Influence of stochastic variations in manufacturing defects on the mechanical performance of textile composites

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Abstract

This paper presents a methodology to evaluate the effects of microscopic manufacturing defects, namely fibre misalignment, waviness and volume fraction, on the mechanical performance. Influences of these defects on the effective elastic properties of composites are quantified by a dual homogenization method. For estimating stochastic characteristics of the properties induced by the variations in these defects, a probabilistic extension of the dual homogenization method is developed and numerically implemented through a perturbation-based stochastic finite element method. It is further incorporated in a multiscale finite element based reliability method to measure the influences of these manufacturing defects on structural performance in terms of reliability. The effectiveness of the proposed method in capturing defects is illustrated initially by investigating the effective elastic properties of a unidirectional fibre composite based yarn and then a plain woven textile

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composite. The capability of the proposed method in quantifying the variations in these defects is further demonstrated through statistical analysis of the effective elastic properties and a woven textile composite and structural reliability analysis of a textile composite laminate. This paper represents a significant advancement in the probabilistic prediction of the behaviour of woven and non-woven composites.

Keywords: manufacturing defects; textile composites; effective elastic properties; homogenization; uncertainty quantification; reliability analysis

1. Introduction

Merits, such as high stiffness to weight ratio, of fibre reinforced composites compared with conventional engineering materials have attracted researchers to devote numerous efforts to get better understanding of their mechanical and physical behaviours and to promote their applications beyond high-value industries to others, such as their emerging use in construction. However, further research, e.g. damage and failure mechanism identification and uncertainty quantification, particularly for civil engineering (Eurocode) applications, are still required to design safer and more economical structures. This is partially due to the complexity of their architectures that govern their mechanical and physical properties, e.g. elastic moduli, strength, thermal conductivity, permeability, which in turn play critical roles in the structural performance of the manufactured components. Despite the complexity and manufacturability of the material architecture, advances in multiscale simulation provide opportunities to review and revise the assumption of perfect or deterministic products without defects. A design approach that enables

17 these manufacturing features to be taken into account is desired to allow the
18 accurate modelling of composite components, by bridging scales and linking
19 uncertainties.

20 Efforts have been devoted to measuring manufacturing defects and to
21 investigating their influences on mechanical behaviours. A variety of tech-
22 niques, e.g. X-ray computed tomography (CT), micro CT, and confocal laser
23 scanning microscopy, have been developed to measure various types of de-
24 fects ([Christensen and Waals, 1972](#); [Creighton et al., 2001](#); [Barwick and Pa-](#)
25 [pathanasiou, 2003](#); [Milani et al., 2007](#); [Kratmann et al., 2009](#); [Sutcliffe et al.,](#)
26 [2012](#); [Mizukami et al., 2016a,b](#)). It has been widely accepted that defects are
27 the main sources of damage initiation in advanced composites ([Budiansky](#)
28 [and Fleck, 1993](#); [Li et al., 2004](#); [Pinho et al., 2006](#); [Gommer et al., 2016](#)).
29 However, a systematic description of the impact of these defects and their
30 randomness is currently not available. In the present study, the effects of
31 uncertainties in fibre volume fraction, fibre misalignment and waviness are
32 investigated in the context of the performance of simple composite struc-
33 tures and their probabilities of failure. More details of their influences on the
34 mechanical behaviours are briefly reviewed in the following.

35 Fibre volume fraction is perhaps one of the earliest and most intensively
36 investigated defects due to its direct link to alterations in mechanical prop-
37 erties. [Ye et al. \(1995\)](#) performed a series of tests on specimens cutting
38 from structural components made from pultruded composites and showed
39 that wide variations exist in the physical properties of pultruded compos-
40 ite materials due to variability of fibre and resin contents. The influence
41 of fibre volume fraction on mechanical behaviours of various composites was

extensively studied numerically or experimentally, such as strengths and elastic properties (Cohen, 1997; de Kok and Meijer, 1999; Cohen et al., 2001), fatigue behaviour of GFRP pultruded rod composites (Allah et al., 1996), fracture toughness (Gu et al., 2000; Li et al., 2004; Davies et al., 2005).

In numerical modelling of long or continuous fibre composites, it is more usual to assume that fibres are perfectly aligned than misaligned. In the manufactured state, they deviate with small angular misalignments about the idealised direction. For instance, using the natural geometry of typically planar composite structures, it is convenient to think of fibre misalignment occurring both (a) in the plane of the laminate, ϕ and (b) out of the plane of the laminate, θ , but they can be treated as independent parameters. The impregnation and mechanical behaviour are affected by the misalignment of the fibre bundles, with initial misalignments possible up to 7-8 degrees (Yurgartis, 1987). These misalignments are suspected of having a significant influence on strengths and elastic properties, e.g. (Yurgartis, 1987; Woo and Whitcomb, 1997; Yang et al., 2002; Liu et al., 2004; Vallons et al., 2011; Bednarczyk et al., 2014).

In the production of composite materials, fibre waviness is a type of manufacturing defect occurring especially during the filament winding process, for instance, caused by volumetric changes during resin bleed-out in thick wound structures. Chun et al. (2001) examined the influences of fibre waviness on the tensile and compressive behaviour of composites. Liu et al. (2004) took fibre waviness into account when predicting compressive strength of fibre composites. Karami and Garnich (2005) considered the situation of periodic fibre waviness in estimating effective moduli and failure for

67 composites. [Tsai et al. \(2011\)](#) also examined the effect of inclusion waviness
68 and waviness distribution on elastic properties of fibre-reinforced compos-
69 ites. [Allison and Evans \(2012\)](#) investigated the effect of fibre waviness on the
70 bending behaviour of GFRP (S-glass/epoxy). [Schmidt et al. \(2012\)](#) anal-
71 ysed the effects of local fibre waviness on damage mechanisms and fatigue
72 behaviour of biaxially loaded tube specimens. [Velmurugan et al. \(2014\)](#) ob-
73 served the detrimental influence of fibre waviness on the effective properties
74 of discontinuous fibre reinforced composites.

75 However, it should be noted that most of the aforementioned studies only
76 investigate these defects in the deterministic context. These manufacturing
77 defects are random in nature as they may be generated during composite
78 production, handling or storage stages ([Piggott, 1995](#); [Potter et al., 2008](#);
79 [Mesogitis et al., 2014](#); [Gommer et al., 2016](#)). Composites produced by resin
80 infusion techniques inevitably suffer from variation in resin distribution due
81 to imprecise fibre placement and distortion of the preform during mould clo-
82 sure and infusion. In the present study, we are principally concerned with
83 variability in textile composites, as compared with pultrusions, for example.
84 Variations in textile composites are associated with waviness, size and shape
85 of yarn, distribution of fibres inside the yarns, resin content, etc. Variation of
86 permeability is one of the mostly studied parameters due to its importance
87 in producing high quality products. Endruweit and his co-workers have con-
88 ducted extensive studies on stochastic analysis of permeability due to geo-
89 metrical uncertainties including fibre volume fraction, fibre angle ([Endruweit
90 and Long, 2006](#); [Endruweit et al., 2006a,b, 2013](#)). Obviously, variations in
91 permeability subsequently lead to uncertainties in mechanical properties, as

92 they directly link with finally manufactured products. Many studies have
 93 also been conducted on the as-manufactured state of composites with tex-
 94 tile reinforcement. [Liu et al. \(2004\)](#) used a power spectral density func-
 95 tion to characterize the fibre misalignment angle and incorpertated it into a
 96 Monte Carlo simulation to study the probability distribution of compressive
 97 strength, and their results showed that strength has Weibull distribution.
 98 [Gill et al. \(2009\)](#) investigated the effect of variation in fibre volume fraction
 99 on model I and II delamination behaviour of woven textile composites. In
 100 mode I, it was found that toughness increased with increasing fibre volume
 101 fraction. [Mendoza Jasso et al. \(2011\)](#) developed a new model, which incor-
 102 porates stochastic variations on the microstructure of a composite material
 103 due to local variations of the fibre volume fraction to predict failure initiation
 104 location of open hole structures. [Tsai et al. \(2011\)](#) investigated the effects
 105 of inclusion waviness and its distribution to the effective composite stiffness.
 106 Komeili and his co-workers conducted research to investigate the uncertain-
 107 ties for woven fabric composites under axial and shear loadings ([Komeili](#)
 108 [and Milani, 2012, 2013](#)). [Turan and Okur \(2012\)](#) investigated the influence
 109 of the cross-sectional shape and size variation of the yarn on the structural
 110 parameters of the fabric. [Ilyani Akmar et al. \(2014\)](#) conducted a study to
 111 evaluate the sensitivity of geometry parameters, including the yarn height,
 112 the yarn width, the yarn spacing and the friction coefficient, on the effective
 113 properties of dry fabric. Vanaerschot and his co-workers conducted studies
 114 to model the variability of internal geometry for textile composites and also
 115 investigated their influences on mechanical properties for textile composites
 116 ([Olave et al., 2012](#); [Vanaerschot et al., 2013a,b, 2016](#)). [Bednarczyk et al.](#)

117 (2014) studied the probabilistic failure envelopes of unidirectional composites
118 with the consideration of randomness of local fibre misalignment integrating
119 probability analysis with the high-fidelity generalized methods of cells. How-
120 ever, a general method, simultaneously considering all these effects remains
121 in its infancy (e.g. Wang and Wang (2015, 2016)).

122 In order to take these manufacturing defects and their related variations
123 into account, a probabilistic homogenization method is proposed in this pa-
124 per. It builds on the method in (Zhou et al., 2016b) developed by the authors
125 for woven textile composites with uncertainties in material properties, which
126 integrates a perturbation technique to propagate uncertainties and a compu-
127 tation homogenization method to bridge micro- and macro-scales. Two new
128 aspects are described here. Firstly, an additional homogenization has been
129 included to form a dual homogenization scheme that enables to consider fibre
130 and matrix separately. In (Zhou et al., 2016b), the fibre yarn is treated as
131 an homogenized material that results in the details of the properties of its
132 constituents being lost, and making it impossible to consider fibre volume
133 fraction and its variation as variables. In contrast, the present study adopts,
134 the Mori-Tanaka method, which has been recognized as an ideal method for
135 the prediction of unidirectional fibre composites. In so doing, a general scale-
136 bridging scheme is established, enabling variations in microscale constituent
137 materials to be propagated naturally. The second new contribution is made
138 in the consideration of variations in manufacturing defects. Three crucial
139 defects, namely fibre misalignment, fibre waviness and fibre volume fraction,
140 were considered in the present study. The fibre volume fraction can be auto-
141 matically taken into account with the introduction of Mori-Tanaka method

142 as fibre volume fraction is one of the input parameters. To investigate the
 143 effect of fibre misalignment, a coordinate transformation is used to calculate
 144 the misalignment angle, while an average strain based technique, initially
 145 proposed in (Hsiao and Daniel, 1996b), is adopted to consider fibre waviness.
 146 Importantly, a probabilistic homogenization approach has been developed to
 147 propagate variations in these defects to the effective elastic properties in the
 148 textile composites. This has been further integrated into multiscale finite
 149 element based reliability analysis method proposed by the authors in (Zhou
 150 et al., 2017) to evaluate the structural performance under variations in these
 151 manufacturing defects. The proposed approach for considering manufactur-
 152 ing defects and their variations is an important addition for the completeness
 153 of the method in (Zhou et al., 2016b), and it also extends the capability of
 154 the multiscale finite element reliability method for composites in (Zhou et al.,
 155 2017). Details of the proposed method are provided in Sections 2 and 3, and
 156 numerical studies are presented in Section 4 for the illustration purpose.

157 **2. Dual homogenization for material characterization of textile com-** 158 **posites with manufacturing defects**

159 A dual homogenization procedure, including an homogenization stage to
 160 estimate the effective elastic properties for the yarn, and another to predict
 161 the counterparts for the complete textile composite, is proposed first. In
 162 textile composites, prepreg yarns consisting of fibres and matrix, are usually
 163 treated as a unidirectional fibre reinforced composites in their own right.
 164 Asymptotic homogenization methods have been shown to be accurate in
 165 predicting the effective elastic properties of unidirectional composites. A

method from this class of problem has been used for the first stage of the homogenization formulation. However, these classic methods are not directly applicable for the second stage, as it needs to deal with textile composites with wavy inclusions. In the present study, computational homogenization has been adopted.

2.1. Mori-Tanaka Method for UD composites

Computational homogenization may also be used to predict the effective elastic properties of yarns, but may prove cumbersome. The Mori-Tanaka method, from the class of asymptotic homogenization approaches, is based on Eshelby's strain-concentration tensor described around an ellipsoidal matrix inclusion, provides an alternative. It is used here for the first homogenisation problem, mainly because it enables the analytical consideration of fibre volume fraction in the stochastic analysis. According to [Mori and Tanaka \(1973\)](#), the estimate of the effective elastic tensor, $\bar{\mathbf{C}}_{\text{Yarn}}$, for the fibre yarn as an unidirectional composite is:

$$\bar{\mathbf{C}}_{\text{Yarn}} = \mathbf{C}^m + V^f (\mathbf{C}^f - \mathbf{C}^m) \mathbf{T} [V^m \mathbf{I} + V^f \mathbf{T}]^{-1} \quad (1)$$

where $\mathbf{S}$ and $\mathbf{C}$ are the compliance and stiffness tensors, respectively; V is the volume fraction; the superscripts f and m are referred to the fibre reinforcement or inclusion and the matrix, respectively; and $\mathbf{T} = [\mathbf{I} + \mathbf{L}\mathbf{S}^m (\mathbf{C}^f - \mathbf{C}^m)]^{-1}$ is the Eshelby's strain-concentration tensor with $\mathbf{I}$ for the identity tensor and $\mathbf{L}$ for the contracted form from the fourth order Eshelby's tensor.

2.2. Consideration of geometry defects in homogenization

As previously mentioned, the defect from fibre volume fraction can be automatically considered with the use of Mori-Tanaka method, while additional

180 operations are required for other two defects, namely fibre misalignment and
 181 fibre waviness. The procedure to augment Mori-Tanaka method to investi-
 182 gate these two defects is described in the following.

183 2.2.1. Fibre misalignment

For the effective elastic tensor obtained from Eq. (1), the influence of fibre misalignment described by an in-plane angle, θ , or out-of-plane angle, ϕ , can be considered through coordination transformation. For illustration purposes, only in-plane misalignment is considered in the present study, whilst the formulation is generalized to consider the out-of-plane misalignment. The transformed stiffness tensor, $\bar{\mathbf{C}}_{\text{Yarn},\theta}$, accounting for in-plane fibre misalignment is,

$$\bar{\mathbf{C}}_{\text{Yarn},\theta} = \mathbf{R}_{\sigma} \bar{\mathbf{C}}_{\text{Yarn}} \mathbf{R}_{\epsilon}^{-1} \quad (2)$$

184 where $\mathbf{R}_{\sigma}$ and $\mathbf{R}_{\epsilon}$ are the standard stress and strain transformation matrices
 185 resulting from the rotation angle, θ , respectively.

186 2.2.2. Fibre waviness

In the present study, the fibres in yarn are assumed to have uniform waviness. The average strain field technique proposed in (Hsiao and Daniel, 1996b) is adopted to take this type of fibre waviness into account in the prediction of the effective elastic properties. In this approach, a yarn having the assumed uniform waviness in fibres is divided into infinitesimally thin slices along the yarn direction. Although there are a bundle of fibres in each yarn slice, they are assumed to be parallel. However, each yarn slice may orient differently due to the waviness. Each yarn slice is thus dealt as an off-axis unidirectional lamina. To obtain an effective constitutive model for the

yarn with wavy fibres, an average strain technique from (Hsiao and Daniel, 1996b) has been used. Assume the yarn is under a prescribed stress, $\boldsymbol{\sigma}$, each slice's strains, $\boldsymbol{\varepsilon}_i = \mathbf{S}_{\text{Yarn}}(\theta_i)\boldsymbol{\sigma}$, can be calculated by considering its off-axis angle, θ_i . The strains from all slices are then integrated over one wavelength to obtain the average strains, $\bar{\boldsymbol{\varepsilon}}$. By doing so, the relation between the applied stress, $\boldsymbol{\sigma}$ and the average strain, $\bar{\boldsymbol{\varepsilon}}$, can be established. Consequently, the compliance of the yarn, $\bar{S}_{\text{Yarn},w}$, with uniform waviness in fibre is obtained, which is in turn used to calculate the effective elastic tensor, $\bar{\mathbf{C}}_{\text{Yarn},w}$. In summary, the procedure to compute the compliances, $\bar{S}_{\text{Yarn},w}^{ij}$, through the average strain technique, is expressed as:

$$\bar{S}_{\text{Yarn},w}^{ij} = \frac{1}{L} \int_0^L \bar{S}_{\text{Yarn},\theta}^{ij} dx = \frac{1}{L} \int_0^L \mathbf{R}_{\varepsilon}(\theta) S_{\text{Yarn}}^{ij} \mathbf{R}_{\sigma}^{-1}(\theta) dx \quad (3)$$

To compute the above integral, it is necessary to relate the off-axis angle θ to wave parameters, namely amplitude A and wavelength L , through $\tan \theta = \frac{dv}{dx} = 2\pi \frac{A}{L} \cos\left(\frac{2\pi x}{L}\right)$ with $v = A \sin \frac{2\pi x}{L}$. Therefore, the elementary terms $\cos \theta$ and $\sin \theta$ in $\mathbf{R}_{\varepsilon}$ and $\mathbf{R}_{\sigma}$ can be expressed as functions of these wave parameters:

$$\begin{aligned} \cos \theta &= \left[1 + \left(2\pi \frac{A}{L} \cos \frac{2\pi x}{L} \right)^2 \right]^{-1/2}, \quad \text{and} \\ \sin \theta &= 2\pi \frac{A}{L} \cos \frac{2\pi x}{L} \left[1 + \left(2\pi \frac{A}{L} \cos \frac{2\pi x}{L} \right)^2 \right]^{-1/2} \end{aligned} \quad (4)$$

187 Substituting Eq. (4) into Eq. (3), the integral related terms of $\bar{S}_{\text{Yarn},w}^{ij}$ can be
188 computed. To avoid repeating, the complete expression for each element in
189 the averaged compliance matrix, $\bar{S}_{\text{Yarn},w}^{ij}$ of wavy yarn with consideration of a
190 uniform waviness can be found in (Hsiao and Daniel, 1996a). It is found that

only elements in $\bar{S}_{\text{Yarn},w}^{ij}$ are even function of $\cos \theta$ and/or $\sin \theta$ are non-zero values. Hence, the connection between the stiffness or compliance of yarn with waviness parameters is established analytically.

2.3. First-order computational homogenization for textile composites

The effective yarn elastic tensor obtained from the Mori-Tanaka method is ready to be integrated into the second stage homogenization with the use of a computational homogenization technique to estimate the effective elastic tensor for the complete textile composite. The computational homogenization method has been found to be both feasible and efficient in predicting the effective mechanical properties for various materials from linear elastic to inelastic (Geers et al., 2010). The authors demonstrated its capabilities on materials with complex architectures through modelling plain woven textile composites (Zhou et al., 2016b). The details of the adopted computational homogenization method can be found, e.g. (Perić et al., 2011). Here, a matrix form for finite element implementation is given for the completeness. For a representative volume of element (RVE) for textile composite (e.g. see Fig. 1), its deformation under a given macrostrain $\bar{\epsilon}$ is numerically calculated when specific boundary conditions, usually including linear displacement leading to a stiff solution, constant traction for a compliant solution, and periodic displacements and anti-periodic forces for a in-between solution, are satisfied to enable the transition between microscale and macroscale displacements. Followed by a generalized boundary condition enforcement approach proposed in (Kaczmarczyk et al., 2008), a discretised system of equations is

written as

$$\begin{bmatrix} \mathbf{K} & \mathbf{P}^T \\ \mathbf{P} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{D}\bar{\boldsymbol{\varepsilon}} \end{Bmatrix} \quad (5)$$

where $\mathbf{K}$ and $\mathbf{u}$ are the stiffness matrix and displacement vector for standard finite element method, and $\boldsymbol{\lambda}$ is the unknown vector for Lagrangian multipliers when solving the system through Lagrangian multiplier method. Matrices $\mathbf{P}$ and $\mathbf{D}$ are defined by

$$\mathbf{P} = \int_{\partial\Omega} \mathbf{H}\mathbf{N}^T \mathbf{N} dA \quad \mathbf{D} = \int_{\partial\Omega} \mathbf{H}\mathbf{N}^T \mathbf{X} dA \quad (6)$$

195 where $\mathbf{H}$ is a matrix associated with the imposed boundary conditions,
 196 $\mathbf{N}$ is the standard shape function matrix, and $\mathbf{X}$ is a position matrix evaluated
 197 at the integration points on the RVE boundary. Details of forming $\mathbf{H}$ and $\mathbf{X}$
 198 can be found in (Zhou et al., 2016b) for three dimensional structures.

With the obtained nodal displacement vector $\{\mathbf{u}\}$ and Lagrangian vector $\{\boldsymbol{\lambda}\}$, the macrostress, $\bar{\boldsymbol{\sigma}}$, is computed by using the volume average theorem as $\bar{\boldsymbol{\sigma}} = \mathbf{D}^T \boldsymbol{\lambda} / V$. This then leads to a straightforward way to numerically compute the effective elastic tensor for the textile composite with:

$$\bar{\mathbf{C}}_{\text{Textile}} = \frac{\bar{\boldsymbol{\sigma}}}{\bar{\boldsymbol{\varepsilon}}} = \frac{1}{V_\mu} \frac{\mathbf{D}^T \boldsymbol{\lambda}}{\bar{\boldsymbol{\varepsilon}}} \quad (7)$$

199 It should be noted that $\bar{\mathbf{C}}_{\text{Textile}}$ is a function of $\bar{\mathbf{C}}_{\text{Yarn}}$, $\bar{\mathbf{C}}_{\text{Yarn},\theta}$ or $\bar{\mathbf{C}}_{\text{Yarn},w}$ as
 200 the responses $\{\mathbf{u}\}$ and $\{\boldsymbol{\lambda}\}$ are directly linked with yarn stiffness. Therefore,
 201 a link between the effective material properties of the textile composite and
 202 its constituents' material properties and manufacturing defects is numerically
 203 established through the proposed dual homogenization scheme.

204 3. Uncertainty Propagation with Perturbation Technique

205 3.1. Propagating uncertainties from microscale to macroscale

An n -dimensional random vector $\mathbf{b} = \{b_1, b_2, \dots, b_n\}^T$ is defined to enable the representation of randomness in material properties and manufacturing defects of the constituents, which may include moduli, Poisson's ratios, fibre misalignment angle, fibre volume fraction, and fibre waviness in the present study. As the effective elastic tensor is a function of material properties of constituent materials and related geometrical parameters as indicated in Eq. (7), the presence of variations in these parameters eventually leads the effective elastic tensor to become a stochastic function. On the base of the dual homogenization procedure, a probabilistic homogenization approach, which is an extension of the technique proposed by the authors for considering uncertainties in microscopic material properties (Zhou et al., 2016b), is developed to take uncertainties in these parameters into account directly. For an arbitrary stochastic function, $\varphi(\mathbf{b})$, it can be approximated via a Taylor series expansion as

$$\varphi(\mathbf{b}) = \varphi(\bar{\mathbf{b}}) + \epsilon \sum_{i=1}^n \frac{\partial \varphi(\bar{\mathbf{b}})}{\partial b_i} \delta b_i + \epsilon^2 \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 \varphi(\bar{\mathbf{b}})}{\partial b_i \partial b_j} \delta b_i \delta b_j + \dots \quad (8)$$

206 where $\bar{\mathbf{b}}$ is the mean value of the random vector $\mathbf{b}$, δb_i denotes the variation
 207 around mean value of the i -th random variable, $\frac{\partial \varphi}{\partial b_i}$ and $\frac{\partial^2 \varphi}{\partial b_i \partial b_j}$ denote the
 208 first- and second-order partial derivatives of $\varphi(\mathbf{b})$ with respect to b_i , and ϵ is
 209 a scalar representing a given small perturbation. When integrating this with
 210 finite element method, it forms the perturbation based stochastic finite ele-
 211 ment method (Kleiber and Hien, 1992). It is a popular solution for uncertain
 212 quantification due to its relatively small computational cost and promising

accuracy. For instance, it has been integrated with homogenization method to conduct stochastic analysis for composites, e.g. (Kamiński and Kleiber, 2000; Kamiski, 2005; Zhou et al., 2016a,b). In the present study, the second-order approximation is considered as it has been found to be sufficient, whilst it should be noted that higher-order approximations, e.g. (Kamiński, 2013; Kamiski and Sokoowski, 2016) should be considered if the stochastic function is a highly nonlinear function of the underlying random variables.

For the effective elastic tensors $\bar{\mathbf{C}}_{\text{Yarn}}$, $\bar{\mathbf{C}}_{\text{Yarn},\theta}$, $\bar{\mathbf{C}}_{\text{Yarn},w}$ obtained from Mori-Takana Method and its augmented versions, their stochastic functions can be directly expanded to obtain the corresponding approximation functions following Eq. (8). This is due to the fact that they are explicitly expressed as a function of underlying constituent material properties as shown in Eqs.(1) - (3). Thus, a second-order approximation of the elastic tensor for yarn can be written as:

$$\bar{\mathbf{C}}_{\text{Yarn}}(\mathbf{b}) = \bar{\mathbf{C}}_{\text{Yarn}}(\bar{\mathbf{b}}) + \epsilon \sum_{i=1}^n \left[\frac{\partial \bar{\mathbf{C}}_{\text{Yarn}}(\bar{\mathbf{b}})}{\partial b_i} \right] \delta b_i + \epsilon^2 \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \left[\frac{\partial^2 \bar{\mathbf{C}}_{\text{Yarn}}(\bar{\mathbf{b}})}{\partial b_i \partial b_j} \right] \delta b_i \delta b_j \quad (9)$$

Similarly, the other two effective elastic tensors $\bar{\mathbf{C}}_{\text{Yarn},w}$ and $\bar{\mathbf{C}}_{\text{Yarn},\theta}$ can be approximated in this form. However, they are relatively complicated compared with $\bar{\mathbf{C}}_{\text{Yarn}}$ due to the inclusion of the geometrical parameters. The calculation of their derivatives with respect to geometrical parameters, such as fibre misalignment angle, fibre waviness amplitude and wavelength, requires special attention. The compliance of yarn with waviness, $\bar{\mathbf{S}}_{\text{Yarn},w}$, is calculated first using the strain average approach Eq.(3). The stiffness, $\bar{\mathbf{C}}_{\text{Yarn},w}$, is then computed as the inverse of the compliance. Consequently, the first- and second-order derivatives of compliance matrix are required. It can be found

from Eq. (3) that the elements of $\bar{\mathbf{S}}_{\text{Yarn},w}$ are simply functions of $\bar{\mathbf{S}}_{\text{Yarn}}$. The calculation of the derivatives of the compliance matrix, $\bar{\mathbf{C}}_{\text{Yarn},w}$, is based on the corresponding terms for $\bar{\mathbf{S}}_{\text{Yarn}}$. They can be obtained from the inverse of the stiffness matrix with the following relationship of derivatives between the original matrix and its inverse. Noting that $\bar{\mathbf{S}}$ is symmetric ($\bar{\mathbf{S}}^T \equiv \bar{\mathbf{S}}$), then,

$$\frac{\partial \bar{\mathbf{S}}}{\partial b_i} = \frac{\partial \bar{\mathbf{C}}^{-1}}{\partial b_i} = -\bar{\mathbf{S}} \frac{\partial \bar{\mathbf{C}}}{\partial b_i} \bar{\mathbf{S}} \quad (10)$$

or

$$\frac{\partial^2 \bar{\mathbf{S}}}{\partial b_i \partial b_j} = \frac{\partial^2 \bar{\mathbf{C}}^{-1}}{\partial b_i \partial b_j} = \bar{\mathbf{S}} \frac{\partial \bar{\mathbf{C}}}{\partial b_j} \bar{\mathbf{S}} \frac{\partial \bar{\mathbf{C}}}{\partial b_i} \bar{\mathbf{S}} - \bar{\mathbf{S}} \frac{\partial^2 \bar{\mathbf{C}}}{\partial b_i \partial b_j} \bar{\mathbf{S}} + \bar{\mathbf{S}} \frac{\partial \bar{\mathbf{C}}}{\partial b_i} \bar{\mathbf{S}} \frac{\partial \bar{\mathbf{C}}}{\partial b_j} \bar{\mathbf{S}}, \quad (11)$$

Combining these formulations with Eq. (3), the stochastic approximation of the compliance matrix considering waviness, can be calculated. Once the derivatives of compliance matrix are known, it remains to perform the inverse calculation to obtain its corresponding constitutive matrix $\bar{\mathbf{C}} = \bar{\mathbf{C}}^{-1}$. Similarly, its first- and second-order derivatives can be found from:

$$\frac{\partial \bar{\mathbf{C}}}{\partial b_i} = \frac{\partial \bar{\mathbf{S}}^{-1}}{\partial b_i} = -\bar{\mathbf{S}}^{-1} \frac{\partial \bar{\mathbf{S}}}{\partial b_i} \bar{\mathbf{S}}^{-1} = -\bar{\mathbf{C}} \frac{\partial \bar{\mathbf{S}}}{\partial b_i} \bar{\mathbf{C}} \quad (12)$$

or

$$\frac{\partial^2 \bar{\mathbf{C}}}{\partial b_i \partial b_j} = \frac{\partial^2 \bar{\mathbf{S}}^{-1}}{\partial b_i \partial b_j} = \bar{\mathbf{C}} \frac{\partial \bar{\mathbf{S}}}{\partial b_j} \bar{\mathbf{C}} \frac{\partial \bar{\mathbf{S}}}{\partial b_i} \bar{\mathbf{C}} - \bar{\mathbf{C}} \frac{\partial^2 \bar{\mathbf{S}}}{\partial b_i \partial b_j} \bar{\mathbf{C}} + \bar{\mathbf{C}} \frac{\partial \bar{\mathbf{S}}}{\partial b_i} \bar{\mathbf{C}} \frac{\partial \bar{\mathbf{S}}}{\partial b_j} \bar{\mathbf{C}} \quad (13)$$

220 3.2. Mesoscale uncertainty propagation

It is recalled that the computational homogenization method is applied to predict the effective elastic properties of textile composites. Comparing with the effective elastic tensor obtained from Mori-Tanaka method for fibre yarn, the effective elastic tensor from computation homogenization is implicitly expressed. Its stochastic function is thus impossible to be approximated

directly using Eq. (8). The deterministic computational homogenization method needs to be incorporated with the perturbation technique to obtain the stochastic expression of the effective elastic tensor for the textile composite. Eq. (5) is first restated in a more compact form for the easiness of deriving the formulations as

$$\left[\hat{\mathbf{K}}\right] \{\hat{\mathbf{u}}\} = \left\{\hat{\mathbf{F}}\right\}. \quad (14)$$

As we only consider uncertainties in material properties and manufacturing defects in constituent materials, the stiffness matrix $\left[\hat{\mathbf{K}}\right]$ is a stochastic function while the load vector is still a deterministic function. However, the nodal displacement vector $\{\hat{\mathbf{u}}\}$ becomes a stochastic function. Hence, these two terms in Eq.(14) need to be expanded in the form of Eq. (8). Substituting them into the equation and equating terms of equal orders of ϵ , we arrive at the following zeroth-, first- and second-order equations:

- The zeroth-order

$$\left[\hat{\mathbf{K}}\right] \{\hat{\mathbf{u}}\} = \left\{\hat{\mathbf{F}}\right\} \quad (15)$$

- The first-order

$$\sum_{p=1}^n \left\{ \left[\hat{\mathbf{K}}\right] \left\{ \frac{\partial \hat{\mathbf{u}}}{\partial b_p} \right\} + \left[\frac{\partial \hat{\mathbf{K}}}{\partial b_p} \right] \{\hat{\mathbf{u}}\} \right\} = \mathbf{0} \quad (16)$$

- The second-order

$$\sum_{p=1}^n \sum_{q=1}^n \left\{ \left[\hat{\mathbf{K}}\right] \left\{ \frac{\partial^2 \hat{\mathbf{u}}}{\partial b_p \partial b_q} \right\} + \left[\frac{\partial \hat{\mathbf{K}}}{\partial b_p} \right] \left\{ \frac{\partial \hat{\mathbf{u}}}{\partial b_q} \right\} + \left[\frac{\partial^2 \hat{\mathbf{K}}}{\partial b_p \partial b_q} \right] \{\hat{\mathbf{u}}\} \right\} = \mathbf{0} \quad (17)$$

Linking with Eq. (5), the uncertainties in the material properties and manufacturing defects are reflected in $\mathbf{K}$, while the block matrix $\mathbf{P}$ is still a

deterministic term. Hence, the first- and second-order derivatives for $\hat{\mathbf{K}}$ will only have contributions from $\mathbf{K}$, while the rest blocks are zeros. The compact displacement vector $\{\hat{\mathbf{u}}\}$ and its first- and second-order partial derivatives can be calculated from Eqs.(15) - (17), and the corresponding terms for $\mathbf{u}$ and $\boldsymbol{\lambda}$ can be obtained from splitting it from $\hat{\mathbf{u}}$. These are then used to calculate the effective elastic moduli and its first- and second-order derivatives using Eq. (7) to form its approximation:

$$\bar{\mathbf{C}}_{\text{Textile}}(\mathbf{b}) = \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}}) + \epsilon \sum_{i=1}^n \left[\frac{\partial \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_i} \right] \delta b_i + \epsilon^2 \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \left[\frac{\partial^2 \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_i \partial b_j} \right] \delta b_i \delta b_j. \quad (18)$$

Given the approximation for $\bar{\mathbf{C}}_{\text{Textile}}(\mathbf{b})$ in Eq. (18), its statistics in terms of mean value and variance can be easily calculated from the following equations for the second-order approximation (Ang and Tang, 1975; Kleiber and Hien, 1992). The mean value is

$$E [\bar{\mathbf{C}}_{\text{Textile}}(\mathbf{b})] = [\bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})] + \frac{1}{2} \sum_r^n \sum_s^n \left[\frac{\partial^2 \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_r \partial b_s} \right] \cdot \text{COV}(b_r, b_s), \quad (19)$$

and the variance is

$$\begin{aligned} \text{COV} ([\bar{\mathbf{C}}_{\text{Textile}}(\mathbf{b})]_r, [\bar{\mathbf{C}}_{\text{Textile}}(\mathbf{b})]_s) &\approx \sum_{r,s}^n \left[\frac{\partial \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_r} \right] \left[\frac{\partial \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_s} \right] \cdot \text{COV}(b_r, b_s) \\ &+ \frac{1}{4} \sum_{r,s,t,w}^n \left[\frac{\partial^2 \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_r \partial b_s} \right] \left[\frac{\partial^2 \bar{\mathbf{C}}_{\text{Textile}}(\bar{\mathbf{b}})}{\partial b_t \partial b_w} \right] E [b_r b_s b_t b_w]. \end{aligned} \quad (20)$$

228 4. Numerical example

229 Three sets of studies have been conducted. The first study aims to show
230 how the effective elastic properties of yarn and textile composites are affected

231 by the investigated manufacturing defects in a deterministic context. In
 232 the second and third studies, uncertainties in textile composites due to the
 233 variations are evaluated through statistical and reliability analyses. The
 234 numerical implementation of the proposed method is carried out on MoFEM
 235 ([Kaczmarczyk et al., 2017](#)), which is an open source finite element analysis
 236 code. Noting the studies are for illustrative purposes, only in-plane fibre
 237 misalignment and waviness are shown, but out-of-plane counterparts can be
 238 easily adopted. More details are given in the following sections, while a brief
 239 description of the studied textile composite is given first.

240 The plain woven textile composite’s microstructure studied in ([Karkkainen](#)
 241 [and Sankar, 2006](#)) is used. To establish the geometrical model, the idealized
 242 periodic microstructure proposed by [Barbero et al. \(2005\)](#) is adopted. It is
 243 assumed that yarn cross-section and path take the form of sinusoidal func-
 244 tion $F(x, y) = A(y) \cdot \sin(B(y) \cdot x + C(y)) + D(y)$ with coefficients A , B , C
 245 and D determined from the geometric parameters of the microstructure. The
 246 geometry of the plain weave composite is shown in Fig. 1 and comprises four
 247 interlaced fibre yarns. It is described through the periodic length of warp
 248 and weft yarns, $4a_1$ and $4a_2$, respectively, crimp amplitude $2a_3$, and spacing
 249 between adjacent warp or weft yarns, a_{g_1} and a_{g_2} . Therefore, the dimensions
 250 of the RVE are $4a_1 \times 4a_2 \times 2a_3$. These parameters are illustrated in Fig.
 251 1 and their values are listed in Table 1. The RVE consists of an isotropic
 252 epoxy matrix and carbon fibre yarns. The warp and weft yarns are made
 253 of identical material. A total of seven independent material parameters are
 254 used to describe the matrix and yarn and their values are presented in Table
 255 2. Due to the flow of epoxy through the fibrous preform during infusion, the

256 yarn is treated as a composite comprising fibre and matrix. In this study,
 257 the fibre volume fraction in the yarn, v_f , is assumed to be 65%. This geo-
 258 metrical model has been programmed in Python as a journal file to be read
 259 by the meshing software, Trelis, a two- and three-dimensional finite element
 260 mesh and geometry tool developed by Sandia National Laboratories in the
 261 US. Further details on the implementation and the preparation to generate
 262 the required model for the computational homogenization analysis are avail-
 263 able in (Zhou et al., 2016b). The representative volume element has been
 264 meshed using 45,427 tetrahedral elements with 18,587 elements representing
 265 the yarns and 26,640 elements the matrix.

266 4.1. *Effects of manufacturing defects on the effective elastic properties*

267 Before considering uncertainty quantification, the capability of the pro-
 268 posed dual homogenization method to capture the manufacturing defects has
 269 been evaluated first in the deterministic analysis context by investigating the
 270 effective elastic properties of fibre yarn and textile composite.

271 4.1.1. *Yarn*

272 Figs. 2 - 4 show the effects of fibre volume fraction, fibre misalignment
 273 and fibre waviness on the effective elastic properties of the unidirectional
 274 fibre yarn, respectively. The results shown in these figures are normalized
 275 against a yarn with a volume fraction of 65% and without misalignment and
 276 waviness. Let us assume the fibre yarn in a coordinate system (1-2-3) with
 277 the 1-axis parallel to the fibres and the 3-axis in the through-thickness di-
 278 rection. It is seen in Fig. 2 that the three Young's moduli increase with
 279 increases of fibre volume fraction. The longitudinal Young's modulus, E_{11} ,

280 shows clearly a linear relation with fibre volume fraction as expected, while
 281 the two Young's moduli in the directions perpendicular to the fibres also in-
 282 crease with slightly nonlinear behaviour, which is also expected as they are
 283 functions of the reciprocal of fibre volume fraction indicated in the inverse
 284 rule of mixture matrix. The three shear moduli, i.e. G_{12} , G_{13} and G_{23} ,
 285 display similar characteristics with changes in fibre volume fraction. Their
 286 values become larger with the increase of fibre volume fraction. Poisson's
 287 ratios also have clear links with fibre volume fraction as shown in Fig. 2.
 288 Compared with the transverse Poisson's ratio, ν_{23} , the two major Poisson's
 289 ratios, ν_{12} and ν_{13} , are affected less by the fibre volume fraction. With the
 290 increase of fibre volume fraction, the dominant role of Poisson's ratio of the
 291 matrix on ν_{23} gradually switches to the transverse Poisson's ratio of the fi-
 292 bre. Fig. 3 gives similar results from the effect of the in-plane (1-2) fibre
 293 misalignment angle from 0° to 10° . The longitudinal Young's modulus signifi-
 294 cantly decreases with the increase of misalignment angle, while the transverse
 295 terms dramatically increase. Similarly, the transverse shear modulus, G_{12} ,
 296 and transverse Poisson's ratio, ν_{12} , show a direct link with the in-plane fibre
 297 misalignment, while the other terms are affected less. Fig. 4 illustrates the
 298 influence of fibre waviness in 1-2 plane on the effective elastic properties. As
 299 expected, the longitudinal Young's modulus seriously degrades with the in-
 300 crease in fibre waviness, while the transverse Young's modulus is also affected
 301 but to a less extent. Transverse shear modulus, G_{12} and transverse Poisson's
 302 ratio, ν_{12} show clearly that they vary as a function of fibre waviness. Other
 303 properties are insensitive to fibre waviness as the results indicated.

304 4.1.2. Textile

305 The three manufacturing defects show significant effects on the effective
306 elastic properties of fibre yarn in the study in the previous section. Their
307 influences on the effective elastic or mechanical properties of the correspond-
308 ing textile composites are further investigated and reported here. Results for
309 textile composites for three different local fibre volume fractions of 50%, 65%
310 and 80% are shown. Fig. 5 illustrates the effects of in-plane fibre misalign-
311 ment on the effective elastic properties. The local volume fraction was set
312 constant at 65%, in isolation of local fibre waviness. Fig. 5a shows that fibre
313 misalignment has a dramatic effect on E_{xx} for the textile composite. A re-
314 duction of about 20% is observed in the value of E_{xx} for a fibre misalignment
315 of 10° . The fibre misalignment also significantly affects the major Poisson's
316 ratio ν_{xy} as shown in Fig. 5d, where the increase of fibre misalignment angle
317 leading to an increase of composite ν_{xy} has been observed. For instance,
318 the misalignment of 10° gives a 36% increase. However, the in-plane fibre
319 misalignment has a much smaller effect on through-thickness Young's mod-
320 ulus, E_{zz} , and in-plane shear modulus, G_{xy} as Fig. 5 b and c indicate. It
321 is thus important to take this defect into account as it is common to have a
322 misalignment of $\pm 3.8^\circ$ (Potter et al., 2008).

323 Similar studies on the effects of fibre waviness have been conducted. Fig.
324 6 illustrates the effective elastic properties as a function of waviness param-
325 eter. It is shown in Fig. 6a that the longitudinal Young's modulus, E_{xx} ,
326 degrades significantly as the fibre waviness increases, while the increase of
327 fibre waviness leads to a dramatic increase of ν_{xy} . Other properties including
328 through thickness Young's modulus E_{zz} and in plane shear modulus G_{xy} are

also affected by the fibre waviness but to less extent compared with E_{xx} and ν_{xy} . These observations agree with similar studies conducted for unidirectional composites in (Hsiao and Daniel, 1996b; Tsai et al., 2011). It thus demonstrates that the proposed dual homogenization method can capture these manufacturing defects correctly.

4.2. *Effect of variations of manufacturing defects on the effective engineering properties*

According to Eqs. (19) and (20), the mean value and variance of the effective elastic tensor due to the variations of constituent material properties and manufacturing defects can be estimated. Subsequently, the statistics of engineering properties of the investigated textile composites in terms of mean value and variance can be found. In the following studies, only the variations of manufacturing defects are investigated, while the relations between the variations of constituent material properties and the effective engineering properties were studied in (Zhou et al., 2016b) for plain woven textile composites. As indicated in Eq. (19), only second-order derivatives contribute to the mean value, and their influences are relatively small, as indicated in Table 3. Mean values and coefficients of variation for E_x , E_z , G_{xy} , and ν_{xy} due to 10% variations in fibre misalignment angle, volume fraction, and waviness are given, and the corresponding values without variations in these defects are given in the parentheses for references. We only show coefficients of variation for the effective elastic properties.

Fig. 7 shows how the coefficients of variation of the four investigated elastic properties, i.e. E_x , E_z , G_{xy} , and ν_{xy} , vary as a function of the coefficient of variation describing uncertainty in the fibre alignment. In these

354 figures, the horizontal axis represents the coefficient of variation of the input,
 355 and the vertical axis represents the resultant coefficient of variation of the
 356 effective engineering properties. Five mean values of fibre angle from 1° to
 357 5° and each with coefficient of variation ranging from 0 to 25% have been
 358 investigated. Although the fibre misalignment angle shows significant influ-
 359 ence on all the four engineering properties (e.g. Fig. 3), its effect on the
 360 coefficient of variation is relatively small. For instance, it is observed that
 361 only a small variation of about 3% is seen in the axial Young's modulus with
 362 a 25% variation in the fibre misalignment angle. Other properties, with the
 363 exception of ν_{xy} show even smaller coefficients of variation.

364 Fig. 8 illustrates the variability of the elastic properties, E_x , E_z , G_{xy} ,
 365 ν_{xy} , due to the variations in fibre volume fraction in the yarn. Significant
 366 increases of variation can be observed in all four properties when variation
 367 in fibre volume fraction becomes severe. For instance, a variation of 25%
 368 in fibre volume fraction leads to around 26% and 28% variations in E_x and
 369 G_{xy} , respectively, but slight smaller variations are seen in E_z and ν_{xy} . Five
 370 mean values from 0.35 to 0.75 have been investigated, and the corresponding
 371 results are shown in the figure. With increases of mean values, a slight non-
 372 linear relation between the coefficients of variation for fibre volume fraction
 373 and the elastic properties can be observed. Variations in the three moduli,
 374 i.e. E_x , E_z , and G_{xy} become larger when the mean value of fibre volume
 375 fraction increases, while the coefficient of variation of ν_{xy} does not show such
 376 clear evidence. This is expected as the moduli are functions of fibre volume
 377 fraction.

378 Results reported in Figs. 9 and 10 show how the coefficients of variation

379 for the elastic properties are affected by the coefficient of variations of wavi-
 380 ness length, L , and amplitude, A , respectively. In the results for variations
 381 in wavelength, calculations have been conducted with different mean value
 382 of wavelength while the amplitude has been kept unchanged. Hence, larger
 383 wavelength is actually led to smaller waviness. Similarly, the wavelength is
 384 constant in the case having variation in amplitude. Five mean values of am-
 385 plitude were considered. Similar observations are found in both wavelength
 386 and amplitude led variations in the four investigated effective elastic prop-
 387 erties. The coefficients of variation for E_x and ν_{xy} are strongly related to
 388 the two variation sources, while only small variation can be seen in the other
 389 two elastic properties, i.e. E_z and G_{xy} . It is expected that in-plane waviness
 390 should have a stronger influence on the in-plane properties, e.g. E_x , while
 391 the out-of-plane counterparts, such as through thickness Young's modulus,
 392 should be less influenced. Hence, the results are intuitively reasonable.

393 *4.3. Effect of variations of manufacturing defects on structural performance* 394 *in terms of reliability*

Results presented in the previous section show clear evidence that the
 effective elastic properties of the investigated composites are functions of the
 investigated manufacturing defects. In the presence of uncertainties in these
 manufacturing defects, significant variations in the elastic properties of com-
 posites can be seen. Therefore, it is desirable to include them in probabilistic
 structural analysis. Here, the uncertainties in manufacturing defects were
 considered together with other parameters, such as material properties and
 strength (see Table 2), to conduct probability-based structural performance
 evaluations through structural reliability analyses. In the reliability analy-

sis for composite structures, it is normal to develop a limit state function from failure criterion. Compared with popularly used unidirectional composites, failure criteria are seldom available for textile composites. Here, a quadratic failure criterion based on direct micromechanics analysis proposed in (Karkkainen et al., 2007) is adopted, such that the limit state function is expressed as:

$$g(\mathbf{x}) = 1 - \left[\gamma_{ij} \frac{F_i(\mathbf{x})}{S_i} \frac{F_j(\mathbf{x})}{S_j} + D_i \frac{F_i(\mathbf{x})}{S_i} \right] \quad (21)$$

where $\mathbf{x}$ are the vector of random variables, e.g. manufacturing defects, F_i s represent the general load terms ($N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$), S_i s denote the corresponding strength terms, and γ_{ij} s or D_i s represent the 27 failure coefficients. For different plain woven textile composites, coefficients, γ_{ij} and D_i need to be found through micromechanics analyses as described in (Karkkainen and Sankar, 2006). For the RVE with the architecture illustrated in Fig. 1 and described in Table 1, corresponding values for γ_{ij} and D_i have been reported in (Karkkainen et al., 2007) and are adopted for the reliability analysis here. For convenience, they are replicated in Table 4.

When considering multiple random variables, e.g. 21 parameters as listed in Table 2 and uncertainties in parameters crossing different length scales, e.g. manufacturing defects in microscale and ply strengths in macroscale in the present study, the probability of failure related to Eq. (21) can not be solved analytically. The multiscale finite element based reliability method proposed in (Zhou et al., 2017) by the authors has been adopted to numerically solve Eq. (21). A stochastic multi-scale finite element method, integrating the probabilistic homogenisation formulation in Eq. (18) with the perturbation-based stochastic finite element method, is used to calculate the stochastic

413 structural responses by propagating uncertainties across scales. Given the
414 first- and second-order partial derivatives are by-products of the perturbation
415 based stochastic finite element method, it is easily combined with the first-
416 order reliability method to numerically approximate Eq. (21). This enables
417 the influences of the investigated uncertainties in manufacturing defects to be
418 reflected on structural performance. The complete implementation procedure
419 can be found in (Zhou et al., 2017).

420 For illustration purposes, a single layer carbon fibre reinforced polymer
421 textile composite laminate has been analysed. Fig. 11 shows the probabil-
422 ities of failure of the laminate under uni- and bi-axial tension with various
423 load cases. The probabilities of failure have been calculated with the use of
424 the first-order reliability method. It can be seen that the laminate is more
425 prone to failure under uniaxial tension than biaxial tension as the cumula-
426 tive distribution function curve for uniaxial tension is shifted to the left side
427 of the corresponding one for biaxial tension. This is also indicated in the
428 critical loading. For a target reliability index of 3.75, the critical loading
429 recovered from Fig. 11 is about 20 MPa for uniaxial loading and 30 MPa
430 for biaxial loading. It is worth noting that the most sensitive parameters are
431 those related to ply strength rather than those contributed to elastic prop-
432 erties under these simple loading conditions. It also points out the necessity
433 of further research to propagate the variations in defects into ply strengths.

434 5. Conclusions

435 In this paper, a probabilistic homogenization approach has been presented
436 that is capable of taking into account the effects of microscopic manufactur-

437 ing defects including fibre misalignment, waviness and volume fraction on the
438 mechanical behaviour of textile composite materials. The unique contribu-
439 tion of the proposed approach is its ability to predict not only variations in
440 the effective elastic or mechanical properties but also structural responses in
441 terms of reliability. Although the importance of manufacturing defects and
442 their inherent variations on composite materials have been well recognized,
443 a systematic analysis method has not been seen to quantify their effects on
444 both mechanical properties and structural performance, especially for textile
445 composites. The proposed approach uses two homogenization techniques,
446 namely Mori-Tanaka and computational homogenization, to link lower scale
447 parameters with upper scale quantities. A Taylor series expansion enables the
448 link between uncertainties in parameters and resultant stochastic responses
449 to be established. These features and capabilities have been implemented
450 within a finite element framework. Uncertainties arising from microscopic
451 manufacturing defects have been successfully propagated into the effective
452 elastic properties of the textile composites. Furthermore, the influences of
453 these uncertainties have been quantified by conducting structural reliabil-
454 ity analyses. Numerical studies have been conducted to demonstrate the
455 capability of the proposed method in quantifying the variations in these mi-
456 croscopic parameters. The effectiveness of the proposed method in capturing
457 microscopic manufacturing defects has been first verified by investigating the
458 effective elastic properties of a fibre yarn which is equivalent to an unidirec-
459 tional composite and then by exploring the performance of a plain woven tex-
460 tile composite. Variations in these manufacturing defects have been further
461 studied through examining the variations in the effective elastic properties

462 of the textile composite under a variety of uncertainties. Finally, variations
463 of these manufacturing defects have been taken into account together with
464 variations in other parameters including material properties and strength
465 to demonstrate the capability of multiscale structural reliability analysis for
466 textile composite laminates.

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Table 1: Geometrical parameters of the plain woven textile RVE, unit: μm , (from (Karkkainen and Sankar, 2006))

Term	Symbol	Value
Warp direction	a_1	420
Warp yarn spacing	a_{g1}	140
Weft direction	a_2	420
Weft yarn spacing	a_{g2}	140
Crimp amplitude	a_3	127
RVE length	l	1680
RVE width	w	1680
RVE thickness	h	254

Table 2: Statistical information of AS4/3501-6 carbon fibre reinforced polymer composite

Scale	Item	Random variables	Unit	Mean*	Coefficient of variation [†]	Distribution type
Micro	Fibre	E_{f1}	GPa	225	0.1	Normal
		E_{f2}	GPa	15	0.1	Normal
		ν_{f12}	-	0.2	0.1	Normal
		ν_{f23}	-	0.0714	0.1	Normal
		G_{f12}	GPa	15	0.1	Normal
	Matrix	E_m	GPa	3.5	0.1	Normal
		ν_m	-	0.35	0.05	Normal
	Defect	θ	°	-5	5	Uniform
		V_f	-	0.65	0.1	Normal
		L	mm	27.9	0.1	Normal
		A	mm	1.359	0.1	Normal
Macro	Strength	N_{X_T}	$Pa \cdot m$	6.4e3	0.1	Log-Normal
		N_{X_C}	$Pa \cdot m$	5.86e3	0.12	Log-Normal
		N_{Y_T}	$Pa \cdot m$	6.4e3	0.06	Log-Normal
		N_{Y_C}	$Pa \cdot m$	5.86e3	0.08	Log-Normal
		N_{XY_T}	$Pa \cdot m$	2.11e3	0.11	Log-Normal
		N_{XY_C}	$Pa \cdot m$	2.11e3	0.11	Log-Normal
		M_{X_C}	$Pa \cdot m^2$	1.85e-4	0.12	Log-Normal
		M_{Y_T}	$Pa \cdot m^2$	1.85e-4	0.06	Log-Normal
		M_{Y_C}	$Pa \cdot m^2$	1.85e-4	0.08	Log-Normal
		M_{XY_T}	$Pa \cdot m^2$	1.85e-4	0.11	Log-Normal
		M_{XY_C}	$Pa \cdot m^2$	1.85e-4	0.11	Log-Normal

* and †: lower and upper limits, respectively, for random variable with uniform distribution.

Table 3: Mean values and coefficients of variation for the effective mechanical properties due to 10% variations in various manufacturing defects

Mechanical properties	$\theta = 0$		$V_f = 0.65$		$A/L = 0.05$	
	MV	COV	MV	COV	MV	COV
E_x (GPa)	19.63 (19.36)	0.1272	19.47 (19.36)	0.0902	13.57 (13.54)	0.0462
E_z (GPa)	6.16 (6.14)	0.0016	6.15 (6.14)	0.0390	5.90 (5.91)	0.0052
ν_{xy}	0.06 (0.06)	0.1107	0.06 (0.06)	0.0514	0.11 (0.11)	0.0709
G_{xy} (GPa)	2.51 (2.52)	0.0035	2.54 (2.52)	0.0871	2.47 (2.47)	0.0002

Table 4: Normalized failure coefficients γ_{ij} for quadratic failure equation, (from (Karkkainen and Sankar, 2006))

i j	1	2	3	4	5	6
1	1.02	-0.81	2.45	0.15	0.15	-0.09
2	-0.81	1.02	2.45	0.15	0.15	-0.09
3	2.45	2.45	9.29	0.15	0.15	-1.28
4	0.15	0.15	0.15	1.00	-0.65	0.29
5	0.15	0.15	0.15	-0.65	1.00	0.29
6	-0.09	-0.09	-1.28	0.29	0.29	1.00

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Figure 1: Geometry of the RVE and the finite element mesh of the reinforcement

Figure 2: Effect of fibre volume fraction on the effective elastic properties of yarn

Figure 3: Effect of fibre misalignment on the effective elastic properties of yarn

Figure 4: Effect of fibre waviness on the effective elastic properties of yarn

(a) Longitudinal modulus E_{xx} (b) Through thickness modulus E_{zz} (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 5: Effect of misalignment angle on the effective elastic properties

(a) Longitudinal modulus E_{xx} (b) Through thickness modulus E_{zz} (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 6: Effect of waviness on the effective elastic properties

(a) Longitudinal modulus E_x (b) Through thickness modulus E_z (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 7: Coefficient of variation of the effective elastic properties due to variation of fibre misalignment angle

(a) Longitudinal modulus E_x (b) Through thickness modulus E_z (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 8: Coefficient of variation of the effective elastic properties due to variation of fibre volume fraction

(a) Longitudinal modulus E_x (b) Through thickness modulus E_z (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 9: Coefficient of variation of the effective elastic properties due to variation of wavelength

(a) Longitudinal modulus E_x (b) Through thickness modulus E_z (c) In-plane shear modulus G_{xy} (d) In-plane Poisson's ratio ν_{xy}

Figure 10: Coefficient of variation of the effective elastic properties due to variation of waviness amplitude

Figure 11: Probability of failure for single layer CFRP laminate under uni- and biaxial tensile loading