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Performance Study of a UCRN Over Nakagami- m Fading Channels in the Presence of CCI

Jamal Ahmed Hussein^{ID}, Said Boussakta, *Senior Member, IEEE*, and Salama S. Ikki, *Member, IEEE*

Abstract—In this paper, the impact of co-channel interference (CCI) on the performance of an underlay cognitive radio network over Nakagami- m fading channels is thoroughly presented and analysed. More precisely, a decode-and-forward relay protocol for a cognitive cooperative network is considered. In this study, the impact of both the primary transmitter interference and CCI on the secondary system performance are considered. First, an exact expression for the equivalent signal-to-interference-plus-noise ratio of the secondary system is obtained. Then, the corresponding exact and asymptotic cumulative distribution functions are derived. From this, the exact outage performance for the secondary network is investigated. Furthermore, the equivalent probability density function is obtained and discussed. In addition, approximate expressions for the average error probability and the system ergodic capacity performances are derived. From the results, it can be inferred that the presence of the CCI and primary network interference severely degrades the system performance. Moreover, a higher value of the shape parameter of the desired fading channel gives better performance and diversity gain. In addition, despite the impact of interferences, the secondary network performance gives better results in comparison to the Rayleigh fading channels scenario.

Index Terms—Underlay cognitive radio network, Nakagami- m fading channels, co-channel interference, error probability, outage probability, ergodic capacity.

I. INTRODUCTION

AN UNDERLAY cognitive radio (CR) scheme [2] is one of the paradigms that has been proposed to realise a cognitive radio network. The potential benefits of this proposed paradigm led to several research investigations [2], [3]. The main goal of proposing a cognitive network is for better utilisation of the current frequency spectrum, resulting in higher data throughput to the communication nodes. In an underlay CR approach, the secondary user (SU) can simultaneously use

the frequency spectrum as the primary user (PU) with the condition that the resulting interference to the primary receivers is below a predefined value, which is known as the interference power constraint [2]. In fact, this is the key protection factor of the primary user's quality of service (QoS). Similar to ordinary cooperative communication [4], an underlay CR network can take advantage of this diversity technique. Indeed, in power limiting environments, such as underlay CR, using cooperative communication for the secondary cognitive network could be an effective way to enhance the overall performance [5].

Co-channel interference (CCI) apparently affects the performance of any wireless communication network. In fact, the phenomenon of a co-channel interference is important to consider in the spectrum sharing network such as underlay cognitive radio [5]. This is due to the coexistence of different kinds of users in the same frequency spectrum, in addition to the interference that might come from outside sources in the neighbouring cells that are working on the similar frequency.

Furthermore, Nakagami- m fading channels are more generalized and practical to consider in the performance investigation of a wireless communication system, as they can better represent the physical channel characteristics than Rayleigh and Rician fading channels [6]. For instance, Nakagami- m fading channels define the envelope of the received signal after maximal ratio combining diversity. In addition, Rayleigh fading channels can be considered to be a special case within the Nakagami model. Moreover, Rician and Nakagami distributions demonstrate almost the same behaviour when approaching their mean values [7].

A. Related Works

Investigations into outage performance for an underlay CR paradigm has been widely studied [6], [8]–[10]. For example, the outage performance of the cooperative decode-and-forward (DF) underlay cognitive network was studied in [6] over Nakagami- m fading channels. Da Costa *et al.* [9] and Duong *et al.* [11] extended the previous work in [6] by considering multi-primary receivers and multi-secondary destinations. In [12], the outage performance of an underlay DF CR network was investigated using the relay selection technique and over Nakagami- m fading channels. In recent work [13], the authors made a comprehensive performance study of a DF cognitive network using the antenna SC technique and by considering proportional and fixed interference power constraint. In the works mentioned above, the impacts of the primary transmission power and CCI on the cognitive radio network

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J. A. Hussein is with the Information Technology Department, College of Science and Technology, University of Human Development, Sulaimaniyah 46003, Iraq (e-mail: barznjy@gmail.com).

S. Boussakta is with the School of Engineering, Newcastle University, Newcastle upon Tyne NE1 7RU, U.K. (e-mail: said.boussakta@ncl.ac.uk).

S. S. Ikki is with the Electrical Engineering Department, Lakehead University, Thunder Bay, ON P7B 5E1, Canada (e-mail: sikki@lakeheadu.ca). Digital Object Identifier 10.1109/TCCN.2017.2768061

were not considered. Deng *et al.* [14] investigated the outage probability of the spectrum sharing network over Rayleigh fading channels considering the impact of general selection combining and outdated channel information.

Some recent studies have examined the impact of primary transmission interference on the secondary network's performance [15], [16]. For instance, in [15], the asymptotic outage performance of the cooperative AF CR network was studied over Rayleigh fading channels. In [17], the transmit antenna selection in conjunction with generalized selection combining technique at the receiver of the secondary network was proposed to examine the outage probability and the symbol error rate of an underlay cognitive radio network and over the Rayleigh fading channels. Finally, Hussein *et al.* [18], [19] have investigated the impact of CCI on an underlay CR network considering Rayleigh fading channels. In addition, detailed performance analyses were investigated in [18].

B. Contribution of This Paper

An elaborate background understanding in the area of the performance study of an underlay CR network (UCRN) has been garnered from the works mentioned above. However, most of them have neglected the interference from the primary transmitter and/or considered only Rayleigh fading channels. In fact, the impact of CCI has to be considered in practical cognitive relaying techniques. This is due to the fact that CCI signals always exist, for example, due to the frequency reuse. Furthermore, the next generation of wireless communication scales toward denser and smaller cells, which means more interference is expected from the neighbouring cells. As a result, it is necessary to investigate the impact of the CCI on an underlay cognitive radio network. To the best of the authors' knowledge, the effects of such CCI on a UCRN over Nakagami- m fading channels has not yet been studied. Similar to other wireless communication networks, when more parameters are considered for investigation of the cognitive radio network, it will add extra complexity during mathematical analysis. In fact, co-channel interference consideration makes the system analysis more difficult compared to previous works, especially when Nakagami- m fading channels are considered.

In this paper, the performance of a UCRN scenario over Nakagami- m fading channels is studied when the CCI signals and the primary network interferences are present. Specifically, the exact equivalent per-hop and end-to-end SINR of the secondary network over Nakagami- m fading channels is addressed. Then, the exact cumulative distribution function (CDF) of the system SINR is obtained. Based on these results, the outage probability (OP), average error probability (AEP) and ergodic capacity (EC) performances of the secondary network are thoroughly investigated.

The remainder of this paper is structured as follows. In Section II the system model is described and represented mathematically. Section III is devoted to the derivation of the statistical and performance metrics. In Section IV, numerical results are presented to validate the derivations and analysis. Section V summarizes the main findings of the research in

Fig. 1. The general network model used for analysis showing cooperative underlay CR network in the existence of the primary transceiver and CCI.

this paper. Finally, the steps of the theoretical derivations are given in Appendixes A–C.

II. SYSTEM MODEL

In the following sections, first, the network parameters and channels are described. Then, mathematical representations of the received signals are presented, and the formula for the equivalent SINR of the network is obtained.

A. Network and Channels Description

The system model under consideration is shown in Fig. 1. S , R , and D are the secondary source, relay and destination nodes, respectively. The existence of an uplink primary transceiver is considered in the network, where T_x is the transmitter user node and R_x is the base station receiver node. Each node in the system has a single antenna and is working in half-duplex mode. Due to the presence of obstacles between the secondary source and destination, it is assumed that there is no direct link between them [20]. In addition, the relay node employs the DF protocol. Moreover, it is assumed that all the channels between the nodes are subject to independent and non-identically distributed Nakagami- m fading channels. The channels have a scale parameter of $\sigma_{X_{ij}}^2$ and a shape parameter of $m_{X_{ij}}$, where X represents the generic channel coefficient; i and j represent the source and destination node for the channel X , respectively.

Furthermore, it is assumed that the CCI links are identical, in terms of their average interference-to-noise ratio (INR), at the secondary user relay and destination nodes, respectively [4], [18]. This is a valid assumption especially when a deterministic number of interference sources are from other neighbouring clusters. In this analysis, L_R and L_D represent the number of co-channel interference sources at both relay and destination node, respectively. For instance, the distance from L_R sources to the relay node is relatively large enough that the interference signals can be assumed to have similar average power gain [18].

In addition, h_{sr} , and h_{rd} represent the first and second hop data channel fading coefficients of the secondary network, respectively. Furthermore, f_{sp} , and f_{rp} represent the interference channel fading coefficients of the secondary source and relay

to the primary receiver, respectively. Moreover, f_{pr} , f_{pd} represent the interference channel fading coefficients of the T_x to the secondary relay and destination, respectively. Consequently, f_{irj} ($j = 1, 2, \dots, L_R$), and f_{idl} ($l = 1, 2, \dots, L_D$) represent the CCI fading coefficients of the j^{th} interference channel at the secondary relay and l^{th} interference channel at the secondary destination, respectively. In fact, the CCI signals affect the primary receiver as well, and therefore, $I_{\max}$ should be defined based on all interferences that the primary receiver can detect. This will ensure the protection of the QoS of the primary network. However, this research focuses on the performance of the secondary network. It is obvious that the performance of the primary network is similar to the extensively studied performance in [21].

Moreover, the corresponding channel power gains for the mentioned Nakagami- m channel coefficients are $|h_{sr}|^2$, $|h_{rd}|^2$, $|f_{sp}|^2$, $|f_{rp}|^2$, $|f_{pr}|^2$, $|f_{pd}|^2$, $|f_{irj}|^2$, and $|f_{idl}|^2$ respectively, which follow the gamma distribution with the scale parameters of $\sigma_{h_{sr}}^2$, $\sigma_{h_{rd}}^2$, $\sigma_{f_{sp}}^2$, $\sigma_{f_{rp}}^2$, $\sigma_{f_{pr}}^2$, $\sigma_{f_{pd}}^2$, $\sigma_{f_{ir}}^2$ and $\sigma_{f_{id}}^2$, respectively, and the corresponding channel fading severity parameters are $m_{h_{sr}}$, $m_{h_{rd}}$, $m_{f_{sp}}$, $m_{f_{rp}}$, $m_{f_{pr}}$, $m_{f_{pd}}$, m_{ir} , and m_{id} , respectively.

B. Mathematical Representation

The transmission in the secondary network is performed in two phases. In the first phase, the source node transmits its message signal to the relay node. The relay node receives the transmitted message from S plus noise, which is represented as additive white Gaussian noise (AWGN), plus the interference from T_x , and the CCI from L_R interference sources.

In the second phase, the relay node decodes the received signal then encodes it and broadcasts the message to the destination node. Similar to the first phase, the received signal at the destination node will be a combination of the desired message, noise and interferences from the primary transmitter and the CCI signals. Thus, the received signal at both secondary relay and secondary destination nodes can be represented as:

$$y_r = \underbrace{\sqrt{E_S} h_{sr} x}_{\text{desired signal}} + \underbrace{\sqrt{E_{IR}} \sum_{j=1}^{L_R} f_{irj} x_{rj}}_{\text{CCI signal}} + \underbrace{\sqrt{E_{PR}} f_{pr} x_{pr}}_{\text{primary interference signal}} + \underbrace{n_r}_{\text{AWGN}}, \quad (1)$$

and

$$y_d = \underbrace{\sqrt{E_R} h_{rd} \hat{x}}_{\text{desired signal}} + \underbrace{\sqrt{E_{ID}} \sum_{l=1}^{L_D} f_{idl} x_{dl}}_{\text{CCI signal}} + \underbrace{\sqrt{E_{PD}} f_{pd} x_{pd}}_{\text{primary interference signal}} + \underbrace{n_d}_{\text{AWGN}}, \quad (2)$$

where E_S , and E_R are the secondary source and relay permitted transmission energy signals, respectively. E_{IR} , and E_{ID} are the CCI energies at the secondary relay and destination nodes, respectively. E_{PR} , and E_{PD} are the interference energy signals of the primary transmitter at the secondary relay and destination nodes, respectively.

Furthermore, x represents the desired signal to be transmitted from the secondary source and $\hat{x}$ is the desired signal to be transmitted from the secondary relay. x_{rj} and x_{dl} are the j^{th} and l^{th} co-channel interferer's signals that are affecting the secondary relay and destination nodes, respectively, x_{pr} , and x_{pd} are the primary interferer's signals that are affecting the secondary relay and destination nodes, respectively. All signals are assumed to have unit energy. Finally, n_r and n_d represent the AWGN terms at the secondary relay and destination nodes, respectively.

In an underlay CR network, the secondary transmitters should adjust their transmission power so that the QoS of the primary network is maintained. Therefore, the transmission powers at S and R are $E_S = \min(\frac{I_{\max}}{|f_{sp}|^2}, P_s)$ and $E_R = \min(\frac{I_{\max}}{|f_{rp}|^2}, P_r)$, respectively, where P_s and P_r are maximum transmission power limits at S and R , respectively. Moreover, $I_{\max}$ is the interference power constraint, which is the maximum level of interference that the secondary network can produce at the primary receiver node. Therefore, the received equivalent SINR in the first time-slot at the relay is represented as:

$$\gamma_{SR}^{\text{eq}} = \frac{\text{signal power}}{\text{CCI power} + \text{primary interference} + \text{noise power}} = \frac{\gamma_{SR}}{\gamma_{IR} + \gamma_{PR} + 1}, \quad (3)$$

where $\gamma_{SR} = \frac{E_S}{N_0} |h_{sr}|^2 = \min(\frac{I_{\max}}{|f_{sp}|^2}, P_s) \frac{|h_{sr}|^2}{N_0}$, $\gamma_{IR} = \sum_{j=1}^{L_R} (\frac{E_{IR}}{N_0} |f_{irj}|^2 \triangleq I_{Rj})$ and $\gamma_{PR} = \frac{E_{PR}}{N_0} |f_{pr}|^2 \triangleq I_{PR}$. In addition, $\gamma_{RD} = \min(\frac{I_{\max}}{|f_{rp}|^2}, P_r) \frac{|h_{rd}|^2}{N_0}$, $\gamma_{PD} = \frac{E_{PD}}{N_0} |f_{pd}|^2 \triangleq I_{PD}$, and $\gamma_{ID} = \sum_{l=1}^{L_D} (\frac{E_{ID}}{N_0} |f_{idl}|^2 \triangleq I_{Dl})$. Moreover, $\bar{I}_R$, $\bar{I}_D$, $\bar{I}_{PR}$, and $\bar{I}_{PD}$ represent the average INR values corresponding to I_R , I_D , I_{PR} , and I_{PD} , respectively.

III. STATISTICAL DERIVATIONS AND PERFORMANCE EVALUATION

In the following sections, first the per-hop and total equivalent CDF and probability density function (PDF) are derived. Then, exact and asymptotic expressions for the outage performance are derived. In addition, multi-hop outage performance is examined. Finally, the system AEP is investigated.

A. The CDF of γ_{eq}^{tot}

For a dual-hop DF cognitive secondary network, the end-to-end equivalent SINR known as γ_{eq}^{tot} can be represented as [21]:

$$\gamma_{eq}^{\text{tot}} = \min(\gamma_{SR}^{\text{eq}}, \gamma_{RD}^{\text{eq}}), \quad (4)$$

where γ_{SR}^{eq} , and γ_{RD}^{eq} are the equivalent SINR for the first and second hop, respectively. Moreover, the total equivalent CDF can be obtained by using the following formula [22]:

$$F_{\gamma_{eq}^{\text{tot}}}(\gamma) = 1 - \left(1 - F_{\gamma_{SR}^{\text{eq}}}(\gamma)\right) \left(1 - F_{\gamma_{RD}^{\text{eq}}}(\gamma)\right), \quad (5)$$

where $F_{\gamma_{SR}^{\text{eq}}}(\gamma)$ and $F_{\gamma_{RD}^{\text{eq}}}(\gamma)$ are the CDFs of the first and second hop equivalent SINR, i.e., γ_{SR}^{eq} and γ_{RD}^{eq} , respectively. The CDFs of γ_{SR}^{eq} and γ_{RD}^{eq} can be found as follows: from

$$\begin{aligned}
F_{\gamma_{SR}^{\text{eq}}}(\gamma) = 1 - \Upsilon \left\{ e^{-\frac{\gamma}{\beta_1}} \left(\frac{\gamma}{\beta_1} \right)^i \left(1 - \frac{\Gamma(m_{fsp}, \beta_5)}{\Gamma(m_{fsp})} \right) \left[\left(\frac{\gamma}{\beta_1} + \beta_3 \right)^{l-j-m_{pr}} \Gamma(j+m_{pr}-l) \right. \right. \\
\left. \left. - \sum_{k=0}^{m_{ir}L_R+l-1} \left(\frac{\gamma}{\beta_1} + \beta_2 \right)^{l-j-k-m_{pr}} \frac{(\beta_4)^k}{k!} \Gamma(j+k+m_{pr}-l) \right] \right. \\
\left. + \frac{(\beta_6/\gamma)^{m_{fsp}}}{\Gamma(m_{fsp})} \left[\Gamma(j+m_{pr}-l)I_{1a} - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{(\beta_4)^k}{k!} \Gamma(j+k+m_{pr}-l)I_{1b} \right] \right\} \quad (11)
\end{aligned}$$

the equivalent SINR formula in (3), the CDF of γ_{SR}^{eq} can be written as:

$$\begin{aligned}
F_{\gamma_{SR}^{\text{eq}}}(\gamma) &= \Pr(\gamma_{SR}^{\text{eq}} \leq \gamma) \\
&= \Pr\left(\min\left(\frac{I_{\max}}{Y}, P_s\right) \frac{X}{(I+P+1)} \leq \gamma\right), \quad (6)
\end{aligned}$$

where X , Y , I , P represent the random variables (RVs) $\frac{|h_{sr}|^2}{N_0}$, $|f_{rp}|^2$, $\frac{E_{IR}}{N_0} \sum_{j=1}^{L_R} |f_{irj}|^2$, and $\frac{E_{PR}}{N_0} |f_{pr}|^2$, respectively, with $\Pr(\cdot)$

denoting probability operator. The PDFs of X , Y , I , P are represented as the following:

$$f_X(x) = \left(\frac{m_{hsr}}{\sigma_{hsr}^2} \right)^{m_{hsr}} \frac{x^{m_{hsr}-1}}{\Gamma(m_{hsr})} \exp\left(-\frac{m_{hsr}}{\sigma_{hsr}^2} x\right), \quad (7)$$

$$f_Y(y) = \left(\frac{m_{fsp}}{\sigma_{fsp}^2} \right)^{m_{fsp}} \frac{y^{m_{fsp}-1}}{\Gamma(m_{fsp})} \exp\left(-\frac{m_{fsp}}{\sigma_{fsp}^2} y\right), \quad (8)$$

$$f_I(\gamma_{IR}) = \left(\frac{m_{ir}}{\bar{I}_R} \right)^{m_{ir}L_R} \frac{\gamma_{IR}^{m_{ir}L_R-1}}{\Gamma(m_{ir}L_R)} \exp\left(-\frac{m_{ir}}{\bar{I}_R} \gamma_{IR}\right), \quad (9)$$

$$f_P(\gamma_{PR}) = \left(\frac{m_{pr}}{\bar{I}_{PR}} \right)^{m_{pr}} \frac{\gamma_{PR}^{m_{pr}-1}}{\Gamma(m_{pr})} \exp\left(-\frac{m_{pr}}{\bar{I}_{PR}} \gamma_{PR}\right), \quad (10)$$

where m_{hsr} , m_{fsp} , m_{ir} , and m_{pr} are the Nakagami- m channel fading severity parameters for the channels between the nodes of the SU source to the SU relay, the SU source to R_x , CCI signals to the SU relay, and T_x to the SU relay, respectively.

Corollary 1: The equivalent CDF of the source to relay SINR can be expressed as in (11), at the top of this page, where Υ is defined in (12). In addition, $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function defined in [23, eq. (8.350.2)]. Furthermore, for the purpose of representing the equations in a simpler form, the following entities have been defined as follows:

$$\beta_1 = \frac{P_s \sigma_{hsr}^2}{m_{hsr}}, \beta_2 = \frac{m_{ir}}{\bar{I}_R}, \beta_3 = \frac{m_{pr}}{\bar{I}_{PR}}, \beta_4 = \frac{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}}{\bar{I}_{PR} \bar{I}_R},$$

$$\beta_5 = \frac{m_{fsp} I_{\max}}{P_s \sigma_{fsp}^2}, \beta_6 = \frac{m_{fsp} \sigma_{hsr}^2 I_{\max}}{m_{hsr} \sigma_{fsp}^2}, \beta_7 = \frac{m_{pr} m_{fsp} \sigma_{hsr}^2 I_{\max}}{m_{hsr} \sigma_{fsp}^2 \bar{I}_{PR}},$$

$$\text{and } \beta_8 = \frac{m_{ir} m_{fsp} \sigma_{hsr}^2 I_{\max}}{m_{hsr} \sigma_{fsp}^2 \bar{I}_R}.$$

$$\begin{aligned}
\Upsilon &= \sum_{i=0}^{m_{hsr}-1} \sum_{j=0}^i \sum_{l=0}^{m_{pr}-1} \binom{m_{pr}-1}{l} \binom{i}{j} \frac{(-1)^l}{i!} \left(\frac{m_{ir}}{\bar{I}_R} \right)^{m_{ir}L_R} \\
&\times \left(\frac{m_{pr}}{\bar{I}_{PR}} \right)^{m_{pr}} \frac{\Gamma(m_{ir}L_R + l)}{\Gamma(m_{ir}L_R) \Gamma(m_{pr})} \left(\frac{\bar{I}_{PR} \bar{I}_R}{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}} \right)^{m_{ir}L_R + l}. \quad (12)
\end{aligned}$$

Proof: See Appendix A. ■

To obtain the second hop CDF, similar derivation steps can be repeated by replacing the following parameters (P_s , m_{hsr} , m_{fsp} , m_{ir} , m_{pr} , σ_{hsr}^2 , σ_{fsp}^2 , $\bar{I}_R$, $\bar{I}_{PR}$, γ_{SR}^{eq} , γ_{SR} , γ_{IR} , γ_{PR}) with the following parameters (P_r , m_{hrd} , m_{frp} , m_{id} , m_{pd} , σ_{hrd}^2 , σ_{frp}^2 , $\bar{I}_D$, $\bar{I}_{PD}$, γ_{RD}^{eq} , γ_{RD} , γ_{ID} , γ_{PD}), respectively. Finally, the dual-hop cognitive network equivalent CDF can be obtained by substituting the derived per-hop CDFs into (5).

B. The PDF of γ_{eq}^{tot}

Another efficient performance indicator of the RVs is the PDF of the total equivalent SINR of the system. By knowing this, the behaviour of the RV over the specified range can be investigated. The PDF can be obtained by taking the first derivative of the CDF; therefore, the total equivalent PDF of the secondary network can be obtained using the following formula:

$$f_{\gamma_{eq}^{\text{tot}}}(x) = f_{\gamma_{SR}^{\text{eq}}}(x) \left(1 - F_{\gamma_{RD}^{\text{eq}}}(x) \right) + f_{\gamma_{RD}^{\text{eq}}}(x) \left(1 - F_{\gamma_{SR}^{\text{eq}}}(x) \right), \quad (13)$$

where $f_{\gamma_{SR}^{\text{eq}}}(x)$ and $f_{\gamma_{RD}^{\text{eq}}}(x)$ are the PDFs of γ_{SR}^{eq} and γ_{RD}^{eq} , respectively. In the previous sections the CDFs for both hops were already derived. The PDFs of γ_{SR}^{eq} and γ_{RD}^{eq} are found as follows:

- Determining $f_{\gamma_{SR}^{\text{eq}}}(x)$: In this section, the exact equivalent PDF of the first hop SINR is derived. $f_{\gamma_{SR}^{\text{eq}}}(x)$ can be obtained by taking the first derivative of the equivalent CDF, i.e., $F_{\gamma_{SR}^{\text{eq}}}(\gamma)$:

$$f_{\gamma_{SR}^{\text{eq}}}(x) = \frac{d}{d\gamma} F_{\gamma_{SR}^{\text{eq}}}(\gamma). \quad (14)$$

Bearing in mind that the derivative of upper incomplete gamma function can be implemented using the chain rule as follows:

$$\frac{d}{dx} \Gamma(s, g(x)) = -(g(x))^{s-1} e^{-g(x)} \frac{d}{dx} g(x). \quad (15)$$

After performing the derivative of each term and some mathematical arrangements, the first hop equivalent PDF can be obtained and written as in (16). Υ_2 , Υ_3 , Υ_4 , and Υ_5 are represented by formulas in (17a)-(17d), respectively. Furthermore, ψ_2 , ψ_3 , ψ_4 , and ψ_5 represent the terms that

are involved with derivatives, and are represented by the formulas (18)–(21), respectively.

$$f_{\gamma_{SR}}^{\text{eq}}(x) = \Upsilon \left\{ (\beta_1)^{m_{pr}+j-l-i} \left(1 - \frac{\Gamma(m_{fsp}, \beta_5)}{\Gamma(m_{fsp})} \right) \times [\Upsilon_3 \psi_3 - \Upsilon_2 \psi_2] - \frac{(\beta_6)^{m_{fsp}}}{\Gamma(m_{fsp})} [\Upsilon_4 \psi_4 - \Upsilon_5 \psi_5] \right\}. \quad (16)$$

$$\Upsilon_2 = \Gamma(j + m_{pr} - l), \quad (17a)$$

$$\Upsilon_3 = \sum_{k=0}^{m_{ir}L_R+l-1} \frac{(\beta_1\beta_4)^k}{k!} \Gamma(j + k + m_{pr} - l), \quad (17b)$$

$$\Upsilon_4 = \Gamma(j + m_{pr} - l) \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} \times (-\beta_3)^{m_{fsp}+i-1-r_1} e^{\beta_3}, \quad (17c)$$

$$\Upsilon_5 = \sum_{k=0}^{m_{ir}L_R+l-1} \sum_{r_2=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_2} \frac{(\beta_4)^k}{k!} \times \Gamma(j + k + m_{pr} - l) (-\beta_2)^{m_{fsp}+i-1-r_2} e^{\beta_2}, \quad (17d)$$

$$\psi_2 = e^{-\frac{x}{\beta_1}} x^i (\beta_1\beta_3 + x)^{l-j-m_{pr}} \times \left(\frac{(l-j-m_{pr})}{(\beta_1\beta_3 + x)} - \frac{1}{\beta_1} + \frac{i}{x} \right), \quad (18)$$

$$\psi_3 = e^{-\frac{x}{\beta_1}} x^i (\beta_1\beta_2 + x)^{l-j-k-m_{pr}} \times \left(\frac{(l-j-k-m_{pr})}{(\beta_1\beta_2 + x)} - \frac{1}{\beta_1} + \frac{i}{x} \right), \quad (19)$$

$$\psi_4 = -\frac{(x)^{r_1+l-j-m_{pr}-m_{fsp}}}{(\beta_6 + x)^{r_1+l-j-m_{pr}+1}} \left\{ e^{-\frac{x}{\beta_1}-\beta_3-\beta_5} \left(\frac{x}{\beta_1} + \frac{\beta_7}{x} + \beta_3 + \beta_5 \right)^{r_1+l-j-m_{pr}} \left(\frac{x}{\beta_1} - \frac{\beta_7}{x} \right) + \Gamma(r_1 + l - j - m_{pr} + 1, \beta_3 + \beta_5 + \frac{x}{\beta_1} + \frac{\beta_7}{x}) e^{\frac{\beta_7}{x}} \left[m_{fsp} + \frac{\beta_7}{x} - (r_1 + l - j - m_{pr} + 1) \left(\frac{\beta_6}{\beta_6 + x} \right) \right] \right\}, \quad (20)$$

$$\psi_5 = -\frac{(x)^{r_2+l-j-k-m_{pr}-m_{fsp}}}{(\beta_6 + x)^{r_2+l-j-k-m_{pr}+1}} \left\{ e^{-\frac{x}{\beta_1}-\beta_2-\beta_5} \left(\frac{x}{\beta_1} + \frac{\beta_8}{x} + \beta_2 + \beta_5 \right)^{r_2+l-j-k-m_{pr}} \left(\frac{x}{\beta_1} - \frac{\beta_8}{x} \right) + \Gamma(r_2 + l - j - k - m_{pr} + 1, \beta_2 + \beta_5 + \frac{x}{\beta_1} + \frac{\beta_8}{x}) e^{\frac{\beta_8}{x}} \left[m_{fsp} + \frac{\beta_8}{x} - (r_2 + l - j - k - m_{pr} + 1) \left(\frac{\beta_6}{\beta_6 + x} \right) \right] \right\}. \quad (21)$$

- Determining $f_{\gamma_{RD}}^{\text{eq}}(x)$: Similar steps can be repeated to obtain the PDF of the second hop. Finally, the total equivalent PDF is obtained by substituting the derived per-hop PDFs and CDFs into (13).

C. Exact Outage Performance

From the derived total equivalent CDF of the secondary network's SINR, the exact OP of a cognitive network can be investigated by replacing the variable γ with γ_{th} (i.e., SNR threshold).

$$P_{\text{out}}(\gamma_{\text{th}}) = \Pr(\gamma_{\text{eq}}^{\text{tot}} \leq \gamma_{\text{th}}) = F_{\gamma_{\text{eq}}^{\text{tot}}}(\gamma_{\text{th}}). \quad (22)$$

The value of γ_{th} for the dual-hop network has the following representation: $\gamma_{\text{th}} = 2^{2R} - 1$, where R is the target data rate and 2 comes from the fact that the transmission is performed within two time-slots.

D. Asymptotic Outage Performance

In this section, two scenarios for the asymptotic CDF are presented, from which the asymptotic outage performance can be investigated.

- **No interference power constraint**, i.e., $I_{\text{max}} \rightarrow \infty$: By substituting this condition in (6), the resulting CDF formula will reduce to the CDF of RV G that has been derived and represented in (48), in Appendix A, with the condition of replacing the RV γ by $\frac{\gamma}{\beta_1}$, i.e., $F_G(\frac{\gamma}{\beta_1})$. Then, the approximate first hop CDF in this case will be:

$$F_{\gamma_{SR}}^{\text{eq}}(\gamma) \approx 1 - \Upsilon \left(\frac{\gamma}{\beta_1} \right)^i e^{-\frac{\gamma}{\beta_1}} \left[\left(\frac{\gamma}{\beta_1} + \beta_3 \right)^{l-j-m_{pr}} \Gamma(j + m_{pr} - l) - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{(\beta_4)^k}{k!} \left(\frac{\gamma}{\beta_1} + \beta_2 \right)^{l-j-k-m_{pr}} \Gamma(j + k + m_{pr} - l) \right]. \quad (23)$$

This scenario can be considered when the primary network is not active. In fact, this can also be considered as a special case of interweave cognitive radio [2], in which the secondary user can use a specific frequency spectrum when this particular spectrum is vacant. In this scenario, the secondary transmission power is the dominant power; however, due to the existence of interference on the secondary network, it is still possible for the performance saturation to occur. Finally, by performing similar steps, the second hop asymptotic CDF can be obtained; then, the asymptotic outage performance can be studied using (5) and (22).

- **No power constraint on the secondary transmitter**, i.e., $P_s \rightarrow \infty$: In this scenario, the first hop conditional equivalent CDF can be represented as:

$$F_{\gamma_{SR}}^{\text{eq}}(\gamma) \Big|_y = \Pr \left(G \leq \frac{\gamma y}{I_{\text{max}}} \right). \quad (24)$$

Therefore, the unconditional CDF can be obtained by taking the expectation of $F_G(\frac{\gamma y}{I_{\max}})$ over the PDF of RV Y .

$$F_{\gamma_{SR}^{\text{eq}}}(\gamma) = \int_0^\infty F_G\left(\frac{\gamma y}{I_{\max}}\right) f_Y(y) dy. \quad (25)$$

The steps for solving the above integral are quite similar to the steps for solving I_1 in (49), in Appendix A. Therefore, for the sake of saving space they have been omitted. Finally, the asymptotic first hop CDF for this scenario can be written as in (26), where I_{1b}^{app} and I_{1a}^{app} are obtained by using the formulas in (27) and (28), respectively.

$$F_{\gamma_{SR}^{\text{eq}}}(\gamma) \approx 1 - \Upsilon\left(\frac{\beta_6}{\gamma}\right)^{m_{fsp}} \frac{1}{\Gamma(m_{fsp})} \left[\Gamma(j + m_{pr} - l) I_{1a}^{\text{app}} - \sum_{k=0}^{a_3-1} \frac{(\beta_4)^k}{k!} \Gamma(j + k + m_{pr} - l) I_{1b}^{\text{app}} \right], \quad (26)$$

$$I_{1a}^{\text{app}} = \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} \frac{(-1)^{m_{fsp}+i-1-r_1}}{(\beta_3)^{r_1+1-m_{fsp}-i}} \left(\frac{\gamma}{\gamma + \beta_6} \right)^{r_1-j-m_{pr}+1+l} e^{\beta_3(1+\frac{\beta_6}{\gamma})} \Gamma\left(l + r_1 - j - m_{pr} + 1, \beta_3\left(1 + \frac{\beta_6}{\gamma}\right)\right), \quad (27)$$

$$I_{1b}^{\text{app}} = \sum_{r_2=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_2} \frac{(-1)^{m_{fsp}+i-1-r_2}}{(\beta_2)^{r_2+1-m_{fsp}-i}} \left(\frac{\gamma}{\gamma + \beta_6} \right)^{r_2-j-k-m_{pr}+1+l} e^{\beta_2(1+\frac{\beta_6}{\gamma})} \Gamma\left(l + r_2 - j - k - m_{pr} + 1, \beta_2\left(1 + \frac{\beta_6}{\gamma}\right)\right). \quad (28)$$

In this scenario, $I_{\max}$ is the dominant power limit for the secondary transmitter nodes. Furthermore, in terms of performance criteria, it can be considered to be the maximum performance limit that the secondary network can achieve. More explanation about the asymptotic results can be found in Fig. 3 in the numerical results section.

E. Multi-Hop Exact Outage Performance

In the case of the multi-hop DF cooperative communication, the end-to-end equivalent SINR $\gamma_{\text{eq}}^{\text{e2e}}$ is calculated based on the weakest per-hop SINR. Mathematically, this can be represented as:

$$\gamma_{\text{eq}}^{\text{e2e}} = \min_{i=1, \dots, N} (\gamma_{\text{eq}}^i), \quad (29)$$

where γ_{eq}^i is the i^{th} hop equivalent SINR, and N is the number of hops in the multi-hop cognitive cooperative network.

Therefore, the exact end-to-end multi-hop outage performance can be obtained by [24]:

$$P_{\text{out}}^{\text{e2e}}(\gamma_{\text{th}}) = 1 - \prod_{i=1}^N \left(1 - F_{\gamma_{\text{eq}}^i}(\gamma_{\text{th}})\right), \quad (30)$$

where $F_{\gamma_{\text{eq}}^i}(\gamma_{\text{th}})$ is the i^{th} hop equivalent CDF. From the previous derivations, it can be noted that the exact multi-hop outage performance can be easily obtained by substituting the per-hop CDF derived in (11) into (30).

F. Average Error Probability

The AEP over slow flat fading channels can be found using different approaches, such as the PDF or CDF of the equivalent SINR of the system. Furthermore, by observing the derived per-hop CDF and PDF in the previous sections, it can be deduced that using CDF is mathematically more convenient. Therefore, the per-hop AEP can be obtained by using [19]:

$$\bar{P}_b^i = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_{\gamma_{\text{eq}}^i}(\gamma) d\gamma, \quad (31)$$

where $\bar{P}_b^i$ is the average symbol error probability for the i^{th} hop. In addition, a and b are modulation constants depending on the constellation used. Finally, the end-to-end AEP can be evaluated by:

$$\bar{P}_b^{\text{e2e}} = \sum_{i=1}^N \bar{P}_b^i \prod_{j=i+1}^N \left(1 - 2\bar{P}_b^j\right), \quad (32)$$

For the case of a dual-hop DF network, the above formula reduces to [25]:

$$\bar{P}_b^{\text{e2e}} = \bar{P}_b^{\text{SR}} + \bar{P}_b^{\text{RD}} - 2(\bar{P}_b^{\text{SR}} \bar{P}_b^{\text{RD}}), \quad (33)$$

where $\bar{P}_b^{\text{SR}}$ and $\bar{P}_b^{\text{RD}}$ are the AEPs for the first and second hop, respectively, which are obtained by using (31). Unfortunately, it is very difficult if not impossible to get the exact expression from the above equation. However, an approximate expression can be obtained using the approximate derived CDF expression in (23).

Corollary 2: The first hop approximate AEP can be represented as in (34).

$$\bar{P}_b^{\text{SR}} \approx \frac{a}{2} - \frac{a}{2} \sqrt{\frac{b}{\pi}} \Upsilon(\beta_1)^{m_{pr}+j-l-i} \left[\Gamma(j + m_{pr} - l) \Omega_1 - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{(\beta_1\beta_4)^k}{k!} \Gamma(j + k + m_{pr} - l) \Omega_2 \right], \quad (34)$$

where Ω_1 and Ω_2 are calculated using formulas represented in (35) and (36), respectively.

$$\Omega_1 = (\beta_1\beta_3)^{l-j-m_{pr}+i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right) U\left(i + \frac{1}{2}, l - j - m_{pr} + i + \frac{3}{2}, \beta_1\beta_3\left(b + \frac{1}{\beta_1}\right)\right), \quad (35)$$

$$C_{erg}^{SR} \approx \Upsilon(\beta_1)^{m_{pr}+j-l-i} \sum_{n_1=0}^i \binom{i}{n_1} (-1)^{i-n_1} \left[\Gamma(j+m_{pr}-l) \left\{ \sum_{n_2=1}^{m_{pr}+j-l} \lambda_{1_{n_2}} (\beta_1 \beta_3)^{i+1-n_2} e^{\beta_3} E_{n_2-n_1}(\beta_3) + \lambda_2 e^{\frac{1}{\beta_1}} E_{1-n_1}\left(\frac{1}{\beta_1}\right) \right\} \right. \\ \left. - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{(\beta_1 \beta_4)^k}{k!} \Gamma(j+k+m_{pr}-l) \left\{ \lambda_4 e^{\frac{1}{\beta_1}} E_{1-n_1}\left(\frac{1}{\beta_1}\right) + \sum_{n_3=1}^{m_{pr}+j+k-l} \lambda_{3_{n_3}} (\beta_1 \beta_2)^{i+1-n_3} e^{\beta_2} E_{n_3-n_1}(\beta_2) \right\} \right] \quad (39)$$

$$\Omega_2 = (\beta_1 \beta_2)^{l-j-k-m_{pr}+i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right) \\ \times U\left(i + \frac{1}{2}, l-j-k-m_{pr}+i+\frac{3}{2}, \beta_1 \beta_2 \left(b + \frac{1}{\beta_1}\right)\right). \quad (36)$$

Proof: See Appendix B. ■

G. Ergodic Capacity

The ergodic capacity (EC) is an alternative significant performance measure for any communication system; its unit is measured in (bits/second/Hz) [18]. It gives an indication of the possible data rate that the considered network can attain under some predefined conditions. In fact, this is important specifically for a cognitive radio network to assess its involvement in providing the amount of data throughput to the intended SU. According to Shannon's theorem for the network capacity measurement, the EC can be defined mathematically as the expected value of the instantaneous mutual information between the source and destination. This can be expressed as $C_{erg} \triangleq \mathbb{E}[B \log_2(1 + \gamma_{eq})]$, where $\mathbb{E}[\cdot]$ is the expectation operator, B is the operating bandwidth and γ_{eq} is the total equivalent SNR. Moreover, the EC can be obtained using the CDF formula of the total equivalent SINR [18]:

$$C_{erg}^i = \int_0^\infty \frac{1}{1+\gamma} (1 - F_{\gamma_{eq}^i}(\gamma)) d\gamma, \quad (37)$$

where $F_{\gamma_{eq}^i}(\gamma)$ represents the CDF of the i^{th} hop of the secondary network. Moreover, the EC for a multi-hop decode-and-forward relay protocol can be obtained as:

$$C_{erg}^{e2e} = \frac{1}{N} \min_{i=1,\dots,N} (C_{erg}^i), \quad (38)$$

where C_{erg}^i represents the i^{th} hop EC of the CR network. In addition, $\frac{1}{N}$ comes from the fact that the transmission in an N hop network is performed within N time slots, which means the overall bandwidth should be divided by the number of hops in the network.

In the following sections, the per-hop EC will be derived. Then, the end-to-end EC, for a multi-hop UCRN scenario, can be obtained by substituting the derived per-hop results into (38).

Corollary 3: The first hop asymptotic EC can be obtained and written as in (39), at the top of this page, where

$\lambda_{1_{n_2}}$, λ_2 , $\lambda_{3_{n_3}}$ and λ_4 are obtained by formulas represented in (40a), (40b), (40c) and (40d), respectively.

$$\lambda_{1_{n_2}} = \frac{1}{(m_{pr}+j-l-n_2)!} \frac{\partial^{m_{pr}+j-l-n_2}}{\partial \gamma^{m_{pr}+j-l-n_2}} \times (1+\gamma)^{-1} \Big|_{\gamma=-\beta_1 \beta_3}, \quad (40a)$$

$$\lambda_2 = (\beta_1 \beta_3 - 1)^{-(m_{pr}+j-l)}, \quad (40b)$$

$$\lambda_{3_{n_3}} = \frac{1}{(m_{pr}+j+k-l-n_3)!} \frac{\partial^{m_{pr}+j+k-l-n_3}}{\partial \gamma^{m_{pr}+j+k-l-n_3}} \times (1+\gamma)^{-1} \Big|_{\gamma=-\beta_1 \beta_2}, \quad (40c)$$

$$\lambda_4 = (\beta_1 \beta_2 - 1)^{-(m_{pr}+j+k-l)}. \quad (40d)$$

Proof: See Appendix C. ■

IV. NUMERICAL RESULTS AND DISCUSSION

This section presents numerical and Monte Carlo simulation results for the purpose of validation of the derived analytical results and to highlight the characteristics of the system performance under the consideration of interference from the primary transmitter and the CCI.

Fig. 2 shows the impact of the desired channel fading parameter values and the value of interference power constraint on the outage performance. For this figure, the following network parameter values are used: the CCI powers are $\bar{I}_R = 2$ dB, $\bar{I}_D = 3$ dB, the primary interference powers are $\bar{I}_{PR} = 3$ dB, $\bar{I}_{PD} = 4$ dB, and $L_R = 2, L_D = 4, P_s = P_r = 15$ dB, the considered power for the primary transmitter is at the secondary receiver node, bearing in mind that the power of the secondary transmitter attenuates with the distance and it is decreased when it reaches the secondary receiver node. In addition, the interference channel fading severity parameters have the following values; $m_{fsp} = m_{fip} = m_{pr} = m_{pd} = m_{ir} = m_{id} = 2$. Furthermore, in all figures, the following channel variance values are assumed: $\sigma_{h_{sr}}^2 = 0.9, \sigma_{h_{rd}}^2 = 0.6, \sigma_{f_{sp}}^2 = 0.4, \sigma_{f_{rp}}^2 = 0.3$.

As expected, a higher value of the fading channel severity parameter, i.e., the shape parameter, and consideration of the desired channel will result in better performance. This shows that having more channel paths provides more diversity; as a result, the diversity gain of the network increases leading to an enhancement of the system performance. It can also be observed that for the case of the Rayleigh fading channel (i.e., $m_{hsr} = m_{hrd} = 1$), the system has the worst performance when compared to other less severe fading parameter values.

Fig. 2. Outage probability as a function of SNR threshold for different values of the channel fading severity parameter and the interference power constraint.

Fig. 3. Outage probability vs. transmission power constraint for a different number of CCI sources.

Furthermore, as the value of the interference power constraint increases the performance improves accordingly. The simulation and analytical results match, which validates the derived formulas.

Fig. 3 shows the outage performance versus transmission power for the different number of CCI sources. For this illustration, the following network parameter values are used: $\bar{I}_R = 2$ dB, $\bar{I}_D = 3$ dB, the primary interference powers are $\bar{I}_{PR} = 3$ dB, $\bar{I}_{PD} = 4$ dB, $I_{\max} = 20$ dB, and $\gamma_{th} = 2$ dB. In addition, the channel fading severity parameters are as follows: $m_{hsr} = 3$, $m_{hrd} = 4$, $m_{fsp} = 2$, $m_{frp} = 3$, $m_{pr} = 3$, $m_{pd} = 2$, $m_{ir} = 2$, $m_{id} = 3$. In this figure, the outage performance is also plotted for the special cases where no CCI exists (as in [26]) and no CCI and primary transmitter exist (as in [11]). It can be observed how the number of CCI signals and the

Fig. 4. Outage probability vs. transmission power constraint for a linear increase of the CCI power and primary transmission power.

existence of both CCI and primary transmitter interference affect the secondary system performance. Furthermore, asymptotic results are presented to better illustrate the performance boundaries of a UCRN. For instance, in the scenario where $P_s \rightarrow \infty$, i.e., no transmission power constraint on the secondary transmitter nodes, $I_{\max}$ is the dominant parameter to limit the outage performance achievement. Moreover, where $I_{\max} \rightarrow \infty$, the secondary user transmission nodes can take full advantage of their transmission power limits.

In Fig. 4 the impact of the linear increase of interference powers on the outage performance of the CR network is investigated. For this figure, the following network parameter values are used: $m_{hsr} = 2$, $m_{hrd} = 2$, $m_{fsp} = 2$, $m_{frp} = 3$, $m_{pr} = 3$, $m_{pd} = 2$, $m_{ir} = 2$, $m_{id} = 3$, $L_R = 3$, $L_D = 4$, $I_{\max} = 25$ dB, and $\gamma_{th} = 2$ dB. The results in Fig. 4 show how the ratio of the interference powers with respect to the secondary network's power limit range can degrade the outage performance behaviour. In addition, an outage degradation phenomenon, instead of outage floor, can be noticed for the relatively higher secondary transmission power. It is worth mentioning that in the worst scenario of the interference on the secondary network, the performance degrades as the secondary transmission power increases, consequently, the diversity gain will lose its advantage. For example, in the case where the considered CCI power increases linearly by the ratio of 1%, the outage performance stops improving and starts degrading at around 4.4×10^{-2} , which means the secondary network performance cannot further increase even if the transmission power increased.

In Fig. 5, the outage performance of the three hops DF cooperative CR network is investigated using the formula in (30). In addition, the effect of different values of the desired channel fading parameter and the primary interference power value are shown on the outage performance. In this figure, the subscripts 1, 2, 3 represent the parameters that belong to that hop index (i.e., first, second, and third hop, respectively). For

Fig. 5. Three hops OP as a function of transmission power constraint for different values of channel fading severity parameter and primary network interference power.

this figure, the following network parameter values are used: $m_{f_1} = 2, m_{f_2} = 1, m_{f_3} = 2, m_{p_1} = 1, m_{p_2} = 2, m_{p_3} = 2, m_{i_1} = 2, m_{i_2} = 2, m_{i_3} = 1, \sigma_{h_3}^2 = 0.8, \sigma_{f_{sp_3}}^2 = 0.5$. In addition, $\gamma_{th} = 2$ dB, $I_{max} = 25$ dB, $\bar{I}_{R_1} = 2$ dB, $\bar{I}_{R_2} = 1$ dB, $\bar{I}_{R_3} = 5$ dB. Similar to the case in Fig. 2, when the fading severity parameters have higher values, better performance will be expected. On the other hand, a higher interference power from the primary network results in relatively lower performance; these can be clearly observed in the results. Moreover, since a DF protocol has been employed at the relay nodes, the outage performance depends on the weakest hop SINR in the secondary network, this means that for a larger number of hops in the network, better performance is not expected. It is worth mentioning that the analytical results match the simulation results which sustain the correctness of the derived formulas.

Fig. 6 illustrates the average symbol error probability performance for the dual-hop secondary network for different modulation schemes. For this figure, the following network parameter values are used: $m_{hsr} = 4, m_{hrd} = 5, m_{fsp} = 3, m_{fip} = 3, m_{pr} = 2, m_{pd} = 2, m_{ir} = 3, m_{id} = 3, L_R = 2, L_D = 3, I_{max} = 25$ dB, $\bar{I}_R = 2$ dB, $\bar{I}_D = 3$ dB, $\bar{I}_{PR} = 5$ dB, and $\bar{I}_{PD} = 4$ dB. Furthermore, formulas in (34), (35) and (36) have been used to calculate and plot the approximate analytical AEP. From this it can be deduced that, in the scenario of QPSK, the error performance saturates at 1.14×10^{-4} , while for 8-PSK, it saturates at around 5.0×10^{-3} . The main reason behind these saturation phenomena is the interference power constraint that limits the transmission power at the secondary transmitter nodes. Furthermore, it can be observed the derived approximate error probability formula gives more accurate results in the secondary transmitter's power dominant region.

Fig. 7 shows the ergodic capacity performance of the three hops DF UCRN. The analytical plots have been obtained using formulas in (38) and (39). Furthermore, the impact of both CCI power and the interference power constraint are considered. In

Fig. 6. Average error probability for the dual-hop CR network for different modulation schemes.

Fig. 7. Three hops OP as a function of transmission power constraint for different values of channel fading severity parameter and primary network interference power.

this figure, the following network parameter values are used: $m_{h_1} = 4, m_{h_2} = 4, m_{h_3} = 3, m_{f_1} = 1, m_{f_2} = 1, m_{f_3} = 2, m_{p_1} = 2, m_{p_2} = 1, m_{p_3} = 2, m_{i_1} = 2, m_{i_2} = 3, m_{i_3} = 1, \sigma_{h_3}^2 = 0.9, \sigma_{f_{sp_3}}^2 = 0.2$. In addition, $\bar{I}_{P_1} = 5$ dB, $\bar{I}_{P_2} = 6$ dB, $\bar{I}_{P_3} = 6$ dB. It can be observed that the derived asymptotic expression for the capacity performance gives quite accurate results, especially for the scenario where the interference power constraint has a relatively higher value, i.e., 40dB. Furthermore, when $I_{max} = 25$ dB, the capacity saturation is expected. Moreover, the higher the value of the CCI power results in lower achievable capacity. These results show how both CCI power and the I_{max} can degrade the capacity performance of the UCRN. It is worth mentioning that the achieved capacity in a DF multi-hop network is calculated based on the minimum hop's capacity divided by the number of hops.

Fig. 8. Dual-hop equivalent PDF characteristics for different channel fading severity parameters and CCI powers.

Finally, Fig. 8 shows the PDF versus RV x for different CCI powers and desired fading channel parameter values. This figure is for the purpose of investigation of the characteristics of the secondary network's equivalent PDF and to better understand its behaviour. For this figure, the following network parameter values are used: $m_{fsp} = 2, m_{frp} = 3, m_{pr} = 3, m_{pd} = 2, m_{ir} = 2, m_{id} = 3, P_s = P_r = 12$ dB, $I_{\max} = 15$ dB, $\bar{I}_{PR} = \bar{I}_{PD} = 1$ dB, $L_R = L_D = 2$. The figure has been plotted by using (13). From this result, it can be observed that the value of the channel fading parameter and the number of CCI signals largely affect the characteristic of the PDF. For example, a lower value of the desired fading channel parameter m_h deteriorates the PDF behaviour characteristic that results in degrading the performance of the system.

In addition, for the Rayleigh fading channel scenario consideration for the desired channels between the secondary transmitter and receiver nodes, i.e., $mh = 1$, the PDF behaviour has its worst scenario, since most of the area under the curve tends to zero. Furthermore, for the purpose of comparison, the function $Q(\sqrt{x})$ has also been plotted. In all plots, it can be seen that the characteristics of the PDF around the origin are always important. For example, when the value of m_h increases, the PDF around zero also decreases, which means better diversity for the network. Similarly, when the interference signal power is increased from 1 dB to 5 dB, the behaviour of the PDF tends to be closer to the origin, the red line plot in Fig. 8, which means degrading the characteristic of the equivalent SINR which results in poorer performance for the secondary network.

V. CONCLUSION

In this paper, outage probability, average error probability and system ergodic capacity for an underlay cooperative cognitive network over Nakagami- m fading channels were extensively studied. In the analysis, the presence of interference from the primary transmitter and CCI sources were considered. First, exact expressions for the CDF and PDF

of the equivalent SINR were derived. Then, the exact and asymptotic OP for the dual-hop and multi-hop DF cognitive network were investigated. Moreover, the asymptotic AEP was examined.

Numerical examples with Monte Carlo simulations have also been given to support the correctness of the theoretical derivations. Results show that the presence of the primary network and CCI interferences apparently degrade the performance of the secondary network. Specifically, in the case of a linear increase of the interference power, this degradation is more noticeable, such that the system performance is expected to get worse when the transmission power increases.

The analysis in this paper is necessary to better understand the performance behaviour of the underlay CR network when both primary transmitter interference and the CCI are considered over Nakagami- m fading channels. Since Massive MIMO is one of the promising techniques for the next generation of wireless communication networks [27], [28], for the future work, it has been planned to investigate the performance of an underlay cognitive radio network over the Nakagami- m fading channel and considering massive MIMO system.

APPENDIX A PROOF OF COROLLARY 1

Using the formula in (6), the first hop CDF can be expanded and written as:

$$\begin{aligned} F_{\gamma_{SR}}^{\text{eq}}(\gamma) &= \Pr\left(G \leq \frac{\gamma Y}{I_{\max}}, Y > \frac{I_{\max}}{P_s}\right) \\ &\quad + \Pr\left(G \leq \frac{\gamma}{P_s}, Y \leq \frac{I_{\max}}{P_s}\right), \\ &= I_1 + I_2, \end{aligned} \quad (41)$$

where $G = \frac{X}{(I+P+1)}$. By observing the above CDF formula, it is easy to find that the second part can be represented as:

$$I_2 = F_G\left(\frac{\gamma}{P_s}\right) F_Y\left(\frac{I_{\max}}{P_s}\right). \quad (42)$$

In the sections below, the derivation steps of the first part, i.e., I_1 is presented. The derivation is started by finding the PDF of the sum of the two RVs (i.e., $Z = I + P$). This can be found by using the following formula:

$$f_Z(z) = \int_0^z f_I(\gamma_{IR}) f_P(z - \gamma_{IR}) d\gamma_{IR}. \quad (43)$$

With the advantage of the Binomial theorem, (43) can be rewritten as:

$$\begin{aligned} f_Z(z) &= \sum_{l=0}^{m_{pr}-1} \binom{m_{pr}-1}{l} \frac{(-1)^l \left(\frac{m_{ir}}{I_R}\right)^{m_{ir}L_R}}{\Gamma(m_{pr})} \left(\frac{m_{pr}}{\bar{I}_{PR}}\right)^{m_{pr}} \\ &\quad \frac{z^{m_{pr}-1-l} e^{-\frac{m_{pr}}{\bar{I}_{PR}}z}}{\Gamma(m_{ir}L_R)} \int_0^z \gamma_{IR}^{m_{ir}L_R+l-1} e^{-\gamma_{IR}\left(\frac{m_{ir}}{I_R} - \frac{m_{pr}}{\bar{I}_{PR}}\right)} d\gamma_{IR}. \end{aligned} \quad (44)$$

The integral in (44) can be solved with the help of [23, eq. (8.350.1)] and written as in (45):

$$f_Z(z) = \sum_{l=0}^{m_{pr}-1} \binom{m_{pr}-1}{l} (-1)^l \left(\frac{m_{ir}}{\bar{I}_R} \right)^{m_{ir}L_R} \left(\frac{m_{pr}}{\bar{I}_{PR}} \right)^{m_{pr}} \frac{z^{m_{pr}-1-l} e^{-\frac{m_{pr}}{\bar{I}_{PR}} z}}{\Gamma(m_{ir}L_R) \Gamma(m_{pr})} \left(\frac{\bar{I}_{PR} \bar{I}_R}{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}} \right)^{m_{ir}L_R+l} \gamma \left(m_{ir}L_R + l, \left(\frac{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}}{\bar{I}_{PR} \bar{I}_R} \right) z \right). \quad (45)$$

In the next step, the CDF of $G = \frac{X}{Z+1}$ is obtained, which is the CDF of the division of two RVs. This can be obtained by using the following formula:

$$F_G(g) = \int_0^\infty F_X(g(z+1)) f_Z(z) dz. \quad (46)$$

The CDF of RV X can be obtained directly from (7), and written as follows:

$$F_X(x) = 1 - Q \left(m_{hsr}, \frac{m_{hsr}}{\sigma_{hsr}^2 \gamma} x \right), \quad (47)$$

where $Q(\cdot, \cdot)$ is the regularized incomplete gamma function [23, eq. (8.350.2)]. Assuming integer values for the fading parameters and by using [23, eq. (8.352.7)], the upper incomplete gamma function in the CDF formula of the $F_X(x)$ can be represented by a sum of a finite series. Then, substituting both CDF and PDF of X and Z into (46), respectively, the integral can be solved and written as:

$$F_G(g) = 1 - \Upsilon e^{-\left(\frac{m_{hsr}}{\sigma_{hsr}^2} g \right)} \left(\frac{m_{hsr}}{\sigma_{hsr}^2} g \right)^i \times \left[\Gamma(j + m_{pr} - l) \left(\frac{m_{hsr}}{\sigma_{hsr}^2} g + \frac{m_{pr}}{\bar{I}_{PR}} \right)^{l-j-m_{pr}} - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{\left(\frac{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}}{\bar{I}_{PR} \bar{I}_R} \right)^k}{k!} \times \left(\frac{m_{hsr}}{\sigma_{hsr}^2} g + \frac{m_{ir}}{\bar{I}_R} \right)^{l-j-k-m_{pr}} \Gamma(j + k + m_{pr} - l) \right], \quad (48)$$

where Υ is represented by the formula in (12). Finally, I_1 in the first hop equivalent CDF can be obtained by taking the expectation of the $F_G(g)$ with respect to $f_Y(y)$. After substituting the entities and doing some mathematical arrangements, I_1 can be expressed as:

$$I_1 = \frac{1}{\Gamma(m_{fsp})} \Gamma \left(m_{fsp}, \frac{m_{fsp}}{\sigma_{fsp}^2} \frac{I_{\max}}{P_s} \right) - \Upsilon \left(\frac{m_{fsp}}{\sigma_{fsp}^2} \right)^{m_{fsp}} \times \frac{1}{\Gamma(m_{fsp})} \left(\frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2} \right)^i$$

$$\left[\Gamma(j + m_{pr} - l) I_{1a} - \sum_{k=0}^{m_{ir}L_R+l-1} \frac{\left(\frac{\bar{I}_{PR} m_{ir} - \bar{I}_R m_{pr}}{\bar{I}_{PR} \bar{I}_R} \right)^k}{k!} \Gamma(j + k + m_{pr} - l) I_{1b} \right], \quad (49)$$

where I_{1a} and I_{1b} have the following expressions, respectively.

$$I_{1a} = \int_{\frac{I_{\max}}{P_s}}^\infty e^{-y \left(\frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2} + \frac{m_{fsp}}{\sigma_{fsp}^2} \right)} \left(\frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2} y + \frac{m_{pr}}{\bar{I}_{PR}} \right)^{-j-m_{pr}+l} y^{m_{fsp}+i-1} dy, \quad (50)$$

$$I_{1b} = \int_{\frac{I_{\max}}{P_s}}^\infty e^{-y \left(\frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2} + \frac{m_{fsp}}{\sigma_{fsp}^2} \right)} \left(\frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2} y + \frac{m_{ir}}{\bar{I}_R} \right)^{-j-k-m_{pr}+l} y^{m_{fsp}+i-1} dy. \quad (51)$$

For part I_{1a} , the integral in its current form is quite difficult to solve. Therefore, it is necessary to change its representation so that it will be easy to compare it with the available standard mathematical functions. First, the following notations are used to make the integral formulas simpler; $\eta_1 = \frac{m_{pr}}{\bar{I}_{PR}}$, $\eta_2 = \frac{m_{hsr} \gamma}{I_{\max} \sigma_{hsr}^2}$, and $\eta_3 = \eta_2 + \frac{m_{fsp}}{\sigma_{fsp}^2}$. Then, let $t = \eta_2 y + \eta_1$, therefore, I_{1a} can be represented as:

$$I_{1a} = \left(\frac{1}{\eta_2} \right)^{m_{fsp}+i} e^{\eta_1 \left(\frac{\eta_3}{\eta_2} \right)} \int_{\eta_2 \frac{I_{\max}}{P_s} + \eta_1}^\infty e^{-t \left(\frac{\eta_3}{\eta_2} \right)} (t)^{-j-m_{pr}+l} (t - \eta_1)^{m_{fsp}+i-1} dt. \quad (52)$$

Using binomial expansion, the above formula can be written as:

$$I_{1a} = \frac{1}{\eta_2^{m_{fsp}+i}} \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} (-\eta_1)^{m_{fsp}+i-1-r_1} e^{\eta_1 \left(\frac{\eta_3}{\eta_2} \right)} \int_{\eta_2 \frac{I_{\max}}{P_s} + \eta_1}^\infty e^{-t \left(\frac{\eta_3}{\eta_2} \right)} (t)^{-j-m_{pr}+l+r_1} dt. \quad (53)$$

The next step is to change the variable of the above integral so that $x = t \left(\frac{\eta_3}{\eta_2} \right)$. After this change of variable, the integral can be written as:

$$I_{1a} = \left(\frac{1}{\eta_2} \right)^{m_{fsp}+i} \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} e^{\eta_1 \left(\frac{\eta_3}{\eta_2} \right)} \frac{1}{(-\eta_1)^{r_1+1-m_{fsp}-i}} \left(\frac{\eta_2}{\eta_3} \right)^{-j-(m_{pr}-1)+r_1} \int_{\left(\eta_2 \frac{I_{\max}}{P_s} + \eta_1 \right) \left(\frac{\eta_3}{\eta_2} \right)}^\infty e^{-x} (x)^{l+r_1-j-m_{pr}} dx. \quad (54)$$

Now, by comparing the above integral representation with [23, eq. (8.350.2)], it is easy to represent I_{1a} in terms of the standard available function, which the upper incomplete gamma function.

$$I_{1a} = \left(\frac{1}{\eta_2}\right)^{m_{fsp}+i} \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} e^{\eta_1 \left(\frac{\eta_3}{\eta_2}\right)} \frac{1}{(-\eta_1)^{r_1+1-m_{fsp}-i}} \left(\frac{\eta_2}{\eta_3}\right)^{r_1-j-(m_{pr}-1-l)} \Gamma\left(l+r_1-j-m_{pr}+1, \left(\eta_2 \frac{I_{\max}}{P_s} + \eta_1\right) \left(\frac{\eta_3}{\eta_2}\right)\right). \quad (55)$$

Similar steps can be repeated to solve part I_{1b} . Finally, both I_{1a} and I_{1b} can be written in a more convenient and compact form as in (56) and (57), respectively.

$$I_{1a} = \sum_{r_1=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_1} \frac{e^{\frac{\beta_7}{\gamma} + \beta_3}}{(-\beta_3)^{r_1+1-m_{fsp}-i}} \left(\frac{\gamma}{\gamma + \beta_6}\right)^{r_1-j-m_{pr}+1+l} \times \Gamma\left(l+r_1-j-m_{pr}+1, \frac{\gamma}{\beta_1} + \frac{\beta_7}{\gamma} + \beta_5 + \beta_3\right), \quad (56)$$

$$I_{1b} = \sum_{r_2=0}^{m_{fsp}+i-1} \binom{m_{fsp}+i-1}{r_2} \frac{e^{\frac{\beta_8}{\gamma} + \beta_2}}{(-\beta_2)^{r_2+1-m_{fsp}-i}} \left(\frac{\gamma}{\gamma + \beta_6}\right)^{r_2-j-k-m_{pr}+1+l} \times \Gamma\left(l+r_2-j-k-m_{pr}+1, \frac{\gamma}{\beta_1} + \frac{\beta_8}{\gamma} + \beta_5 + \beta_2\right), \quad (57)$$

where $\beta_1 = \frac{P_s \sigma_{hsr}^2}{m_{hsr}}$, $\beta_2 = \frac{m_{ir}}{I_R}$, $\beta_3 = \frac{m_{pr}}{I_{PR}}$, $\beta_5 = \frac{m_{fsp} I_{\max}}{P_s \sigma_{fsp}^2}$, $\beta_7 = \frac{m_{pr} m_{fsp} \sigma_{hsr}^2 I_{\max}}{m_{hsr} \sigma_{fsp}^2 I_{PR}}$, and $\beta_8 = \frac{m_{ir} m_{fsp} \sigma_{hsr}^2 I_{\max}}{m_{hsr} \sigma_{fsp}^2 I_R}$. Then, I_1 can be obtained by substituting (56) and (57) in (49). Finally, the exact equivalent CDF of the first hop can be obtained by adding both derived parts, i.e., I_1 and I_2 , and it can be represented as in (11).

APPENDIX B PROOF OF COROLLARY 2

After substituting the approximate CDF formula derived in (23) into (31), three integral terms appear. The first part can be directly evaluated as:

$$\bar{P}_b^{SR_1} = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} d\gamma = \frac{a}{2}. \quad (58)$$

The second and third integral parts are represented as follows:

$$\bar{P}_b^{SR_2} \triangleq \Omega_1 = \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} (\gamma)^i e^{-\frac{\gamma}{\beta_1}} (\gamma + \beta_1 \beta_3)^{l-j-m_{pr}} d\gamma, \quad (59)$$

$$\bar{P}_b^{SR_3} \triangleq \Omega_2 = \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} (\gamma)^i e^{-\frac{\gamma}{\beta_1}} (\gamma + \beta_1 \beta_2)^{l-j-k-m_{pr}} d\gamma. \quad (60)$$

It can be observed that the second and third part, i.e., $\bar{P}_b^{SR_2}$ and $\bar{P}_b^{SR_3}$, are identical in terms of the integral evaluation structure. Therefore, only the second part integral, i.e., Ω_1 , is derived in the sections below. For part Ω_1 , first, the variable of the integral is exchanged so that $t = \frac{\gamma}{\beta_1 \beta_3}$. The resulting integral has the following expression:

$$\Omega_1 = (\beta_1 \beta_3)^{l-j-m_{pr}+i+\frac{1}{2}} \int_0^\infty e^{-t \beta_1 \beta_3 (b + \frac{1}{\beta_1})} (t)^{i-\frac{1}{2}} (t+1)^{l-j-m_{pr}} dt. \quad (61)$$

Now, by comparing the above formula with [29, eq. (13.2.5)], it is easy to come up with the formula in (35).

APPENDIX C PROOF OF COROLLARY 3

After substituting the formula of the asymptotic first hop equivalent CDF in (23) into (37), the following expression is obtained:

$$\begin{aligned} C_{erg}^{SR} &\approx \Upsilon(\beta_1)^{m_{pr}+j-l-i} \left[\Gamma(j+m_{pr}-l) \int_0^\infty \frac{(\gamma)^i e^{-\frac{\gamma}{\beta_1}}}{(1+\gamma)(\gamma + \beta_1 \beta_3)^{m_{pr}+j-l}} d\gamma \right. \\ &\quad - \sum_{k=0}^{m_{ir} L_R + l - 1} \frac{(\beta_1 \beta_4)^k}{k!} \Gamma(j+k+m_{pr}-l) \\ &\quad \left. \int_0^\infty \frac{(\gamma)^i e^{-\frac{\gamma}{\beta_1}}}{(1+\gamma)(\gamma + \beta_1 \beta_2)^{m_{pr}+j+k-l}} d\gamma \right]. \quad (62) \end{aligned}$$

In the following section, the solution of integral of I_{erg}^1 , i.e., first part, is presented. The second integral has similar steps, therefore, it has been omitted for saving space. Using partial fraction decomposition, the formula of I_{erg}^1 can be written in simpler form as:

$$I_{erg}^1 = \int_0^\infty (\gamma)^i e^{-\frac{\gamma}{\beta_1}} \left[\sum_{n_2=1}^{m_{pr}+j-l} \frac{\lambda_{1n_2}}{(\gamma + \beta_1 \beta_3)^{n_2}} + \frac{\lambda_2}{(1+\gamma)} \right] d\gamma, \quad (63)$$

where λ_{1n_2} and λ_2 are obtained by formulas represented in (40a), and (40b), respectively. Therefore, the expression for I_{erg}^1 can be represented within two integral terms, named as

I_{erg}^{1a} and I_{erg}^{1b} , as follows:

$$I_{erg}^{1a} = \sum_{n_2=1}^{m_{pr}+j-l} \lambda_{1n_2} \int_0^{\infty} \frac{(\gamma)^i e^{-\frac{\gamma}{\beta_1}}}{(\gamma + \beta_1\beta_3)^{n_2}} d\gamma, \quad (64)$$

$$I_{erg}^{1b} = \lambda_2 \int_0^{\infty} \frac{(\gamma)^i e^{-\frac{\gamma}{\beta_1}}}{(1+\gamma)} d\gamma. \quad (65)$$

For part I_{erg}^{1a} , first the variable in the integral formula is exchanged such that $x = \gamma + \beta_1\beta_3$. In addition, by exploiting the Binomial expansion, the formula can be written as:

$$I_{erg}^{1a} = \sum_{n_2=1}^{m_{pr}+j-l} \sum_{n_1=0}^i \binom{i}{n_1} \lambda_{1n_2} (-\beta_1\beta_3)^{i-n_1} e^{\beta_3} \int_{\beta_1\beta_3}^{\infty} \frac{e^{-\frac{x}{\beta_1}}}{(x)^{n_2-n_1}} dx. \quad (66)$$

Then, in order to obtain a closed-form expression for the above integral formula, the variable is changed as $y = \frac{x}{\beta_1}$. Finally, by comparing the resulting integral formula with [30, eq. (8.19.2)], the desired expression can be obtained and written as:

$$I_{erg}^{1a} = \sum_{n_2=1}^{m_{pr}+j-l} \sum_{n_1=0}^i \binom{i}{n_1} \lambda_{1n_2} (-\beta_1\beta_3)^{i-n_1} (\beta_1)^{n_1+1-n_2} e^{\beta_3} (\beta_3)^{1-n_2+n_1} E_{n_2-n_1}(\beta_3), \quad (67)$$

where $E_p(z)$ is the exponential integral function defined by $E_p(z) = z^{p-1} \int_z^{\infty} \frac{e^{-t}}{t^p} dt$, [30, eq. (8.19.2)].

Similarly, part I_{erg}^{1b} can be expressed as:

$$I_{erg}^{1b} = \sum_{n_1=0}^i \binom{i}{n_1} \lambda_2 (-1)^{i-n_1} e^{\frac{1}{\beta_1}} E_{1-n_1}\left(\frac{1}{\beta_1}\right). \quad (68)$$

Similar steps can be repeated to solve the integral of part I_{erg}^{2a} . Therefore, the resulting expressions for this part can be written as:

$$I_{erg}^{2a} = \sum_{n_3=1}^{m_{pr}+j+k-l} \sum_{n_1=0}^i \binom{i}{n_1} \lambda_{3n_3} (-\beta_1\beta_2)^{i-n_1} (\beta_1)^{n_1+1-n_3} (\beta_2)^{1-n_3+n_1} e^{\beta_2} E_{n_3-n_1}(\beta_2), \quad (69)$$

$$I_{erg}^{2b} = \sum_{n_1=0}^i \binom{i}{n_1} \lambda_4 (-1)^{i-n_1} e^{\frac{1}{\beta_1}} E_{1-n_1}\left(\frac{1}{\beta_1}\right), \quad (70)$$

where λ_{1n_3} and λ_4 are obtained by formulas represented in (40c), and (40d), respectively. Finally, the first hop closed-form expression for the ergodic capacity formula can be obtained by combining the derived parts of I_{erg}^{1a} and I_{erg}^{1b} and putting them into (62), and after some mathematical arrangements, it can be written as in (39).

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Jamal Ahmed Hussein received the B.Sc. degree (First Class Hons.) in electrical engineering from the Electrical Department, College of Engineering, University of Salahaddin, Erbil, Iraq, in 2001, the M.Sc. degree in electrical engineering from the Electrical Department, College of Engineering, University of Sulaimani, Sulaimani, Iraq, in 2007, and the Ph.D. degree in electrical and electronic engineering from Newcastle University, Newcastle Upon Tyne, U.K., in 2017.

He is currently a Senior Communication Engineer with the Directorate of Transportation and Communications, Ministry of Transportation and Communications, Kurdistan Regional Government, Sulaimaniyah, Iraq. He works as a Researcher with the Information Technology Department, College of Science and Technology, University of Human Development, Sulaimaniyah. His research interests include wireless communication, cooperative communication, cognitive radio, NOMA, 5G, and energy harvesting. He also acts as a reviewer for several IEEE journals and the IET Communications.

Said Boussakta (S'89–M'90–SM'04) received the "Ingenieur d'Etat" degree in electronic engineering from the National Polytechnic Institute of Algiers, Algeria, in 1985 and the Ph.D. degree in electrical engineering from Newcastle University, U.K., in 1990.

From 1990 to 1996, he was with Newcastle University as a Senior Research Associate in digital signal processing. From 1996 to 2000, he was with the University of Teesside, U.K., as a Senior Lecturer in communication engineering. From 2000 to 2006, he was with the University of Leeds as a Reader in digital communications and signal processing. He is currently a Professor of communications and signal processing with the School of Engineering, Newcastle University, where he is lecturing in Communication Networks and Signal Processing subjects. He has authored and co-authored over 200 publications. His research interests are in the areas of fast DSP algorithms, digital communications, communication network systems, cryptography, and digital signal/image processing. He has served as the Chair for Signal Processing for Communications Symposium in ICC06, ICC07, ICC08, ICC2010, and ICC2013. He is a fellow of the IET and a Senior Member of the IEEE Communications and Signal Processing Societies.

Salama S. Ikki received the B.S. degree from Al-Isra University, Amman, Jordan, in 1996, the M.Sc. degree from the Arab Academy for Science and Technology and Maritime Transport, Alexandria, Egypt, in 2002, and the Ph.D. degree from the Memorial University of Newfoundland, St. John's, in 2009, all in electrical engineering. He was a Research Assistant with the INRS, University of Quebec, Montreal, from 2010 to 2012 and a Post-Doctoral Fellow with the University of Waterloo, Waterloo, ON, Canada, from 2009 to 2010. He

is currently an Associate Professor in wireless communications with the Department of Electrical Engineering, Lakehead University.

He has been carrying out research in communications and signal processing for over ten years. He is widely recognized as an Expert in the field of wireless communications. He has authored or co-authored over 100 papers in peer-reviewed IEEE international journals and conferences with over 3000 citations and with an H-index of 30. He was a recipient of the Best Paper Award published in the *EURASIP Journal on Advances in Signal Processing*, the IEEE COMMUNICATION LETTERS and the IEEE WIRELESS COMMUNICATION LETTERS Exemplary Reviewer Certificate in 2012, and the Top Reviewer Certificate from the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY in 2015. His Ph.D. student received the second place for Best Poster from the School of Electrical and Electronic Engineering, Newcastle University, U.K., Annual Research Conference, in 2014. He serves on the Editorial Board of the IEEE COMMUNICATION LETTERS and the IET Communications Proceeding.