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Multi-mode flow-induced vibrations of two side-by-side slender flexible cylinders in a uniform flow

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Abstract: Flow-induced vibration (FIV) of multiple marine risers frequently occurs in deepwater applications and might result in serious structural failure due to fatigue damage accumulation. It is known that long marine risers may experience high modes of vibration and behave multi-mode vibration features. Moreover, the interactions of multiple risers subject to FIV are very complex and still unclear. In this paper, a series of experimental tests were carried out to investigate FIV of two side-by-side flexible cylinders with high aspect ratio (length to diameter, $L/D=350$) in a towing tank. Four cases of different spacing ratios (center-to-center separation distances to cylinder diameter, $S/D = 3.0, 4.0, 6.0$ and 8.0) were adopted to examine the effect of spacing on the multi-mode FIV of the two flexible cylinders. The maximum dominant modes are 4th and 6th in cross-flow (CF) and in-line (IL) directions for both side-by-side cylinders, as well as the single one. In the switching region of the adjacent modes of vibration, higher-order mode vibrations are less difficult to excite for side-by-side cylinders. The IL displacement amplitudes of the two cylinders could be enhanced by the remarkably strong interaction between cylinders, even with a center-to-center distance of up to 8.0 cylinder diameter. In addition, the IL FIV behaviors are much more complicated than those in CF direction, for instance the response spectra in IL direction exhibit several large peaks and lots of small spikes around. The IL and CF interactions of the two side-by-side flexible cylinders were also investigated by using the response trajectories collected from seven measurement points at different reduced velocities.

Keywords: Marine riser; Flow-induced vibration; Side-by-side arrangement; flexible cylinder; Spacing ratio;

1. Introduction

As a key equipment for extracting oil and gas from oilfields located at deep sea, marine riser system is widely used to transport oil and gas drilled from underneath ocean floor to the production

platforms in the oil industry. This system generally consists of a group of slender, flexible cylinder structures connecting floaters on the surface and wellheads at the seabed. It is well recognized that flow-induced vibration (FIV) is an important source of fatigue damage for marine risers. Compared with the case of a single cylinder, the oscillation characteristic and wake flow of multiple cylinders are much more complicated and haven't been fully understood due to the interactions of a bundle of cylinders [1-5]. In order to further comprehend the FIV of multiple risers, some research work has been reported over the past decade [6-9].

Two cylinders in a side-by-side arrangement can be viewed as one of the simplest cases of multiple cylinders. For a better understanding of the interference effects, experimental investigations of the flow around two side-by-side fixed cylinders have been performed by a large number of researchers. It was reported that there are three flow regimes for such an arrangement based on the spacing ratio, S/D (S is the center-to-center separation distance and D is the cylinder diameter). At very small spacing ratios ($S/D < 1.1-1.2$), one single vortex street is formed and no vortex is generated in the gap between cylinders [10]; at intermediate cylinder spacing ratios ($1.1-1.2 \leq S/D < 2.2-2.7$), a narrow wake behind one cylinder as well as a wide wake behind the other is observed and a bistable biased gap flow emerges [11]; at large spacing ratios ($S/D \geq 2.2-2.7$), a coupled wake street appears [12-13]. When the center-to-center separation distance is even greater than 4.0-5.0 cylinder diameters, there is no obvious interaction between the two wakes of the cylinders [14].

When the two side-by-side cylinders are free to oscillate in cross-flow (CF) direction, the wake flow may be different from those of fixed cylinders. The response behaviors, flow patterns and fluid forces are strongly dependent upon the arrangement of structures and the distance between them. FIV of two elastically mounted side-by-side cylinders has attracted much interest in recent years. Wang et al. [15] experimentally studied CF vortex-induced vibration (VIV) of two side-by-side elastically mounted cylinders in a closed loop wind tunnel at $Re=5000-41000$ with the spacing ratio (S/D) varying from 1.17 to 1.90. It was found that VIV was suppressed due to wake interference when $S/D < 1.50$. At $S/D=1.70$, the response amplitudes of cylinders were larger than that of a single cylinder in the high reduced velocity range. The VIV behavior was similar to that of a single cylinder with spacing ratio increasing further. Cui et al. [16] numerically investigated CF VIV of two elastically coupled side-by-side circular cylinders by the finite element method (FEM) at $Re=5000$ with $S/D=3.0$. The simulations

of the two cases corresponding to the symmetric and asymmetric configurations were performed for a wide range of reduced velocities covering the whole lock-in regime. It was reported that five response regimes i.e., the first-mode lock-in regime, the second-mode lock-in regime, the sum-frequency lock-in regime and two transition regimes, were found in the numerical simulation cases. [Chen et al. \[17\]](#) numerically studied the FIV of two elastically mounted side-by-side circular cylinders in a uniform flow with $S/D=2.0-5.0$ and $Re=100$ by the immersed boundary method (IBM). It was found that the interaction between the two cylinders could be ignored when $S/D \geq 4.0$. Also six distinct wake flow patterns were observed in the parametric plane of reduced velocity and spacing ratio. In addition, they employed a similar numerical simulation method to further investigate the asymmetric vibration and symmetric hysteresis phenomena by focusing on the near-wake patterns and the interaction between cylinders [\[18\]](#). [Kim & Alam \[19\]](#) carried out free vibration tests in a closed-circuit rectangular wind tunnel, to examine the FIV characteristics at $Re=4365-74200$ for two identical circular cylinders that were elastically mounted in a side-by-side arrangement. The spacing ratio (S/D), which can cover all possible flow regimes, was in the range of 0.1-3.2. Four characteristic vibration patterns were identified at $0.1 \leq S/D < 0.2$, $0.2 \leq S/D < 0.9$, $0.9 \leq S/D < 2.1$ and $2.1 \leq S/D \leq 3.2$, the vibration behaviors in these four regimes were described in detail.

Because the initial spacing between two side-by-side flexible cylinders during oscillation may change with time, the FIV characteristics of flexible cylinders are much more complicated than those of elastically mounted ones. Some attention has been paid to FIV of two flexible cylinders by several researchers. [Zhou et al. \[20\]](#) carried out experiments on the interference behavior of two fixed-fixed flexible cylinders in a cross flow within a suction-type wind tunnel at $Re=800-10000$. Three spacing ratio cases, $S/D=1.13, 1.70$ and 3.0 were tested. The experimental results indicated the occurrence of first-, second- and third-mode resonance. In addition, the third-mode resonance was far more violent because of the combined effect of higher flow energy and smaller effective damping ratio. These findings proved that structural flexibility plays a significant role in the dynamic analysis for two side-by-side cylinders, which has so far been ignored in most previous studies. [So & Wang \[21\]](#) numerically studied VIV of two flexible beams in a side-by-side arrangement with the same three spacing ratios as by [Zhou et al. \[20\]](#) at $Re=800$. It was reported that in-phase and out-phase relations between fluid forces and their corresponding response amplitudes were observed. Recently, [Huera-Huarte & Gharib](#)

[7] conducted some excellent experimental studies on FIV of two side-by-side flexible circular cylinders with $S/D=2.0, 2.5, 3.0, 4.0$ and 5.0 in a free-surface water channel. The two cylinder models, partly emerged in the flow to simulate the stepped flow, with the same aspect ratio (cylinder length to cylinder diameter, L/D) of 93.75 and different mass ratios (total structural mass to displaced fluid mass, $m^* = 4m_s/\rho\pi D^2$, where m_s is the mass of cylinder per unit length and ρ is the fluid density) of 1.80 and 1.45 , were allowed to oscillate in CF and IL directions. It was found the interaction between the two flexible cylinders through the wake was very weak when $S/D \geq 3.5$ and strong proximity interference existed when $S/D < 3.5$. More recently, Sanaati & Kato [8] experimentally investigated the effects of spacing on FIV behaviors of two side-by-side identical flexible slender circular cylinders at $Re=1400$ - 20000 in a towing tank. The cylinder models were mounted at two spacing ratios, i.e. $S/D=2.75$ and 5.5 , and had an aspect of 162.0 and a mass ratio of 1.17 . It was observed that out-of-phase synchronization existed during high amplitude vibration for $S/D=5.5$. This finding is conflicted with that of Huera-Huarte & Gharib [7].

Due to designing and installing risers in deep water, the dynamics of marine risers subject to ocean currents become more and more complex in the offshore oil industry. It is well known that long marine risers may experience high modes of vibration and behave multi-mode vibration features. While a few studies have been published concerning FIV of two flexible cylinders in a side-by-side arrangement, little information is available on the multi-mode and multi-frequency response behaviors of two side-by-side flexible cylinders with a high aspect ratio in CF and IL directions. The main purpose of this paper is to conduct an experimental investigation on FIV of two identical flexible cylinders that are side-by-side arranged in a towing tank. Four spacing ratios ($S/D=3.0, 4.0, 6.0$ and 8.0) were selected in the experiments to study the effects of center-to-center separation distance on multi-mode responses of two flexible cylinders undergoing vortex shedding.

This paper is organized into the following sections. The description of the experimental set-up is presented in detail in Section 2. The approach used for analyzing the test data is introduced in Section 3. The experimental results, including time history and amplitude spectra of displacement, response amplitude, dominant frequencies and modes, multi-mode response feature and x - y trajectories at different measurement points, are shown and discussed in Section 4. Some conclusions are drawn based on present experimental results in final Section.

2. Experimental set-up

As shown in Fig.1, the two cylinders in a side-by-side arrangement were free to vibrate in both CF and IL directions. The upper and lower cylinders were labeled “cylinder #1” and “cylinder #2”, respectively. Four center-to-center separation distances, i.e. $S/D = 3.0, 4.0, 6.0$ and 8.0 , were adopted

Fig.1 Two side-by-side cylinders in steady uniform flow (cross-sectional view)

for our tests. The spacing ratios chosen herein are in the ranges with and without proximity interference as suggested by Zdravkovich [2]. Based on the previous study by Sanaati & Kato [8], the wakes of two flexible cylinders were strongly synchronized in the region of large vibration amplitudes when $S/D = 5.5$. In order to investigate if this FIV behavior exists or not with increasing spacing, a much larger spacing ratio of $S/D=8.0$ was selected for our experiments. In addition, a set of tests for $S/D=2.0$ were also conducted, but a collision between the two cylinder models was observed for some cases, hence the experimental results were not presented in this paper.

The experimental investigations were carried out in a towing water tank with dimensions of $137.0\text{m} \times 7.0\text{m} \times 3.3\text{m}$. The experimental apparatus was mainly composed of a supporting frame, vertical supporting rods, supporting plates, guide plates, cylinder models and pre-tension adjustment and measurement systems (see Fig. 2). The two cylinder models were mounted horizontally and towed by a carriage in one direction along the tank to generate a uniform fluid flow. The towing velocity ranged from 0.05m/s to 1.0m/s with an interval of 0.05m/s . The corresponding Reynolds number was approximately in the range of 800 - 16000 . The guide plate was mounted on the supporting plate by several long bolts to avoid the disturbing flow generated by the supporting plates and supporting rods and keep the incoming flow two-dimensional. The universal joints were used to connect the ends of

cylinder models with supporting plates. More details can be found in Fig.3.

Fig.2 Schematic diagram of the experimental set-up

The two cylinder models were the same size, property and axial pre-tension. Main properties of the cylinders are listed in Table 1. The flexible cylinder with a length of 5.60 m was made up of an internal copper pipe and an outer silicone tube. The inner copper pipe with a wall thickness of 1.0 mm and an outer diameter of 8.0 mm was covered by a silicone tube with an external diameter of 16mm to provide a smooth external surface. The mass per unit length of the cylinder model was 0.3821 kg/m. The mass ratio and aspect ratio were 1.90 and 350, respectively.

Table 1. Main parameters of the cylinder model

Total length, L	5.60 m
Outer diameter, D	0.016 m
Aspect ratio, L/D	350
Bending stiffness, EI	17.45 Nm ²
Axial stiffness, EA	2.793×10^6 N
Mass per unit length, m_s	0.3821 kg/m
Mass ratio, m^*	1.90
Axial tension, T	450 N

The upper cylinder was mounted 1.0 m below the still water level, which was deep enough to eliminate the free-surface effect. The initial axial pretension of the cylinder was equal to 450N and adjusted by a tensioner. The variation of axial tension was monitored using a load cell. A total of

fourteen pairs of strain gages were installed along the internal copper pipe of the flexible cylinder model at seven positions that were uniformly distributed to directly measure the FIV responses in

Fig.3 Guide plate and flexible cylinder models

both CF and IL directions. G1-G7 were employed to represent the locations of the gage pairs. Fig.4 shows a sketch of strain gages on the cross section and their arrangement along the cylinder axis. The sampling duration of each test run was 50s, counting after the carriage reached a stable towing velocity. The waiting time between two consecutive runs was more than 15minutes so that the disturbed water could calm down. The sampling frequency of the measuring instrument was 100 Hz which is sufficient to avoid aliasing problems. The experimental tests in our work consisted of more than 100 runs, including the cases of single cylinder and two side-by-side cylinders with four spacing ratios.

Fig.4. A sketch of strain gages on the cross section and arrangement along the cylinder axis

3. Data analysis

For obtaining the displacement responses of the two flexible cylinders, a modal analysis approach was employed to estimate the displacements in CF and IL directions by using measured strain signals at measurement positions. Similar modal analysis approach has been used successfully in previous experiments on VIV of flexible cylinders [22-25].

The dynamical behavior of flexible cylinder could be assumed to be approximately linear, thus the time-varying shape of the cylinder can be composed as a series of mode-shapes. Herein, only the CF displacement reconstruction was introduced as an example. The response displacement in CF direction can be expressed as

$$y(z, t) = \sum_{n=1}^{\infty} \varphi_n(z) w_n(t) \quad (1)$$

where $y(z, t)$ is the CF displacement, z is the coordinate along the axis of the cylinder, t is time, $\varphi_n(z)$ is the mode-shape, $w_n(t)$ is the modal weight and n is the order of vibration mode.

The cylinder with a pretension of 450N in our tests can be simplified as a Bernoulli-Euler beam with the pinned-pinned boundary condition, thus the mode-shape of displacement can be written as

$$\varphi_n(z) = \sin \frac{\pi n z}{L} \quad (2)$$

The displacement response was reconstructed based on the relationship between curvature and strain,

$$\frac{\varepsilon(z, t)}{R} = - \sum_{n=1}^{\infty} \left(\frac{\pi n}{L} \right)^2 \sin \frac{\pi n z}{L} w_n(t) \quad (3)$$

where R is the outer radius of the copper pipe. In Eq. (3), only modal weight $w_n(t)$ is unknown. It can be calculated using the processed strain signals at measurement points. Note that the strain signals cannot be used directly in Eq. (3), the unneeded frequency components should be removed by applying the filtering in the frequency domain. In our experiments, the strain signal was high-pass filtered with a cutoff of 1.0 Hz to eliminate the low-frequency effect caused by carriage motion, and it was also low-pass filtered with a cutoff of 40.0 Hz to remove other inevitable high-frequency noise caused by strain gages.

In order to illustrate how to use the modal analysis approach to process the measured data in our experiments, a typical example of CF modal weights and corresponding amplitude spectra of cylinder #1 with spacing ratio $S/D=3.0$ and towing velocity $U=0.7$ m/s is shown in Fig.5. It can be clearly seen that the 1st, 2nd and 3rd modes are excited, and the amplitudes of modal weights from mode 4 to mode 7 have very small values. It indicates that the oscillation in CF direction is mainly dominated by the 3rd mode in this case. In addition, it can be found that the three excited modes, i.e. mode 1-3, oscillate at the same dominant frequency. According to the results of modal weight presented in Fig.5, the

displacement response could be reconstructed by using Eq. (1).

Fig.5 An example of dimensionless CF modal weights and corresponding amplitude spectra of cylinder #1 with $S/D=3.0$ and $U=0.7$ m/s.

4. Results and discussion

In this section, the experimental results, including dominant frequencies, dominant modes, displacement amplitudes, space-time varying displacement contour, time histories of displacement at measurement points and x - y trajectories along the flexible cylinders are presented and discussed to investigate the FIV behavior of the two side-by-side cylinders undergoing vortex shedding.

Herein, the reduced velocity V_r is defined as

$$V_r = \frac{U}{f_1 D} \quad (4)$$

where f_1 is the fundamental frequency (1st mode frequency) of the cylinder in still water, calculated from

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{m} + \left(\frac{n\pi}{L}\right)^2 \frac{EI}{m}} \quad (5)$$

where n is the order of the mode, f_n is the n th-order eigenfrequency of the cylinder, and m is the total mass per unit length, including structural mass m_s and added mass $m_a (= \rho \pi D^2 / 4)$.

[Fig.6](#) and [Fig.7](#) show the dimensionless dominant frequencies, f_y/f_1 and f_x/f_1 , in both CF and IL directions with respect to reduced velocity. Note that f_1 is the theoretical value of the first natural frequency calculated by Eq. (5). The dominant frequencies were obtained by means of a fast Fourier transform (FFT) of the time-varying displacement (in the time windows chosen for each run). Two additional lines were drawn in these figures, the diagonal dash line represents the Strouhal frequency corresponding to the vortex-shedding frequency for the stationary cylinder. The horizontal solid lines represent the natural frequency for the cases $f_y=f_1$ in [Fig.6](#) and $f_x=2f_1$ in [Fig.7](#). It indicates that the variation of dominant frequencies increases linearly with the increasing reduced velocity in both CF and IL directions. The ratios of IL to CF dominant frequencies of the two side-by-side cylinders both roughly equal to a constant number of two. A similar trend has been observed in this and previous experimental studies on the VIV of a single cylinder [[24](#), [26](#), [27](#)]. As shown in [Fig.6](#) and [Fig.7](#), dominant frequencies vary unapparently with increasing spacing ratio in both CF and IL directions. In addition, it can be clearly seen that the dominant frequencies of the two side-by-side cylinders are approximately same as that of the single cylinder when reduced velocity is larger than 3.76. This means that the influence of center-to-center separation distance on dominant frequencies in CF and IL directions is insignificant in the lock-in regime for the spacing ratios tested in our experiment.

Fig.6 Dimensionless dominant frequencies in CF direction versus reduced velocity

Fig.7 Dimensionless dominant frequencies in IL direction versus reduced velocity.

Fig.8 and Fig.9 show the maximum dimensionless RMS (Root Mean Square) displacement amplitudes in CF and IL directions for the two side-by-side flexible cylinders with four center-to-center separation distance. The experimental results of the single cylinder are also presented in the figures for the sake of comparison. It should be pointed out that the effect of mean IL deflection was removed from Fig.9.

Table 3. Dominant modes in IL direction

Towing velocity $U(\text{m/s})$	Reduced velocity V_r	Single cylinder	Cylinder #1				Cylinder #2			
			$S/D=3.0$	$S/D=4.0$	$S/D=6.0$	$S/D=8.0$	$S/D=3.0$	$S/D=4.0$	$S/D=6.0$	$S/D=8.0$
0.05	1.25	1	2	1	1	1	1	1	1	1
0.10	2.51	1	2	1	2	1	1	1	2	1
0.15	3.76	2	2	2	2	2	2	2	2	2
0.20	5.01	3	2	3	3	2	2	2	2	3
0.25	6.26	2	2	2	2	2	2	2	2	2
0.30	7.52	2	2	2	2	4	4	4	4	2
0.35	8.77	4	4	4	4	3	3	3	2	4
0.40	10.02	3	3	3	3	3	3	3	3	3
0.45	11.27	3	3	3	3	3	3	3	3	3
0.50	12.52	3	3	3	3	3	4	3	3	3
0.55	13.78	3	4	4	3	3	4	4	4	4
0.60	15.03	3	4	4	4	4	4	4	4	4
0.65	16.28	4	4	4	4	4	4	4	4	4
0.70	17.54	4	5	4	4	4	5	5	5	4
0.75	18.79	4	5	4	5	4	5	5	5	5
0.80	20.04	5	5	5	5	5	5	5	5	5
0.85	21.29	5	5	5	5	5	5	5	5	5
0.90	22.55	5	5	5	5	5	6	6	6	5
0.95	23.80	5	5	6	6	5	6	6	6	6
1.0	25.05	6	6	6	6	6	6	6	6	6

At first sight, the data appear to be quite widely scattered in Fig.8. It can be found both cylinder #1 and cylinder #2, as well as the single cylinder exhibit typical upper branch within the first lock-in region in CF direction, even with a much smaller spacing ratio $S/D=3.0$. This finding is different from the experimental results by Sanaati & Kato [8] of FIV of two side-by-side flexible cylinders with $S/D=2.75$. In their experimental investigation, the upper cylinder showed no obvious upper branch and the lower one had an upper branch response as the first lock-in occurred. This difference might be attributed to the gap flow biased towards the upper cylinder when the gap between two cylinders

decreased because of the anti-phase vibration of cylinders. The peak values of CF displacement amplitudes are up to $1.79D$ for cylinder #1 and $1.65D$ for cylinder #2. It indicates that the CF maximum attained amplitude of cylinder #1 is slightly larger than that of cylinder #2 in our research work. It can also be observed that the CF displacement amplitudes of both cylinders reach their maxima at the same reduced velocity ($V_r = 12.53$). This behavior is in contrast to the results from Huera-Huarte & Gharib [7]. In their experiment, the two side-by-side flexible cylinders reached their maximum amplitudes at a reduced velocity between 5.0 and 6.0, but the exact reduced velocities where the maxima for the upper and lower cylinders occurred were not the same. The reason might be the mass ratios of the cylinders were different from each other in the experiments of Huera-Huarte & Gharib [7]. In addition, for reduced velocities smaller than 6.26, the CF response amplitude increases with increasing V_r (up to its peak value). When reduced velocity continues increasing, the second mode gradually appears in the CF response, and leading to a new resonance region. Moreover, it should be noticed that there are several obvious drops in the maximum CF RMS amplitude of the two cylinders, e.g. when $V_r \approx 8.77$ and 17.54. These drops could be contributed to the switching of vibration modes of the cylinder model. A similar trend is also observed in the FIV experiment of the single cylinder.

Fig.8 Max dimensionless RMS displacement amplitude in CF direction versus reduced velocity

As shown in Fig.9, the data in IL direction are as scattered as that in CF direction. The peak values of IL response amplitudes for cylinder #1 and cylinder #2 are $0.67D$ at $V_r = 22.55$ and $0.63D$ at $V_r = 12.53$ respectively. It indicates that cylinder #1 and cylinder #2 nearly have the same maximum displacement amplitude in IL direction. For the single cylinder case, the maximum value of response amplitude is

0.42D at $V_r=12.53$. This reveals that the side-by-side arrangement of two cylinders could enhance their displacements in IL direction. Also, some sharp drops exist in Fig 9. This phenomenon could be attributed to the same reason as that of CF. To the best of the authors' knowledge, this is the first time to illustrate the displacement amplitude behaviors of two side-by-side flexible cylinders in IL direction.

Fig. 9 Max dimensionless RMS displacement amplitude in IL direction versus reduced velocity

Fig.10 and Fig 11 further present the maximum response amplitude ratio of the two cylinders to the single one versus reduced velocity. The main aim is to compare the experimental results of the two side-by-side cylinders with those of the single one. There are four subplots in each of the figure showing the response amplitude ratios of cylinder #1 and cylinder #2 with different four spacing ratios ($S/D=3.0, 4.0, 6.0$ and 8.0). The CF response amplitudes of the two cylinders are both smaller than that of the single one when reduced velocity $V_r < 3.76$. Beyond $V_r=5.01$, it can be found the two side-by-side cylinders oscillated differently from the single cylinder with $S/D=3.0, 4.0$ and 6.0 . The data appear much more scattered as the two cylinder is much closer to each other. Experimental results indicate that strong proximity interference exists when the spacing is smaller 6.0 diameter. Huera-Huarte & Gharib [7] reported that for spacing ratio larger than 3.5, the two side-by-side flexible cylinders behaved no synchronized vibrations, showing independent CF VIV due to the weak interaction between the two cylinder models through the wake. In contrast to their conclusion, in our experiments, the CF vibrations of the two cylinders strongly interact with each other even for a spacing up to $6.0D$. Similar behavior has been reported in the experimental work by Sanaati & Kato [8]. They found that violent interaction between two side-by-side flexible cylinders exhibited with $S/D=5.5$. In addition,

Fig. 10 Max CF displacement amplitude ratio of side-by-side cylinders to single cylinder versus reduced velocity

Fig.11 Max IL displacement amplitude ratio of side-by-side cylinders to single cylinder versus reduced velocity

it can be observed the effect of spacing on CF response amplitude becomes much more insignificant with the increase of S/D . And both cylinders have similar amplitude responses and behave similarly to the single cylinder with $S/D=8.0$. As shown in Fig.11, the response amplitudes of the two side-by-side cylinders are generally much larger than that of the single one in the range of $1.25 \leq V_r \leq 25.05$, except in several mode transition regions, i.e. $6.26 \leq V_r \leq 7.52$, $13.78 \leq V_r \leq 15.03$ and $22.55 \leq V_r \leq 23.80$. Different feature in CF response amplitudes is observed, as presented in Fig.10. It indicates that the two flexible cylinders in a side-by-side arrangement could enhance the displacement amplitude of each other in IL direction. It could be attributed to the proximity interference and synchronization between the two side-by-side flexible cylinders. Sanaati & Kato [8] obtained fluctuating drag coefficients for the FIV of two cylinders in a side-by-side arrangement with $S/D=2.75$ and 5.5 , the amplitude of fluctuating drag coefficient for the single cylinder has a much lower value than that of the two side-by-side ones when reduced velocity is larger than 6.74 . It infers that the IL amplitude response behavior in our experiments might be identical with that given by Sanaati & Kato [8] in spite of the absence of experimental results in their paper. Comparing the two figures, it is apparent that the two side-by-side cylinders vibrate similarly with an isolated cylinder in CF direction at a large spacing ratio $S/D=8.0$. However, the IL vibration of the two side-by-side cylinders still has remarkably strong interaction with each other at $S/D=8.0$.

From the engineering application point of view, the cylinder structures in many offshore engineering situations, e.g. marine risers and cables, have a high aspect ratio and a low mass ratio. When a slender flexible cylinder structure is placed in a fluid flow, some complicated phenomena such as multi-mode vibration, higher harmonic responses and traveling wave dominant response may appear [24, 28-32]. The side-by-side arrangement of two flexible cylinders with a relatively small spacing can cause strong proximity interference. This phenomenon has been experimentally observed by Huera-Huarte & Gharib [7] and Sanaati & Kato [8]. However, the multi-mode FIV characteristics of two side-by-side flexible cylinders subjected to a uniform flow were not investigated in their research works due to limitations of the experimental device. Herein, a typical spacing case with $S/D=6.0$ is discussed. It is the first time that the multi-mode response is investigated in experimental tests on FIV of two side-by-side flexible cylinders. Fig.12 and Fig.13

Fig.12 Space-time varying dimensionless CF displacement contours with $S/D=6.0$.

show the space-time varying dimensionless CF and IL displacement contours for some representative reduced velocities, e.g. $V_r=8.77$, 15.03, 17.54 and 22.55, corresponding to the flow velocities of 0.35m/s, 0.60m/s, 0.70m/s and 0.90m/s respectively. It can be observed that the 1-order oscillation is dominated in CF direction for both cylinders at $V_r=8.77$, and the CF dominant modes of the two cylinders are 2-order at $V_r=15.03$. With reduced velocity increasing to 22.55, much higher modes of vibration are gradually excited in CF direction. In addition, it can also be found the overall FIV response of the side-by-side cylinders is standing wave at a lower reduced velocity ($V_r=8.77$), and then a combining behavior of standing wave and traveling wave appears at higher reduced velocities ($V_r=15.03$, 17.54 and 22.55). These trends are consistent with the results of CF dominant modes illustrated in Table.2. Compared with the CF FIV, the IL vibration of two flexible side-by-side cylinders is much easier to excite higher mode of vibration (see Fig.13). As for a reduced velocity of around 8.55, the IL dominant modes are 4-order and 2-order for cylinder #1 and cylinder #2 respectively, and the

typical standing wave responses are observed for both cylinders. With an increasing V_r , the IL FIV responses of two side-by-side cylinders are gradually changing from standing wave to traveling wave. This trend is also found in CF direction, as shown in Fig.12. In addition, the IL responses of cylinder #1 and cylinder #2 are dominated by 6th mode when $V_r=22.55$. It should be pointed out that the 6th mode is the maximum IL dominant mode in our experiment. The feature of IL dominant mode can also be found in Table.3.

Fig.13 Space-time varying dimensionless IL displacement contours with $S/D=6.0$.

In order to study the time-varying displacements and vibration frequency spectra of the two cylinders at measurement positions, G1-G7. Some representative spacing ratios and reduced velocities should be investigated in detail. However, it is impossible to show the time histories for all the reduced velocities and spacing ratios within the space of a journal paper. Herein, a typical case with $S/D=4.0$ and $V_r=15.03$ is selected as an example, and is shown in Fig.14 and Fig.15. The first column graph gives the time history of the dimensionless displacement for the time range of 20-40s. The second

column graph presents the displacement response spectra which was produced by the FFT analysis of the displacement data. The beating phenomenon can be observed for both cylinder #1 and #2 in Fig.14. Similar results were found by Cui et al. [16] in their numerical investigation on the VIV of two elastically coupled cylinders in a side-by-side arrangement with $S/D=3.0$. The spectral plot shows a

Fig.14 Time history and amplitude spectra of CF displacement at measurement points with $S/D=4.0$ and $V_r=15.03$.

large peak at $f_y \approx 6.3\text{Hz}$ and several small spikes. The same maximum amplitude of up to $2.0D$ are obtained for the two cylinders. These experimental results indicate that the two cylinders vibrate with nearly the same maximum amplitude and vibration frequency in CF direction. In Fig.15, it can be observed that the maximum vibration amplitudes of the two cylinders in IL direction with $S/D=4.0$ and

$V_r=15.03$ are pretty close to each other. Both flexible cylinders vibrate regularly at some periods of time and then the vibration becomes irregular. This phenomenon happens repeatedly at some measurement points, e.g. G5 and G7. The spectral plot shows several large peaks at frequencies of $f_x \approx 6.05\text{Hz}$ and 12.15Hz and lots of small spikes are around, indicating some unsteadiness in the vortex shedding. This further reveals that the FIV of two side-by-side flexible cylinders in IL direction is much more complex than those in CF direction. In addition, it can be clearly seen that different IL vibration frequencies exist at different positions along the flexible cylinders, arising as the multi-mode response.

Fig.15 Time history and amplitude spectra of IL displacement at measurement points with $S/D=4.0$ and $V_r=15.03$

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414 Fig. 16 x-y trajectories taken from seven measurement positions along flexible cylinders versus reduced velocity with
 415 spacing ratio $S/D=3.0$ and 6.0 .

416

417 For a single long flexible cylinder, it has been known that the IL response will influence CF vortex
 418 shedding forces, and vice versa [33-34]. However, so far as the authors know, the IL-CF interaction of
 419 two side-by-side flexible cylinders is still unknown. Fig.16 shows the response trajectories of the two

flexible cylinders at seven measurement points along the length with different reduced velocities. And two spacing cases, i.e. $S/D=3.0$ and 6.0 , were chosen from the ranges with and without proximity interference, respectively, as defined by Zdravkovich [2]. Based on the findings in the experiment by Huera-Huarte & Gharib [7], $S/D=3.0$ belongs to the region of in-phase or out-of-phase synchronization and $S/D=6.0$ to the region of non-synchronization. The top and bottom subplots correspond to the x - y trajectories of cylinder #1 and cylinder #2 respectively. For small spacing ratio, such as $S/D=3.0$, the trajectories for both cylinders exhibit good symmetry in CF direction at $V_r=5.01$ and 10.02 , and distinct eight-shape figures are presented at each measurement point due to the 2:1 ratio of IL to CF dominant frequency. As reduced velocity increases to 15.03 , the trajectories of cylinder #1 and cylinder #2 are both slightly irregular owing to the appearance of the mode transition. Continuing to increase reduced velocity, the typical eight-shape figures are observed again at $V_r=20.04$, and gradually distort and evolve to the half-moon or C-shape form at $V_r=25.05$. Similar behaviors of the x - y trajectories were observed in the experiment of a slender flexible riser subjected to VIV by Vandiver et al. [33]. For the large spacing ratio $S/D=6.0$, the response trajectories of the two cylinders behave the same way as that when $S/D=3.0$, except that the distorted trajectories of cylinder #2 can be clearly found at $V_r=5.01$. This phenomenon could be explained by that the IL dominant vibration frequency of the cylinder is not two but three times the CF dominant vibration frequency, as shown in Fig.6 and Fig.7.

5. Conclusions

The multi-mode FIV of two side-by-side flexible slender cylinders with identical sizes, properties and pretensions were experimentally investigated in a towing tank for four spacing ratios ($S/D = 3.0, 4.0, 6.0$ and 8.0). The influence of spacing ratio on the characteristics of vibration responses was examined by analyzing the dominant frequencies, dominant modes, maximum RMS displacement amplitudes and x - y trajectories at measurement positions.

Variation of dominant frequencies of the two cylinders in a side-by-side arrangement keeps an approximate linear upward trend with the increasing of reduced velocity in both CF and IL directions. The IL and CF dominant frequencies roughly follow a doubled relationship. This trend is consistent with that of the single flexible cylinder undergoing vortex shedding. Up to 4th and 6th dominant modes are observed in CF and IL directions for both side-by-side cylinders and the single

one. Compared with the single cylinder, higher-order mode vibrations easily occur for the case of two side-by-side cylinders in the switching region of vibration modes.

Amplitude responses of the two side-by-side cylinders in CF and IL directions are different from that of the single cylinder for the four cases with different spacing ratios in our experiments. The CF displacement amplitudes of the two side-by-side cylinders reach their maxima at the same reduced velocity. The apparent proximity interference exists in CF direction when the spacing is smaller 6.0 diameter in our tests. Moreover, the side-by-side arrangement for the two flexible slender cylinders could enhance the IL displacement. The two side-by-side cylinders still have remarkably strong interaction with each other in IL direction as the spacing ratio $S/D=8.0$.

The multi-mode response is observed in experimental tests on FIV for the two side-by-side flexible cylinders. Different vibration frequencies exist at different positions along the length of the cylinder leading to the presence of multi-mode response in the behavior of the two flexible cylinders subjected to VIV.

Distinct eight-shape figures are observed at measurement points because of the 2:1 ratio of the IL dominant frequency to CF dominant frequency. In addition, the response trajectories with half-moon and C-shape are also found in some cases of reduced velocities. This feature agrees well with that of a single cylinder undergoing an oncoming flow by [Vandiver et al. \[33\]](#).

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