Heat flux and flow topology statistics in oblique and head-on quenching of turbulent premixed flames by isothermal inert walls

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ABSTRACT

Direct Numerical Simulations (DNS) of oblique wall quenching of a turbulent V-flame and head-on quenching of a statistically planar flame by isothermal inert walls have been utilised to analyse the statistics of wall heat flux, flame quenching distance in terms of the distributions of flow topologies and their contributions to the wall heat flux. The flow topologies have been categorised into 8 generic flow configurations (i.e. S1-S8) in terms of three invariants of the velocity gradient tensor (i.e. first, second and third P , Q and R respectively). It has been found that nodal (i.e. strain rate dominated) flow topologies are major contributors to the wall heat flux when it attains large magnitude in the head-on quenching configuration, whereas focal (i.e. vorticity dominated) topologies contribute significantly to the wall heat flux in the case of oblique flame quenching. These differences in the heat transfer mechanisms contribute to the differences in wall heat flux and flame quenching distance between head-on quenching and oblique quenching configurations. The maximum wall heat flux magnitude in the case of oblique flame quenching has been found to be greater than that in the corresponding turbulent head-on quenching case. By contrast, the minimum wall Peclet number, which quantifies the flame quenching distance, in the case of oblique quenching has been found to be smaller than that in the case of head-on quenching.

Keywords: Flow topology, wall heat flux, wall Peclet number, oblique flame quenching, head-on quenching, Direct Numerical Simulations

1. INTRODUCTION

Modern combustors are increasingly made smaller in size than earlier versions in order to achieve higher **energy-density** and a more cost-effective light-weight design. Furthermore, the development of miniaturised devices such as micro-robots, micro-aerial vehicles (e.g. drones), or **notebook** computers are increasingly becoming indispensable parts of modern life, and these devices could potentially be powered by micro-combustors because of their higher energy-density and shorter recharge time than normal electrochemical batteries (Fernandez Pello, 2012). However, the operation and durability of small and micro-combustors are severely limited by flame quenching caused by high heat transfer rate through the cold wall due to high surface to volume ratio. The flame-wall interaction (FWI) plays a key role in determining the efficiency of combustion systems, **the** formation of pollutants (e.g. unburned hydrocarbons), wall cooling for safe operation and overall durability (Heywood, 1998; Fritz *et al.*, 2004). Moreover, flame propagation in turbulent boundary layers has been identified as one of the mechanisms of flashback into the mixing zone from the combustion chamber (Fritz *et al.*, 2004). **In FWI, a large temperature variation ($\sim 400 - 800$ K) takes place in a thin layer ($\sim 1.0 \mu\text{m}$) next to the wall so the near-wall physical mechanisms need to be resolved in addition to the resolution of the smallest turbulent vortical and flame structures. This makes its DNS more computationally challenging than DNS simulations of non-reacting wall-bounded turbulent flows, and reacting flows without walls. Since** Investigations based on DNS allow for obtaining both temporally and spatially resolved three-dimensional data without any physical approximation regarding the turbulent fluid motion, whereas this information is either impossible or extremely expensive to access using experimental means.

DNS based analyses have contributed to the fundamental understanding and modelling of turbulent fluid flows in last two decades. However, to date, relatively limited effort has been

directed to DNS of FWI (Poinsot *et al.*, 1993; Bruneaux *et al.*, 1996, 1997; Popp *et al.*, 1996; Alshaalan and Rutland, 1998, 2002; Gurber *et al.*, 2010; Gruber *et al.*, 2012; Dabireau *et al.*, 2003; Lai and Chakraborty, 2016a-d; Sellmann *et al.*, 2017; Lai *et al.*, 2017a-d) in comparison to the vast body of literature on DNS of turbulent reacting flows away from the walls. Poinsot *et al.* (1993) used two-dimensional DNS to analyse the flame-vortex interaction in head-on quenching (HOQ) of premixed flames where the mean direction of flame propagation is normal to the wall. This two-dimensional analysis also provided the quantification of the maximum wall heat flux and the minimum wall Peclet number (i.e. normalised quenching distance) in the case of HOQ, which has been confirmed recently by a three-dimensional DNS based analysis by Lai *et al.* (2016a). The analysis of Lai *et al.* (2016a) further demonstrated the influences of the characteristic Lewis number on the maximum wall heat flux and the flame quenching distance. Lai *et al.* (2017a,b) also analysed turbulent kinetic energy and enstrophy transports in the near-wall region for HOQ of premixed turbulent flames based on simple chemistry DNS data, and revealed that the presence of wall and wall-induced quenching significantly affect the statistical behaviours and modelling of the unclosed terms of the turbulent kinetic energy equation and baroclinic torque and dilatation contributions to the enstrophy transport. Recently, Sellmann *et al.* (2017) and Lai *et al.* (2017c) have addressed the modelling of the FSD and turbulent scalar flux transports using DNS data of HOQ of statistically planar premixed turbulent flames. The algebraic closures of FSD in the context of Large Eddy Simulations (LES) have also recently been analysed by Lai *et al.* (2017d). Lai and Chakraborty (2016b,c) and Lai and Chakraborty (2016d) analysed the near-wall behaviours of scalar dissipation rate and variance respectively using DNS data of HOQ of premixed turbulent flames by inert isothermal walls.

The analyses by Poinso *et al.* (1993), Lai and Chakraborty (2016a-d), Sellmann *et al.* (2017) and Lai *et al.* (2017a-d) dealt with transient HOQ configuration where the wall normal aligns collinearly with the mean direction of flame propagation. However, in several engineering applications the mean direction of flame propagation may not align with the wall normal and thus it is necessary to analyse FWI in other configurations where the flame interacts with the wall at an angle. Bruneaux *et al.* (1996) analysed side wall quenching of premixed turbulent flames in a channel flow configuration using DNS and this data has subsequently been used to propose near-wall modifications to the unclosed terms of the Flame Surface Density (FSD) transport equation (Bruneaux *et al.*, 1998). Alshaalan and Rutland (1998, 2002) analysed oblique wall quenching (OWQ) of a V-flame by isothermal walls using three-dimensional DNS and used this data to analyse FSD and scalar flux closures in the context of FWI.

Poinso *et al.* (1993), Bruneaux *et al.* (1996, 1997), Alshaalan and Rutland (1998, 2002), Lai and Chakraborty (2016a-d), Sellmann *et al.* (2017) and Lai *et al.* (2017a-d) considered a single-step irreversible Arrhenius type chemical mechanism for the purpose of computational economy. However, some recent analyses utilised the advancements in high-performance computing to conduct multi-dimensional simulations of FWI in the presence of detailed chemical mechanisms. Two-dimensional detailed chemistry unsteady simulations of FWI of $\text{H}_2 + \text{O}_2$ mixtures have been carried out Dabireau *et al.* (2003), and the statistical behaviours of the wall heat flux and flame quenching distance obtained from detailed chemistry simulations have been found to be qualitatively similar to those obtained from simple chemistry simulations (Poinso *et al.*, 1993; Lai and Chakraborty, 2016a). However, the intermediate species such as H_2O_2 and HO_2 have been found to play pivotal roles in the near-wall heat release and species distributions in the case of FWI of $\text{H}_2 + \text{O}_2$ systems, which cannot be predicted by simple chemistry simulations. Gruber *et al.* (2010) and Gruber *et al.* (2012)

carried out three-dimensional detailed chemistry DNS of FWI for turbulent premixed flames for OWQ of a V-flame and side-wall quenching in a channel flow configuration, respectively. Although all the aforementioned analyses contributed significantly to the fundamental understanding and modelling of FWI, the nature of dominant flow topologies in the case of FWI and their contributions to the wall heat flux are yet to be analysed in detail.

Perry and Chong (1987) and Chong *et al.* (1990) categorised all the possible turbulent flow topologies in terms of the invariants of the velocity gradient tensor (i.e. first- P , second- Q and third- R) with the components given by $\partial u_i / \partial x_j$ where u_i is the i^{th} component of the velocity vector. The topologies, denoted S1 – S8, distinguish 8 regions in the three-dimensional $P - Q - R$ phase space, as described in Fig. 1. Perry and Chong (1987) and Soria *et al.* (1994) indicated that within incompressible (where $P = 0$) turbulence, the S4 topology is most likely to occur for positive values of Q , whereas the topologies S2 and S4 remain dominant in the regions away from the wall (Blackburn *et al.*, 1996). Furthermore, the joint probability density function (PDF) between Q and R demonstrates a universal ‘teardrop’ structure in the $Q - R$ plane (Chong *et al.*, 1990; Chacin and Cantwell, 2000; Ooi *et al.*, 1999), which has been explained by Elsinga and Marusic (2010). However, this ‘teardrop’ structure of the $Q - R$ joint PDF exists only in the fully turbulent region and not in the interface between turbulent and non-turbulent regions (Da Silva, and Pereira, 2008; Khashehchi *et al.*, 2010). Tsinober (2000) provided arguments that the enstrophy and strain rate productions are associated with S4 and S1 topologies. The interaction between flow topology and scalar surface topology for passive scalar mixing has been analysed by Dopazo *et al.* (2007). All these aforementioned analyses were carried out for incompressible fluids, but in compressible flow turbulence the first invariant of the velocity gradient tensor P plays a key role in addition to Q and R .

Chen *et al.* (1989) used the critical point theory in terms of P , Q and R in order to analyse the structure of a compressible wake, whereas scatter plots of P , Q and R were used by Sondergaard *et al.* (1991) to analyse the local flow geometry of a turbulent shear flow. Maekawa *et al.* (1999) and Suman and Girimaji (2010) demonstrated that S2 and S4 topologies dominate $Q - R$ plane for decaying isotropic compressible turbulence. Wang and Lu (2012) analysed topology distributions in the inner and outer layers in turbulent compressible boundary layers. Relatively limited body of work has been directed to the statistical distributions of compressible flow topologies and their roles in turbulent combustion processes.

The flow topology in turbulent premixed flames was analysed for the first time by Tanahashi *et al.* (2000) and they used the analysis to distinguish strain- and vorticity-dominated regions in the reacting flow field. Grout *et al.* (2011) analysed the flow topology of a reactive transverse fuel jet in a cross-flow, and demonstrated that the regions of the highest heat release rates of the flame are associated with the S8 topology. Recently, Cifuentes and his co-workers (Cifuentes *et al.*, 2014; Cifuentes *et al.*, 2016) showed that the probability of finding the focal (nodal) flow topologies decreases (increases) across the flame front using turbulent premixed combustion DNS. Wacks and Chakraborty (2016) analysed flow topology distributions in flame-droplet interaction and indicated qualitative similarities to the previous findings by Cifuentes *et al.* (2014) and Grout *et al.* (2011). Wacks *et al.* (2016) analysed the differences in flow topology distributions in premixed turbulent flames in different combustion regimes and indicated that the likelihood of obtaining the flow topologies, which are obtained only for positive values of dilatation rate, decreases significantly in the flames representing nominally the broken reaction zones regime combustion.

Although the previous analyses (Tanahashi *et al.*, 2000; Grout *et al.*, 2011; Cifuentes *et al.*, 2014; Cifuentes *et al.*, 2016; Wacks and Chakraborty, 2016; Wacks *et al.*, 2016) on flow topologies in turbulent reacting flows provided useful insights into the flow structures and the roles they play in the combustion processes away from the wall, the evolution of flow topologies in the wall-induced premixed flame quenching and their contributions to the wall heat flux are yet to be reported. This gap in existing literature has been addressed here by carrying out three-dimensional compressible DNS of OWQ of a turbulent V-flame by inert isothermal walls. The flame holder is kept closer to one of the walls (here the bottom wall) for the purpose of inducing FWI as previously done by Alshaalan and Rutland (1998, 2002). The evolution of flow topologies in the near-wall region with the progress of flame quenching has been analysed here in detail. Moreover, their contributions to wall heat flux have been identified and physical explanations have been provided for the observed findings. Furthermore, in order to compare wall heat flux, quenching distance and flow topology statistics between turbulent HOQ of statistically planar flames and OWQ of turbulent V-flames, both laminar and turbulent statistically planar turbulent premixed flames in the HOQ configuration have been considered for the same thermo-chemistry and identical inert wall temperature as that of the V-flame case. In this respect, the main objectives of the current analysis are:

- (a) To demonstrate the differences in the statistical behaviours of first- P , second- Q and third- R invariants of the velocity gradient tensor and flow topology distributions between the near-wall region and away from the wall for both HOQ and OWQ of premixed turbulent flames.
- (b) To indicate the differences in wall heat flux, quenching distance and flow topology statistics between HOQ of a statistically planar flame and OWQ of a turbulent V-flame.
- (c) To provide physical explanations for the observed differences and indicate the implications of the findings.

The rest of the paper is organised as follows. The mathematical framework and numerical implementation pertinent to this paper will be provided in Section 2. This will be followed by the discussion of results in Section 3. Finally, main findings will be summarised and conclusions will be drawn.

2. MATHEMATICAL BACKGROUND & NUMERICAL IMPLEMENTATION

A well-known three-dimensional compressible DNS code SENGGA (Jenkins and Cant, 1999) has been used to simulate OWQ of a V-flame by an inert isothermal sidewall. In SENGGA, the governing equations of mass, momentum, energy and species are solved in a coupled manner in non-dimensional form. A single step irreversible chemistry (i.e. Fuel + s Oxidiser $\rightarrow$ (1 + s)Products) is used for the purpose of computational economy. The simulation domain for the OWQ configuration is schematically shown in Fig. 2a. The simulation domain is taken to be a rectangular box of size $175.8 \delta_Z \times 58.5 \delta_Z \times 58.5 \delta_Z$ (the long-side of the domain is aligned with the x_1 -direction), where $\delta_Z = \alpha_{T,0}/S_L$ is the Zel'dovich thickness with $\alpha_{T,0}$ and S_L being the thermal diffusivity of the unburned gas and the unstrained laminar burning velocity respectively. The computational domain is discretised by a uniform Cartesian mesh of size $900 \times 300 \times 300$, which ensures at least 10 grid points across the thermal flame thickness $\delta_{th} = (T_{ad} - T_0)/\max|\nabla\hat{T}|_L$, where T_{ad} , T_0 and $\hat{T}$ are the adiabatic, unburned and instantaneous temperature respectively, and the sub-script L denotes the steady unstrained planar flame values. Furthermore, this resolution ensures that the normalised grid size $\rho_0 u_\tau \Delta x / \mu_0$ remains smaller than unity, where u_τ and μ_0 are the friction velocity and unburned gas viscosity respectively. No-slip isothermal inert walls with temperature $T_w = T_0$, with zero wall-normal mass flux is specified at $x_2 = 0$ (i.e. lower wall) and $x_2 = L_2$ (for upper wall where L_2 is the domain length in x_2 -direction). A turbulent inflow condition with specified

velocity components and density at $x_1 = 0$, and a partially non-reflecting outflow for the boundary opposite to the isothermal inert wall are specified using the Navier-Stokes Characteristic Boundary Conditions (NSCBC) technique (Poinsot and Lele, 1992). The boundaries in the x_3 -direction are taken to be periodic. A plane is scanned through a frozen field of turbulent velocity fluctuations in order to specify inlet turbulent velocity fluctuations (Chakraborty and Cant, 2004) using Taylor's hypothesis. A flame holder with a radius of $R_{fh} \approx 1.5 \delta_{th}$ is placed at a distance $44 \delta_z$ from the inlet and $14.6 \delta_z$ from the lower wall to ensure the flame interacts more readily with the bottom wall. At the flame holder, the species, temperature and mean velocity distributions were imposed using presumed Gaussian function following Dunstan *et al.* (2011). The formation of boundary layer on the flame holder and the effects of shear generated turbulence due to the flame holder are not considered in this analysis following Dunstan *et al.* (2011) (and also references therein where V-flame was numerically simulated). This paper concentrates on FWI which takes place sufficiently away from the flame holder. Furthermore the locations at which the statistics are taken are far away from the flame holder so that this assumption does not play a significant role in this analysis. The inlet values of u'/S_L and l/δ_{th} are taken to be 5.0 and 1.67 respectively, and the corresponding values of Da and Ka are given by 0.33 and 8.65 respectively for the OWQ V-flame case. These values are representative of the values encountered in typical Spark Ignition engines (see Fig. 4.20 of Poinsot and Veynante, 2001). The mean inlet velocity U_{mean} is taken to be $12.0 S_L$ for the V-flame simulation. The V-flame simulation has been carried out for more than two complete flow-through times (i.e. $2.39 t_{ft} = 2.39 L_1/U_{mean}$, where L_1 is the domain length in x_1 -direction). Standard values are considered for the Zel'dovich number $\beta = T_{ac}(T_{ad} - T_0)/T_{ad}^2$ and the ratio of specific heats γ (i.e. $\beta = 6.0$ and $\gamma = 1.4$), T_{ac} is the activation temperature. The oxidiser to fuel ratio by mass s , heat release parameter $\tau = (T_{ad} - T_0)/T_0$ and equivalence ratio ϕ are taken to be 4.0, 2.3 and 1.0 respectively. The Lewis number Le of

all the species are taken to be unity for all cases considered here. The value of $s = 4.0$ is representative of methane-air combustion. A 10th order central difference scheme is used for spatial discretisation of the internal points and the order of differentiation drops gradually to a one-sided 2nd order scheme at the non-periodic boundaries (Jenkins and Cant, 1999). A low storage 3rd order Runge-Kutta scheme has been used for explicit time-marching (Wray, 1990). The time step size Δt is taken to be $2.1 \times 10^{-4} \alpha_{T,0}/S_L^2$ which leads to a Courant number of the order of 0.1.

In order to compare wall heat flux, quenching distance and flow topology statistics a HOQ configuration for a statistically planar premixed flame has been considered which has the same thermo-chemistry and numerical methodology as that of the V-flame case. The simulation domain for the HOQ configuration is schematically shown in Fig. 2b. The simulation domain for the HOQ case is taken to be $58.5 \delta_Z \times 58.5 \delta_Z \times 58.5 \delta_Z$ which is discretised by a uniform Cartesian mesh of size $300 \times 300 \times 300$. Moreover, the initial values of u'/S_L and l/δ_{th} are taken to be 5.0 and 1.67 respectively. In HOQ configuration, an isothermal inert non-slip wall with $T_w = T_0$ is specified at $x_1 = 0$, and the mass flux is specified to be zero in the wall normal direction. The boundary opposite to the wall is taken to be partially non-reflecting, whereas the transverse boundaries are taken to be periodic. Initially the isosurface corresponding to $(\hat{T} - T_0)/(T_{ad} - T_0) = 0.9$ is kept $30 \delta_Z$ away from the wall and the simulations have been continued until the maximum and minimum value of wall heat flux following the flame quenching assume same values, which correspond to $t \geq 20 \delta_Z/S_L$. The time step size of the HOQ configuration has been taken to be the same as that of the V-flame configuration. The numerical implementation related details for the HOQ configuration have been provided elsewhere (Lai and Chakraborty, 2016a-d; Lai *et al.*, 2017a-d) in detail, and thus are not repeated here for the

sake of conciseness. The OWQ simulation needed about 100,000 CPU hours, whereas the HOQ simulation required about 20,000 CPU hours before the statistics were extracted.

3. RESULTS & DISCUSSION

The instantaneous distribution of normalised vorticity magnitude (i.e. $\sqrt{\omega_i \omega_i} \times \delta_Z / S_L$ with ω_i being the i^{th} component of vorticity) is shown in Fig. 3a where the instantaneous distributions of non-dimensional temperature T and fuel mass fraction Y_F on the bottom wall surface for the OWQ V-flame case are also shown. Figure 3a shows a significant decay in the magnitude of $\sqrt{\omega_i \omega_i} \times \delta_Z / S_L$ with the increases (decreases) in T (Y_F) across the flame front. The flame quenches due to heat loss through the bottom wall which leads to diffusion of remaining fuel from the near-wall region to the gaseous mixture at the interior of the domain and thus the magnitude fuel mass fraction Y_F drops in the region where the flame interacts with the wall. The distributions of $\sqrt{\omega_i \omega_i} \times \delta_Z / S_L$, Y_F and T at two different time instants for the HOQ are shown in Fig. 3b, which also reveals that Y_F at the wall depletes with time. Moreover, a comparison between the $\sqrt{\omega_i \omega_i} \times \delta_Z / S_L$, Y_F and T fields reveals that the normalised vorticity magnitude $\sqrt{\omega_i \omega_i} \times \delta_Z / S_L$ decreases from the unburned gas side to the burned gas side of the flame also in the HOQ case.

For the present analysis, the reaction progress variable c is defined in terms of fuel mass fraction Y_F as: $c = (Y_{F0} - Y_F) / (Y_{F0} - Y_{F\infty})$ where the subscripts 0 and ∞ denote the values in the unburned gas and fully burned products, respectively (here $Y_{F0} = 0.055$ and $Y_{F\infty} = 0$). The contours of T and c are shown for the $x_1 - x_2$ mid-plane in Figs. 4a and 4b for the OWQ and HOQ cases respectively. Three different locations A1, B1 and C1 corresponding to $x_1 = 60 \delta_Z$, $100 \delta_Z$ and $140 \delta_Z$ respectively has been considered for the V-flame OWQ case, so that, different stages of OWQ can be compared to the evolution of HOQ of the statistically planar

flame. In the OWQ case, location A1 represents the flame growth region with no interaction with the wall, whereas the flame begins to interact with the wall and starts to quench in the vicinity of the wall at location B1 and the flame quenching effects are prominent at location C1. Additionally, the influence of the walls on the lower (facing towards the bottom wall) and the upper (facing towards the top wall) branches of the V-flame are different and the nature of this interaction changes with the distance from the flame holder. It can be seen from Figs. 3b and 4b that in the HOQ case the flame propagates without FWI at $t = 5 \delta_z/S_L$, whereas at $t = 10 \delta_z/S_L$ the flame begins to interact with the wall, and $t = 15 \delta_z/S_L$ represents a stage when the flame quenching is an advanced stage.

A careful comparison between c and T reveals that $c = T$ where the flame is away from the wall. However, an inequality between the reaction progress variable and non-dimensional temperature (i.e. $c \neq T$) is obtained in the vicinity of the wall for both OWQ and HOQ cases. The difference in boundary condition (i.e. Dirichlet boundary condition for temperature and Neumann boundary condition for species mass fractions) leads to the inequality between c and T . This inequality between c and T is consistent with previous analyses on FWI (Poinsot *et al.*, 1993; Brunenux *et al.*, 1997; Alshaalan and Rutland, 1998, 2002; Lai and Chakraborty, 2016a-d; Lai *et al.*, 2017a-d).

The temporal evolutions of the maximum, mean and minimum values of non-dimensional wall heat flux $\Phi = |q_w|/[\rho_0 C_{p0} S_L (T_{ad} - T_0)]$ and the minimum value of Peclet number $Pe = X/\delta_z$ for top and bottom walls for the V-flame case is shown in Fig. 5, where X is the wall normal distance of the nearest $T = 0.9$ isosurface (Poinsot *et al.*, 1993) and $q_w = -\lambda(\partial\hat{T}/\partial n)_w$ is the wall heat flux with ρ_0, C_{p0}, λ and n being the unburned gas density, specific heat at constant pressure of the unburned gas, thermal conductivity and wall normal

direction respectively. The wall normal distance of the $T = 0.9$ isosurface is considered for defining the wall Peclet number because the chemical reaction rate for a freely propagating premixed laminar flame attains its maximum value for a value of T , which is close to 0.9 for this thermo-chemistry (Poinsot *et al.*, 1993; Lai and Chakraborty, 2016a-d; Lai *et al.*, 2017a-d). Only the minimum value of Peclet number Pe_{min} is shown for the OWQ V-flame case because the maximum and mean values of Peclet number are of limited relevance in this case because of two flame branches in this configuration.

The temporal variations of the maximum, mean and minimum values of Φ and Peclet number Pe for HOQ of a statistically planar flame with initial values of u'/S_L and l/δ_{th} equal to the inlet values for the turbulent V-flame case are also shown in Fig. 5. The value of Pe drops with time as the flame advances towards the wall, whereas the value of Φ increases as Pe decreases with time. The maximum value of normalised wall heat flux Φ_{max} is obtained at a time when the minimum Peclet number Pe_{min} is attained. The values of Φ_{max} and Pe_{min} for laminar flame HOQ are given by 0.39 and 2.53 respectively. The magnitude of $|q_w|$ can be scaled as: $|q_w| \sim \lambda (T_{ad} - T_0)/X$, which leads to $\Phi \sim 1/Pe$ and accordingly one obtains the following relation: $\Phi_{max} \sim 1/Pe_{min}$. The values of Φ_{max} and Pe_{min} in the laminar HOQ case are consistent with previous experimental (Vosen *et al.*, 1984; Huang *et al.*, 1986; Jarosinsky, 1986) and computational (Poinsot *et al.*, 1993; Lai and Chakraborty, 2016a-d; Lai *et al.*, 2017a-d) analyses. For turbulent HOQ one obtains $\Phi_{max} = 0.42$ and $Pe_{min} = 2.16$ which suggest that the maximum heat flux and the minimum Peclet number values in turbulent HOQ remain almost equal to the corresponding values for laminar HOQ. By contrast, Pe_{min} for the turbulent V-flame case is found to be 1.71 whereas Φ_{max} assumes a value of 0.63. According to the scaling $\Phi_{max} \sim 1/Pe_{min}$, a smaller value of Pe_{min} in the turbulent V-flame case than in the HOQ of statistically planar flames leads to a higher value of normalised wall heat flux Φ_{max} .

in the turbulent V-flame case. It can be seen from Fig. 5 that Pe values are higher for the top wall than for the bottom wall because the flame holder is placed closer to the bottom wall so that FWI takes place more readily for the bottom wall. For the turbulent HOQ case, the maximum value of Peclet number increases initially with time due to the flame wrinkles, which are concavely curved towards the reactants. The maximum, mean and minimum values of Peclet number decrease as the flame advances towards the wall and this trend continues until flame quenching. The decreases of the maximum, mean and minimum values of Peclet number are associated with the increases in minimum, mean and maximum values of Φ with time in the HOQ configuration. After flame quenching the isotherms move away from the wall (Lai and Chakraborty, 2016a) in the HOQ configuration. By contrast, $Pe_{\min}$ does not change much following flame quenching in the V-flame configuration. In the case of oblique flame quenching the fluid velocity remains small in the near-wall region and thus the flame can propagate closer to the wall before quenching than in the corresponding HOQ case. It can be seen from Fig. 5 that the maximum normalised heat flux $\Phi_{\max}$ for both top and bottom walls remain comparable but $\Phi_{\max}$ for the lower wall attains higher value than the value obtained for the top wall. The smaller values of the minimum Peclet number for the bottom wall are reflected in the higher value of $\Phi_{\max}$ than the top wall. Moreover, the mean heat flux is greater for the bottom wall than that for the top wall because the flame remains closer to the bottom wall. It can further be seen from Fig. 5 that the extent of the fluctuation of $\Phi_{\max}$ for the top wall is greater than the bottom wall. The wall heat flux rises sharply when the turbulent fluid motion brings flame elements close to the wall and similarly heat flux drops when either the flame quenches or the flame moves away from the wall under the influence of turbulence. As the bottom wall remains closer to the flame, it interacts more readily with the flame than the top wall, which leads to relatively less rapid changes of $\Phi_{\max}$ for the bottom wall.

In order to understand the wall heat flux behaviour conditional upon local flow topologies, the descriptions of different flow topologies need to be presented. The local flow topologies are characterised by the invariants of the velocity-gradient tensor (Chong *et al.*, 1990): $A_{ij} = \partial u_i / \partial u_j = S_{ij} + W_{ij}$, where $S_{ij} = 0.5(A_{ij} + A_{ji})$ is the symmetric strain-rate tensor and $W_{ij} = 0.5(A_{ij} - A_{ji})$ is the anti-symmetric rotation rate tensor. The eigenvalues of the velocity gradient tensor A_{ij} are λ_1 , λ_2 and λ_3 , which are the solutions of the characteristics equation $\lambda^3 + P\lambda^2 + Q\lambda + R = 0$ with the invariants P , Q and R as specified below (Chong *et al.*, 1990):

$$P = -\text{tr}(A_{ij}) = -(\lambda_1 + \lambda_2 + \lambda_3) = -S_{ii} \quad (1)$$

$$Q = 0.5 \left([\text{tr}(A_{ij})]^2 - \text{tr}(A_{ij}^2) \right) = 0.5(P^2 - S_{ij}S_{ij} + W_{ij}W_{ij}) = Q_s + \underbrace{W_{ij}W_{ij}/2}_{Q_w} \quad (2)$$

$$R = -\det(A_{ij}) = (-P^3 + 3PQ - S_{ij}S_{jk}S_{ki} - 3W_{ij}W_{jk}S_{ki})/3 \quad (3)$$

The discriminant D , as shown below, divides the $P - Q - R$ phase space into two regions: A_{ij} shows a focal topology for $D > 0$ and it displays a nodal topology for $D < 0$ (Chong *et al.*, 1990):

$$D = [27R^2 + (4P^3 - 18PQ)R + 4Q^3 - P^2Q^2]/108 \quad (4)$$

The surface $D = 0$ leads to two surfaces r_{1a} and r_{1b} in the $P - Q - R$ phase space:

$$r_{1a} = P(Q - 2P^2/9)/3 - 2(-3Q + P^2)^{3/2}/27 \quad (5)$$

$$r_{1b} = P(Q - 2P^2/9)/3 + 2(-3Q + P^2)^{3/2}/27 \quad (6)$$

Additionally, A_{ij} has purely imaginary eigenvalues on the surface r_2 which is given by $R = PQ$. The surfaces r_{1a} , r_{1b} and r_2 , where r_2 is described by $PQ - R = 0$, divide the $P - Q - R$ phase space into the eight flow topologies as shown in Fig. 1b. Equations 1-6 and Fig. 1b indicate that the statistics of the invariants P , Q and R of the velocity gradient tensor $A_{ij} =$

$\partial u_i / \partial x_j$ need to be analysed in detail prior to the discussion of the distributions of flow topologies and their contributions to the normalised wall heat flux magnitude Φ .

The mean values of $P^* = P \times \delta_Z / S_L$ conditional upon c (i.e. $\langle P^* \rangle$ where the angled brackets are used to indicate the mean values conditional on bins of c , and the superscript $*$ is used to indicate the normalised quantities in this paper) at $t = 2.39 t_{ft}$ ¹ for locations A1, B1 and C1 in the OWQ V-flame case are shown in Fig. 6a, but the qualitative nature of the distributions of all the invariants (i.e. P , Q and R) remain unchanged since $t = 1.0 t_{ft}$. It can be seen from Fig. 6a that $\langle P^* \rangle$ remains negative, since the dilatation rate $\nabla \cdot \vec{u} = -P$ assumes predominantly positive values within the flame as the result of the thermal expansion due to chemical heat release. However, $\langle P^* \rangle$ decreases from the location A1 to the location C1.

Figure 4 reveals that the extent of FWI is greater at location C1 than at location A1, and thus, the effects of flame quenching are more prominent at location C1 than at location A1. The magnitude of $\langle P^* \rangle$ based on the samples from the bottom branch of the V-flame turns out to be comparable to the corresponding value evaluated based on the samples from the top branch at locations A1 and B1, where the flame either does not interact or begins to interact with the wall. However, $\langle P^* \rangle$ magnitude from the bottom branch is smaller than the corresponding value from the top branch at location C1, because the effects of thermal expansion and dilatation rate $\nabla \cdot \vec{u}$ are weak for the bottom branch as a result of flame quenching.

¹ The statistics for the OWQ case in this paper are presented at $t = 2.39 t_{ft}$ but the configuration reaches a statistically stationary state since one through pass time (i.e. $t = 1.0 t_{ft}$) so the statistics remain the same (see Fig. 11 where the statistics remain qualitatively the same) for $t \geq 1.0 t_{ft}$. Here $t = 2.39 t_{ft}$ is explicitly mentioned in the figure captions for the purpose of completeness.

The variations of $\langle P^* \rangle$ with c at different time instants for the HOQ case are shown in Fig. 6b which shows that $\langle P^* \rangle$ assumes predominantly negative values because of overwhelmingly positive values of $\nabla \cdot \vec{u}$ in premixed turbulent flames. However, the magnitude of the negative value of $\langle P^* \rangle$ decreases with time as the quenching progresses. The effects of heat release weaken as the flame quenching progresses with time, which reduces the magnitude of positive dilatation rate. It is worth noting that $\nabla \cdot \vec{u}$ becomes negative close to the wall due to the reversal of flow direction at the advanced stages of HOQ, which leads to positive values of $\langle P^* \rangle$ at later times.

The normalised mean values of the second invariant of velocity gradient $\partial u_i / \partial x_j$ tensor $\langle Q^* \rangle = \langle Q \rangle \times (\delta_z / S_L)^2$, second invariant of the symmetric part of the velocity gradient tensor $\langle Q_s^* \rangle = \langle P^2 - S_{ij}S_{ij} \rangle / 2 \times (\delta_z / S_L)^2$ and only invariant of the rotation-rate tensor $\langle Q_w^* \rangle = \langle W_{ij}W_{ij} \rangle / 2 \times (\delta_z / S_L)^2$ conditional upon c at $t = 2.39t_{ft}$ for locations A1, B1 and C1 are shown in Fig. 7a for the OWQ V-flame case. The second invariant Q of the velocity gradient tensor provides a measure of the relative importance of the symmetric (i.e. strain rate part) and anti-symmetric (i.e. rotation rate part) parts of the velocity gradient tensor A_{ij} . Thus a positive value of Q is indicative of the dominance of the rotational part of velocity gradient tensor over its symmetric part, and *vice versa*. The value of the invariant of the rotation-rate tensor $\langle Q_w^* \rangle$ remains deterministically positive throughout the flame front for all locations. However, the qualitative behaviour of $\langle Q_w^* \rangle$ is different for locations A1, B1 and C1. The value of $\langle Q_w^* \rangle$ decreases from the unburned to the burned gas side of the flame front at location A1, whereas at location B1 $\langle Q_w^* \rangle$ increases from the fresh reactants to a peak value close to the unburned gas side of the flame front before decaying towards the burned gas side. By contrast, $\langle Q_w^* \rangle$ increases with c before decreasing in magnitude close to the burned gas side of the flame front at location C1. It can be seen from Figs. 3 and 4 that the flame does not interact significantly at location A1,

and thus, the contribution of dilatation rate remains significant. As a result of this, the contribution of $\langle P^2 \rangle = \langle (\nabla \cdot \vec{u})^2 \rangle$ at location A1 overcomes the negative values of $\langle -(S_{ij}S_{ij})/2 \rangle$ to yield positive values of the normalised second invariant $\langle Q_S^* \rangle$ of the strain rate tensor at the middle of the flame where P assumes large magnitudes (see Fig. 6). The values of $\langle P^2 \rangle$ remain small for both unburned and burned gas side of the flame, and thus the normalised second invariant $\langle Q_S^* \rangle$ of the strain rate tensor assumes negative values due to the predominance of $\langle -(S_{ij}S_{ij})/2 \rangle$. At locations B1 and C1, the flame reaches close to the wall and starts to quench. The presence of the flame in the vicinity of the wall gives rise to an increase in velocity gradient magnitude and a weakening of dilatation rate as a result of flame quenching. The strengthening of velocity gradient magnitude due to the increase in the magnitude of the negative value of $\langle -(S_{ij}S_{ij})/2 \rangle$ along with the weakening of $\langle P^2 \rangle$ contribution leads to negative values of the second invariant $\langle Q_S^* \rangle$ of the strain rate tensor throughout the flame at locations B1 and C1. The strong velocity gradient magnitude in the reacting thermal boundary layer yields higher magnitudes of the positive (negative) values of $\langle Q_W^* \rangle$ ($\langle Q_S^* \rangle$) at locations B1 and C1 than at location A1. The decay of enstrophy $\Omega = \omega_i \omega_i / 2 = W_{ij}W_{ij}$ on the burned gas side leads to a decay in the normalised invariant $\langle Q_W^* \rangle$ of the rotation rate tensor at the burned gas region at locations B1 and C1. There is no flame normal acceleration in the fully burned region, which acts to reduce the magnitudes of $W_{ij}W_{ij}$ and $S_{ij}S_{ij}$, which along with the increase in the boundary layer thickness reduces the magnitude of velocity gradients at location C1 in comparison to that at location B1 (because the probability of obtaining burned gas is greater at location C1 than at location B1). At locations B1 and C1 the negative values of the normalised second invariant $\langle Q_S^* \rangle$ of the symmetric part of the velocity gradient tensor are almost balanced by positive values of the normalised invariant $\langle Q_W^* \rangle$ of the anti-symmetric part, and thus the magnitude of the normalised second invariant $\langle Q^* \rangle$ of $A_{ij} = \partial u_i / \partial x_j$ remains smaller than the magnitudes of $\langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ but $\langle Q^* \rangle$ remains

positive at these locations. The invariants $\langle Q_S \rangle$ and $\langle Q_W \rangle$ of the symmetric and anti-symmetric parts of the velocity gradient tensor remain negligible at A1 location, in comparison to the statistics at B1 location.

The variations of the **normalised second invariant of $\langle Q^* \rangle$ of the velocity gradient tensor**, and its components, $\{\langle Q_S^* \rangle, \langle Q_W^* \rangle\}$ with c for the HOQ case are shown in Fig. 7b for different time instants. It is evident from Fig. 7b that high values of the normalised **invariant $\langle Q_W^* \rangle$ of the rotation rate tensor** are obtained at the wall but also increases within the flame due to flame generated vorticity due to baroclinic torque (Lai *et al.*, 2017b). Figure 6b shows that **the normalised second invariant $\langle Q_S^* \rangle$ of the strain rate tensor** predominantly assumes negative values and assumes locally small positive values away from the wall within the flame where $\langle P^2 \rangle / 2$ dominates over $\langle -S_{ij}S_{ij} \rangle / 2$. This is qualitatively consistent with the behaviours of $\langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ in the OWQ V-flame case at location A1. The effects of dilatation rate $\nabla \cdot \vec{u} = -P$ are weak in the vicinity of the wall and thus $\langle -S_{ij}S_{ij} \rangle / 2$ dominates over $\langle P^2 \rangle / 2$ to give rise to a negative value of Q_S . The magnitude of $\langle S_{ij}S_{ij} \rangle / 2$ decays within the flame, whereas $\langle P^2 \rangle / 2$ increases in reaction zone so under some conditions $\langle P^2 \rangle / 2$ may overcome $\langle -S_{ij}S_{ij} \rangle / 2$ in this region and yield positive values of **the second invariant $\langle Q_S \rangle$ of the strain rate tensor**. In the HOQ case, the invariants $\langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ mostly balance each other and as a result the magnitude of $\langle Q^* \rangle$ remains negligible in comparison to those of $\langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ especially at the advanced stages of HOQ, which is qualitatively consistent with the behaviours of $\langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ in the OWQ V-flame case at locations B1 and C1. The magnitudes of $\langle Q^* \rangle, \langle Q_S^* \rangle$ and $\langle Q_W^* \rangle$ decay with time as quenching progresses in the HOQ case.

The second invariant Q_S of the strain rate tensor can be expressed as: $Q_S = Q_{S1} + Q_{S2} = P^2/3 - E/4\nu$ with $E = (\tau_{ij}\partial u_i/\partial x_j)/\rho$ and ν being the dissipation rate of instantaneous

kinetic energy (i.e. $u_i u_i / 2$) and kinematic viscosity respectively. Thus, $Q_S > 0$ ($Q_S < 0$) corresponds to dilatation (dissipation) dominated regions. It was suggested by Wacks *et al.* (2016) that $Q_{S1} / |Q_{S2}| \sim \tau^2 K a^{-2}$, using $Q_{S1} = P^2 / 3 \sim \{\tau S_L / \delta_{th}\}^2$ (Chakraborty and Swaminathan, 2007) and $|Q_{S2}| = | -E / 4\nu | \sim 1 / \tau_\eta^2$ with τ_η being the Kolmogorov time scale. The flames considered here belong to $Ka > 1$ combustion and thus $|Q_{S2}|$ dominates over Q_{S1} to yield predominantly negative values of the second invariant of Q_S of the strain rate tensor.

The third invariant R of the velocity gradient tensor $A_{ij} = \partial u_i / \partial x_j$ can be recast in terms of the sum of the terms which play roles in dissipation rate generation ($-S_{ij} S_{jk} S_{ki} / 3$) and enstrophy production ($PQ_W - \omega_i S_{ij} \omega_j / 4$):

$$\begin{aligned} R &= \frac{1}{3} (-P^3 + 3PQ - S_{ij} S_{jk} S_{ki}) - \frac{1}{4} \omega_i S_{ij} \omega_j \\ &= \underbrace{\frac{1}{3} (-P^3 + 3PQ_S - S_{ij} S_{jk} S_{ki})}_{R_S} + PQ_W - \frac{1}{4} \omega_i S_{ij} \omega_j \end{aligned} \quad (7)$$

The variations of the normalised third invariant $\langle R \rangle \times (\delta_Z / S_L)^3$ of the velocity gradient tensor, and its components $\langle R_S \rangle \times (\delta_Z / S_L)^3$, $\langle PQ_W \rangle \times (\delta_Z / S_L)^3$ and $\langle -\omega_i S_{ij} \omega_j / 4 \rangle \times (\delta_Z / S_L)^3$ conditional upon c at $t = 2.39 t_{ft}$ for locations A1, B1 and C1 are shown in Fig. 8a for the OWQ case. The third invariant R of the velocity gradient tensor includes the third invariant R_S of the strain rate tensor S_{ij} , and the terms PQ_W and $-\omega_i S_{ij} \omega_j / 4$, which are related to the enstrophy transport (Cifuentes *et al.*, 2014, Cifuentes, 2015). Figure 8a indicates that $\langle R_S \rangle \times (\delta_Z / S_L)^3$ remains positive, whereas $\langle PQ_W \rangle \times (\delta_Z / S_L)^3$ assumes negative value throughout the flame front. The positive values of $\langle R_S \rangle \times (\delta_Z / S_L)^3$ almost balance the negative values of $\langle PQ_W \rangle \times (\delta_Z / S_L)^3$ and the magnitudes of $\langle -\omega_i S_{ij} \omega_j / 4 \rangle \times (\delta_Z / S_L)^3$ and $\langle R \rangle \times (\delta_Z / S_L)^3$ remain negligible in comparison to those of $\langle R_S \rangle \times (\delta_Z / S_L)^3$ and $\langle PQ_W \rangle \times (\delta_Z / S_L)^3$. The

combination of positive values of the invariant Q_W of the rotation rate tensor and predominantly negative values of the first invariant $P = -\nabla \cdot \vec{u}$ of $A_{ij} = \partial u_i / \partial x_j$ leads to overwhelming probability of obtaining negative values of PQ_W . In order to explain positive mean value of the normalised third invariant $\langle R_S \rangle \times (\delta_Z / S_L)^3$ of the strain rate tensor, the contributions of its components (i.e. $\{-P^3/3, \langle PQ_S \rangle, \langle -S_{ij}S_{jk}S_{ki}/3 \rangle\} \times (\delta_Z / S_L)^3$) conditional upon c are also shown in Fig. 8b at $t = 2.39t_{ft}$ for locations A1, B1 and C1 for the OWQ case. It can be seen from Fig. 8b that the positive value of third invariant $\langle R_S \rangle$ of the strain rate tensor arises principally due to $\langle -P^3/3 \rangle$, which predominantly assumes positive values due to overwhelmingly negative values of $P = -\nabla \cdot \vec{u}$ in premixed turbulent flames. The contribution of $\langle -S_{ij}S_{jk}S_{ki}/3 \rangle$ assumes negative value throughout the flame front at all locations considered here. It is possible to express $(\nabla \cdot \vec{u})E$ as $(\nabla \cdot \vec{u})E = 4\nu(PQ_S - P^3/3)$. The relative magnitudes of $\langle PQ_S \rangle$ and $\langle -P^3/3 \rangle$ in Fig. 8b show that $\langle PQ_S - P^3/3 \rangle = \langle E/4\nu \rangle$ remains positive throughout the flame. The combination of predominantly positive value of dilatation rate $\nabla \cdot \vec{u} = -P$ and positive value of the dissipation rate of instantaneous kinetic energy $E = 2\nu(S_{ij}S_{ij} - P^2/3)$ indicates that the predominance of positive net values of $(PQ_S - P^3/3)$ because of the weak correlation between $\nabla \cdot \vec{u}$ and E . It is worth noting that the third invariant $R_S = (-P^3 + 3PQ_S - S_{ij}S_{jk}S_{ki})/3$ of the strain rate tensor contains a contribution to the dissipation rate generation (i.e. $S_{ij}S_{jk}S_{ki}$), whereas $(PQ_W - \omega_i S_{ij} \omega_j / 4)$ contributes to the enstrophy transport (Cifuentes *et al.*, 2014, Cifuentes, 2015, Wacks and Chakraborty, 2016; Wacks *et al.*, 2016). Figure 7a indicates that $(PQ_W - \omega_i S_{ij} \omega_j / 4)$ dominates over R_S in the V-flame configuration and this tendency weakens in the downstream direction.

The variations of the normalised third invariant $\langle R^* \rangle = \langle R \rangle \times (\delta_Z / S_L)^3$ of the velocity gradient tensor $A_{ij} = \partial u_i / \partial x_j$, and its components, $\{\langle R_S \rangle, \langle PQ_W \rangle, \langle -\omega_i S_{ij} \omega_j / 4 \rangle\} \times (\delta_Z / S_L)^3$, with c for the HOQ case are shown in Fig. 8c for different time instants and the corresponding

variations of the normalised values of the components of the **normalised third invariant $\langle R_S \rangle \times (\delta_Z/S_L)^3$ of the strain rate tensor** (i.e. $\{\langle -P^3/3 \rangle, \langle PQ_S \rangle, \langle -S_{ij}S_{jk}S_{ki}/3 \rangle\} \times (\delta_Z/S_L)^3$) are shown in Fig. 8d. Qualitatively similar to the OWQ V-flame case, $\langle -P^3/3 \rangle$ in the HOQ case assumes predominantly positive values within the flame due to overwhelming probability of obtaining negative values of P when the flame remains away from the wall but the magnitude of $\langle -P^3/3 \rangle$ decreases significantly with the progress of HOQ. Moreover, predominantly negative values of the invariants P and Q_S (see Figs. 6 and 7) lead to positive values of $\langle PQ_S \rangle$ and its magnitude also decays with time. The contribution of $\langle -S_{ij}S_{jk}S_{ki}/3 \rangle$ shows large negative values within the flame front for the HOQ case, which is qualitative similar to the behaviour observed in the OWQ V-flame case. It can be seen from Fig. 8c that the magnitudes of $\langle -\omega_i S_{ij} \omega_j / 4 \rangle$ and $\langle R \rangle$ are negligible in comparison to the predominantly positive $\langle R_S \rangle$ and negative $\langle PQ_W \rangle$ contributions when the flame remains away from the wall (i.e. at the early stages of flame quenching, e.g. $t = 2\delta_Z/S_L$), which is consistent with the behaviour observed for the OWQ case in Fig. 8a, but $\langle R_S \rangle$ attains negative values towards the burned gas side of the flame front at later stages of the HOQ case (e.g. $t = 10\delta_Z/S_L$).

It is worth noting that a negative value of $\langle -S_{ij}S_{jk}S_{ki}/3 \rangle$ is not sufficient to indicate that the strain rate destruction instead of its generation. The combination of the terms $\langle -S_{ij}S_{jk}S_{ki}/3 - \omega_i \omega_j S_{ij}/4 - S_{ij}(\partial^2 p / \partial x_i \partial x_j) \rangle$ represent the inviscid production of total strain rate. Thus, the behaviour of $\langle -\omega_i \omega_j S_{ij}/4 - S_{ij}(\partial^2 p / \partial x_i \partial x_j) \rangle$ along with the distribution of $\langle -S_{ij}S_{jk}S_{ki}/3 \rangle$ governs the overall strain rate production statistics but this aspect is beyond the scope of the current analysis and will not be discussed further.

The contours of joint pdf between the normalised second and third invariants (i.e. Q^* and R^*) of the velocity gradient tensor for the OWQ V-flame case are shown in Fig. 9a for $c = 0.3, 0.5$ and 0.7 at $t = 2.39t_{ft}$ whereas the corresponding joint pdf for the HOQ case are shown for different time instants in Fig. 9b. A dominant negative correlation between Q^* and R^* characteristic of teardrop shape has been observed towards the unburned gas side of the flame front when the flame remains away from the wall for both cases. Furthermore, for both OWQ and HOQ cases, the joint pdf between Q^* and R^* reduces to a scatter around the origin and the teardrop structure is lost as turbulence decays across the flame. It can be seen from Fig. 9a that the negative correlation between Q^* and R^* on the unburned gas side weakens from location A1 to C1 in the OWQ case, as quenching progresses and the flame interacts the wall. The same qualitative behaviour has been observed for the HOQ case where the negative correlation between Q^* and R^* weakens with time as the flame approaches the wall and gets increasingly quenched. At advanced stages of flame quenching, the joint pdf between Q^* and R^* does not show a weak correlation which is qualitatively similar to the weak correlation observed at location C1 for the OWQ V-flame case. However, the distributions of Q^* for top and bottom branches of V-flames are qualitatively different. The probability of finding negative values of Q^* increases from location A1 and location C1 for the bottom branch, whereas an opposite behaviour has been observed for the top branch. The effects of $P = -\nabla \cdot \vec{u}$ are weak for the bottom branch of the V-flame and thus the probability of finding negative $Q = 0.5(P^2 - S_{ij}S_{ij}) + 0.5W_{ij}W_{ij}$ increases from location A1 to location C1 because of decay of enstrophy (and also $W_{ij}W_{ij}$) and increasingly small values of P due to the progress of flame quenching from the location A1 to C1. By contrast, the top branch of the V-flame reaches increasingly close to the wall from location A1 to location C1 and thus the wall-induced vorticity magnitude increases from location A1 to location C1, which reduces the probability of obtaining negative values of Q .

The aforementioned statistical behaviours of P , Q and R determine the distribution of flow topologies and provide insights into the flow topologies which contribute significantly to the heat transfer to the wall and thereby play key roles in flame quenching. In order to understand the flow contribution to the wall heat flux, the distributions of the volume fractions VF of the flow topologies conditional on reaction progress variable c at locations A1, B1 and C1 are shown in Fig. 10a at $t = 2.39t_{ft}$. The variations of volume fractions VF conditional on c for the HOQ case at different time instants are shown in Fig. 10b following the previous analyses by Cifuentes *et al.* (2014) and Cifuentes (2015). It can be seen from Fig. 10a that the S2 and S7 topologies are the leading contributors within the flame front at location A1 where the V-flame does not interact with the wall. This is consistent with the topology variation in the corresponding turbulent HOQ case at early times when the flame is away from the wall (see $t = 5\delta_z/S_L$ in Fig. 10b). The topologies which are typical of negative dilatation rate (i.e. S5 and S6) are rare at location A1 but the volume fraction of S5 topology assumes non-negligible value at locations B1 and C1 where the flame quenches as a result of heat transfer through the wall. A similar increase of VF of S5 and S6 topologies can be observed at later times in the turbulent HOQ case during advanced stages of flame quenching (e.g. $t = 20\delta_z/S_L$). A comparison between locations A1, B1 and C1 reveals that FWI and flow development in the downstream of the flame holder significantly affect the distribution of flow topologies. The relative contribution of the S7 topology decreases from location A1 to location C1, whereas the relative contribution of S8 topology increases in the downstream direction, and it becomes a dominant contributor at location C1. A qualitatively similar transition in behavior can also be observed in the turbulent HOQ case, as quenching progresses with time. It is worth noting that the S8 topology is obtained only for a combination of small positive values of Q and small negative values of R , whereas the S7 topology is favoured for large positive values of Q (see

Fig. 1). As the probability of finding small magnitudes of R increases, whereas the magnitude of Q decays with the progress of flame quenching (see Fig. 7), the probability of finding the S8 topology increases at the cost of reduction of the likelihood of obtaining the S7 topology. The S2 topology remains a dominant flow feature and the flow configurations corresponding to S1, S3 and S4 topologies remain significant especially towards both unburned and burned gas sides of the flame front where the effects of heat release are relatively weak in the both OWQ and HOQ cases.

The percentages of wall heat flux contribution by individual flow topologies for top and bottom walls for the V-flame and HOQ cases are shown in Fig. 11 for different time instants. Figure 11 shows that the S1 and S4 topologies contribute significantly to the wall heat flux for both walls. However, the S1 topology is the leading contributor for the bottom wall, whereas the S4 topology is the leading contributor for the top wall. It can be seen from Fig. 10 that VF values for the S1 and S4 topologies increase in the unburned and burned gas regions ($c = 0$ and $c = 1$). As either unburned gas before the flame quenching or burned gas after flame quenching is predominantly found at the wall, the topologies S1 and S4 contribute significantly to the wall heat flux. Moreover, the presence of the wall generates vorticity in the near-wall region and thus the focal topologies S1 and S4 contribute to wall heat flux in the OWQ V-flame case. A comparison between Figs. 10 and 11 reveals that all topologies except for S5 and S6 have comparable wall heat flux contributions for the HOQ case when the flame begins to interact with the wall. A comparison between Figs. 5 and 10 reveals that the nodal topologies S2 and S3 become the major contributors to the wall heat flux when the maximum value of $\Phi_{\max}$ attains its peak value. The contributions of all flow topologies to the wall heat flux become comparable when the maximum, mean and minimum values of Φ approach each other but the contributions of the S1, S3-S8 topologies remain greater than the S2 topology in both HOQ

and OWQ configurations. The above findings suggest that the focal topologies such as S1 and S4 topologies contribute the wall heat flux magnitude in the case of OWQ, whereas the nodal topologies S2 and S3 are the major contributors to wall heat flux in HOQ when the high magnitude of Φ is obtained.

The effects of dilatation rate $\nabla \cdot \vec{u}$ weaken with the progress of flame quenching (see Fig. 6) so the topologies S1-S4 which are obtained for all values of $\nabla \cdot \vec{u}$ (see Fig. 1) play important roles in determining the wall heat flux during FWI. In the OWQ case, the velocity gradient in the wall tangential direction sustains vorticity in the flame quenching zone. This can be substantiated from Fig. 3, which shows that vorticity magnitude remains significant in the near wall region for the bottom wall although the vorticity magnitude drops from the unburned to the burned gas side of the flame front. Thus, the focal topologies S1 and S4 play key roles in determining the wall heat flux in this configuration. By contrast, the mean velocity gradient is aligned with the wall normal direction in the HOQ configuration, which is similar to the canonical configurations represented by S2 and S3 topologies (see Fig. 1). Moreover, Fig. 3 shows that the magnitude of vorticity drops as quenching progresses (see also Lai *et al.*, 2017a) and thus S2 and S3 play key roles in determining the peak value of normalised heat flux magnitude Φ in the HOQ case. As the HOQ configuration does not have any mechanism for sustaining vorticity in the quenching zone and the mean velocity gradient drops significantly due to flame extinction, no preferential contribution of flow topologies to the overall heat flux magnitude has been found at the final stage of flame quenching.

Thus, the fluid-dynamic processes which govern wall heat transfer to the wall for OWQ are fundamentally different from HOQ, and thus the maximum value of $\Phi_{\max}$ and the minimum value of $Pe_{\min}$ (which quantifies the normalised flame quenching distance) in the case of V-

flame wall interaction has been found to be different from the corresponding values obtained for HOQ. The dominant focal topologies in the case of OWQ may locally push the flame elements closer to the wall than in the HOQ case where the nodal topologies play pivotal roles, and this leads to greater (smaller) value of $\Phi_{\max}$ ($Pe_{\min}$) in the case of OWQ than in HOQ.

4. CONCLUSIONS

The statistics of wall heat flux, flame quenching distance in the case of oblique wall quenching (OWQ) of a turbulent V-flame flame by isothermal inert walls have been analysed in terms of the distributions of flow topologies and their contributions to the wall heat flux magnitude using DNS data. Moreover, an additional DNS of Head-on Quenching (HOQ) of a turbulent premixed flame by an isothermal inert wall has been considered for the same thermo-chemistry and also for initial turbulence intensity and integral length scale to flame thickness values same as that of inlet in the V-flame configuration in order to compare the statistics of wall heat flux, quenching distance and flow topology distributions. The flow topologies have been categorised into 8 generic flow configurations (i.e. S1-S8) in terms of the values of three invariants of velocity gradient $\partial u_i / \partial x_j$ (i.e. P , Q and R respectively). The magnitude of predominantly negative $P = -\nabla \cdot \vec{u}$ decreases as a result of the weakening of thermal expansion due to flame quenching. This reduction of the magnitude of P also affects the components of second and third invariants, Q and R . The magnitudes of Q and R also decay with the progress of flame quenching in both OWQ and HOQ configurations. It has been found that the maximum wall heat flux in the case of OWQ assumes greater value than in the corresponding turbulent HOQ case. By contrast, the minimum wall Peclet number, which quantifies the flame quenching distance, for OWQ has been found to be smaller than that in the case of HOQ. Although the volume fractions of S2 and S7 topologies assume high values within the flame front, the focal topologies S1 and S4 have been found to be major contributors to the wall heat flux magnitude

in the case of OWQ. By contrast, nodal topologies S2 and S3 have been found to be major contributors to the wall heat flux magnitude when it attains large magnitude in the HOQ case but all topologies contribute comparably to the wall heat flux at the advanced stages of flame quenching. These differences in the heat transfer mechanisms contribute to the differences in wall heat flux and flame quenching distance between HOQ and OWQ configurations.

Furthermore, this analysis reveals the canonical flow configurations which make dominant contributions to the wall heat flux in the case of turbulent FWI. This, in turn, helps to design simplified configurations representing dominant flow topologies to gain further physical insights into the flame-turbulence interaction and wall heat transfer, and thus these simple configurations can be used for experiments and engineering calculations involving Reynolds Averaged Navier-Stokes (RANS) and Large Eddy Simulations (LES) for obtaining fundamental physical understanding and model validation for premixed flame wall interaction. Thus, the information obtained from this analysis can offer guidance for choosing representative simple flow geometries for the development of models and their validation based on both experimental and computational analyses. The high-fidelity models identified based on the aforementioned exercise will give rise to the development of methodologies for accurate quantitative predictions of wall heat transfer in future combustors.

It is worth noting that the present analysis has been carried out using simple chemistry. Although a recent analysis demonstrated that the flame quenching and heat flux statistics obtained from simple chemistry DNS remain qualitatively similar to those from corresponding detailed chemistry simulations (Lai *et al.*, 2017e), further analyses with detailed chemistry and transport will be necessary for accurate predictions of wall heat flux and flame quenching distance.

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FIGURE CAPTIONS

Figure 1: Classification of S1 – S8 topologies (UF = unstable focus, UN = unstable node, SF = stable focus, SN = stable node, S = saddle, C = compressing, ST = stretching) in the $Q - R$ plane with the lines r_{1a} , r_{1b} and r_2 dividing the topologies, and black dark line indicates $Q = 0$ and $R = 0$ (reproduced from Wacks and Chakraborty (2016) and Wacks et al. (2016)).

Fig. 2: Schematic diagram of (a) OWQ and (b) HOQ configurations.

Fig. 3: (a) Instantaneous distributions of normalised vorticity magnitude (i.e. $\sqrt{\omega_i \omega_i} \times \delta_z / S_L$), non-dimensional temperature (i.e. $T = (\hat{T} - T_0) / (T_{ad} - T_0)$) on the $x_1 - x_2$ side-plane), and fuel mass fraction Y_F on the bottom wall surface for the OWQ V-flame case at $t = 2t_{ft}$. (b) Instantaneous distributions of $\sqrt{\omega_i \omega_i} \times \delta_z / S_L$, Y_F and T on the $x_1 - x_2$ mid-plane at $t = 5\delta_z / S_L$ and $20\delta_z / S_L$ for the HOQ case. In this figure the magnitude increases from blue to red.

Fig. 4: (a) Contours of T and c on the $x_1 - x_2$ mid-plane for the OWQ case at $t = 2t_{ft}$. The locations A1, B1 and C1 correspond to $x_1 = 60\delta_z$, $100\delta_z$ and $140\delta_z$ respectively. (b) Contours of T and c on the $x_1 - x_2$ mid-plane at $t = 5\delta_z / S_L$, $10\delta_z / S_L$ and $15\delta_z / S_L$ for the HOQ case. The contours of $c = 0.1 - 0.9$ (in steps of 0.1) are shown by broken black lines.

Fig. 5: (a) Temporal variations of the maximum, mean and minimum values of normalised wall heat flux magnitude Φ and the minimum value of wall Peclet number Pe_{min} for top and bottom walls for the OWQ case, (b) Temporal variations of the maximum, mean and minimum values of normalised wall heat flux magnitude Φ and wall Peclet number Pe for the HOQ case.

Figure 6: (a) Variations of the normalised mean values of the first invariant $\langle P^* \rangle = \langle P \rangle \times \delta_z / S_L$ of the velocity gradient tensor $A_{ij} = \partial u_i / \partial x_j$ conditional upon c at $t = 2.39t_{ft}$ for locations A1, B1 and C1 for the OWQ case. The line with circle (broken line) indicates mean values conditional upon c based on samples from the bottom (top) branches of the flame, and same

is applicable for Figs. 7 and 8. (b) Variations of the normalised mean values of the first invariant $\langle P^* \rangle = \langle P \rangle \times \delta_Z/S_L$ of the velocity gradient tensor $A_{ij} = \partial u_i/\partial x_j$ conditional upon c at $t = 5\delta_Z/S_L$, $10\delta_Z/S_L$ and $20\delta_Z/S_L$ for the HOQ case.

Fig. 7: (a) Variations of the mean values of the normalised second invariant $\langle Q^* \rangle = \langle Q \rangle \times (\delta_Z/S_L)^2$ of the velocity gradient tensor, normalised second invariant $\langle Q_S^* \rangle = (\langle P^2 \rangle - \langle S_{ij}S_{ij} \rangle)/2 \times (\delta_Z/S_L)^2$ of the strain rate tensor and the normalised invariant $Q_W^* = \langle W_{ij}W_{ij} \rangle/2 \times (\delta_Z/S_L)^2$ of the rotation rate tensor conditional upon c at $t = 2.39t_{ft}$ for locations A1, B1 and C1 of the OWQ case, (b) Variations of mean values of the normalised second invariant $\langle Q^* \rangle = \langle Q \rangle \times (\delta_Z/S_L)^2$ of the velocity gradient tensor, normalised second invariant $\langle Q_S^* \rangle = \langle P^2 - S_{ij}S_{ij} \rangle/2 \times (\delta_Z/S_L)^2$ of the strain rate tensor and the normalised invariant $\langle Q_W^* \rangle = \langle W_{ij}W_{ij} \rangle/2 \times (\delta_Z/S_L)^2$ of the rotation rate tensor conditional upon c at $t = 5\delta_Z/S_L$, $10\delta_Z/S_L$ and $20\delta_Z/S_L$ for the HOQ case.

Fig. 8: Variations of the mean values of (a) normalised third $\langle R \rangle \times (\delta_Z/S_L)^3$ invariant of the velocity gradient tensor and its components (i.e. $R_S \times (\delta_Z/S_L)^3$, $\langle PQ_W \rangle \times (\delta_Z/S_L)^3$ and $\langle -\omega_i S_{ij} \omega_j/4 \rangle \times (\delta_Z/S_L)^3$), (b) normalised third invariant $R_S \times (\delta_Z/S_L)^3$ of the strain rate tensor and its components $\{ \langle -P^3/3 \rangle, \langle PQ_S \rangle, \langle -S_{ij}S_{jk}S_{ki}/3 \rangle \} \times (\delta_Z/S_L)^3$ conditional upon c at $t = 2.39t_{ft}$ for locations A1, B1 and C1 for the OWQ case, Variations of (c) normalised third $\langle R \rangle \times (\delta_Z/S_L)^3$ invariant of the velocity gradient tensor and its components (i.e. $\langle R_S \rangle \times (\delta_Z/S_L)^3$, $\langle PQ_W \rangle \times (\delta_Z/S_L)^3$ and $\langle -\omega_i S_{ij} \omega_j/4 \rangle \times (\delta_Z/S_L)^3$), (d) normalised third invariant $\langle R_S \rangle \times (\delta_Z/S_L)^3$ of the strain rate tensor and its components $\{ \langle -P^3/3 \rangle, \langle PQ_S \rangle, \langle -S_{ij}S_{jk}S_{ki}/3 \rangle \} \times (\delta_Z/S_L)^3$ conditional upon c at $t = 5\delta_Z/S_L$, $10\delta_Z/S_L$ and $20\delta_Z/S_L$ for the HOQ case.

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Fig. 11: Percentages of wall heat flux contributions from individual flow topologies S1-S8 in the (a) OWQ case from $0.24t_{ft}$ to $2.39t_{ft}$ and (b) HOQ case from $t = 2\delta_z/S_L$ to $20\delta_z/S_L$.

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OWQ

HOQ

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OWQ

(a)

HOQ

(b)

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