Analytical modeling of shear behaviors of rockbolts perpendicular to joints

Shuqi Ma^{1,2}, Zhiye Zhao², Jun Peng³, Yilin Gui^{4*}

¹Key Laboratory of Transportation Tunnel Engineering, Ministry of Education, School of Civil Engineering, Southwest Jiaotong University, Chengdu 610031, China

²School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639789, Singapore

³State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China

⁴School of Civil Engineering and Geosciences, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom

Abstract:

Rockbolts have been used to minimize the deformation of underground excavations where the surrounding rock masses contain weak planes such as fractures, joints or faults. Rockbolts installed across rock joints are able to resist the opening and shearing movements of rock joints. One of the rockbolt failure modes encountered in the field is caused by the excessive shear loads. A simple analytical model based on the Beam on Elastic Foundation (BEM) method is proposed in this study to predict the shear responses of a bolt installed perpendicularly to rock joint. The shear load-displacement curve of a double shear test can be divided into three stages: the elastic stage, the elasto-plastic stage and the plastic stage. The foundation stiffness for each respective stage are varied with the curvature influencing zone L_c . The pretension effects are taken into account in the proposed analytical model. The model agrees well with the experimental shear tests, suggesting that the analytical model has the capability to predict the shear load-displacement curve of bolts crossing rock joints.

* Corresponding author.

E-mail address: yilin.gui@newcastle.ac.uk

Keywords: Beam on Elastic Foundation (BEM) method; rockbolt; double shear test; dowel shear behavior; analytical model.

1. Introduction

Rock masses surrounding underground excavations usually contain weak planes such as fractures, joints or faults. Rock masses become deformable due to the presences of these weak planes. Rock masses might open and slide along the fractures or joints due to the excavations. When rockbolts are used to reinforce a fractured rock mass, rockbolts are subjected to tension, shear and compressive forces. Fig. 1 shows several possible force types which might be encountered during the rock block movement [1]. The force types of *a*, *b* and *c* are dominated by the bolt tensile loads whereas the force types of *d*, *e* and *f* are dominated by the bolt shear loads. Less tensile stresses were developed in the perpendicular bolts (force type of *e*) than the inclined rockbolts (force types of *d* and *f*), as the axial displacement (see Fig. 2a) is smaller than the inclined rockbolts (see Figs. 2b and 2c). The tensile stress in the bolt is calculated based on the bolt axial displacement. Azuar in 1977 reported that the perpendicular bolts do not cause considerable axial stress [2].

a: tension and shear

b: tension only

c: tension and shear

d: shear and tension

e: shear only

f: shear and compression

Fig. 1 The types of forces developed in rockbolts [1]

Fig. 2 The axial displacement of inclined and perpendicular rockbolts subjected to shearing loads.

Rockbolt failure in the field could be attributed to the tension (joint opening) or the shear along the joint. Majority of rockbolts fail because of shearing loads when bolts are subjected to rock burst conditions [3] or within high stress rock masses [4]. Many studies have been carried out on the axial behaviors of rockbolts [5-10]. In comparison, relatively less attentions have been paid to the shear behaviors of rockbolts [11-12].

The shear responses of a bolted rock joint are complex and influenced by many factors, such as joint roughness, rock strength, grout properties, bolt properties, bolt installation angles and bolt pretension. These influencing factors have been investigated by many researchers [13-24]. It is generally accepted that the properties of rock and rockbolts affect the shear resistances of bolted joint. Rock with higher compressive strength leads to higher shear resistances than soft rock [16, 22, 23]. Spang and Egger [16] showed that the increase of the bolt diameter could result in higher shearing stiffness of the bolted joint. Joint roughness would cause dilation in the normal direction during shearing movement and higher tensile stress could be mobilized in the bolts due to the joint opening. Joint dilation increases the shear stiffness of the bolted joint [2]. Inclined bolts could also increase the shear stiffness of the joint, leading to higher shear resistance than the perpendicular ones [2, 13, 20, 25]. However, the shear displacement of the inclined bolts prior to failure was smaller than perpendicular bolts. Chen and Li [26] conducted shear tests on fully grouted rebar bolt and D-Bolt under various displacing angles. The displacing angle is defined as the angle between the transversal shear displacement and axial pullout displacement. In their tests, axial and shear loads were applied simultaneously. They found that for the rebar bolt and D-Bolt, the ultimate load of the bolt is independent upon the displacing angle; however, the ultimate displacement is affected by the displacing angles.

Haas [2] concluded that pretension does not have positive impact on the bolt shear behaviors. McHugh and Signer [27] found that shear loads contributed greatly to the bolt failure and the axial load (i.e. pretension) had little impact on the shear resistance of a joint. However, these findings on pretension effects contradict the experimental results from Jalalifar and Aziz [23]. They found that the pretension forces increase the shear stiffness and results in higher shear loads.

In addition to experimental studies, numerical and analytical methods have also been used to study the shear responses of rockbolts. Haile [28] carried out numerical studies on the contributions of rockbolts to the shear resistance of discontinuities. In his study, factors such as steel types, bolt diameter, and bolt inclination angles were examined. Grasselli [20] numerically modelled double shear tests using Finite Element Model code (ZSOIL_3D). Jalalifar and Aziz [23] simulated the shear behaviors of rockbolts in double shear tests using FEM code ANSYS. Li et al. [12] numerically investigated the influences of various factors in the double shear tests using FLAC3D and found that the bolt shear behavior is influenced by concrete strength, bolt installation angle and the bolt diameter. Wei et al. [29] proposed a beam-element-based rockbolt model in the framework of the numerical manifold method (NMM) and this model is able to simulate the shear responses of rockbolts. Bahrani and Hadjigeorgiou [30] numerically investigated behaviors of bolts under pure pull and pure shear loadings using the reinforcement elements in universal distinct element code (UDEC). The difference between “Rockbolt” elements and “Cable” elements were investigated in the numerical experiments and they found that the “Cable” elements tend to underestimate the shear capacity of the bolt. Pellet and Egger [18] proposed an analytical model to simulate the shear behaviors of a bolted rock joint when subjected to shearing loads. Li et al. [11] proposed an analytical model which is able to predict the joint shear strength and joint shear displacement for double shear tests.

This study will focus on the shear behaviors of rockbolts which are installed at an angle of 90° to the rock joint, i.e. the type of e as shown in Fig. 1. Jalalifar et al. [31] and Jalalifar and Aziz [23] carried out experimental studies examining factors influencing the shearing behaviors of a rockbolt installed perpendicularly to the joints. A specially constructed double shearing apparatus as shown in Fig. 3 was used by Aziz et al. [19], Jalalifar et al. [31] and Jalalifar and

Aziz [23] to study the bolt shear behavior. Jalalifar and Aziz [23] concluded that the shear resistance of rock joints was influenced by the concrete strength, the bolt properties and the pretension.

Fig. 3 Configuration of double shear tests used in the studies of Aziz et al. [19], Jalalifar et al. [31] and Jalalifar and Aziz [23].

When the bolted joint is sheared, the shear movement is counteracted by the crossing bolt. Dowel action is defined as the counteraction of a bolt to joint shearing movement. The Beam on Elastic Foundation analogy (BEF) theory has been used as a tool to analytically model the dowel shear behaviors of bolts crossing joints [32-34]. Maekawa and Qureshi [33] presented a model based on the BEF theory to predict the behaviors of bolts when subjected to the combined axial

pullout and shear loads. In the experimental setup of Maekawa and Qureshi [33], the axial displacement was induced due to the joint dilation and the tensile forces were developed due to the pullout mechanism. In the theoretical studies of Maekawa and Qureshi [33] and Moradi et al. [34], the overall shear loads of the bolted joint are considered as the dowel shear forces. The model of Maekawa and Qureshi [33] was further extended to path-dependent cyclic loading case by Soltani and Maekawa [35]. The most relevant parameter in the BEF method is the stiffness of the host concrete. Marcus [36], Soroushian et al. [37] and Dei Poli et al. [38] experimentally studied the foundation stiffness (k) and various empirical equations have been proposed for k . However, these equations just give the elastic value for the foundation stiffness and could not simulate the stiffness changes during the shear loading. Based on studies of Maekawa and Qureshi [33], Moradi et al. [34] proposed the elasto-plastic formulation of spring stiffness which is able to capture the local crushing nearby the joint interface.

In the studies [33, 34, 37, 38], the maximum shear displacement ranges from 1/5 to 1/3 of the bolt diameter [38], which is much lower than that occurs in double shear tests. The maximum shear displacement in double shear tests ranges from 1 to 4 times of the bolt diameter [12, 23]. The previous proposed models could not be simply applied to predict the shear behaviors in double shear tests.

In this study, a simple analytical model is presented based on the Beam on Elastic Foundation analogy (BEF) theory, with the objective to simulate the dowel action of rockbolts perpendicular to joints. This paper focuses the dowel shear forces developed in the bolt. **The pretension effects are taken into account in the study while the axial force developed in the bolt during shearing is not considered.** The proposed analytical model as well as its experimental verification will be presented in the following sections.

2. Analytical model for double shear tests

As the double shear test is symmetric (see Fig. 3), only half of the test is analyzed here as shown in Fig. 4a. A crossing bolt at the joint under shear loads could be treated as a semi-infinite beam on an elastic foundation. The host concrete is considered as elastic foundation which is represented by springs as shown in Fig. 4. Timoshenko and Lessels [32] proposed the analytical solution for the BEF theory, and the differential equation of the deflection of a beam resting on an elastic foundation is expressed as:

$$E_b I_b \frac{d^4 w}{dx^4} = -kw \quad (1)$$

where k denotes the modulus of the elastic foundation (i.e. the spring stiffness); w denotes the bolt deflection; E_b is Young's modulus of the bolt; I_b is the moment of the inertia of the bolt, which is calculated by: $I_b = \frac{\pi D^4}{64}$; D is the diameter of the bolt.

The solution to Eq. (1) is expresses as:

$$w(x) = e^{\lambda x} (A \cos \lambda x + B \sin \lambda x) + e^{-\lambda x} (C \cos \lambda x + D \sin \lambda x) \quad (2)$$

$$\lambda = \sqrt[4]{\frac{k}{4E_b I_b}} \quad (3)$$

where A , B , C and D are constants of integration, which are determined by the boundary conditions.

Fig. 4 (a) Half of the double shear test; (b) Semi-infinite beam on elastic foundation.

The half of the double shear test could be further simplified to that as shown in Fig. 4b. The shear load $V_0 = \frac{P}{2}$ and moment M_0 are applied at the joint interface (i.e. $x = 0$) as shown in Fig. 4b. The bending moment M_0 is equal to 0 at the joint interface.

Under this boundary condition (see Appendix), constants A , B and D are calculated to be zero and C is computed by:

$$C = \frac{V_0}{2\lambda^3 E_b I_b} \quad (4)$$

Hence Eq. (2) becomes:

$$w(x) = \frac{e^{-\lambda x}}{2\lambda^3 E_b I_b} V_0 \cos \lambda x \quad (5)$$

Substituting Eq. (3) into Eq. (5) gives:

$$w(x) = \frac{2\lambda V_0 e^{-\lambda x} \cos \lambda x}{k} \quad (6)$$

The slope of the deflection curve can be obtained by differentiating Eq. (6) with respect to x :

$$\frac{dw}{dx} = -\frac{2\lambda^2 V_0 e^{-\lambda x} (\cos \lambda x + \sin \lambda x)}{k} \quad (7)$$

Setting $x = 0$ in Eq. (6), the relationship between the applied shear force V_0 and local deflection w_0 at the joint is expressed as:

$$V_0 = \frac{k}{2\lambda} w_0 = \frac{k}{4\lambda} \Delta \quad (8)$$

where Δ is the shear displacement; w_0 is the local deflection of the bolt at $x = 0$; $\Delta = 2w_0$, which is illustrated in Fig. 4a.

In this paper, the shear displacement Δ is treated as an input and the incremental form of the Eq. (8) is derived as follows:

$$dV_0 = \frac{k}{4\lambda} d\Delta \quad (9)$$

It can be seen from Eq. (9) that the shear behavior is directly associated with the mechanical properties of the host concrete (the spring stiffness k), the applied shear displacement $d\Delta$, as well as the bolt properties (λ : E_b and I_b).

The values of k suggested in the literature [34, 36, 37, 38] are very scattered as different testing conditions were used in their respective tests. The formula of k proposed by Moradi et al. [34] is used here:

$$k(\Delta) = 181 \frac{E_b I_b}{L_c^4} \quad (10)$$

where: E_b is Young's modulus of the bolt; I_b is the moment of the inertia of the bolt; L_c is the length of the curvature influencing zone.

The foundation stiffness k is a function of the curvature influencing zone (L_c). Maekawa and Qureshi [33] introduced the concept of a curvature influencing zone (L_c) based on the BEF theory to account for the effect of the localized curvature of the bolt. The shape of the curvature profile $\phi(x)$ within the curvature influencing zone is shown in Fig. 5.

Fig. 5 Schematic distribution profiles along the embedded rockbolt, after Maekawa and Qureshi [33]

For a bolt crossing a joint under shear loads, a hinge point will be formed in the bolt. The hinge point is defined as the point where the maximum bending moment is reached and the shear load is equal to zero. According to the BEF theory, the hinge point location is expressed as:

$$L_h = \frac{\pi}{4} \sqrt[4]{\frac{4E_b I_b}{150 f_c'^{0.85}}} \quad (11)$$

where: f_c' is the compressive strength of concrete. According to Eq. (11), the hinge point location only depends on the bolt properties and the concrete strength.

Jalalifar and Aziz [23] studied the hinge point location and concluded that the hinge point distance from the joint interface L_h is influenced by the concrete strength, the pretension, bolt diameter and the shear displacement. In addition, the hinge point location is also affected by the test setup such as the size of the concrete block and the confining conditions. The curvature influencing zone L_c is also dependent upon these factors in the same manner as the hinge point location L_h .

In Eq. (11), the hinge point location is not associated with the axial force (pretension) and the test setup. In other words, the hinge point location computed by Eq. (11) remains constant once the properties of bolts and concrete are given, which is not the case in reality. As there does not exist an formula which could account for the influences of all the factors, Eq. (11) is adopted in this paper due to its simplicity and a coefficient a is introduced in the formula of L_c to incorporate the effects of the experimental conditions such as the test setup and pretension effects.

During the initial shear loading, the shear displacement is small and the rockbolt and surrounding concrete are in elastic state. The initial curvature influencing zone L_{c0} is assumed as a times the size of the hinge point location:

$$L_c = L_{c0} = a \cdot L_h \quad (12)$$

where: L_{c0} is the initial curvature influencing zone; a is the coefficient depending on the experimental conditions. The value of the coefficient a cannot be easily obtained from experiments and the curve fitting technique is used to determine its value.

Jalalifar and Aziz [23] conducted a series of physical double shear tests and concluded that the typical shear load-displacement profile consists of three stages: elastic stage, elasto-plastic stage and plastic stage, which are shown in Fig. 6. During the initial loading stage, the bolt and the host concrete deform elastically, which corresponds to the first section in Fig. 6. The curvature influencing zone L_c is assumed to remain constant as L_{c0} and the foundation stiffness k is held constant in the elastic stage. Eq. (9) coupled with a constant stiffness k have a fair accuracy in describing the linear shearing behaviors in the early shear loading.

Fig. 6 Typical shear load-displacement relationship of double shear tests, after Jalalifar and Aziz [23].

The increasing shear displacement results in local crushing of the concrete. The host concrete close to the joint interface transforms from the elastic stage to the plastic stage. This stage is

referred as the elasto-plastic stage, which corresponds to the second stage in Fig. 6. In the elasto-plastic stage, the curvature influencing zone L_c increases with the concrete crushing and in the meanwhile, the foundation stiffness k decreases. The non-linear shear behavior in this stage could be captured by gradually changing the spring stiffness, which is a convenient method in simulating the damages in the concrete nearby the joint.

During the elasto-plastic stage, the curvature influencing zone L_c is gradually increased:

$$L_c = L_{c0}[1 + 3(DI(\Delta) - b)^{0.8}] \quad (13)$$

where $DI(\Delta)$ is a non-dimensional damage index. In this study, the damage index DI is defined as,

$$DI(\Delta) = \frac{\Delta}{D} \quad (14)$$

Note that in the studies of Maekawa and Qureshi [33] and Moradi et al. [34], the damage index DI was defined as $\frac{\Delta}{2D}$. In this study, the damage index definition in Eq. (14) is more suitable for double shear tests.

The parameter b in Eq. (13) is dependent on the test setup and is defined as:

$$b = \frac{\Delta_1}{D} \quad (15)$$

where Δ_1 is the displacement of the point at the beginning of the elasto-plastic stage as shown in Fig. 6. The constant value $b = 0.02$ is used throughout in the studies of Maekawa and Qureshi [33] and Moradi et al. [34], while in this study, the parameter b is considered as a variable depending upon the experimental setup.

242 Afterwards, the host concrete enters into the plastic stage, during which the rock reaction is
 243 assumed to be constant and the curvature influencing zone L_c reaches an ultimate value and
 244 remains constant throughout this stage.

$$245 \quad L_c = L_{c0}[1 + 3(c - b)^{0.8}] \quad (16)$$

$$246 \quad c = \frac{\Delta_2}{D} \quad (17)$$

247 where Δ_2 is the displacement of the point at the beginning of the plastic stage as shown in Fig. 6.

248 The bolt failure at the joint interface occurs when the tensile and shear forces satisfy the
 249 following failure criteria [18]:

$$250 \quad \left(\frac{N_0}{N_f}\right)^2 + \left(\frac{V_0}{V_f}\right)^2 = 1 \quad (18)$$

251 where N_0 is the tensile force developed in the bolt; N_f is the ultimate tensile force at bolt failure
 252 and equal to $A\sigma_f$; A is the bolt cross section area; σ_f is the ultimate tensile stress; V_f is the
 253 ultimate shear force at bolt failure; $V_f = A\tau_f$; according to the Von Mises criterion in plane stress
 254 state, $\tau_f = \frac{\sigma_f}{\sqrt{3}}$.

255 As the tensile stress of bolt is neglected in this study and hence, the bolt is assumed to break
 256 when $V_0 = V_f$ (19)

257 Eq. (19) will overestimate the bolt shear force as the tensile stress N_0 is not taken into account.

258 The calculation procedure for the proposed model is simple and is summarized below:

259 a) Incremental displacement $d\Delta$ is considered as input.

- b) The cumulative displacement Δ can be computed and $DI(\Delta)$ is obtained by Eq. (14).
- c) The foundation stiffness k is computed by Eq. (10), in which the curvature influencing zone L_c is computed by Eq. (12) when $DI(\Delta) \leq b$; or by Eq. (13) when $b < DI(\Delta) < c$; or by Eq. (16) when $DI(\Delta) \geq c$.
- d) The incremental shear force is computed by Eq. (9), and the cumulative shear force can be then obtained.

3. Experimental verification by double shear tests

Jalalifar and Aziz [23] carried out a series of double shear tests under various concrete strength (f'_c) and pretensions (T_{pre}). Firstly, the double shear tests under the condition of $T_{pre} = 0$ are used to verify the proposed model. Afterwards, the model's ability to capture the pretension effects are verified by the double shear tests under various pretension loads.

3.1 No pretension

Four types of bolts were used in the double shear tests of Jalalifar and Aziz [23]. Bolts T1, T2 and T3 have the same core diameter of 21.7 mm and the same Young's Modulus of 200 GPa, which are shown in Table 1. These three types of bolts have different profiles (i.e. the rib spacing and rib height). Bolt T4 has smaller core diameter and lower Young's Modulus than the other three bolts.

Table 1. Mechanical properties of bolt materials

Bolt type	T1	T2	T3	T4
Bolt core diameter, mm	21.7	21.7	21.7	10.7
Young's Modulus, E_b , GPa	200	200	200	70
Rib spacing, mm	12	12.5	25	-
Rib height, mm	0.65	1.4	1.25	-

Yield stress, MPa	683	673	552	365
Ultimate tensile stress, MPa	862	900	942	490
Ultimate shear stress, MPa	498	520	545	283

Two concrete compressive strengths (20 MPa and 40 MPa) were used in their tests. Bolts T1 and T2 were grouted in concrete with compressive strength of 20 MPa and 40 MPa. Bolts T3 and T4 were grouted in concrete with $f'_c = 40$ MPa. A total number of 6 double shear tests were carried out under the pretension of $T_{pre} = 0$. Table 2 shows the number of tests conducted for each bolt type.

Table 2. The number of tests for each bolt type.

Bolt type	T1	T2	T3	T4
Concrete, $f'_c = 20$ MPa	1	1	0	0
Concrete, $f'_c = 40$ MPa	1	1	1	1

The loading rate of 0.75 mm/step was used in the analytical analyses. The parameter a for the four bolts was chosen as 4. For bolt types T1, T2 and T3, Δ_1 and Δ_2 are determined as 3 mm and 14 mm, respectively. For bolt T4, the used Δ_1 and Δ_2 in the analytical analysis are 3 mm and 7 mm, respectively. These values were selected to best fit the experimental data.

Fig. 7 shows the comparison of the analytical model and the double shear tests for bolts T1 and T2 installed in concrete ($f'_c = 20$ MPa). The whole shear load-displacement curve was divided into three distinct stages as shown in Fig. 7. Foundation stiffness k was computed for each respective stage. It can be seen that the proposed analytical model matches well with the experimental shear behaviors of bolts T1 and T2.

It is noted that the bolt profile could slightly affect the bolt performances (see the discrepancy between the shear load-displacement curves for bolts T1 and T2 in Fig. 7). However, this effect

is negligible when compared to the influence of the concrete strength. The bolt profile effect is not taken into account in the analytical model of this study.

Fig. 7 Comparison of analytical and experimental shear load-displacement curves for bolts T1 and T2 ($f_c' = 20$ MPa).

Fig. 8 shows the comparison of the analytical model and the double shear tests for bolts T1, T2 and T3 for $f_c' = 40$ MPa. The analytical model produces good agreement with the experimental tests. It can be seen from Figs. 7 and 8 that the concrete could significantly affect the shear resistances and the increasing concrete strength leads to higher shear resistances of the joint. The proposed analytical model successfully capture the concrete influence.

Fig. 8 Comparison of analytical and experimental shear load-displacement curves for bolts T1, T2 and T3 ($f'_c = 40$ MPa).

Fig. 9 shows the results of the analytical model and the shear test for bolt T4 installed in concrete with $f'_c = 40$ MPa. The proposed analytical model agrees well with the experimental results. It can be seen from Figs. 8 and 9 that under the same concrete strength ($f'_c = 40$ MPa), bolt T4 generates much smaller shear resistances than bolts T1, T2 and T3 as bolt T4 has much smaller Young's Modulus and diameter. In addition to the concrete strength, the bolt diameter and Young's Modulus could also affect the bolt shear behaviors. The proposed model is able to simulate the effects of the bolt properties.

Fig. 9 Comparison of analytical and experimental shear load-displacement curves for bolts T4 ($f'_c = 40$ MPa).

The stiffness evolutions of analytical solutions in Figs. 7-9 are shown in Fig. 10. It can be seen that foundation stiffness remains still in the initial loading stage, which is followed by a gradual decrease in the elasto-plastic stage. Afterwards, the foundation stiffness reached an ultimate value in the plastic stage. For T1, T2 and T3 having the same diameter and Young's Modulus, the chosen parameters Δ_1 and Δ_2 are the same for both concrete strengths of 20 and 40 MPa. For T4 which has smaller diameter and lower Young's Modulus, the parameter Δ_1 is the same as other bolts while Δ_2 has a smaller value. These parameters are selected based on trial and error methods in order to best model the experimental curves.

Fig. 10 The foundation stiffness evolutions.

3.2. Pretension effects

Up until now, the complete shear load-displacement curve of the double shear test can be properly represented by the proposed analytical model. However, the BEF theory does not take into account the axial forces developed in the bolt and hence the proposed model could not reflect the effects of pretension on the shear behaviors.

Jalalifar [21] and Jalalifar and Aziz [23] found that the hinge point location is also affected by the applied pretension loads. Smaller L_h will be formed when bolts are subjected to higher pretension forces. In the same manner, the curvature influencing zone L_c is supposed to decrease with the increasing pretension forces. In this paper, the pretension effects are simply represented by varying the curvature influencing zone L_c . It means that the parameter a in Eq. (12) depends not only on the test setup but also on the pretension forces.

In the double shear tests of Jalalifar and Aziz [23], pretensions of 0 kN, 20 kN, 50 kN and 80 kN were used. In the verification of pretension effects, the used bolt was bolt T1 as shown in Table 1.

For the pre-tensioned bolts, double shear tests were also conducted in two concrete strength of 20 and 40 MPa. Table 3 shows the parameters used in the analytical analysis. The used parameters in the following analytical analysis are the same for both concrete strength ($f'_c = 20 \text{ MPa and } 40 \text{ MPa}$). In addition, the parameters for the case of $T_{pre} = 0 \text{ kN}$ are the same as those used for bolts T1, T2, and T3 in Section 3.1.

The parameter Δ_2 is different for un-pretensioned bolt ($T_{pre} = 0 \text{ kN}$) and pretensioned bolt ($T_{pre} = 20, 50 \text{ and } 80 \text{ kN}$). The curvature influencing zone L_c (represented by the parameter a in Table 3) is decreased with the increasing pretension forces.

Table 3. The parameters used in modeling the behaviors of bolts in 20 MPa and 40 MPa concrete.

Parameters	a in Eq. (12)	Δ_1 (mm)	Δ_2 (mm)
$T_{pre} = 0 \text{ kN}$	4	3	14
$T_{pre} = 20 \text{ kN}$	3.8	3	8
$T_{pre} = 50 \text{ kN}$	3.5	3	8
$T_{pre} = 80 \text{ kN}$	3.2	3	8

Fig. 11 shows the comparison between the analytical model and the double shear tests for concrete strength of 20 MPa. It can be seen that the analytical model is able to predict the shear load-displacement curves of bolts with different pretensions. This model successfully takes into account the pretension effects. The pretension increases the shear stiffness, leading to higher shear resistance than un-pretensioned bolts at a same displacement.

Fig. 11 Comparison of analytical model and the double shear tests under the concrete strength of 20 MPa.

The shear behaviors of bolts in 40 MPa concrete are also predicted by the analytical model where the used parameters are the same as those used for the 20 MPa concrete. It can be seen from Fig. 12 that the analytical model has good agreement with the shear behaviors of bolts subjected to various pretensions in 40 MPa concrete. This indicates that the analytical model accounts for the effects of pretension as well as the effects of concrete strength.

Fig. 12 Comparison of analytical model and the double shear tests under the concrete strength of 40 MPa.

4. Experimental verification by single shear tests

In the above section, the proposed analytical model is verified by the double shear tests. Except for double shear tests, single shear tests have also been used to study the shear behaviors of rock bolts. In this section, single shear tests conducted by previous researchers are used to verify the analytical model. The parameters used in the analytical modelling under various conditions are discussed later.

Note that when analytically modelling the double shear tests in Section 3, the overall shear load is equal to two times the cumulative shear load obtained via Eq. (9); however, in this section, the overall shear load of the single shear tests is equal to the computed shear load by Eq. (9), as there is only one bolted joint in the single shear test.

Schubert [39] carried out a single shear test on bolts installed perpendicular to the rock joint. The compressive strength of the concrete is 40 MPa, the bolt diameter is 24 mm and the Young's modulus of the bolt is 210 GPa. The three parameters are selected as: $a = 4$; $\Delta_1 = 3 \text{ mm}$; and $\Delta_2 = 7 \text{ mm}$. Fig. 13 shows the comparison between the analytical model and the experimental shear test.

Fig. 13 Comparison of analytical model and the shear test of Schubert [39]

Stjern [40] carried out a single shear test on bolts installed in concrete with compressive strength of 65 MPa. The bolt is 18 mm in diameter and the Young's modulus is 200 GPa. The three parameters are selected as: $a = 4$; $\Delta_1 = 3 \text{ mm}$; and $\Delta_2 = 7 \text{ mm}$. Fig. 14 shows the comparison of the analytical model and the experimental shear test.

Fig. 14 Comparison of analytical model and the shear test of Stjern [40]

Goris et al. [41] carried out single shear tests on cable bolts installed in concrete with compressive strength of 68.9 MPa. The cable bolt is 15.24 mm in diameter and the Young's modulus is estimated as 200 GPa. The three parameters are selected as: $a = 4$; $\Delta_1 = 3 \text{ mm}$; and $\Delta_2 = 5 \text{ mm}$. Fig. 15 shows the comparison of the analytical model and the experimental shear test.

Fig. 15 Comparison of analytical model and the shear test of Goris et al. [41]

McHugh and Signer [27] carried out single shear tests on bolts installed in concrete with compressive strength of 85.5 MPa. The bolt is 22 mm in diameter. As two 6.4-mm-wide by 3.2-mm-deep slots were milled on the two sides of the bolt, the actual diameter of the bolt should be smaller than 22 mm and is chosen as 19 mm in the analytical modelling. The bolt has the Young's modulus of 200 Gpa. The three parameters are selected as: $a = 4$; $\Delta_1 = 3 \text{ mm}$; and $\Delta_2 = 7 \text{ mm}$. Fig. 16 shows the comparison of the analytical model and the experimental shear test.

Fig. 16 Comparison of analytical model and the shear test of McHugh and Signer [27]

Based on the above four single shear tests shown in Figs. 13-16, it can be seen that the proposed model is able to predict the global shear load evolution of the single shear tests. The parameters used in the analytical modelling are listed in Table 4. The parameters such as the bolt diameter D , the bolt Young's modulus E_b and the concrete compressive strength f'_c can be obtained directly from the test setup. The parameters a , b (Δ_1) and c (Δ_2) need to be solved via the curve-fitting technique. As can be seen, the values of a and b (Δ_1) remain constant for the double shear tests and single shear tests under various bolt properties and concrete strength. However, the parameter Δ_2 of double shear tests tend to have larger values than those in single shear tests.

Table 4 The parameters used in the analytical modelling for the experimental shear tests with the pretension $T_{pre} = 0$ kN.

Parameters	Concrete compressive strength, f'_c (MPa)	Bolt diameter (mm)	Young's Modulus (GPa)	a	Δ_1 (mm)	Δ_2 (mm)
Jalalifar and Aziz [23], $T_{pre} = 0$	20	21.7	200	4	3	14
Jalalifar and Aziz [23], $T_{pre} = 0$	40	21.7	200	4	3	14
Jalalifar and Aziz [23], $T_{pre} = 0$	40	10.7	70	4	3	7
Schubert (1984)[39]	40	24	210	4	3	7
Stjern [40]	65	18	200	4	3	7
Goris et al. [41]	68.9	15.24	200	4	3	5
McHugh and Signer [27]	85.5	22	200	4	3	7

5. Discussion

The proposed analytical model is for bolts installed perpendicularly to the rock joint. This model provides a simple but effective way to predict the overall shear load-displacement curves of bolts. The limitation of the proposed model lies in that the model does not take into account the axial forces developed in bolts. The factors such as bolt surface profiles (rib spacing and rib height), grout strength, the interaction between bolt and grout, and the interaction between grout and host concrete, which could affect the axial behaviors of the bolt, are not considered in the proposed model.

The failure mechanism of a bolt crossing a joint is complex as it includes the damage of the host concrete which provides bearing support to the bolt, the yielding of the bolt, the formation and evolution of the hinge point along the bolt, and the deterioration of grout between the bolt and

the concrete. Few studies in the available literature have discussed the failure mechanisms of the two materials. The experimental tests mainly focused on the global shear load-displacement curves. Even for the shear tests where strain gauges are attached on the bolt, strain gauges close to the joint interface are usually damaged due to the shear displacement and consequently, the bolt failure mechanism cannot be measured properly. The failure mechanism of these two materials are unclear. In this study, the analytical model is based on the Beam on Elastic Foundation (BEM) method. The global shear load-displacement curve is simply described by the foundation stiffness degradation, which represents the crushing and damage of the concrete nearby the joint interface.

According to Eq. (19), the total failure shear load of the double shear test for T4 bolts is predicted as: $2V_f = 50$ kN for T4 bolts. Most tests of Jalalifar and Aziz [23] were terminated before the bolt failure. The failure shear load of the T4 bolt is 75 kN as shown in Fig. 9, which is larger than 50 kN. Hence, in reality Eq. (19) underestimates the failure shear load. The predicted failure load for the T1 bolt is $2V_f = 378$ kN. Fig. 12 shows that the T1 bolt does not fail as the applied shear load reaches 500 kN. It can be seen that the failure criteria defined by Eq. (19) tends to give a conservative prediction of the failure shear load. Eq. (18) gives an even more conservative value as the axial stress is accounted for. In comparison, Eq. (19) provides a closer prediction on the failure load of bolts under double shear tests.

6. Conclusions

A simple analytical model is proposed in this study based on the Beam on Elastic Foundation (BEM) method, with the objective to predict the shear load-displacement curve of the bolt subjected to shearing loads. The shear load-displacement curve is divided into three distinct

stages: the elastic stage, the elasto-plastic stage and the plastic stage. The foundation stiffness is associated with the curvature influencing zone L_c . The foundation stiffness is a constant value in elastic stage whereas the stiffness in the elasto-plastic stage is mobilized downwards with the increasing shear displacement. The non-linear shear behavior of rockbolts is successfully modelled by gradually changing the foundation stiffness in the BEM. In the plastic stage, the foundation stiffness reaches the ultimate value and remains constant afterwards. The analytical model is in an incremental form and its capability of representing the dowel shear behaviors of bolts is evaluated by shear tests. Physical shear tests show that pretension could decrease the curvature influencing zone. The pretension effects are simply incorporated in the proposed model by varying the size of the curvature influencing zone.

The experimental verifications suggest that the analytical model is able to predict the complete shear load-displacement curve of bolts. In addition, the model successfully takes into account the effects of concrete strength and bolt properties, as well as pretension effects.

Appendix:

By differentiation of Eq. (2), we have:

$$w'(x) = A_1 e^{\lambda x} \cos \lambda x + B_1 e^{\lambda x} \sin \lambda x + C_1 e^{-\lambda x} \cos \lambda x + D_1 e^{-\lambda x} \sin \lambda x \quad (A1)$$

where $A_1 = A\lambda + B\lambda$; $B_1 = -A\lambda + B\lambda$; $C_1 = -C\lambda + D\lambda$; $D_1 = -C\lambda - D\lambda$.

Differentiating Eq. (A1) leads to

$$w''(x) = A_2 e^{\lambda x} \cos \lambda x + B_2 e^{\lambda x} \sin \lambda x + C_2 e^{-\lambda x} \cos \lambda x + D_2 e^{-\lambda x} \sin \lambda x \quad (A2)$$

where $A_2 = A_1\lambda + B_1\lambda$; $B_2 = -A_1\lambda + B_1\lambda$; $C_2 = -C_1\lambda + D_1\lambda$; $D_2 = -C_1\lambda - D_1\lambda$.

474 Differentiating Eq. (A2) gives

475 $w'''(x) = A_3 e^{\lambda x} \cos \lambda x + B_3 e^{\lambda x} \sin \lambda x + C_3 e^{-\lambda x} \cos \lambda x + D_3 e^{-\lambda x} \sin \lambda x$ (A3)

476 where $A_3 = A_2 \lambda + B_2 \lambda$; $B_3 = -A_2 \lambda + B_2 \lambda$; $C_3 = -C_2 \lambda + D_2 \lambda$; $D_3 = -C_2 \lambda - D_2 \lambda$.

477 At $x \rightarrow \infty$, $w(x \rightarrow \infty) = e^{\lambda x} (A \cos \lambda x + B \sin \lambda x) = 0$. Hence, we have $A = B = 0$.

478 Knowing that $w''(x) = \frac{M(x)}{E_b I_b}$ and $w'''(x) = \frac{V(x)}{E_b I_b}$, at $x = 0$,

479 $w''(0) = M_0 = -C_1 \lambda + D_1 \lambda = -(-C \lambda + D \lambda) \lambda + (-C \lambda - D \lambda) \lambda$. (A4)

480 The bending moment M_0 is equal to 0 at the joint interface (i.e. $x = 0$) and hence, $D = 0$.

481 At $x = 0$, $w'''(0) = \frac{V_0}{E_b I_b} = A_2 \lambda + B_2 \lambda - C_2 \lambda + D_2 \lambda = (2C + 2D) \lambda^3$ (A5)

482 Hence, $C = \frac{V_0}{2 \lambda^3 E_b I_b}$

483 **References:**

- 484 [1] Windsor. Rock reinforcement systems. Int J Rock Mech Min Sci Geomech Abstr. 1997;34:919-951
- 485 [2] Haas CJ. Analysis of rockbolting to prevent shear movement in fractured ground Mining engineering.
486 1981;33:698-704
- 487 [3] Haile A. Observation of the dynamic support performance of South African tunnel support systems. In:
488 In: Proceedings rock support and reinforcement practice in mining, Kalgoorlie, Australia, 1999a.
489 pp 335-341
- 490 [4] Li CC. Field Observations of Rock Bolts in High Stress Rock Masses Rock Mechanics and Rock
491 Engineering. 2010;43:491-496 doi:10.1007/s00603-009-0067-8
- 492 [5] Deb D, Das KC. Modelling of fully grouted rock bolt based on enriched finite element method.
493 International Journal of Rock Mechanics and Mining Sciences. 2011; 48:283-293
494 doi:<http://dx.doi.org/10.1016/j.ijrmms.2010.11.015>
- 495 [6] Ma S, Nemcik J, Aziz N. An analytical model of fully grouted rock bolts subjected to tensile load
496 Construction and Building Materials. 2013;49:519-526
497 doi:<http://dx.doi.org/10.1016/j.conbuildmat.2013.08.084>
- 498 [7] Ma S, Nemcik J, Aziz N, Zhang Z. Analytical model for rock bolts reaching free end slip Construction
499 and Building Materials. 2014;57:30-37 doi:<http://dx.doi.org/10.1016/j.conbuildmat.2014.01.057>

- [8] Nie W, Zhao ZY, Ning YJ, Sun JP. Development of Rock Bolt Elements in Two-Dimensional Discontinuous Deformation Analysis. *Rock Mechanics and Rock Engineering*. 2014b;47:2157-2170 doi:10.1007/s00603-013-0525-1
- [9] He L, An XM, Zhao ZY. Fully Grouted Rock Bolts: An Analytical Investigation. *Rock Mechanics and Rock Engineering*. 2014;48:1181-1196 doi:10.1007/s00603-014-0610-0
- [10] Chen J, Saydam S, Hagan PC. An analytical model of the load transfer behavior of fully grouted cable bolts. *Construction and Building Materials*. 2015. 101, Part 1:1006-1015 doi:<http://dx.doi.org/10.1016/j.conbuildmat.2015.10.099>
- [11] Li X, Nemcik J, Mirzaghobanali A, Aziz N, Rasekh H. Analytical model of shear behaviour of a fully grouted cable bolt subjected to shearing *International Journal of Rock Mechanics and Mining Sciences*. 2015;80:31-39 doi:<http://dx.doi.org/10.1016/j.ijrmms.2015.09.005>
- [12] Li L, Hagan PC, Saydam S, Hebblewhite B, Li Y. Parametric Study of Rockbolt Shear Behaviour by Double Shear Test *Rock Mechanics and Rock Engineering*. 2016a; 49:4787-4797 doi:10.1007/s00603-016-1063-4
- [13] Bjurström S. Shear strength of hard rock joints reinforced by grouted untensioned bolts. In: *Proceedings of the Congress of the International Society for Rock Mechanics*, 1974. pp 1194-1199
- [14] Dight PM. Improvements to the stability of rock walls to open pit mines. 1983. Ph.D Thesis, Monash
- [15] Dulacska H. Dowel action of reinforcement crossing cracks in concrete. *Journal of the American Concrete Institute*. 1972; 69:754-757
- [16] Spang K, Egger P. Action of fully-grouted bolts in jointed rock and factors of influence. *Rock Mechanics and Rock Engineering*. 1990;23:201-229 doi:10.1007/bf01022954
- [17] Ferrero AM. The shear strength of reinforced rock joints *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*. 1995;32:595-605 doi:[http://dx.doi.org/10.1016/0148-9062\(95\)00002-X](http://dx.doi.org/10.1016/0148-9062(95)00002-X)
- [18] Pellet F, Egger P. Analytical model for the mechanical behaviour of bolted rock joints subjected to shearing. *Rock mechanics and rock engineering*. 1996;29:73-97
- [19] Aziz N, Pratt D, Williams R. Double shear testing of bolts. In: Aizz N (ed) *Coal 2003: Coal Operators' Conference*, University of Wollongong & the Australasian Institute of Mining and Metallurgy, 2003. pp 154-161
- [20] Grasselli G. 3D Behaviour of bolted rock joints: experimental and numerical study. *International Journal of Rock Mechanics and Mining Sciences*. 2005;42:13-24 doi:<http://dx.doi.org/10.1016/j.ijrmms.2004.06.003>
- [21] Jalalifar H. A new approach in determining the load transfer mechanism in fully grouted bolts. University of Wollongong. 2006
- [22] Jalalifar H, Aziz N. Analytical Behaviour of Bolt-Joint Intersection Under Lateral Loading Conditions. *Rock Mechanics and Rock Engineering*. 2010a;43:89-94 doi:10.1007/s00603-009-0032-6
- [23] Jalalifar H, Aziz N. Experimental and 3D Numerical Simulation of Reinforced Shear Joints. *Rock Mechanics and Rock Engineering*. 2010b;43:95-103 doi:10.1007/s00603-009-0031-7
- [24] Li X, Aziz N, Mirzaghobanali A, Nemcik J. Behavior of Fiber Glass Bolts, Rock Bolts and Cable Bolts in Shear *Rock Mechanics and Rock Engineering*. 2016b;49:2723-2735 doi:10.1007/s00603-015-0907-7
- [25] Ludvig B. Shear Tests on Rockbolts. In: *Proceedings of the International Symposium on Rockbolting*, Abisko, Balkema, 1983. pp 113-123
- [26] Chen Y, Li CC. Performance of fully encapsulated rebar bolts and D-Bolts under combined pull-and-shear loading. *Tunnelling and Underground Space Technology*. 2015; 45:99-106 doi:<https://doi.org/10.1016/j.tust.2014.09.008>
- [27] McHugh E., Signer SD. Roof bolt response to shear stress: laboratory Analysis. In: *Proceedings 18th international conference on ground control in mining*, Morgantown, WV, 1999. pp 232-238

- [28] Haile A. A mechanistic evaluation and design of tunnel support systems for deep level South African mines. University of Natal, Durban, South Africa. 1996b.
- [29] Wei W, Jiang Q, Peng J. New Rock Bolt Model and Numerical Implementation in Numerical Manifold Method. *International Journal of Geomechanics*. 2016;0:E4016004 doi:doi:10.1061/(ASCE)GM.1943-5622.0000669
- [30] Bahrani N, Hadjigeorgiou J. Explicit reinforcement models for fully-grouted rebar rock bolts. *Journal of Rock Mechanics and Geotechnical Engineering*. 2017;9:267-280 doi:https://doi.org/10.1016/j.jrmge.2016.07.006
- [31] Jalalifar H, Aziz N, Hadi M. The effect of surface profile, rock strength and pretension load on bending behaviour of fully grouted bolts. *Geotechnical and Geological Engineering*. 2006;24:1203-1227 doi:10.1007/s10706-005-1340-6
- [32] Timoshenko S, Lessels J. *Applied elasticity*. Pennsylvania:Westinghouse Technical Night School Press. 1925.
- [33] Maekawa K, Qureshi J. Computational model for reinforcing bar embedded in concrete under combined axial pullout and transverse displacement. *Proceeding of JSCE*. 1996;31:227-239
- [34] Moradi AR, Soltani M, Tasnimi AA. A Simplified Constitutive Model for Dowel Action across RC Cracks *Journal of Advanced Concrete Technology*. 2012;10:264-277 doi:10.3151/jact.10.264
- [35] Soltani M, Maekawa K. Path-dependent mechanical model for deformed reinforcing bars at RC interface under coupled cyclic shear and pullout tension. *Engineering Structures*. 2008;30:1079-1091 doi:<http://dx.doi.org/10.1016/j.engstruct.2007.06.013>
- [36] Marcus H. Load carrying capacity of dowels at transverse pavement joints *SCI Journal Proceedings*. 1951; 48:169-184
- [37] Soroushian P, Obaseki K, Rojas MC. Bearing strength and stiffness of concrete under reinforcing bars. *ACI Materials Journal*. 1987;84:179-184 doi:10.14359/1885
- [38] Dei Poli S, Di Prisco M, Gambarova PG. Shear response, deformation, and subgrade stiffness of a dowel bar embedded in concrete. *ACI Journal Structural Journal*. 1992; 89:665-675
- [39] Schubert P. *Das Tragvermögen des mörtelversetzten Ankers unter aufgezwingener Kluftverschiebung*. PhD Thesis, Montan-Universität, Leoben. 1984.
- [40] Stjern G. *Practical performance of rock bolts*. Trondheim, Norway: University of Trondheim. 1995.
- [41] Goris JM, Martin LA, Curtin RP. Shear behavior of cable bolt supports in horizontal, bedded deposits. In: *15th International Conference on Ground Control in Mining*, Colorado School of Mines, 1996. pp 511-521