal and the measurement plane of the SMI. This angle deviation computed from the involute intersection model, for the gear geometry under discussion, is shown in figure 8. The typical measurement range of interest is between SAP and EAP, for which the angle deviation is below 0.5° . Such small angle deviations will incur minimal errors and therefore satisfies the SMI requirement discussed above.

Figure 8: Angle deviation between the measurement plane and the gear surface normal.

In this section the coordinate transformation, from length vs amplitude x_m - z_m of SMI measurement plane, to roll angle vs deviation of involute helical gear transverse plane, is made. In order to achieve this, the theoretical trace is calculated using the involute intersection model described in section 4. The involute part of the measurement is then fitted to the theoretical trace, in order to allow the coordinate transformation.

First step of the analysis is to extract the involute region of the measurement trace. The involute region is bound by the root trochoid and the tip of the gear, as shown in figure 9 (a). Curvature analysis of the trace can reveal these boundaries.

The root trochoid region has positive curvature and the involute section has negative curvature, in the orientation presented. Firstly, roughness features of the raw data were filtered out in order to clearly reveal the curvature inflection point. In this case a Gaussian low pass filter was applied with cut of wavelength $\lambda_c = 0.8$ mm [17]. The inflection of the curvature was then found by taking first derivative of the amplitude z with respect to length x , see figure 9 (b). Positive and negative slopes of the first derivative indicated positive and negative curvature of the trace, respectively. The curvature inflection point is the point of zero slope. Since the zero slope position is not unique we need additional information to fully define this point. It can be seen that the positive slope rises until the inflection point is reached. Then the slope inverts and lowers most of the way along the trace, apart from the edge effects seen at the right end. Therefore the curvature inflection point was found as the highest point of the first derivative curve, with attention shown to any edge effects. This point is labelled as ‘start of involute form’ in figure 9 (a).

Figure 9: Extracting involute area of the trace. (a) SMI raw data. (b) First derivative of the SMI trace amplitude z with respect to length x . Raw data was filtered by a Gaussian low pass filter with cut off wavelength $\lambda_c = 0.8$ mm. highest point of this trace indicates the start of involute form. (c) second derivative of the SMI trace amplitude z with respect to length x . Raw data was filtered by a Gaussian low pass filter with cut off wavelength $\lambda_c = 0.04$ mm. Lowest point of this trace indicates the tip of the gear tooth.

The tip was also found by similar analysis. Since the tip has sharp change of curvature, this part of the signal contains relatively high frequency content, which needs to be kept post filtering. In this case a Gaussian low pass filter with cut off wavelength of $\lambda_c = 0.04$ mm was applied to the raw data. Then the second derivative of the filtered signal was taken, see figure 9 (c). The magnitude of the second derivative correlates with magnitude of the curvature. The small radius of the tip has large negative curvature which can be seen as the minimum point on the second derivative curve. This point is labelled as ‘tip’ in figure 9 (a).

The analysis method for finding the boundaries of the involute works well for the geometry of the gear considered and was tested on a number of different gear teeth. However this was not tested on gear geometries significantly different to the example considered. It is envisaged that similar analysis can be followed with perhaps amendment to filter cut off wavelengths, which is likely in cases of sharp protuberance or notches being present in the root region.

Using the boundaries that have been found, the involute region of the measurement was extracted. This was needed since the model computes theoretical trace of the involute geometry. The extracted involute region of the measurement was then fitted to the theoretical trace. Point to point absolute orientation least squares fitting method was used, two dimensional adaptation of the three dimensional method as described in the reference [15]. In order to employ this fitting method some preparation of the theoretical data was necessary.

Each measurement point must have a corresponding theoretical point to which the data is fitted. The rotation of the measurement data, about y_m measurement axis as defined in figure 1, is not exact with respect to the theoretical trace. Hence the length coordinate x cannot be reliably used to define the positions for theoretical data points. Instead it was decided to use length along the trace. Length along the trace does not change with varying rotation of the trace, therefore this is a more robust quantity for this purpose. The form of the measurement trace was approximated by a high order polynomial (polynomial order fifteen was used), which is needed to approximate the form close to the discontinuity at the start of tip relief. Length along the form for each point was found. The tip serves as an origin from which the length is calculated. The theoretical data points are computed at the corresponding lengths along the trace. After these preparation the measurements were fitted.

The gear geometry in the example has linear tip relief, therefore the measurement data was fitted to a theoretical trace which included the tip relief. The deviation from the involute was found as the amplitude difference between the fitted data and the theoretical trace without tip relief. Roll length information for the theoretical trace is computed from the model, through interpolation the roll length vector for the corresponding measurement data points was subsequently calculated. As was mentioned previously, these steps constitute a coordinate system transformation, from length vs amplitude of the SMI coordinate system to roll length vs deviation from involute of the gear coordinate system. The result of such transformation for the raw trace presented in figure 9 (a), is shown in figure 10.

Figure 10: SMI data transformed into gear coordinate system. ‘GMI’ trace has been filtered by a Gaussian low pass filter with cut off wavelength $\lambda_c = 0.6577$ mm by the GMI software prior to output. ‘SMI’ trace has been filtered as part of analysis by the same filter, the result labelled ‘SMI filtered’. Each trace has been arbitrarily spaced vertically to aid the presentation.

5.2 GMI bench mark comparison

Consider the result of SMI data transformation into the gear coordinate system as presented in figure 10. For comparison the example gear was measured using the GMI on the selected tooth at mid face width. Profile was scanned in transverse plane tangential to base diameter, refer to figure 6. The deviations are filtered by the GMI software prior to evaluation. A Gaussian low pass filter was applied with cut off wavelength 3.33% of the profile measurement length of 19.75 mm between EAP and SAP, which equates to $\lambda_c = 0.6577$ mm. This filter is specified for gear inspection by ISO 1328-1:2013 standard [11].

It can be seen from visual inspection of figure 10 that SMI filtered trace is similar in shape and phase to GMI trace. However, there are a number of differences that can be observed:

- The SMI trace contains higher frequency content than the GMI trace, which is partly due to the smaller stylus tip diameter of the SMI as compared to GMI.
- There is a slope difference in the tip relief region between STR and EAP. This difference exists between measurement in the inclined SMI measurement plane, as defined by the alignment fixture, and the transverse plane, in which GMI measurement is made. Details about this difference were outlined in section 4.4. The tip region least squares slopes of the GMI and SMI traces are $-7.46 \mu\text{m/mm}$ and $-6.69 \mu\text{m/mm}$, respectively. It was previously outlined that the difference between these two slopes is the cosine of the incline angle, which is base helix angle $\beta_b = 26.27^\circ$. $\cos(26.27^\circ) = 0.897$, by way of comparison the ratio between SMI and GMI tip relief slopes is 0.897. This result validates the hypothesis for the tip relief difference, as measured in the two planes, as outline in section 4.4.
- In the evaluation region between SAP and STR, the profile slopes are different by about $2 \mu\text{m}$. The measured SMI data is fitted to a theoretical trace which describes the form of the measured data details such as tip relief. However the real form of the measured surface is likely to be different from the theoretical trace due to manufacturing error. The real form of the surface can be seen in the GMI trace, without considering the GMI measurement uncertainty. It is possible to extract the form from the GMI trace and modify the nominal geometry by the slope deviation caused by the manufacturing error. Performing the analysis with this form deviation slope correction, yields result in figure 11. It can be seen that the more accurate description of the theoretical trace, reduces the difference of slopes in the evaluation region.

Figure 11: Theoretical form corrected by the GMI measurement. Each trace has been arbitrarily spaced vertically to aid the presentation.

A comparison of the measurements in terms of gear quality and surface waviness parameters [5, 11, 17] is summarized in table 2. In order to calculate the waviness parameters, high pass filters were applied to remove the longwave content of the signals and isolate the waviness. Gaussian high pass filter was used, with cut off wavelength equal to the evaluation length $\lambda_f = 19.75$ mm. All parameters are calculated for the signals in the evaluation range between SAP and STR as illustrated in figure 12 (a). The calculated parameters are presented in table 2.

Table 2: Comparison of parameters. SMI theoretical form corrected by the GMI measured form.

Parameter name	Parameter symbol	GMI result	SMI result	Difference
Total profile deviation	F_a [μm]	6.81	7.18	+0.38
Profile form deviation	f_{fa} [μm]	2.47	2.89	+0.43
Profile slope deviation	f_{Ha} [μm]	-4.55	-4.66	+0.12
Waviness RMS deviation	Wq [μm]	0.52	0.53	+0.01
Waviness mean width	WSm [μm]	3.32	2.66	-0.66
Waviness RMS slope	$W\Delta q$ [$^\circ$]	0.077	0.101	+0.024

It can be seen that the difference between parameters is small and within the traceable measurement uncertainty of the GMI for the gear form parameters. The biggest differences are seen in the waviness mean width WSm and waviness root mean square (RMS) slope $W\Delta q$. It was noted previously that the SMI trace contains more of higher frequency content than the GMI trace. Since there are more of the shorter wavelength oscillations in the signal which cross the mean line, hence reducing the overall mean

width result WSm . Similarly, the shorter wavelength components have sharper slopes since the width is reduced but the amplitude is similar, resulting in an increased RMS slope $W\Delta q$ value.

Figure 12: Comparison analysis. (a) SMI filtered and GMI signals in the evaluation range between SAP and STR. (b) FFT spectrum of wavelengths between 1 mm and 20 mm. (c) FFT spectrum of wavelengths between 0.5 mm and 2 mm. (d) Cross correlation of the SMI filtered and GMI signals.

Frequency decomposition of the signals is shown in figure 12 (b) and (c). Frequency decomposition was calculated by means of fast Fourier transform (FFT) discrete algorithm [17]. It can be seen that the amplitudes of the filtered SMI and GMI signals are mostly similar and less than $0.05\mu m$. However in the region of the short wavelengths between 0.5 mm and 1.3 mm, a number of distinct SMI frequencies have higher amplitudes, than that of GMI frequencies. The observation stated earlier, that the SMI signal has more of the higher frequency content, is supported and probably due to the difference in stylus radius. GMI and SMI resolution is also different which may affect the results.

Cross correlation analysis [18], in figure 12 (d), is also useful to substantiate this comparison. It can be seen that the lag between the two signals was calculated to be zero to the nearest 10 nm, at the maximum cross correlation factor. This confirms earlier observation that the SMI measurement is in phase with the GMI measurement. This validates that the fitting method and subsequent transformation into the gear coordinate system is successful, allowing accurate mapping from arbitrary measurement coordinate system onto the roll length. This is very useful since measurement positions on the roll length, are directly linked with the functionality of the gear.

5.3 SMI measurement reproducibility

The requirement of the measurement system is to be able to measure similar traces along a gear flank, repeatedly. The alignment fixture provides a reference by means of a crest on a precision sphere, so the position and orientation of the gear being measured can be defined fully, in the measurement coordinate system. Location between the SMI table and the alignment fixture is also fully defined by locating pins. Refer to section 3.2 for details.

Reproducibility of this measurement method, is of interest because it is likely to be a significant contribution to overall measurement uncertainty. Contribution that have been investigated and quantified include:

- Reproducibility error caused by the positioning of the gear with the aid of the alignment fixture. The effect of this depends on the variation in gear form deviations over the gear face width.
- Reproducibility of the fitting and coordinate transformation of the data into gear coordinate system.

There are also other contributors such as the probe calibration, SMI axes straightness and system noise from the axes drive system.

Due to its significant contribution reproducibility of defining the alignment fixture datum is important. For this purpose the stylus was moved to offset in both x_m and y_m directions relative to the reference precision sphere. Then the stylus was moved back to the vicinity of the reference sphere, after which a crest finding measurement was made. This was performed six times and the reproducibility results for finding the datum are shown in table 3.

Table 3: Reference finding reproducibility

Offset [mm]		Reference position [mm]	
x_m	y_m	x_m	y_m
-6.929	45.433	0.045	-0.017
-14.363	63.561	0.001	-0.070
-24.360	73.561	0.001	-0.004
0.661	28.561	0.017	0.011
5.664	18.561	0.013	-0.091
10.663	8.561	0.004	-0.012
Maximum		0.045	0.011
Mean		0.014	-0.031
Minimum		0.001	-0.091
Range		0.044	0.102
Standard deviation		0.017	0.040

The error of finding the datum in the x_m direction is not important since the measurement overlaps the involute section with a portion of gear tip and root. The error in locating the reference in the y_m direction is directly related to the gear face width position of measurement. From the range of the results in table 3, it can be concluded that two measured traces on the same gear flank may be offset by ± 0.1 mm in the y_m direction, from each other, as the worst case.

Since the gear under investigation is form ground, see section 3.1 for further information, the surface texture features are effectively extruded along the helix. From this reasoning it can be expected that measurements that are slightly offset in the y_m direction, will have minimal difference on the measured trace and the calculated surface texture parameters, as compared to each other. To investigate this hypothesis eleven measurements were taken, one in the middle of the gear at the ideal position of measurement and others offset by 0.1 mm steps in both positive and negative y_m directions.

In order to analyse the variability, a selection of surface texture roughness and waviness parameters were analysed. Amplitude parameters: arithmetic mean deviation Ra and Wa , RMS deviation Rq and Wq , maximum height Rz and Wz . Spacing parameters: mean width RSm and WSm , using lateral and amplitude discrimination of 1% of sampling length and 10% of Rz or Wz , respectively. Hybrid parameters: RMS slope $R\Delta q$ and $W\Delta q$, were chosen. Roughness parameters Ra , Rq and $R\Delta q$ have been calculated as average of twenty four sampling lengths of 0.8 mm which make up the whole profile length. The surface texture roughness and waviness parameters of these results are shown in tables 4 and 5, respectively. These results characterize variability in the manufactured gear across the offset range and can be used to estimate uncertainty contribution caused by datum definition.

Table 4: Offset roughness surface texture parameters.

Offset [mm]	Ra [μm]	Rq [μm]	Rz [μm]	RSm [μm]	$R\Delta q$ [$^\circ$]
-0.5	0.35	0.44	2.33	0.11	3.732
-0.4	0.35	0.44	2.31	0.11	3.777
-0.3	0.36	0.44	2.28	0.10	3.755
-0.2	0.36	0.44	2.28	0.11	3.703
-0.1	0.36	0.44	2.31	0.10	3.750
0	0.36	0.45	2.28	0.12	3.772
0.1	0.36	0.45	2.27	0.11	3.798
0.2	0.36	0.44	2.24	0.12	3.777
0.3	0.35	0.43	2.23	0.09	3.800
0.4	0.35	0.43	2.25	0.10	3.767
0.5	0.35	0.43	2.25	0.10	3.861
Maximum	0.36	0.45	2.33	0.12	3.861
Mean	0.35	0.44	2.27	0.11	3.772
Minimum	0.35	0.43	2.23	0.09	3.703
Range	0.01	0.02	0.09	0.03	0.158
Standard deviation	0.01	0.01	0.03	0.01	0.041

From the reproducibility results of finding the alignment fixture reference, the worst case offset to be expected is approximately ± 0.1 mm. At this level of offset, roughness and waviness amplitude parameters Rz and Wz , are both shown to vary by 10 - 30 nm. The results show good stability across a 1 mm offset range. This supports the idea that this form ground flank of the gear has extruded surface texture features in the direction of the helix. The parameter that shows the biggest variation is WSm . This is due to small number of valid zero crossing intervals, used to calculate WSm , which ranges from 3 to 5 resulting in large variation.

Table 5: Offset waviness surface texture parameters.

Offset [mm]	W_a [μm]	W_q [μm]	W_z [μm]	W_{Sm} [μm]	$W\Delta q$ [$^\circ$]
-0.5	0.43	0.54	2.26	3.32	0.083
-0.4	0.44	0.55	2.28	4.42	0.083
-0.3	0.40	0.51	2.25	3.33	0.082
-0.2	0.44	0.54	2.30	4.45	0.083
-0.1	0.42	0.53	2.09	3.32	0.081
0	0.41	0.51	2.06	3.32	0.080
0.1	0.44	0.54	2.06	2.65	0.081
0.2	0.44	0.55	2.15	3.31	0.082
0.3	0.42	0.52	2.13	4.43	0.081
0.4	0.42	0.52	2.15	4.42	0.081
0.5	0.43	0.53	2.23	4.43	0.082
Maximum	0.44	0.55	2.30	4.45	0.083
Mean	0.43	0.53	2.18	3.76	0.082
Minimum	0.40	0.51	2.06	2.65	0.080
Range	0.04	0.04	0.23	1.79	0.003
Standard deviation	0.01	0.01	0.09	0.67	0.001

To investigate reproducibility of the measurement method, seven repeated measurements along the same flank were taken. Between each measurement the gear was removed from the fixture. The fixture itself was removed from the moving stage. The moving stage was removed from the SMI table. The SMI probe was translated arbitrarily to reset its position. The measurement system was reassembled. The coordinate system reference was relocated. Then the consecutive measurement was taken.

Figure 13: Repeated measurements. Seven repeated measurements are shown, with the black lines indicating the waviness of the gear surface. Amplitude variation about the waviness line is the roughness of the surface. Each trace has been arbitrarily spaced vertically to aid the presentation.

The outlined method was applied, in order to transform the measurement from SMI coordinate system into gear coordinate system. In line with standard measurement practice Gaussian band pass filter was applied with short and long cut off wavelengths $\lambda_s = 2.5 \mu\text{m}$ and $\lambda_f = 19.75 \text{ mm}$, respectively. Waviness and roughness were then separated by means of a Gaussian filter with cut off wavelength $\lambda_c = 0.8 \text{ mm}$. The repeated measurements within the evaluation range between SAP and STR are presented in figure 13. In order to characterize the reproducibility of the method, surface texture roughness and waviness parameters were analysed. Computed values are shown in tables 6 and 7.

Table 6: Reproducibility roughness surface texture parameters.

Measurement number	$Ra [\mu\text{m}]$	$Rq [\mu\text{m}]$	$Rz [\mu\text{m}]$	$RSm [\mu\text{m}]$	$R\Delta q [^\circ]$
1	0.37	0.45	2.35	0.10	4.025
2	0.37	0.45	2.28	0.12	3.945
3	0.37	0.45	2.33	0.11	3.916
4	0.37	0.45	2.31	0.11	3.917
5	0.37	0.45	2.35	0.11	3.961
6	0.37	0.45	2.33	0.11	3.899
7	0.37	0.46	2.35	0.10	3.930
Maximum	0.37	0.46	2.35	0.12	4.025

Mean	0.37	0.45	2.33	0.11	3.942
Minimum	0.37	0.45	2.28	0.10	3.899
Range	<0.01	0.01	0.07	0.03	0.127
Standard deviation	<0.01	<0.01	0.03	0.01	0.042

It is notable that the variation between each repeated measurement is fairly small, and is indicated by the range of each parameter. Particular interest is given to the maximum height roughness parameter R_z . This parameter captures the amplitude range of the surface roughness features. Investigation of failure development on super finished or coated gears, requires discrimination of surface texture changes at the level of 1-2 μm . It is notable that the range of R_z parameter is 0.07 μm which is about 3% of the mean value of R_z . Such small difference between repeated measurements will not significantly affect the discrimination of surface texture changes at the roughness level. This shows ability of the measurement method to produce repeatable results even when the system has been removed and reassembled.

Table 7: Reproducibility waviness surface texture parameters.

Measurement number	W_a [μm]	W_q [μm]	W_z [μm]	W_{Sm} [μm]	$W_{\Delta q}$ [$^\circ$]
1	0.41	0.50	2.06	2.66	0.082
2	0.46	0.57	2.26	2.64	0.082
3	0.41	0.51	2.11	2.65	0.082
4	0.48	0.60	2.47	2.63	0.084
5	0.42	0.51	2.05	2.66	0.081
6	0.46	0.56	2.24	2.64	0.083
7	0.41	0.51	2.05	2.66	0.081
Maximum	0.48	0.60	2.47	2.66	0.084
Mean	0.43	0.54	2.18	2.65	0.082
Minimum	0.41	0.50	2.05	2.63	0.081
Range	0.07	0.09	0.42	0.03	0.004
Standard deviation	0.03	0.04	0.16	0.01	0.001

5.4 Measurement uncertainty

The Klingelnberg P65 GMI provided reference data for the measurement of gear profile form total deviation F_a , form deviation slope f_{Ha} and profile form deviation f_{fa} , ISO 1328-1:2013 parameters [11]. The measurement uncertainty of data from this machine is assessed as part of the UKAS accredited UK National Gear Metrology Laboratory (UKAS accreditation number 0250). The measurement uncertainty (U_{95}) or CMC values are $\pm 1.2 \mu\text{m}$ for total deviation F_a , and profile form deviation f_{fa} and $\pm 1.0 \mu\text{m}$ for profile form deviation slope f_{Ha} . Differences between the Form Talysurf Intra for the ISO

ISO 1328-1 profile evaluated parameters are thus within the measurement uncertainty at a 95% confidence level.

The measurement uncertainty of the Form Talysurf Intra has not been estimated because of the lack of suitable calibrated artefacts with the required form deviations to provide surrogate artefact that approximates an involute helicoid. The uncertainty of roughness evaluated parameters on gears using SMIs and GMIs with roughness attachments requires further study. The SMI reproducibility results in tables 5 to 7 provide contributions to this uncertainty assessment.

6 Conclusions

A method has been described which allows the transformation of the general SMI measurement results from an arbitrary coordinate system of the SMI onto the involute gear length of roll, inherent to involute gear coordinate system. This method enables the study of surface roughness characterisation parameters with respect to gear meshing, which affect gear functionality and surface fatigue and scuffing failure modes.

The paper proposes and validates a robust method of establishing the start and end of active profile for the example test gear but acknowledges that adjustment to the method may be needed for other gear shapes.

The method also allows for the designer specified micro geometry modifications to the nominal involute profile such as tip relief and profile crowning which are applied to reduce stress and minimize gear noise during operation. A discussion of the proposed procedure's sensitivity to manufacturing deviations is presented and a method to minimize these effects is proposed.

The success of this method has been validated by comparison with a Klingelnberg P65 GMI measurement for involute form measurement parameters defined in ISO 1328-1:2013. Differences between the SMI and the traceably calibrated P65 are within the accredited measurement uncertainty.

By contrast, the presented method allows mapping of the roughness features onto roll length, hence allowing surface texture to be revealed in higher resolution than a standard GMI measurement. The method may be used for quantifying the damage or wear of gears during service using a portable instrument without removal of the gears from a gearbox, with further refinement.

Ability of the method to obtain repeatable results, of the similar position along the gear flank, has been quantified using a fixture designed with suitable datum features. This measurement methodology could be applied to monitor roughness scale surface texture changes, resulting from wear and other types of failure development, at different stages of operation. Authors plan to present such results of failure development monitoring in the future.

While the reproducibility of the process has been quantified, the measurement uncertainty for roughness parameters has yet to be completed and is planned for future research. When this is completed the process will be suitable for establishing traceable measurement uncertainty on existing GMIs equipped with surface roughness attachments.

Acknowledgements

The authors acknowledge the European Metrology Research Programme (EMRP). The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union. This work was done as part of EMRP collaborative project ENG56.

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