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On resistance of a rectangular thin plate under lateral indentation by a wedge indenter

Min Zhang ^{a,b}, Jingxi Liu ^{a,b,*}, Zhiqiang Hu ^c, Yao Zhao ^{a,b}

^aSchool of Naval Architecture and Ocean Engineering, Huazhong University of
Science and Technology, Wuhan 460074, China

^bCollaborative Innovation Center for Advanced Ship and Deep-Sea
Exploration(CISSE), Shanghai 200240, China

^cSchool of Engineering, Newcastle University, NE1 7RU, UK

*Corresponding Author

e-mail: liu_jing_xi@hust.edu.cn

Tel.: +86 13638654101

On resistance of a rectangular thin plate under lateral indentation by a wedge indenter

ABSTRACT

This paper addresses a study on the resistance of a clamped rectangular plate quasi-statically punched at the mid-span by a rigid indenter with a knife and a flat edge shape, through simplified analytical method. A new membrane stretching pattern is proposed and the resistance-penetration response is obtained. To validate this analytical method, a series of quasi-static tests on specimens with three length-width ratios and indenters with six different scantlings are conducted. The well-matching of the resistance-penetration curves from analytical calculations and those from experiments proves the validity of the proposed simplified analytical method. In addition, the experimental observations also unveil the rupture behavior of the specimens. Based on the analytical method, the influences of the indenter size and the plate scantlings on the resistance are evaluated. Moreover, the applications of the proposed method in more complex structure and in dynamic impact situation are also assessed. The comparison results show that the analytical method proposed in this paper can be applied to predict the quasi-static response for a wide range of plate length-width ratios as well as to forecast low-velocity impact dynamic responses.

Keywords: Impact; Simplified analytical method; Rectangular plate; Experiment; Resistance-penetration response

1. Introduction

A large number of impact problems are relevant to a metal plate punched by a rigid object, as summarized by Simonsen and Lauridsen (2000). In ship collision or grounding accidents, the plate on the ship side or bottom could be the most efficient energy dissipation member no matter for the conventional ship structure or for the novel sandwich structure (Wolf 2003; Ehlers et al. 2012; He et al. 2016).

In order to evaluate the load-carrying of a plate under lateral accidental loads, extensive experiments, analytical methods and finite element simulations have been conducted (Paik 2007a; Paik 2007b; Paik and Won 2007; Haris and Amdahl 2012; Zeng et al. 2014; Storheim and Amdahl 2017; Zhang and Pedersen 2017). Among them, model tests are usually applied to obtain the deformation and fracture patterns of the structural components aiming at establishing reliable analytical method. Generally in the experiments, the topologies of ship bow and seabed obstacle in the scenario of ship collisions and groundings are usually simplified as a wedge and a cone, respectively (Amdahl et al. 1995). For the cone indenter, experiments on the deformation process and onset of fracture of the plate are extensively conducted and the corresponding simplified analytical methods are also established. In the theoretical analysis of a rectangular plate punched at the mid-span, the mathematical models are usually treated as axis-symmetric, thus the problem can be simplified as two dimensional and the plate is commonly treated as circular. For example, Wang et al. (1998) derived the resistance of thin-walled plates pressed by a sphere prior to fracture with the assumption of an idealized rigid-plastic material and deformation. The prediction results corresponded well with the experimental results and it revealed that the resistance depends strongly on the roundness of the indenter. An analytical method for the resistance-penetration behavior of a plate subjected to a plastic membrane up to failure was proposed by Simonsen and Lauridsen (2000). The point of plate failure was determined by a global stability criterion. Lee et al. (2004) argued that the fracture prediction of a plate indented by a sphere has relation with both the

accumulated equivalent plastic strain and the stress triaxiality. Gong et al. (2015) established a semi-analytical approach to predict the energy dissipation up to failure and it was validated by a series of experiments including clamped circular, square and rectangular plates indented by conical indenters with various radius. Besides, Shen (1995), Shen (2002), and Shen et al. (2002) made much research on the theoretical analysis of dynamic response of a thin circular plate struck transversely by a mass striker with a conical head or a spherical nose and the predicted responses agreed well with the experiment results.

Compared with the cone indenter, the wedge indenter is non-blunt, which would induce much lower resistance and lead to reduction of safety margin with the blunt ones (Wang et al. 1998; Wang 2002). Earlier studies were mainly focused on the deformation mode of the plate indented by a wedge. For instance, Zhu et al. (1994a) investigated the minor collision experiments by an energy method proposed by Jones and Wood's model (Jones 1971; Wood 1961). The numerical work further revealed their deformation patterns (Zhu et al. 1994b), as shown in Figure 1a. Shen (1997) presented a theoretical analysis for the dynamic plastic response of a thin rectangular plate struck transversely by a wedge indenter, in which a pure membrane model with two travelling hinge phases were employed and the strain-rate effects were considered. The maximum permanent deflection and deformation profile agreed well with the experiments performed in Shen et al. (2003). Moreover in the experiments, the failure process was described in detail. Figure 1b shows the deformation of the plate at the moment of crack therein. The deformation mode depicted in Figure 1 also reveals that the deformation profile on the plate indented by a wedge is quite inhomogeneous where the curvature near the denting lines is extremely large and the change of curvature in the path parallel to the denting line is more obvious than that of the path perpendicular to the denting line. Thus the deformation mode is quite different from that of a plate punched by a sphere. And other set of analytical methods should be established to solve this problem. Cho and Lee (2009) developed a simplified

analytical method to predict the damage extent of stiffened plates subjected to lateral dynamic collisions. The rectangular plate was separated into individual regions by the denting lines formed at the edge of the indenter and the plastic hinge lines generated at the intersection lines of two adjacent regions. The total resistance of the plate was the sum of the individual parts. The energy absorbed by the plate was aroused by the effect of the plate membrane stretching and the plastic hinge rotation. In addition, Zhang (1999) proposed a novel simplified analytical method to obtain the resistance-deflection relationship of a shell plate subjected to lateral rectangular area load deduced from the analytical solution of the plate suffering point load, in which the deformation of the plate was expressed as a polynomial. The consistency of the above solution methods is that the expressions derived were built on the basis of the rectangular coordinate systems. While in the case of a plate indented by a sphere indenter, the analytical expressions are established in the polar coordinate systems as described above. Therefore, the present paper attempts to obtain the resistance of a rectangular plate punched by an indenter with a sharp or a flat top in the latter coordinate.

In this study, a simplified analytical solution for the resistance of a rectangular plate indented by a wedge indenter in the mid-span is proposed. A new deformation pattern is proposed and the resistance-penetration relation is derived by theoretical calculation. In addition, experimental tests are conducted for plates with different length-width ratios quasi-statically punched by various indenters including knife and flat edge shapes. The experimental results validate the feasibility of the proposed method and even reveal the rupture behavior of the specimens. Moreover, based on the analytical method, the factors that could influence the reaction force are assessed. Furthermore, the applications of the proposed analytical method in predicting the impact response of more complex structure and in dynamic impact situations are evaluated and some conclusions are finally presented.

2. Simplified analytical method

2.1. Deformation pattern of the plate

Generally, the deformation pattern should originate from the actual deformation shape observed from the experiments, as shown in Figure 1b. Nevertheless, the deformation profile is difficult to express in a mathematic way and the individual part of the deformed plate is generally treated as planar for simplicity in theoretical analysis. Figure 2a depicts the commonly used deformation mode of a plate indented by an indenter with a flat top (Cho and Lee 2009; Liu et al. 2015). The plate constitutes four planar parts and the bending effect is taken into account by considering the plastic hinges forming at the edges of the plate and the indenter as well as the intersection lines between two adjacent parts. Comparatively, the membrane stretching effect plays the most important role in the total resistance. The direction of the membrane stretching strain is perpendicular to the edge of the plate as the strip lines marked with dashed lines. While in the deformation pattern of a circular plate punched by a spherical indenter, the strip lines are axis-symmetrical, Figure 2b. Thus the problem is transformed into two-dimensional problem and only one strip line is used to derive the whole resistance (Simonsen and Lauridsen 2000; Wang et al. 1998; Lee et al. 2004; Gong et al. 2015). Therefore, a rectangular plate subjected to line or rectangular area load in the middle position is proposed, where the straight strip lines representing the membrane stretching mapped from the edge of the indenter to the edge of the plate and the plate bending is omitted, Figure 2c.

2.2. Analytical expressions

In order to make the derived equations more versatile, the geometry defined in Figure 3a is built for the indentation by an indenter with a rectangular top. The corresponding half-length and half-width are a and b , respectively. If the top of the indenter is a sharp edge, then b should be zero. The indenter is displaced a vertical distance $w_{\max}$ into the middle span of a rectangular plate with initial thickness t and dimensions $2a_0 \times 2b_0$.

Due to the symmetry of the deformed plate, only the area in the first quadrant of

the rectangular coordinate is used to derive the whole resistance of the plate. Moreover, the contact area between the indenter and the plate is assumed to be intact. Thus there are two remaining regions divided by the line connected by the corners of the indenter and the plate. Therefore, two trapezoidal regions can be considered to represent the whole deformation of the plate. The area with dashed lines determined by the half-length of the rectangular edge and the corresponding plate edge is employed to derive its lateral resistance. As shown in Figure 3b, this area is regarded as axis-symmetric and point O_1 is the origin formed at the intersection of the two sides of the trapezoid. Therefore, the plate deformation can be described in a (ρ, θ, w) cylindrical coordinate system where ρ is the radial coordinate on the strip line, θ is the angle between the y -axis and the strip line, w is the lateral deflection of an arbitrary point on the strip line. It should be noted that the minimum and maximum radial length (ρ_{min} and ρ_{max}) are varying accompanied with the angle θ . Hence, given an arbitrary angle θ , the deflection of a point on the strip line of the plate is $w(\rho)$ and it is regarded as linear. Thus the deflection can be expressed as

$$w = C_1\rho + C_2 \quad \rho_{min} \leq \rho \leq \rho_{max} \quad (1)$$

where C_1 and C_2 are constants, and ρ_{min} and ρ_{max} are

$$\begin{aligned} \rho_{min} &= m / \cos\theta, \\ \rho_{max} &= (b_0 - b + m) / \cos\theta \end{aligned} \quad (2)$$

where m is length between point O_1 and point O_2 , and it can be obtained according to the geometry relation, which is

$$m = \frac{ab_0 - ab}{a_0 - a} \quad (3)$$

Moreover, the angle θ is limited by the maximum value α and it can be expressed as

$$\tan \alpha = \frac{a_0}{b_0 - b + m} \quad (4)$$

The energy absorption of the plate is mainly dissipated by the membrane stretching response and the relevant strain tensors is the radial strain ε_ρ . It is defined by

$$\varepsilon_{\rho} = \left(\frac{\partial u}{\partial \rho} \right) + \frac{1}{2} \left(\frac{\partial w}{\partial \rho} \right)^2 \quad (5)$$

where u is in-plane displacement of the deformed plate. In order to obtain a mathematically tractable system of equations, u is assumed to be zero (Lee et al. 2004). Thus the first derivative term of radial strain determined by u is ignored.

Since $w_{\max}$ is the maximum deflection at the contact point of the plate and the indenter. The constants C_1 and C_2 in Equation (1) can be determined by the following boundary constraints

$$\begin{aligned} \rho = \rho_{\min}, \quad w &= w_{\max} \\ \rho = \rho_{\max}, \quad w &= 0 \end{aligned} \quad (6)$$

Substituting Equations (2), (3) and (6) into Equation (1) and it yields

$$w = -\frac{w_{\max} \cos \theta}{b_0 - b} \rho + \frac{(b_0 - b + m) w_{\max}}{b_0 - b} \quad \rho_{\min} \leq \rho \leq \rho_{\max} \quad (7)$$

Based on Equation (7), the expression for the radial strain can be expressed as

$$\varepsilon_{\rho} \approx \frac{1}{2} \left(\frac{dw}{d\rho} \right)^2 = \frac{1}{2} \frac{w_{\max}^2 \cos^2 \theta}{(b_0 - b)^2} \quad (8)$$

Therefore, the average strain can be obtained by integrating the strains on the area of region one

$$\varepsilon_{\rho 1 \text{avg}} = \frac{1}{A_1} \int_{A_1} \varepsilon_{\rho 1} dA_1 \quad (9)$$

Substituting Equations (2)-(4) into Equation (9), it becomes

$$\varepsilon_{\rho 1 \text{avg}} = \frac{w_{\max}^2}{2(a_0 - a)(b_0 - b)} \cdot \arctan \left(\frac{a_0 - a}{b_0 - b} \right) \quad (10)$$

The corresponding virtual strain is given by:

$$\delta \varepsilon_{\rho 1 \text{avg}} = \frac{w_{\max}}{(a_0 - a)(b_0 - b)} \cdot \arctan \left(\frac{a_0 - a}{b_0 - b} \right) \delta w_{\max} \quad (11)$$

Therefore, the virtual work with the reaction force P_1 and the corresponding deflection $\delta w_{\max}$ in region one is given by:

$$P_1 \cdot \delta w_{\max} = \iint_{A_1} \sigma_0 t \delta \varepsilon_{\rho 1 \text{avg}} dA_1 \quad (12)$$

where σ_0 is the flow stress of the plate. For simplicity, $\sigma_0 = (\sigma_y + \sigma_u)/2$ (Hong 2009; Jones 2014). σ_y and σ_u are initial yield stress and ultimate tensile stress, respectively.

Substituting Equations (2)-(4) and (11) into Equation (12), the reaction force P_1 can be obtained and expressed as

$$P_1 = w_{\max} \sigma_0 t \frac{(a_0 + a)}{2(a_0 - a)} \arctan\left(\frac{a_0 - a}{b_0 - b}\right) \quad (13)$$

Furthermore, the absorbed energy with the maximum deflection can be achieved

$$E_1 = \int_0^{w_{\max}} P_1 dw = w_{\max}^2 \sigma_0 t \frac{(a_0 + a)}{4(a_0 - a)} \arctan\left(\frac{a_0 - a}{b_0 - b}\right) \quad (14)$$

Generally, the strain-hardening stress σ_{eq} can be easily obtained by the uniaxial tension test. Therefore, substituting the stress σ_{eq} to the flow stress, a more precise analytical result can be acquired

$$P_1 = w_{\max} \sigma_{eq} t \frac{(a_0 + a)}{2(a_0 - a)} \arctan\left(\frac{a_0 - a}{b_0 - b}\right) \quad (15)$$

$$E_1 = w_{\max}^2 \sigma_{eq} t \frac{(a_0 + a)}{4(a_0 - a)} \arctan\left(\frac{a_0 - a}{b_0 - b}\right) \quad (16)$$

In addition, the reaction force of region two can be obtained by exchanging a and b , a_0 and b_0 because Equations (13) and (14) are symmetric and the solution process is similar to the process in region one. Therefore, the reaction force P_2 , the absorbed energy E_2 , and the average strain in region two can be directly generated, respectively.

$$P_2 = w_{\max} \sigma_0 t \frac{(b_0 + b)}{2(b_0 - b)} \arctan\left(\frac{b_0 - b}{a_0 - a}\right) = w_{\max} \sigma_{eq} t \frac{(b_0 + b)}{2(b_0 - b)} \arctan\left(\frac{b_0 - b}{a_0 - a}\right) \quad (17)$$

$$E_2 = w_{\max}^2 \sigma_0 t \frac{(b_0 + b)}{4(b_0 - b)} \arctan\left(\frac{b_0 - b}{a_0 - a}\right) = w_{\max}^2 \sigma_{eq} t \frac{(b_0 + b)}{4(b_0 - b)} \arctan\left(\frac{b_0 - b}{a_0 - a}\right) \quad (18)$$

$$\varepsilon_{\rho 2 \text{avg}} = \frac{w_{\max}^2}{2(a_0 - a)(b_0 - b)} \cdot \arctan\left(\frac{b_0 - b}{a_0 - a}\right) \quad (19)$$

Then, the total resistance force P and absorbed energy E versus maximum

deflection of the rectangular plate $w_{\max}$ can be expressed as Equations (20) and (21).

$$P = 4(P_1 + P_2) \quad (20)$$

$$E = 4(E_1 + E_2) \quad (21)$$

3. Experimental details

In order to validate the analytical method, a series of experiments are conducted in Huazhong University of Science and Technology. The experimental setup is presented in Figure 4. The thin mild steel rectangular plates with dimension of $2a_0 \times 2b_0$ are firmly clamped between two heavy flanges and are loaded by an indenter with dimension of $2a \times 2b$. The dimension of the device is shown schematically in Figure 5. The upper and lower flanges are 25 mm in thickness and fixed by a host of M20 bolts. The diameters of bolt holes in the tested plates and flanges are 2 mm larger than the bolts for assembly requirements. This fixation method has been applied by Gong et al. (2015), and Cho and Lee (2009) and is proved to provide clamped boundary condition.

In order to investigate the influence of specimen and indenter dimensions on the resistance and fracture modes, 18 model tests are designed, as listed in Table 1, in which plates with three kinds of dimension are punched by indenters with six scantlings. Figure 6 illustrates the test cases in each plate. The material used for the specimens is cold-rolled normal structural steel with 1.455 mm in thickness. Standard tensile tests are performed to characterize the mechanical properties of the steel. The tension test piece is depicted in Figure 7 as well as the corresponding dimensions. The mechanical properties are summarized in Table 2 and the engineering stress-strain curve is plotted in Figure 8. The true stress-strain relation prior to local necking can be obtained by the formulae proposed by Dieter (1986). And strain hardening is given by the Voce model (Voce 1955):

$$\sigma_{eq} = \sigma_y + R_0 \cdot \varepsilon_{eq} + R_\infty \left(1 - e^{-n\varepsilon_{eq}}\right) \quad (22)$$

where σ_y is the initial yield stress, ε_{eq} is the average strain of the deformed plate at

each region and the other Voce parameters R_0 , R_∞ and n are obtained from the true stress-strain curve. The resulting parameters are: $R_0=383.6$ MPa, $R_\infty=95.89$ MPa and $n=22.08$. The true stress-strain curve is also plotted in Figure 8.

The indenter is positioned at the mid-span of the specimens. The plate deformation is enforced by a hydraulic cylinder at a rate of ~ 10 mm/min. The resistance-time and displacement-time curves are recorded using a load cell (20 tonnage) and two displacement transducers (750 mm), respectively. The deformation of the specimen is visualized by the grid lines (40×40 mm) drawn on both sides. Moreover, a video camera is used to capture the fracture process of the plate.

4. Experimental results

The analytical approximations are compared with the experimental resistance-penetration responses (P - w curves) in Figures 9-11. The results match well in the case of the plate punched by an indenter with a flat top. Moreover, it is indicated that better results can be obtained when strain-hardening material is used instead of rigid-strain material, in the case of a plate punched by an indenter with a sharp top. This is attributed to the local denting effect. Figure 12 depicts the three-dimensional deformation of specimen B2-s and specimen B2-f extracted by a 3D scanner at the moment before plate fracture. It can be observed that the local denting is more obvious in specimen B2-s than in specimen B2-f because the transition of curvature is more severe especially near the denting line from the side view. Generally, local denting happens in the initial stage, resulting in a lower resistance value. Thus the slopes of the P - w curves are lower in the initial stage. Moreover, the strain distribution is more complex due to local denting. These bring the distinction of the analytical results from the experimental one. Nonetheless, it can be found that the slopes of the experimental and analytical response curves are very close with larger indentation depth when a constant flow stress is applied.

In addition, the fracture behavior of the plate punched by a wedge indenter is worth of investigation. Researchers have put much efforts on the fracture prediction of a

plate punched by a sphere (Simonsen and Lauridsen 2000, Lee et al. 2004, Gong et al. 2015, Liu et al. 2017). But it is still a challenge for the plate indented by a wedge indenter because the failure mode is more complex. The rupture processes of two typical specimens are illustrated in Figure 13. Figure 13a shows that the resistance forces will not drop immediately when reaching the peak value and this phenomenon is more obvious for the indenter with a sharp top. The resistance-penetration response is quite different from the situation of a plate punched by a conical indenter (Alsos et al. 2009), in which the resistance force will drop instantly when fracture happens on the plate. The difference can be attributed to the different fracture mode. Figure 13b and Figure 13c display the fracture process in these two typical model tests. Stage a and stage b are the moments of fracture initiated and formed on the rectangular plate respectively. Step c represents the cracks propagate along the edge of the indenter. At stage d, the cracks are torn open. The corresponding moments of these steps are also marked on the resistance-penetration curves in Figure 13a. It is found that the initiation of fracture on the plate can decrease the load-carrying of the plate and the load will not drop instantly until the plate is penetrated by the indenter.

The values of the critical indentation depth (stage b) and the indentation depth at load decrease (stage d) as well as the corresponding resistance and energy dissipated in each test are summarized in Table 3. The critical indentation depths and the corresponding resistance forces of the plates punched by indenters with rectangular top are larger than that of the indenters with sharp top. This can ascribe to the disparities of the failure characteristic. In the cases of the specimens punched by indenters with sharp edges, the shear effect is severer than the case when the plates are punched by indenters with flat tops. In addition, the critical indentation depth in plates with dimensions of 400×400 mm and 400×600 mm will decrease with the increase of the length of the indenter edge while the law cannot be found in plates with dimension of 600×400 mm. Besides, compared with the moment of load decrease, the indentation depth and the energy dissipated at the moment of plate

fracture are generally smaller. This discrepancy is obvious in the specimens punched by an indenter with sharp edge and it exhibits larger with the increase of the length of indenter edge. It demonstrates that there is considerable portion of energy after plate fracture appears and the out-of-plane tearing effect is deserved to be investigated.

5. Discussions

5.1. Parametrical analysis

In order to investigate the influence of the dimensions of the indenter and the plate on the resistance response, the following dimensionless forms are assumed

$$\Omega_p = \frac{P}{w_{\max} \sigma_0 t} \quad \eta = \frac{a_0}{b_0} \quad \lambda = \frac{a}{a_0} \quad \mu = \frac{b}{b_0} \quad 0 \leq \lambda, \mu < 1 \quad (23)$$

where Ω_p is the dimensionless resistance force at a given indentation depth, η is the length-width ratio of the plate, λ and μ are the ratios of the indenter edge to the related plate edge, respectively.

Substituting Equation (23) into Equation (20), the dimensionless resistance force Ω_p can be obtained and expressed as

$$\Omega_p = \frac{2(1+\lambda)}{1-\lambda} \arctan \frac{(1-\lambda)\eta}{1-\mu} + \frac{2(1+\mu)}{1-\mu} \arctan \frac{1-\mu}{(1-\lambda)\eta} \quad (24)$$

If the length-width ratio (η) of a rectangular plate is 1.5, the influence of the indenter size on the resistance can be obtained, as shown in Figure 14a. It reveals that the resistance force is quite sensitive to the indenter size and it will become larger with the increase of the length of the indenter edge. Moreover, the resistance performs higher sensitiveness with the increase of the indenter edge perpendicular to the short edge of the plate. Figure 14b is a two-dimensional section example of Figure 14a and it represents the influence of one indenter edge size on the resistance. It shows that with the increase of one indenter size, the raise of the resistance force is more remarkably when the other indenter size is larger. Besides, the relation between Ω_p and $1/\lambda$ can illustrate the influence of the plate size on the resistance force. Based on Figure 14b, it is obvious that the resistance force will become smaller when one side

of plate size becomes larger.

5.2. Application in the stiffened plate

The stiffened plate can be the primary structural form that equipped on the ship side or ship bottom. In an accident of ship collision or grounding, the total resistance of the stiffened plate is the sum of the contribution of the plate and its attached stiffeners in analytical prediction. Therefore, the applicability of the proposed analytical method on the resistance of the stiffened plate is assessed in this section.

Liu et. al (2015) presented a simplified method to evaluate the energy absorption of the stiffened plates quasi-statically punched at the mid-span by a rigid indenter with a knife and a flat edge shape by considering the contribution of the plate and the stiffeners individually, in which the analytical method for the plate is referred to Cho and Lee (2009) and the analytical approximation for the stiffener is derived. The reliability of the method was validated by the experiments and numerical simulations. Therefore, the experimental results are also utilized to validate the present method by substituting the energy-penetration response of the plate in Liu et al. (2015) by the present expressions. The compared curves shown in Figure 15 indicates that the only slight distinction is found between the results forecasted by the present method and predicted by the method in Liu et al. (2015). This can validate the accuracy of the present method. Note that the stress applied in the present method is the flow stress which can cause deviation when predicting the response of the plate indented by a wedge indenter with a knife edge due to the effect of local denting as depicted in Figures 9-11. However, well agreement compared with the experimental result is found in the stage before plate rupture as shown in Figure 15a. It illustrates that the local denting of the plate can be confined by the attached stiffeners and the flow stress can directly predict the resistance of the plate well.

The length-width ratio of the plate in Liu et al. (2015) is moderate (length/width=1.2). While the proposed method is validated to be feasible by a series of experiments including specimens and indenters with different dimensions. In order

to investigate the sensitivity of the plate size on the energy dissipation response, the relation between the energy dissipation and the dimension of one plate side is obtained, Figure 16, in which the lateral penetration is assumed to be 60 mm and the plate side parallel to the sharp edge of the indenter is constant. Moreover, the length-width ratio of the rectangular plate ranges from 0.67 to 1.5. The compared results reveal that the trends are different with the increase of the one side of the plate. In general, the consistency of these two analytical methods is that the resistance of the plate all depends on the variables including the stress (σ_0 or σ_y), the volume of the solid body (V), the indentation depth ($w_{\max}$) and the membrane tensile strain (ε). Among them, the solution method of the strain can result in the difference. In the present method, the strain will gradually decrease from the middle strip line to the side strip line of the plate with the decrease of the slope of the projection line (line O_3P in Figure 3b). While the strain assumed by Cho and Lee (2009) is uniform in the individual region for the rotation angles of the strip lines are the same. This can be the reason that gives rise to the difference in the resistance responses predicted by these two methods. In general, the method proposed in this paper shows its ability to predict the resistance of a rectangular plate punched by a wedge indenter in a wide range of plate size and indenter size.

5.3. Application in dynamic impact response

A typical ship bow-side collision scenario should consider the strain-rate effect of the material when research the impact behavior by analytical method and numerical simulation. Generally, the strain-rate effect is taken into account by multiplying the yield stress or the flow stress by a strain-rate strengthen factor (Shen et al. 2003; Cho and Lee 2009). Thus the dynamic response of the rectangular plate impacted by a wedge can be obtained by substituting the flow stress σ_0 as the dynamic flow stress σ_{d0} according to Shen et al. (2003).

In the experiments conducted by Shen et al. (2003), 60 experiments of rectangular plates impact by wedge indenters with knife edge were carried out. While only 19 of

them are available for the maximum impact force and the related maximum permanent deflection. These data are utilized to verify the applicability of the present method in low velocity impact scenario. The comparisons between the theoretical calculations and experimental results are listed in Table 4, in which the mass of striker G , the initial velocity of striker V_0 , and the dimensionless impact force $P^*=P/(\sigma_{d0}t^2)$. The comparison is also shown in Figure 17. The results predicted by the present method are approximate to the experiments although there are some deviations. It illustrates that the present method is capable of forecasting the situation of low velocity impact.

6. Conclusions

The behavior of rectangular plates punched by a series of indenters with sharp top and flat top at the mid-span are investigated by experimental and analytical method. The deformation pattern of a rectangular plate punched by indenters with a sharp top and a flat top is proposed based on previous contributions. The resistance-penetration response is derived and they match well with the experimental results.

The rupture behaviors of the plates are investigated by the experiments. In the tests of plates punched by a sharp indenter, the resistance force will gradually decrease when plate fracture happens while this phenomenon is not obvious in the response of the rectangular plates punched by an indenter with flat top.

The influence of the indenter size on the resistance of the plate is quite obvious. When a rectangular plate is subjected to an area load, the resistance force is more sensitive to the indenter edge perpendicular to the longer edge of the plate. Moreover, with the increase of one indenter edge, the resistance force is also more sensitive if the other edge is larger.

The proposed method is available for predicting the resistance of stiffened plate when the flow stress is applied and it is able to assess the response with a wide range of plate size and indenter size. Moreover, the method shows its feasibility in predicting a low velocity impact response by considering the effect of material strain

rate.

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Table Captions

Table 1. Experimental cases.

Table 2. Mechanical properties of material.

Table 3. Summary of experimental results.

Table 4. Comparison of predicted results with experimental results in Shen et al.
(2003)

Tables

Table 1. Experimental cases.

Test Model	Plate $2a_0 \times 2b_0$ (mm)	Indenter $2a \times 2b$ (mm)
A1-s	400×400	80×0
A2-s		160×0
A3-s		240×0
A1-f		80×80
A2-f		160×80
A3-f		240×80
B1-s	600×400	80×0
B2-s		160×0
B3-s		240×0
B1-f		80×80
B2-f		160×80
B3-f		240×80
C1-s	400×600	80×0
C2-s		160×0
C3-s		240×0
C1-f		80×80
C2-f		160×80
C3-f		240×80

Table 2. Mechanical properties of material.

Property	Symbol	values
Young's modulus	E	201 GPa
Yield stress	σ_y	183 MPa
Ultimate tensile stress	σ_u	298.5 MPa
Fracture strain	ε_f	0.33

Table 3. Summary of experimental results.

Test model	Values at initiation of fracture			Values at load decrease rapidly		
	Penetration [mm]	Force [KN]	Energy [J]	Penetration [mm]	Force [KN]	Energy [J]
A1-s	20.63	21.10	197.50	23.49	17.48	258.32
A2-s	20.58	26.68	244.14	25.83	21.63	385.29
A3-s	19.43	31.59	277.78	29.76	22.58	623.02
A1-f	42.67	73.60	1490.4	44.87	63.51	1649.0
A2-f	41.70	89.72	1709.3	43.64	87.49	1895.3
A3-f	37.07	96.30	1673.5	37.94	93.34	1756.6
B1-s	19.98	19.69	179.03	21.91	17.74	217.04
B2-s	21.82	24.75	247.37	26.00	19.94	349.16
B3-s	21.67	31.45	315.70	29.86	20.91	575.16
B1-f	45.29	64.89	1391.1	46.16	64.07	1447.5
B2-f	45.81	82.96	1755.0	47.99	80.96	1935.9
B3-f	47.24	106.3	2302.2	48.47	104.3	2433.1
C1-s	19.79	19.46	167.54	21.60	16.71	199.19
C2-s	19.65	23.02	202.42	25.65	16.70	346.17
C3-s	18.88	27.44	232.19	29.20	18.18	530.13
C1-f	44.29	63.66	1312.9	46.37	64.07	1447.5
C2-f	41.46	76.91	1492.5	42.91	73.99	1604.2
C3-f	36.87	82.11	1396.5	38.02	78.84	1490.5

Table 4. Comparison of predicted results with experimental results in Shen et al. (2003)

Specimen	$G(\text{kg})$	$V_0(\text{m/s})$	$2a$ (mm)	$2a_0$ (mm)	$2b_0$ (mm)	t (mm)	P^*_{test}	P^*_{theo}
ASa 1	25.4	2.45	40	280	130	1	64.47	38.58
ASa 2	25.4	2.53	40	280	130	1	67.18	39.35
ASa 3	25.4	2.68	40	280	130	1	61.12	42.44
ASb 2	25.4	4.23	40	280	130	1.5	44.77	47.57
ASb 3	25.4	3.73	40	280	130	1.5	42.28	37.31
ALa 1	32.43	5.92	120	280	130	1	112.4	139.37
ALa 3	12.16	6.67	120	280	130	1	81.30	108.88
ALb 2	32.43	6.96	120	280	130	1.5	78.46	100.64
ALb 4	32.43	6.32	120	280	130	1.5	76.49	91.00
BSc 1	31.45	4.62	40	280	190	2	47.14	42.10
BSc 2	31.45	4.36	40	280	190	2	43.65	35.74
BSc 4	31.45	4.24	40	280	190	2	37.96	30.87
BSd2	42	5.35	40	280	190	3	12.79	26.57
BSd3	42	4.57	40	280	190	3	12.18	18.08
BSd4	42	4.69	40	280	190	3	12.18	19.09
BLc 2	32.43	6.64	120	280	190	2	54.03	64.21
BLc 3	32.43	7.5	120	280	190	2	66.33	70.76
BLc 4	32.43	8.44	120	280	190	2	71.69	77.99
BLd2	43	7.15	120	280	190	3	21.85	35.64

Figure Captions

Figure 1. Deformation mode of a plate struck by a wedge indenter: (a) in numerical simulation (Zhu et al. 1994b), (b) in impact experiment (Shen et al. 2003).

Figure 2. Deformation patterns of the plate suffering lateral indentation: (a) a rectangular plate indented by a wedge indenter (Cho and Lee 2009; Liu et al. 2015), (b) a circular plate indented by a sphere indenter (Simonsen and Lauridsen 2000), (c) a rectangular plate indented by wedge indenters.

Figure 3. Assumed deformation shape of the rectangular plate: (a) overall plate, (b) region one.

Figure 4. Experimental setup: (a) schematic fixture, (b) test photo. (This figure is available in colour online.)

Figure 5. Dimension of the experimental setup.

Figure 6. Dimension of the plate and the indenter.

Figure 7. Standard dimensions of the machined test pieces (ASTM,E8).

Figure 8. Stress-strain curves.

Figure 9. P - w curves of plate 400×400 mm.

Figure 10. P - w curves of plate 600×400 mm.

Figure 11. P - w curves of plate 400×600 mm.

Figure 12. Experimental deformation shape from three-dimensional view: (a) specimen B2-s, (b) specimen B2-f. (This figure is available in colour online.)

Figure 13. Rupture process of two typical specimens: (a) P - w curves, (b) specimen B2-s, (c) specimen B2-f. (This figure is available in colour online.)

Figure 14. Influence of the indenter size on the resistance. (This figure is available in colour online.)

Figure 15. Energy-penetration response compared with Liu et al. (2015): (a) Knife specimen, (b) Flat specimen.

Figure 16. Influence of the length of the plate on the results: (a) Knife indenter, (b)

Flat indenter.

Figure 17. Resistance force compared with the experiments conducted by Shen et al. (2003).

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G-Gauge length	50 mm
W-Width	12.5 mm
T-Thickness	1.455 mm
R-Radius of fillet	12.5mm
L-Overall length	200 mm
A-Length of reduced section	85 mm
B-Length of grip section	~50 mm
C-Width of grip section	20 mm

Figure 7. Standard dimensions of the machined test pieces (ASTM,E8).

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