Numerical investigation of aspect ratio influences on Rayleigh-Bénard convection of power-law fluids in vertical cylindrical annuli

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KEYWORDS: Natural convection, aspect ratio, Power-law model, cylindrical enclosures, constant wall temperature, constant wall heat flux.

ABSTRACT

The influences of aspect ratio on laminar Rayleigh-Bénard convection of power-law fluids in cylindrical annular enclosures have been numerically investigated for Neumann and Dirichlet boundary conditions for the differentially heated horizontal walls. The axisymmetric simulations have been carried out for a range of different Rayleigh number (i.e. $Ra = 10^3 - 10^5$), aspect ratio (i.e. $AR = H/L$ where H is the enclosure height and L is the difference between outer and inner radii) (i.e. $AR = 0.25 - 4$), power-law index (i.e. $n = 0.6 - 1.8$) and normalised inner radius (i.e. $r_i/L = 0 - 16$ where r_i is internal cylinder radius) for a nominal representative Prandtl number (i.e. $Pr = 10^3$). It has been found that thermal convection weakens with an increase in AR and conductive thermal transport becomes dominant for $AR > 3$ irrespective of the values of Ra , n and r_i/L for both boundary conditions. The mean Nusselt number $\overline{Nu}_{cy}$ exhibits a complex non-monotonic behaviour with the variations of above parameters due to the changes in flow pattern (e.g. number of cells) for shear-thinning fluids (i.e. $n < 1$). However, this tendency is much weaker for shear-thickening fluids (i.e. $n > 1$) for both boundary conditions. Moreover, the critical Rayleigh number Ra_{crit} for the onset of convection of power-law fluids is found to be largely independent of r_i/L for both Neumann and Dirichlet boundary conditions.

NOMENCLATURE

AR	[-]	Aspect ratio (H/L)
c_p	[J/kgK]	Specific heat at constant pressure
g	[m/s ²]	Gravitational acceleration
h	[W/m ² K]	Heat transfer coefficient
k	[W/mK]	Thermal conductivity
L	[m]	height of the enclosure and difference between inner and outer radius
n	[-]	Power-law index
$\overline{Nu}$	[-]	Mean Nusselt number
$\overline{Nu}_{cy}$	[-]	Mean Nusselt number for cylindrical annular enclosure
P	[Pa]	Pressure
Pr	[-]	Prandtl number
q	[W/m ²]	Heat flux
r	[m]	Radius
r_g	[-]	Grid expansion ratio
r_i	[m]	Inner radius
r_o	[m]	Outer radius
Ra	[-]	Rayleigh number
T	[K]	Temperature
u_i	[m/s]	i^{th} velocity component
U, W	[-]	Dimensionless radial ($U = u L / \alpha$) and vertical velocity ($W = w L / \alpha$)
U_{ref}	[m/s]	Reference velocity scale
$\mathcal{U}$	[m/s]	Characteristic velocity
x_i	[m]	Coordinate in i^{th} direction
α	[m ² /s]	Thermal diffusivity
β	[1/K]	Coefficient of thermal expansion
δ, δ_{th}	[m]	Velocity and thermal boundary-layer thickness
θ	[-]	Dimensionless temperature
μ	[Ns/m ²]	Dynamic viscosity
ν	[m ² /s]	Kinematic viscosity
ρ	[kg/m ³]	Density
$\tau_{ij} (\tau)$	[Pa]	Stress tensor (stress)
ϕ	[-]	Azimuthal co-ordinate
Ψ	[-]	Dimensionless stream function

Subscripts

C	Cold wall
cen	Geometrical centre
cy	Cylindrical Annular Enclosure
eff	Effective value
H	Hot wall
nom	Nominal value
rec	Rectangular
ref	Reference value

Special characters

ΔT	[K]	Temperature difference
$\Delta_{min,cell}$	[m]	Minimum cell distance

1. INTRODUCTION

Shear-thinning fluids are a special type of non-Newtonian fluids, which show a decrease in viscosity with increasing shear rate, whereas viscosity increases with increasing shear rate for shear-thickening fluids. Majority of synthetic and biological fluids exhibit shear-thinning (e.g. ketchup, blood, silicone oils and coatings) and shear-thickening (e.g. mixtures of corn starch and water) behaviours. Rayleigh-Bénard convection (i.e. the buoyancy-driven flow of a fluid heated from below and cooled from above, which gives rise to convection cells between the differentially heated horizontal walls) of shear-thinning and shear-thickening fluids in enclosed spaces has a broad range of applications in food and chemical processing, pharmaceutical, biomedical and biological process, oil industry, nuclear and solar power systems [1]. Therefore, a number of studies concentrated on Rayleigh-Bénard convection of shear-thinning and shear-thickening fluids [2-16] in the existing literature. Recently, Darbouli *et al.* [2] reported that 0.1-0.2% aqueous solutions of xanthan gum exhibit shear-thinning behaviour and analysed the onset of Rayleigh-Bénard convection for shear-thinning fluids in cylindrical enclosures. A significant enhancement of convective heat transfer rate due to shear-thinning behaviour has been observed in the experimental investigation by Inaba *et al.* [3] and also in numerical simulations by Ohta *et al.* [4], which focused on Rayleigh-Bénard convection of shear-thinning fluids in square enclosures.

Power-law model of viscosity is often used to model the shear rate dependence of viscosity for numerical modelling of the Rayleigh-Bénard convection of both shear-thinning and shear-thickening fluids [5-16]. Therefore, these fluids have been termed as power-law fluids in the existing literature. The main findings of analytical and numerical investigations of the Rayleigh-Bénard convection of power-law fluids are summarised in Table 1. Furthermore, all previous studies (excluding references [2] and [16]) have concentrated on rectangular

enclosures. Amongst these, references [13] and [14] indicate that the convective transport weakens with an increase in aspect ratio (height: length ratio) for Rayleigh-Bénard convection of power-law fluids in rectangular enclosures. As the convective transport rate weakens, the diffusive transport rate strengthens with increasing aspect ratio [13, 14].

In fact, in most engineering applications, Rayleigh-Bénard convection of power-law fluids takes place in cylindrical annular spaces (e.g. canned foods, chemical and biological fluid storages, solar collectors, heat exchangers, etc.). Recently, it has been reported by Yigit *et al.* [16] that the convective thermal transport strengthens with increasing internal cylinder radius for laminar Rayleigh-Bénard convection of power-law fluids in square cross-sectional cylindrical annular enclosures. Thus, laminar Rayleigh-Bénard convection behaviour is expected to be different in cylindrical annular enclosures in comparison to that in the rectangular enclosures due to the influences of wall curvature. However, the effects of aspect ratio (i.e. $AR = H/L$ where H is the enclosure height and L is the difference between outer and inner radii) in vertical cylindrical annuli with differentially heated horizontal walls is yet to be analysed in the existing literature.

This gap is addressed here by numerically analysing natural convection of power-law fluids in vertical cylindrical annuli with differentially heated horizontal walls for different values of nominal Rayleigh number (i.e. $Ra = 10^3 - 10^5$), aspect ratio (i.e. $AR = 0.25 - 4$) and normalised inner radius (i.e. $r_i/L = 0 - 16$, where r_i is the internal cylinder radius), power-law index (i.e. $n = 0.6 - 1.8$) for a representative nominal Prandtl number $Pr = 10^3$. Moreover, previous findings of Refs. [13, 14] indicated that the boundary conditions of the differentially heated horizontal walls significantly affect the aspect ratio AR dependence of the mean Nusselt number $\overline{Nu}$ in rectangular enclosures and thus, both constant wall temperature

(CWT) and constant wall heat flux (CWHF) boundary conditions have been considered for this analysis. In this respect, the main objectives of this analysis are:

1. To analyse the influences of AR , Ra and r_i/L on the mean Nusselt number $\overline{Nu}$ for Rayleigh-Bénard convection of power-law fluids in cylindrical annular spaces.
2. To provide physical explanations for the above influences using scaling arguments and numerical results.

The mathematical background and numerical implementation pertaining to this analysis are presented in next two sections. Following this, the results will be presented and discussed before the conclusions are drawn.

2. MATHEMATICAL BACKGROUND

According to the power-law model the viscous stress tensor τ_{ij} is expressed as [7]:

$$\tau_{ij} = \mu_a e_{ij} = K(e_{kl}e_{kl}/2)^{(n-1)/2} e_{ij} \quad (1)$$

Here, $e_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the rate of strain tensor, K is the consistency and n is the power-law index and $\mu_a = K(e_{kl}e_{kl}/2)^{(n-1)/2}$ is the apparent viscosity. The apparent viscosity μ_a decreases with increasing shear rate for $n < 1$, whereas for $n > 1$ the apparent viscosity increases with increasing shear rate. Thus, fluids with $n < 1$ and $n > 1$ are referred to as shear-thinning and shear-thickening fluids, respectively, whereas n is unity (i.e. $n = 1$) for Newtonian fluids. According to the Buckingham's pi theorem in this configuration one gets: $Nu = f(Ra, Pr, n, r_i/L, AR)$ where the nominal Rayleigh and Prandtl numbers for power-law fluids are defined as [8, 9, 13, 14]:

$$Ra = \frac{g\beta\Delta T_{ref}H^{2n+1}}{\alpha^n(K/\rho)}; Pr = (K/\rho)\alpha^{n-2}H^{2-2n} \quad (2)$$

where $g, \beta, \Delta T_{ref}$ are the acceleration due to gravity, volume expansion coefficient and characteristic temperature difference. In Eq. 2, Rayleigh number (i.e. Ra) represents the ratio

of heat transfer due to fluid flow induced by buoyancy to thermal transport due to heat conduction, whereas Prandtl number (i.e. Pr) represents the ratio of momentum diffusion to thermal diffusion. The characteristic temperature difference ΔT_{ref} can be considered as the difference between hot and cold wall temperatures (i.e. $\Delta T_{ref} = T_H - T_C$) in the CWT boundary condition, whereas ΔT_{ref} is taken to be $\Delta T_{ref} = qH/k$ for CWHF boundary condition.

For the current investigation, the local heat transfer coefficient h is defined as:

$$h = |-k(\partial T / \partial z)_{z=0} \times 1 / (T_{z=0} - T_{z=H})| \quad (3)$$

The mean heat transfer coefficient and the mean Nusselt number (i.e. ratio of convective and conductive heat transfer) are evaluated as:

$$\bar{h} = \int_{r_i}^{r_i+L} 2\pi r h dr / [\pi(r_i + L)^2 - \pi r_i^2], \quad \overline{Nu}_{cy} = \bar{h}H/k \quad (4)$$

The spatial co-ordinates, velocity components, pressure and temperature are expressed in the dimensionless form for steady-state, incompressible flows with constant thermophysical properties (e.g. k, c_p, n and K):

$$r^+ = r_i/H; \quad z^+ = z/H; \quad u_i^+ = u_i/U_{ref}; \quad P^+ = P/\rho U_{ref}^2; \quad \tau_{ij}^+ = \tau_{ij}H/\rho \alpha U_{ref}; \quad \Theta = (T - T_{ref})/\Delta T_{ref} \quad (5)$$

where $U_{ref} = \alpha/H$ is the reference velocity scale, T_{ref} is the reference temperature. The cold wall temperature T_C is expressed as reference temperature T_{ref} for the CWT boundary condition, whereas the temperature at the centre of the domain T_{cen} is taken to be the reference temperature for the CWHF boundary condition. The reference temperature difference ΔT_{ref} is taken to be $(T_H - T_C)$ (qH/k) for the CWT (CWHF) boundary condition. The incompressible steady-state mass, momentum and energy equations under the assumption of axisymmetry can be stated as:

Non-dimensional mass conservation equation

$$\frac{1}{r^+} \frac{\partial(r^+ u^+)}{\partial r^+} + \frac{\partial w^+}{\partial z^+} = 0 \quad (6)$$

Non-dimensional momentum conservation equations

Radial direction

$$u^+ \frac{\partial u^+}{\partial r^+} + w^+ \frac{\partial u^+}{\partial z^+} = -\frac{\partial P^+}{\partial r^+} + \frac{1}{r^+} \frac{\partial(r^+ \tau_{rr}^+)}{\partial r^+} - \frac{\tau_{\phi\phi}^+}{r^+} + \frac{\partial(\tau_{rz}^+)}{\partial z^+} \quad (7i)$$

Vertical direction

$$u^+ \frac{\partial w^+}{\partial r^+} + w^+ \frac{\partial w^+}{\partial z^+} = -\frac{\partial P^+}{\partial z^+} + RaPr \Theta + \frac{1}{r^+} \frac{\partial(r^+ \tau_{rz}^+)}{\partial r^+} + \frac{\partial(\tau_{zz}^+)}{\partial z^+} \quad (7ii)$$

Non-dimensional energy conservation equation

$$u^+ \frac{\partial \Theta}{\partial r^+} + w^+ \frac{\partial \Theta}{\partial z^+} = \frac{1}{r^+} \frac{\partial}{\partial r^+} \left(r^+ \frac{\partial \Theta}{\partial r^+} \right) + \frac{\partial^2 \Theta}{\partial z^+ \partial z^+} \quad (8)$$

Here, the radial coordinate is r and z axis is taken to align with the vertical direction, whereas the flow is assumed to be independent of the azimuthal direction ϕ . The simulation domain is schematically shown in Fig. 1, which indicates that the two vertical walls of a cylindrical annular enclosure are insulated, whereas the horizontal boundaries are taken to be differentially heated. Equations 6-8 are subjected to the following boundary conditions in this configuration:

Velocity boundary conditions

$u^+ = 0$ and $w^+ = 0$ at $r^+ = r_i/H$ and $r^+ = r_i/H + (1/AR)$ due to no-slip condition and impenetrability on vertical walls and

$u^+ = 0$ and $w^+ = 0$ at $z^+ = 0.0$ and $z^+ = 1.0$ due to impenetrability and no-slip condition on horizontal walls.

Temperature boundary conditions

$\partial \Theta / \partial r^+ = 0$ at $r^+ = r_i/H$ and $r^+ = r_i/H + (1/AR)$ due to adiabatic vertical walls

$\Theta = 1$ at $z^+ = 0.0$ and $\Theta = 0$ at $z^+ = 1.0$ for the CWT configuration

$-\partial \Theta / \partial z^+ = 1$ at $z^+ = 0.0$ and $z^+ = 1.0$ for the CWHF configuration.

3. NUMERICAL IMPLEMENTATION

The governing equations of mass, momentum and energy have been numerically solved in the context of finite-volume methodology using a commercial package ANSYS-FLUENT. This commercial package was previously used successfully for simulating power-law fluid flows [8, 9, 13, 14]. A second-order central difference scheme is used for the discretisation of the diffusive terms and a second-order up-wind scheme is used for the convective terms. The well-known SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm [17] is used for coupling of the pressure and velocity components. The convergence criteria were set to 10^{-6} for all the relative (scaled) residuals.

The mean Nusselt numbers $\overline{Nu}$ for laminar Rayleigh- Bénard convection of Newtonian fluids in square enclosures for $10^3 \leq Ra \leq 10^6$ and $Pr = 0.71$ have been compared to the benchmark data [18] in Table 2. It is evident from Table 2 that an excellent agreement (i.e. maximum difference in mean Nusselt number is found to be less than 1%) has been achieved between the present results and the benchmark data [18]. Moreover, the variation of $\overline{Nu}/\overline{Nu}_{n=1}$ with n for natural convection of power-law fluids in square enclosures with differentially heated vertical side walls subjected to CWT boundary condition has also been compared to the corresponding results presented by Ref. [19] and an excellent agreement (i.e. maximum difference in mean Nusselt number is found to be less than 1%) was also obtained (see Fig. 2 of Ref. [20]). This difference might have occurred due to a different mesh density between the studies.

It is also worth noting that minimum and maximum levels of μ are taken to be $\mu_{min} = 10^{-4}\mu_{n=1}$ and $\mu_{max} = 10^4\mu_{n=1}$ respectively where $\mu_{n=1}$ is the viscosity of the Newtonian fluid for the same nominal values of Rayleigh and Prandtl numbers. The chosen values of μ_{min}

and μ_{max} allow for a large range of μ values without compromising the fidelity of the simulations. It has been checked that the range of μ obtained from the simulation remains within μ_{min} and μ_{max} , and the results remain independent of the choices of μ_{min} and μ_{max} .

Three different meshes (i.e. M1, M2 and M3) for each AR values have been utilised to ensure grid independence of the results for both Newtonian and power-law fluids. The details of the non-uniform Cartesian meshes, which have been used in the current analysis, are listed in Table 3. The maximum numerical uncertainty associated with the mean Nusselt number $\overline{Nu}_{cy}$ for both Newtonian (i.e. $n = 1$) and power-law fluids (i.e. $n = 0.6 - 1.8$) cases has been found to be smaller than 1% between M2 and M3 meshes for the range of parameters (i.e. $0 \leq r_i/L \leq 16$, $0.25 \leq AR \leq 4$, $0.6 \leq n \leq 1.8$ and $10^3 \leq Ra \leq 10^5$ at $Pr = 10^3$) considered here (see Table 4). The M2 mesh configuration has been used for each AR for the sake of sensitivity of results and computational economy. Moreover, it has been ensured that a change in the computational grid does not lead to a change in flow pattern within the enclosure during the course of the grid independency analysis. This is extremely important because the flow pattern changes significantly depending on all the parameters considered here.

Three-dimensional simulations of laminar Rayleigh-Bénard convection of power-law fluids cylindrical annular enclosures have been conducted for selected parameters to assess the validity of the assumption of axisymmetry. The distributions of non-dimensional temperature in the vertical plane, as obtained from three-dimensional simulations, are exemplarily shown in Fig. 2 for $n = 0.8, 1$ and 1.2 at $Ra = 10^4, Pr = 10^3$, $AR = 1$ and $r_i/L = 1$ in the CWT configuration. The isotherms in Fig. 2 indicate the assumption of axisymmetry remains valid. It is worth noting that three-dimensional simulations are considerably more expensive than two-dimensional calculations. For example, three-dimensional simulations with grid size $50 \times$

50×100 is typically 10 times more expensive than the corresponding two-dimensional simulations with mesh M2 for shear-thinning (i.e. $n < 1$) fluids (e.g. CPU time is 36 minutes and 10 hours for two- and three-dimensional simulations is for $n = 0.8$, $Ra = 10^4$, $Pr = 10^3$ $AR = 1$ and $r_i/L = 1$ in the CWT configuration). Here, it is worth noting that the mesh is considerably finer in two-dimensional analysis (e.g. number of nodes 57600 (240×240)) than that in the corresponding three dimensional simulation (e.g. total number of nodes 2.5×10^5 (50×50 in the vertical mid-plane)). The three-dimensional simulations yielded $\overline{Nu}_{cy} = 3.02$, 2.206, 1.571 for $n = 0.8, 1$ and 1.2 fluids for $Ra = 10^4, Pr = 10^3$ $AR = 1$ and $r_i/L = 1$ in the case of CWT boundary condition. The corresponding values in the case of two-dimensional simulations are 2.985, 2.189 and 1.55, respectively. The grid for the three-dimensional simulations are not sufficiently optimised and has much smaller mesh density than used for two-dimensional simulations. Furthermore, the grid for three-dimensional simulations has curvature in the azimuthal direction, whereas the grid for two-dimensional simulations is free from any curvature effects. All these effects lead to slight differences between the mean Nusselt number values between two- and three-dimensional simulations.

It has been demonstrated earlier [13, 14, 16, 21, 22] that the initial conditions significantly affect the steady two-dimensional solutions for Rayleigh-Bernard convection of Newtonian (i.e. $n = 1$) and shear-thinning (i.e. $n < 1$) fluids for both square cross sectional cylindrical and rectangular enclosures. Therefore, two types of initial conditions have been considered in the current analysis. These initial conditions referred as the “quiescent flow (Q.F.)” and “established flow (E.F.)” initial conditions, respectively. The Q.F. condition considers zero velocity initial condition, whereas the steady state simulation results obtained for a smaller value of Ra is used for the initial condition for the cases with $Ra \geq 1 \times 10^3$ in the case of E.F. for Newtonian fluid simulations. The steady-state solution obtained for Newtonian (i.e. $n = 1$)

fluids based on E.F. initial condition for a given set of values of Ra and Pr has in turn been used as an initial condition for first set of simulations involving non-Newtonian power-law fluids (i.e. E.F. initial condition for power-law fluids). This category of simulations are referred to as the established flow (E.F.) simulations in this paper. The quiescent condition is used as an initial condition (i.e. Q.F. initial condition) for the second category of power-law fluid simulations. This category of simulations will henceforth be called as the quiescent flow (Q.F.) simulations.

Yigit *et al.* [13] reported the nature of the instability is sub-critical for shear-thinning fluids (i.e. $n < 1$) in the configuration analysed here. The viscosity assumes quite large value for shear-thinning fluids (i.e. $n < 1$) in the quiescent condition. Shear-thinning fluids behave solid-like and viscosity sharply decreases with increasing shear rate from quiescent condition. Thus, it is important to analyse the influences of the initial conditions in the current configuration.

4. SCALING ANALYSIS

A detailed scaling analysis is conducted to explain the influences of $Ra, Pr, AR, n, r_i/L$ on the mean Nusselt number $\overline{Nu}_{cy}$. Equating the order of magnitudes of inertial and viscous terms in the vertical (radial) direction yields:

$$\rho w^2/H \sim [(K/\delta)(w/\delta)^n] \quad (9i)$$

$$\rho u^2/L \sim [(K/\delta_1)(u/\delta_1)^n] \quad (9ii)$$

It is possible to estimate hydrodynamic boundary layer thickness for $r_i \neq 0$, where w and u are the vertical and horizontal velocity components respectively and accordingly δ and δ_1 are the hydrodynamic boundary layer thickness on the vertical and horizontal walls, respectively. Similarly, equating the order of magnitudes of inertial and buoyancy terms yields $w \sim \sqrt{g\beta(T_H - T_C)H}$ and $w \sim \sqrt{g\beta q \delta_{th} H/k} \sim \sqrt{g\beta q \delta H/f_1 k}$ for the CWT and CWHF boundary

conditions, where the function $f_1(Ra, Pr, AR, n, r_i/L)$ represents the ratio of hydrodynamic and thermal boundary layer thicknesses (i.e. δ/δ_{th}) on the horizontal walls. Using the above scaling of w in Eq. 9i yields estimates of δ in the following manner:

$$\delta \sim H Ra_{CWT}^{(n-2)/(2n+2)} Pr^{n/(2n+2)} \quad (\text{for CWT}) \quad (10i)$$

$$\delta \sim H Ra_{CWHF}^{(n-2)/(n+4)} Pr^{n/(n+4)} f_1^{(2-n)/(n+4)} \quad (\text{for CWHF}) \quad (10ii)$$

Using continuity equation for axisymmetric geometry, one gets:

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} \sim \left(\frac{u}{r} + \frac{u}{L} \right) \sim \frac{\partial w}{\partial z} \sim \frac{w}{H} \quad (11)$$

which leads to scaling estimate of u :

$$u \sim \frac{w}{AR(1+L/r_i)} \sim \frac{\alpha}{L} \frac{1}{AR^2} \frac{\sqrt{Ra_{CWT} Pr}}{(1+L/r_i)} \quad (\text{for CWT}) \quad (12i)$$

$$u \sim \frac{w}{AR(1+L/r_i)} \sim \frac{\alpha}{L} \frac{1}{AR^2} \frac{Ra_{CWHF}^{(n+1)/(n+4)} Pr^{(n+2)/(n+4)} f_1^{-(n+1)/(n+4)}}{(1+L/r_i)} \quad (\text{for CWHF}) \quad (12ii)$$

Using the above scalings of u in Eq. 9ii gives:

$$\delta_1 \sim H Ra_{CWT}^{(n-2)/(2n+2)} Pr^{n/(2n+2)} AR^{(1-n)/(n+1)} (1 + L/r_i)^{(2-n)/(n+1)} \quad (\text{for CWT}) \quad (13i)$$

$$\delta_1 \sim H Ra_{CWHF}^{(n-2)/(n+4)} Pr^{n/(n+4)} AR^{(1-n)/(n+1)} f_1^{(2-n)/(n+4)} (1 + L/r_i)^{(2-n)/(n+1)} \quad (\text{for CWHF}) \quad (13ii)$$

Using Eqs. 10-13, it is possible to estimate the effective viscosity in the vertical and horizontal boundary layers:

$$\mu_{eff}^V \sim K(w/\delta)^{n-1} \quad (14i)$$

$$\mu_{eff}^H \sim K(u/\delta_1)^{n-1} \quad (14ii)$$

in order to predict the effective Rayleigh numbers:

$$Ra_{eff} = \rho^2 c_p g \beta \Delta T_{ref} H^3 / \mu_{eff} k \quad (15)$$

in the vertical and horizontal boundary layers in the following manner:

$$Ra_{CWT,eff}^V \sim Ra_{CWT}^{(5-n)/(2n+2)} Pr^{(1-n)/(2n+2)} \quad (16i)$$

$$Ra_{CWHF,eff}^V \sim Ra_{CWHF}^{(7-2n)/(n+4)} Pr^{(2-2n)/(n+4)} f_1^{3(n-1)/(n+4)} \quad (16ii)$$

$$Ra_{CWT,eff}^H \sim Ra_{CWT}^{(5-n)/(2n+2)} Pr^{(1-n)/(2n+2)} AR^{(2n-2)/(n+1)} (1 + L/r_i)^{(3n-3)/(n+1)} \quad (17i)$$

$$Ra_{CWHF,eff}^H \sim Ra_{CWHF}^{(7-2n)/(n+4)} Pr^{(2-2n)/(n+4)} AR^{(2n-2)/(n+1)} f_1^{(3n-3)/(n+4)} (1 + L/r_i)^{(3n-3)/(n+1)} \quad (17ii)$$

Equations 16 and 17 show that the effective value of Rayleigh number attains significantly larger value than its nominal value for a decrease in values of n for shear-thinning (i.e. $n < 1$) fluids. Furthermore, $Ra_{CWT,eff}^H$ and $Ra_{CWHF,eff}^H$ decrease with decreasing r_i/L for shear-thinning (i.e. $n < 1$) fluids. However, $Ra_{CWT,eff}^H$ and $Ra_{CWHF,eff}^H$ increase with decreasing r_i/L for shear-thickening (i.e. $n > 1$) fluids.

The mean Nusselt number $\overline{Nu}_{cy}$ can be estimated by using the wall heat flux scaling $q \sim k\Delta T/\delta_{th}$ as:

$$\overline{Nu}_{cy,CWT} \sim Ra_{CWT}^{\frac{2-n}{2n+2}} Pr^{-\frac{n}{2n+2}} AR^{\frac{n-1}{n+1}} f_1^{-1} (1 + L/r_i)^{\frac{n-2}{n+1}} \quad (18i)$$

$$\overline{Nu}_{cy,CWHF} \sim Ra_{CWHF}^{\frac{2-n}{n+4}} Pr^{-\frac{n}{n+4}} AR^{\frac{n-1}{n+1}} f_1^{\frac{6}{n+4}} (1 + L/r_i)^{\frac{n-2}{n+1}} \quad (18ii)$$

Equations 18i and 18ii imply that the exponent of Rayleigh number increases with decreasing n but the exponent remains positive for the range of the values of n considered here. Thus, the mean Nusselt number $\overline{Nu}_{cy}$ is expected to increase with an increase in nominal Rayleigh number. Moreover, the mean Nusselt number $\overline{Nu}_{cy}$ is expected to increase with a decrease in power-law exponent.

5. RESULTS & DISCUSSION

5.1. Effect of nominal Rayleigh number Ra and power-law index n

The variation of mean Nusselt number $\overline{Nu}_{cy}$ with nominal Ra for $0.6 \leq n \leq 1.8$ at $r_i/L = 1.0$, $AR = 0.5$ and $Pr = 10^3$ is shown in Fig. 3 with representative streamline patterns and isotherms for both established (E.F.) and quiescent (Q.F.) flow initial conditions in the case of

both CWT and CWHF boundary conditions. Figure 3 shows that $\overline{Nu}_{cy}$ monotonically increases with an increase in Ra for both shear-thickening (i.e. $n > 1$) and Newtonian (i.e. $n = 1$) fluids regardless of the initial and boundary conditions. By contrast, the variation of $\overline{Nu}_{cy}$ with Ra is sensitive to the both initial and boundary conditions for shear-thinning (i.e. $n < 1$) fluids. The variation of $\overline{Nu}_{cy}$ with Ra is sensitive to the both initial and boundary conditions for shear-thinning (i.e. $n < 1$) fluids. This can be substantiated from Fig. 3 where different values of $\overline{Nu}_{cy}$ and flow patterns have been shown for different initial conditions for shear-thinning (i.e. $n < 1$) fluids. A non-monotonic trend for the variation of $\overline{Nu}_{cy}$ with Ra is observed for shear-thinning (i.e. $n < 1$) fluids for both choices of initial and boundary conditions. This behaviour can be explained by the change in the streamline patterns with the alteration of Ra , as it shown in Fig. 3. This change in flow pattern modifies the distance between the isotherms in the vicinity of the active walls (i.e. hot and cold walls), and is responsible for the change in the values of $\overline{Nu}_{cy}$. Furthermore, it can be seen in Fig. 3 that $\overline{Nu}_{cy}$ decreases with increasing n due to augmented viscous resistance before assuming a value equal to unity (indicating conduction-driven transport) for high values of n . Moreover, the streamline patterns change with the alteration of n for both choices of initial and boundary conditions, especially for shear-thinning (i.e. $n < 1$) fluids, which is consistent with previous analyses [13, 16, 22], which demonstrated transition between different preferred flow patterns for shear-thinning (i.e. $n < 1$) fluids. By contrast, a single roll pattern is observed in the case of shear-thickening (i.e. $n > 1$) fluids irrespective of the choice of initial and boundary conditions, which is consistent with the observations from Fig 3. It is also worth noting that the findings of Fig. 3 can be confirmed by the scaling estimation given by Eq. 18 (i.e. $\overline{Nu}_{cy}$ increases with an increase in nominal Rayleigh number, whereas $\overline{Nu}_{cy}$ is expected to decrease with increasing power-law exponent).

5.2. Effect of aspect ratio AR

The variation of $\overline{Nu}_{cy}$ with AR for different values of Ra and n is shown in Fig. 4 at $r_i/L = 1.0$ and $Pr = 10^3$ for both E.F. and Q.F. initial conditions in both CWT (left column) and CWHF (right column) boundary conditions. It can be seen from Fig. 4 that $\overline{Nu}_{cy}$ shows a mostly decreasing trend with increasing AR for different values of Ra and n . Moreover, Fig.4 shows that $\overline{Nu}_{cy}$ exhibits a non-monotonic trend with the variation of AR mainly for shear-thinning (e.g. $n = 0.8$) and Newtonian (i.e. $n = 1$) fluids in the $AR \leq 1$ cases due to the change in flow structure within the enclosure. This can be seen in Fig. 5, which shows streamline patterns (i.e. Ψ) for different values of Ra and AR for shear-thinning fluids (e.g. $n = 0.8$), Newtonian (i.e. $n = 1$) and shear-thickening fluids (e.g. $n = 1.4$) at $r_i/L = 1.0$ and $Pr = 10^3$ for the E.F. initial condition in the case of both CWT and CWHF configurations. It is worth noting that the streamline patterns in Fig. 5 are in good agreement with the findings of Leong [23] on Rayleigh-Bénard convection of Newtonian (i.e. $n = 1$) fluids in a vertical cylinder (i.e. $r_i/L = 0$). Figure 4 further shows that the variation of $\overline{Nu}_{cy}$ with AR is sensitive to the initial conditions and the steady-state solution is not obtained for the Q.F. initial condition especially for $AR < 1$ for shear-thinning (i.e. $n < 1$) fluids in the CWT configuration.

Figure 4 further shows that $\overline{Nu}_{cy}$ eventually attains a value equal to unity (indicating conduction-driven transport) for tall enclosures (i.e. $AR > 1$). This also can be confirmed from Fig. 6, which shows that the isotherms for the $AR = 4.0$ case remain parallel to the horizontal walls indicating a conduction-dominated transport. Moreover, a conduction-dominated transport is obtained for $AR > 3$ irrespective of the values of Ra and n for all the initial and boundary conditions considered here.

The AR dependence of $\overline{Nu}_{cy}$ for CWT (CWHF) boundary condition can be explained in the following manner using the energy flux integral between hot and cold walls. The energy flux integral along the horizontal line can be written as follows:

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{cond} = \int_0^H \rho c_p T u dz - \int_0^H k \left(\frac{\partial T}{\partial r} \right) dz \quad (19)$$

where the first term on the right-hand side represents the effects of convective transport, whereas the second term on the right-hand side accounts for thermal conduction. Here, $\dot{Q}_{conv}$ and $\dot{Q}_{cond}$ can be scaled in the following manner by using Eqs. 9-13:

$$\dot{Q}_{conv,CWT} \sim \rho c_p \Delta T u \delta_{th} \sim k \Delta T Ra_{CWT}^{\frac{2n-1}{2n+2}} Pr^{\frac{2n+1}{2n+2}} AR^{-\frac{2n}{n+1}} / (1 + L/r_i)^{\frac{2n-1}{n+1}} \quad (20i)$$

$$\dot{Q}_{conv,CWHF} \sim \frac{\rho c_p u q \delta_{th} \delta_1}{k} \sim q L Ra_{CWHF}^{\frac{3n-3}{n+4}} Pr^{\frac{3n+2}{n+4}} AR^{\frac{2-2n}{n+1}} f_1^{\frac{-4n-1}{n+4}} / (1 + L/r_i)^{\frac{3n-3}{n+1}} \quad (20ii)$$

$$\dot{Q}_{cond,CWT} \sim (k \Delta T) AR \quad (21i)$$

$$\dot{Q}_{cond,CWHF} \sim q AR \delta_{th} \sim q AR H Ra_{CWHF}^{\frac{n-2}{n+4}} Pr^{\frac{n}{n+4}} AR^{\frac{1-n}{n+1}} f_1^{\frac{-2(n+1)}{n+4}} / (1 + L/r_i)^{(n-2)/(n+1)} \quad (21ii)$$

Eqs. 20 and 21 suggest relative weakening (strengthening) of the contribution of $\dot{Q}_{conv}$ ($\dot{Q}_{cond}$) to the overall thermal transport with increasing AR for both shear-thinning (i.e. $n < 1$) and shear-thickening (i.e. $n > 1$) fluids for both CWT and CWHF boundary conditions. The weakening of the convective transport can also be explained by the reduction of the magnitude of horizontal velocity component with increasing AR (see Eq. 12), which suggests that the first term on the right hand side vanishes for large AR and under that condition the heat flow rate $\dot{Q}$ in the horizontal direction vanishes altogether because of adiabatic boundary condition for the vertical walls. This suggests that no heat transfer within the fluid takes place in the horizontal direction and thermal transport only takes place in the vertical direction, which is only realisable under pure conductive transport.

For shear-thinning (i.e. $n < 1$) fluids, the exponent of AR remains negative in Eq. 18 (i.e. $\overline{Nu}_{cy} \sim AR^{(n-1)/(n+1)}$) and this exponent (i.e. $(n-1)/(n+1)$) becomes increasingly negative

with decreasing n , which suggests that $\overline{Nu}_{cy}$ is expected to attain higher values than that obtained for Newtonian (i.e. $n = 1$) fluids for the combination of small values of n at AR (i.e. $n < 1$ and $AR < 1$) for a given set of values of Ra and Pr . By contrast, the exponent of AR assumes increasingly positive values with increasing n value for shear-thickening (i.e. $n > 1$) fluids independent of boundary conditions. Therefore, $\overline{Nu}_{cy}$ for shear-thickening (i.e. $n > 1$) fluids is expected to attain smaller values than that obtained for Newtonian (i.e. $n = 1$) fluids for a given set of values of Ra , AR and Pr for both CWT and CWHF boundary conditions.

5.3. Effect of r_i/L

The variations of $\overline{Nu}_{cy}$ with r_i/L for different values of n and AR at $Ra = 5 \times 10^4$ and $Pr = 10^3$ are shown along with representative flow patterns in Figs. 7-9 for both E.F. and Q.F. initial conditions in the case of both CWT and CWHF boundary conditions. It can be seen from Figs. 7-9 that the values of $\overline{Nu}_{cy}$ and flow patterns are not significantly affected by the modification of r_i/L for shear-thickening fluids (i.e. $n > 1$) for both E.F. and Q.F. initial conditions in the case of CWHF configuration. However, the values of $\overline{Nu}_{cy}$ and flow patterns are sensitive to the choices of r_i/L and initial conditions especially for $r_i/L < 1$ and $AR < 1$ in the CWHF configuration for shear-thinning fluids (i.e. $n < 1$) and Newtonian (i.e. $n = 1$) fluids. Furthermore, $\overline{Nu}_{cy}$ variation with r_i/L , shows several sudden jumps due to the changes in flow pattern for shear-thinning (i.e. $n < 1$), Newtonian (i.e. $n = 1$) and shear-thickening fluids (i.e. $n > 1$) for a given set of values of Ra and Pr for both E.F. and Q.F. initial conditions in the case of CWT boundary condition. This tendency is prevalent for small values of r_i/L and AR for cases with a small values of n . It is worth noting that only the cases for which steady two-dimensional solution exists are shown in Figs. 7-9. For example, the steady state solution does not exist for $n = 0.6$, $AR = 0.5$ and $r_i/L > 1$ in the case of CWT boundary condition for both

initial conditions considered here. The effective Rayleigh number Ra_{eff} (see. Eqs. 16-17) becomes increasingly larger than the nominal Rayleigh number for decreasing values of AR and also for increasing values of r_i/L in the case of shear-thinning fluids (i.e. $n < 1$) regardless of the boundary condition. Thus, the steady state solution corresponding to a particular type of flow pattern associated with small values of Ra_{eff} , may not be possible to obtain for large values of Ra_{eff} , where the steady state solution yields a different flow pattern. It has been demonstrated in Yigit et al. [13] that it may not be possible to obtain a steady state solution for some combinations of Ra and n for shear-thinning fluids and transient simulations reveal time-dependent Rayleigh-Bénard cells and complex temporal variations of the mean Nusselt number.

It is also worth indicating that the flow patterns presented here for the current analysis are not artefacts of convergence criteria. This can be substantiated from Fig. 10, which shows that reducing the convergence tolerance by an order of magnitude did not make any difference to the magnitudes of the stream functions for some of the selected cases but this is valid for all cases considered here.

Moreover, the mean Nusselt number $\overline{Nu}_{cy}$ assumes higher values for the CWT boundary condition than for the CWHF boundary condition for large values of Ra and for small values of n for a given set of values AR, Pr , and r_i/L . However, an opposite behaviour is observed for small values of Ra and large values of n provided the flow pattern remains unchanged with the variations of Ra and n . This can be explained by the scaling estimates of $\overline{Nu}_{cy}$ as given by Eq. 18. The exponent of Ra_{CWT} (i.e. $(2 - n)/(2n + 2)$) assumes greater values than that of Ra_{CWHF} (i.e. $(2 - n)/(n + 4)$) and this difference widens with decreasing n for the same set values of Pr, AR and r_i/L . By contrast, the exponent of Pr assumes greater negative value for

CWT boundary condition (i.e. $-n/(2n + 2)$) than the corresponding value in the case of CWHF boundary condition (i.e. $-n/(n + 4)$). The effects of higher exponent of nominal Rayleigh number dominate over the influences of higher magnitude of negative exponent of Prandtl number for large values of Ra and small values of n for the same set values of AR and r_i/L , which leads to higher values of mean Nusselt number $\overline{Nu}_{cy}$ in the CWT configuration than in the case of CWHF boundary condition as long as flow pattern remains unchanged. However, the effects of higher magnitude of negative exponent of Prandtl number overcome the influences of higher exponent of nominal Rayleigh number for small values of Ra and large values of n for the same set values of AR and r_i/L , which leads to higher values of mean Nusselt number $\overline{Nu}_{cy}$ in the CWHF configuration than in the case of same flow pattern under CWT boundary condition.

Although $\overline{Nu}_{cy}$ exhibits a non-monotonic variation with r_i/L for small values of r_i/L and AR (see Fig. 7), the mean Nusselt number $\overline{Nu}_{cy}$ and the flow pattern for large values of r_i/L approach those for rectangular enclosures ($r_i/L \rightarrow \infty$) irrespective of the choices of initial and boundary conditions provided the steady-state solution remains valid.

Turan *et al.* [9] proposed correlation for mean Nusselt number $\overline{Nu}_{sq,CWHF}$ for laminar Rayleigh-Bénard convection of power-law fluids in square enclosures for CWHF boundary condition (i.e. $AR = 1$) for $10 \leq Pr \leq 10^4$ based on scaling relations given by Eq. 18 as follows:

$$\overline{Nu}_{sq,CWHF} = \overline{Nu} \text{ when } \overline{Nu} = 0.289 Ra_{CWHF}^{0.014} \frac{Pr^{0.217}}{(1+Pr)^{0.017}} \left(\frac{Ra_{CWHF}^{1-n/2}}{Pr^{n/2}} \right)^{1/(n/2+2)} A > 1; \quad (22i)$$

$$A = \exp[(n - 1)c_1 Ra^{c_2} Pr^{c_3}] \quad (22ii)$$

$$c_1 = 0.345, c_2 = 0.129 \text{ and } c_3 = 0.103 \text{ for } n \leq 1 \text{ and } 5 \times 10^3 \leq Ra \leq 1 \times 10^5; \quad (22iii)$$

$$c_1 = 0.014, c_2 = 0.458 \text{ and } c_3 = 0.097 \text{ for } n > 1 \text{ and } 5 \times 10^3 \leq Ra \leq 1 \times 10^4; \quad (22iv)$$

$$c_1 = 0.259, c_2 = 0.153 \text{ and } c_3 = 0.073 \text{ for } n > 1 \text{ and } 10^4 \leq Ra \leq 1 \times 10^5; \quad (22v)$$

$$\overline{Nu}_{sq,CWHF} = 1 \text{ when } \overline{Nu} \leq 1. \quad (22vii)$$

This correlation was improved by Yigit *et al.* [16] for shear-thickening (i.e. $n > 1$) fluids, which is given as follows:

$$A = n^d \text{ for } n > 1; \quad (23i)$$

$$d = [0.136Pr^{0.04}\ln Ra]^{1.5} \text{ for } 5 \times 10^3 \leq Ra \leq 1 \times 10^4; \quad (23ii)$$

$$d = [0.145Pr^{0.04}\ln Ra]^{1.5} \text{ for } 1 \times 10^4 < Ra \leq 1 \times 10^5; \quad (23iii)$$

$$\overline{Nu}_{sq,CWHF} = 1 \text{ when } \overline{Nu} \leq 1. \quad (23iv)$$

It is worth noting that Eqs. 22 and 23 are valid for where convection takes place as one cell flow pattern. Equations 22 and 23 have been modified by Yigit and Chakraborty [14] to include AR dependence of $\overline{Nu}_{Rec,CWHF}$ in the following manner for single cell flow pattern for $0.25 \leq AR \leq 4$, $5 \times 10^3 \leq Ra \leq 1 \times 10^5$ and $10 \leq Pr \leq 10^4$:

$$\overline{Nu}_{Rec,CWHF} = \overline{Nu}_{sq,CWHF} + [(1.987 - 0.157\ln Ra)(1 - AR)] \text{ for } n \leq 1; \quad (24i)$$

$$\overline{Nu}_{Rec,CWHF} = \overline{Nu}_{sq,CWHF} + \left[\left(\frac{1 - 0.124\ln Ra}{1.452 - 0.183\ln Ra} \right) (1 - AR) \right] \text{ for } n \geq 1. \quad (24ii)$$

Equation 24 is also valid for where single cell flow pattern is realised and can be utilised for cylindrical annular enclosures with $r_i/L > 1$ in the CWHF configuration. It can be seen from Fig. 7 that Eq. 24 satisfactorily predicts $\overline{Nu}_{cy,CWHF}$ in the CWHF configuration. By contrast, the non-monotonic variation of the mean Nusselt number in response to the changes in $Ra, AR, r_i/L, n$ and initial condition severely limit the usefulness of a correlation for $\overline{Nu}_{cy}$ for

CWT boundary condition for both rectangular [13] and cylindrical annular enclosures, and therefore is not attempted here for CWT configuration.

5. 4. Onset of convection of power-law fluids

It is well-known that a threshold value of nominal Rayleigh number needs to be exceeded for the onset of fluid motion in Rayleigh-Bénard configuration. The critical Rayleigh number Ra_{crit} for which natural convection starts to affect the thermal transport appreciably is estimated by a linear regression analysis of $(\overline{Nu} - 1)$ variation with Ra for Newtonian (i.e. $n = 1$) fluids where the instability is super-critical (not shown here but refer to Fig. 8a in Yigit *et al.* [13]). The criterion $(\overline{Nu} - 1) \approx 0$ instead of $\overline{Nu} \approx 1.0$ has been chosen for the ease of linear regression analysis which has been used to evaluate Ra_{crit} . In the case of shear-thinning (i.e. $n < 1$) fluids, where the instability is sub-critical, the critical Rayleigh number is estimated by reducing nominal Rayleigh number Ra in steps of 50, 250, 1000 for $AR \leq 1$, $AR = 2$ and $AR = 4$ respectively from an established flow condition where $\overline{Nu} \gg 1$ [13]. The quantity $(\overline{Nu} - 1)$ jumps abruptly from zero value with an increase in Ra for shear-thinning fluids (i.e. $n < 1$) according to the expectation of sub-critical instability (not shown here but refer to Fig. 8a in Yigit *et al.* [13]). An increase in Ra induces high magnitudes of shear rate due to enhanced buoyancy force, which in turn weakens the resistance to fluid motion for shear-thinning fluids (i.e. $n < 1$) due to reduced viscosity and strengthens the convective transport. This is responsible for an abrupt increase of $(\overline{Nu} - 1)$ from 0 with an increase in Ra during the onset of convection and also for a rise in $(\overline{Nu} - 1)$ values with increasing Ra .

For shear-thickening (i.e. $n > 1$) fluids, $(\overline{Nu} - 1)$ cannot be used for a sufficiently accurate measure for the onset of fluid motion in the case of shear-thickening (i.e. $n > 1$) fluids since $(\overline{Nu} - 1)$ remains close to 0.0 for a large range of values of Ra . It has been found that fluid

flow initiates for any non-zero value of nominal Rayleigh number by examining the variation of the maximum values of stream function ψ_{max} with Rayleigh number (not shown here but refer to Fig. 8b in Yigit et al. [13]). Thus, the critical Rayleigh number for shear-thickening (i.e. $n > 1$) fluids can be considered as $Ra_{crit} = 0$ since viscosity remains very small at low shear rate values for shear-thickening fluids (i.e. $n > 1$). Therefore, shear-thickening fluids do not experience any resistance to fluid motion once the nominal Rayleigh approaches zero. This is found to be consistent with previous findings [11].

Table 5 lists the critical Rayleigh number Ra_{crit} for the onset of convection for shear-thinning and Newtonian fluids and the threshold Rayleigh number $Ra_{\overline{Nu}_{cy}=1}$ for which $\overline{Nu}_{cy}$ values deviate from unity in the third decimal place (i.e. $\overline{Nu}_{cy} \approx 1.001$) for shear-thickening fluids which are consistent with previous findings of Refs. [13, 14] for the CWT and CWHF boundary conditions, respectively. It is also worth noting that all the simulations have been used to find Ra_{crit} had a simple single roll flow pattern within the flow domain. Table 5 indicates that Ra_{crit} and $Ra_{\overline{Nu}_{cy}=1}$ remains largely independent of r_i/L for different AR in the both CWT and CWHF boundary conditions but Ra_{crit} for $r_i/L = 0$ has been found to be slightly smaller in the case of CWT boundary condition. Therefore, the correlations proposed in Refs. [13, 14] for Ra_{crit} and $Ra_{\overline{Nu}_{cy}=1}$ in the case of laminar Rayleigh-Bénard convection of power-law fluids in rectangular enclosures (i.e. $r_i/L \rightarrow \infty$) are listed in Table 6, which shows that these correlations can also be used to predict Ra_{crit} and $Ra_{\overline{Nu}_{cy}=1}$ in the case of cylindrical annular enclosures for both CWT and CWHF configurations.

CONCLUSIONS

The laminar Rayleigh-Bénard convection of power-law fluids in cylindrical annular enclosures has been analysed for different values of nominal Rayleigh number (i.e. $Ra = 10^3 - 10^5$),

aspect ratio (i.e. $AR = 0.25 - 4$) and normalised inner radius (i.e. $r_i/L = 0 - 16$), power-law index (i.e. $n = 0.6 - 1.8$) for a representative nominal Prandtl number $Pr = 10^3$ for different initial conditions in the case of both CWT and CWHF wall boundary conditions for the horizontal walls. The effects of Ra , AR , r_i/L and n on the mean Nusselt number have been analysed in detail and the observed variations have been explained by physical principles and scaling relations. It has been found that thermal convection strengthens with increasing Ra and with decreasing n regardless of the boundary condition. Moreover, the mean Nusselt number $\overline{Nu}_{cy}$ decreases with increasing AR and attains a value of unity (indicating pure conduction) for $AR > 3$ for shear-thinning fluids (i.e. $n < 1$), Newtonian (i.e. $n = 1$) and shear-thickening (i.e. $n > 1$) fluids irrespective of the choices of Ra , r_i/L , and initial and boundary conditions. Furthermore, $\overline{Nu}_{cy}$ assumes higher values for CWT boundary condition than that in the case for CWHF boundary condition for large values of Ra , and also for small values of n , whereas an opposite trend is observed for small values of Ra , and also for large values of n , provided the qualitative nature of the flow pattern remains unchanged. Additionally, the mean Nusselt number $\overline{Nu}_{cy}$ and flow patterns for large values of r_i/L approach those for rectangular enclosures ($r_i/L \rightarrow \infty$) for both CWT and CWHF boundary conditions. Moreover, the mean Nusselt number $\overline{Nu}_{cy}$ dependences of Ra , Pr , AR , r_i/L , n and initial condition show complex non-monotonic trends due to the changes in flow pattern (i.e. number of cells) for shear-thinning fluids (i.e. $n < 1$). However, this behaviour is much weaker for shear-thickening fluids (i.e. $n > 1$) for both CWT and CWHF configurations. The non-monotonic variation of the mean Nusselt number in response to the changes in Ra , AR , r_i/L , n and initial condition severely limit the usefulness of a correlation for the mean Nusselt number in the case of CWT boundary condition. However, in the case of CWHF boundary condition, both computational results and scaling estimations have been utilised to develop mean Nusselt number correlations for laminar Rayleigh-Bénard convection of power-law fluids in cylindrical annular enclosures.

Finally, the critical Rayleigh number Ra_{crit} for the onset of convection of power-law fluids has been found to be largely independent of r_i/L for both CWT and CWHF boundary conditions.

ACKNOWLEDGEMENTS

The authors are grateful to Mr. Lee Maddison and Mr. Xubin Han from Newcastle University for his help in developing the computational database.

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Table 1: Summary of the findings of existing analyses on Rayleigh-Bénard convection in enclosures involving non-Newtonian fluids obeying power-law model of viscosity.

Ref.	Type	Enclosure	Configuration & Boundary conditions	$AR = H/L$	Model & power-law exponent	Ra, Pr	Correlation
[4]	E,N	-	Diff. heated horizontal layers (CWT)	-	Power-law $0.4 \leq n \leq 1$	Ra_{crit} for $\overline{Nu} > 1$	-
[5]	N	-	Diff. heated horizontal layers (CWT)	-	Power-law $1 \leq n \leq 4$	Ra_{crit} for $\overline{Nu} > 1$ $\mu_{\infty} = f(T)$	-
[6]	N	Square	Diff. heated horizontal wall (CWT)	1	Power-law $0.4 \leq n \leq 1$	$Ra = 6 \times 10^3$ $Pr = 0.1 - 1 - 10$	-
[7]	N	Square	Diff. heated horizontal wall (CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^6$	$\overline{Nu} = f(Ra, Pr, n)$
[8]	N	Square	Diff. heated horizontal wall comparison (CWT-CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^5$	$\overline{Nu} = f(Ra, Pr, n)$
[9]	N	Square	Diff. heated horizontal wall (CWHF)	1	Power-law $0.6 \leq n \leq 1.4$	$0 \leq Ra \leq 10^5$ $\mu_{\infty} = f(T)$	-
[10]	A,N	Rectangular	Diff. heated horizontal wall (CWHF)	1/6	Power-law $0.8 \leq n \leq 1.2$	Ra_{crit} for $\overline{Nu} > 1$	-
[11]	A, N	Rectangular	Diff. heated horizontal wall (CWHF)	1/12	Power-law $0.6 \leq n \leq 1.4$	$10^3 \leq Ra \leq 5 \times 10^3$ $Pr = 1 - 10 - 100$	-
[12]	N	Rectangular	Diff. heated horizontal wall (CWT)	$1/4 \leq AR \leq 4$	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $Pr = 10^3$	$Ra_{crit} = f(Ra, Pr, AR, n)$

[13]	N	Rectangular	Diff. heated horizontal wall (CWHF)	$1/4 \leq AR \leq 4$	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $Pr = 10^3$	$\overline{Nu} = f(Ra, Pr, AR, n)$ $Ra_{crit} = f(Ra, Pr, AR, n)$
[14]	N	Rectangular $0^\circ \leq \phi \leq 90^\circ$	Diff. heated inclined horizontal wall (CWT)	$1/8 \leq AR \leq 1$	Power-law $0.6 \leq n \leq 1.4$	$10^4 \leq Ra \leq 10^5$ $10^2 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, AR, \phi, n)$
[15]	N	Cylindrical Annular $0 \leq r_i/L \leq 24$	Diff. heated horizontal wall comparison (CWT-CWHF)	1	Power-law $0.6 \leq n \leq 1.8$	$10^3 \leq Ra \leq 10^5$ $10 \leq Pr \leq 10^4$	$\overline{Nu} = f(Ra, Pr, r_i/L, n)$

A: analytical; E: experimental; N: numerical

Table 2: Comparison of the mean Nusselt number $\overline{Nu}$ for Newtonian fluid (i.e. $n = 1.0$) with the benchmark data [17] for square enclosure ($AR = 1$) at $Pr = 0.71$.

Ra	Present study	[17]
1×10^3	1.000	1.000
1×10^4	2.154	2.158
1×10^5	3.907	3.910
1×10^6	6.363	6.309

Table 3: Summary of the non-uniform Cartesian meshes used in the current analysis for $0.125 \leq r_i/L \leq 16$, $0.25 \leq AR \leq 4$, $0.6 \leq n \leq 1.8$ and $10^3 \leq Ra \leq 10^6$ at $Pr = 10^3$ with non-dimensional minimum cell distance ($\Delta_{min,cell}/L$) and grid expansion ratio (r_e) values.

$AR = 0.25$			
Grid	M1 (160×180)	M2 (180×220)	M3 (200×240)
$\Delta_{min,cell}/L$	6.270×10^{-4}	5.575×10^{-4}	5.018×10^{-4}
r_e	1.01	1.009	1.008
$AR = 0.5$			
Grid	M1 (180×220)	M2 (200×240)	M3 (220×260)
$\Delta_{min,cell}/L$	1.115×10^{-3}	1.003×10^{-3}	9.127×10^{-4}
r_e	1.009	1.008	1.007
$AR = 1.0$			
Grid	M1 (180×180)	M2 (240×240)	M3 (260×260)
$\Delta_{min,cell}/L$	2.230×10^{-3}	1.673×10^{-3}	1.545×10^{-3}
r_e	1.009	1.006	1.006
$AR = 2.0$			
Grid	M1 (140×300)	M2 (160×320)	M3 (200×400)
$\Delta_{min,cell}/L$	2.678×10^{-3}	2.508×10^{-3}	2.007×10^{-3}
r_e	1.01	1.01	1.008
$AR = 4.0$			
Grid	M1 (140×400)	M2 (160×480)	M3 (200×600)
$\Delta_{min,cell}/L$	2.865×10^{-3}	2.508×10^{-3}	2.007×10^{-3}
r_e	1.01	1.01	1.008

Table 4: Numerical uncertainty for the mean Nusselt number $\overline{Nu}_{cy}$ for shear-thinning (e.g. $n = 0.8$), Newtonian (i.e. $n = 1$) and shear-thickening (e.g. $n = 1.4$) fluids are shown for different AR and ri/L at $Ra = 5 \times 10^4$ and $Pr = 10^3$ for CWT boundary condition.

		$ri/L = 0$			$ri/L = 16.0$		
n	AR	M1	M2	M3	M1	M2	M3
0.8	0.25	*	*	*	4.22	4.25	4.26
1		3.08	3.074	3.074	3.65	3.68	3.68
1.4		2.04	2.04	2.04	2.24	2.24	2.24
0.8	0.5	2.64	2.64	2.64	4.36	4.36	4.36
1		3.19	3.19	3.19	3.55	3.56	3.56
1.4		2.00	2.00	2.00	2.14	2.14	2.14
0.8	1	3.09	3.09	3.09	4.56	4.56	4.56
1		1.96	1.96	1.96	3.30	3.30	3.31
1.4		2.02	2.02	2.02	1.98	1.98	1.98
0.8	2	3.01	3.01	3.01	4.19	4.19	4.19
1		3.11	3.11	3.11	2.97	2.97	2.97
1.4		1.08	1.08	1.08	1.05	1.05	1.05

*Steady-state solution does not exist.

Table 5: Values of critical Rayleigh number Ra_{crit} for different values of n and r_i/L for both CWT and CWHF boundary conditions at $Pr = 10^3$. Values of Rayleigh number $Ra_{\overline{Nu}_{cy}=1}$ for which $\overline{Nu}_{cy}$ values deviate from unity in the third decimal place (i.e. $\overline{Nu}_{cy} \approx 1.001$) for shear-thickening fluids (i.e. $n = 1.8$) or both CWT and CWHF boundary conditions at $Pr = 10^3$.

CWT									
r_i/L	$n = 0.6$			$n = 1.0$			$n = 1.8$		
	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 0.5$	$AR = 1$	$AR = 2$
0	700	750	2500	1850	2250	11000	4000	5000	65000
1	650	800	2750	2000	2600	12250	4000	5000	65000
16	650	800	2750	2000	2600	12250	4000	5000	65000
Rec.	650	800	2750	2000	2600	12250	4000	5000	65000
CWHF									
r_i/L	$n = 0.6$			$n = 1.0$			$n = 1.8$		
	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 0.5$	$AR = 1$	$AR = 2$	$AR = 0.5$	$AR = 1$	$AR = 2$
0	400	650	3000	850	1700	11250	950	2500	40000
1	400	650	3000	850	1700	11250	950	2500	40000
16	400	650	3000	850	1700	11250	950	2500	40000
Rec.	400	650	3000	850	1700	11250	950	2500	40000

Table 6: Summary of Ra_{crit} and $Ra_{\overline{Nu}_{cy}=1}$ correlation for both CWT and CWHF boundary conditions [12,13].

CWT	$n \leq 1$	$Ra_{crit} = 1620 \exp(0.48AR) n^{2.25} \text{ for } 0.25 \leq AR \leq 1$ $Ra_{crit} = [1964 \exp(1.08AR) - 4168] n^a \text{ for } 1 < AR \leq 4$ $a = [534 / (1 + \exp((-6.23 - AR)/1.28))] - 530$
	$n > 1$	$Ra_{\overline{Nu}_{cy}=1} = 660 \exp(0.5AR + 0.85n) \text{ for } 0.25 \leq AR \leq 1$ $Ra_{\overline{Nu}_{cy}=1} = a_1 \exp(b_0 n) \text{ for } 1 < AR \leq 4$ $a_1 = 1400; b_0 = 0.64AR^{1.73} \text{ for } 1 < AR \leq 2$ $a_1 = 5600; b_0 = 1.15 + 0.08 \exp(0.8AR) \text{ for } 2 < AR \leq 4$
CWHF	$n \leq 1$	$Ra_{crit} = 1700n^2 + [(AR - 1)(a_0 n^b)] \text{ for } 0.25 \leq AR \leq 4$ $a_0 = 65860 / (1 + \exp((3.387 - AR)/0.789)); b = 2.38 + 0.27AR$
	$n > 1$	$Ra_{\overline{Nu}_{cy}=1} = 720 + \exp(3.85AR) [13.79 / (1 - 0.352n)] \text{ for } 0.25 \leq AR \leq 1$ $Ra_{\overline{Nu}=1} = [1370 \exp(a_2 AR)] - 4000 \text{ for } 1 < AR \leq 4$ $a_2 = 1.281 + 0.462 / (1 + \exp((1.462 - n)/0.09))$

FIGURE CAPTIONS

Fig. 1: Schematic diagram of the simulation domain: a) CWT, b) CWHF configurations.

Fig. 2: Variation of the non-dimensional temperature in the vertical mid-plane of three dimensional cylindrical annular enclosures for $n = 0.8, 1.0$ and 1.2 at $Ra = 10^4, Pr = 10^3$ $AR = 1$ and $r_i/L = 1$ in the CWT configuration.

Fig. 3: Variation of $\overline{Nu}_{cy}$ with nominal Ra for different values of n at $r_i/L = 1.0, AR = 0.5$ and $Pr = 10^3$ for with representative streamline and isotherm patterns for both established (E.F.) and quiescent (Q.F.) flow initial conditions in both a) CWT and b) CWHF boundary conditions. The values of n (i.e. 0.6, 0.8, 1, 1.4, 1.8) have been shown with black, red, blue, green, grey respectively.

Fig. 4: Variation of $\overline{Nu}_{cy}$ with AR for a) shear-thinning fluids (e.g. $n = 0.8$), b) Newtonian fluids (i.e. $n = 1$) and c) shear-thickening fluids (e.g. $n = 1.4$) at $r_i/L = 1.0$ and $Pr = 10^3$ for both established (E.F.) and quiescent (Q.F.) flow initial conditions in the case of both CWT (left column) and CWHF (right column) boundary conditions.

Fig. 5: Contours of non-dimensional stream function Ψ for different values of Ra and AR for a) shear-thinning fluids (e.g. $n = 0.8$), b) Newtonian fluids (i.e. $n = 1$) and c) shear-thickening fluids (e.g. $n = 1.4$) at $r_i/L = 1.0$ and $Pr = 10^3$ for established flow (E.F.) initial condition in the case of both CWT (left column) and CWHF (right column) boundary conditions.

Fig. 6: Contours of θ and Ψ for different AR at $r_i/L = 1, Ra = 10^5$ and $Pr = 10^3$ for shear-thinning (e.g. $n = 0.8$) fluid case for established flow initial condition in the case of CWT boundary condition.

Fig. 7: Variation of $\overline{Nu}_{cy}$ with r_i/L for different values of n and a) $AR = 0.5$, b) $AR = 2.0$ at $Ra = 5 \times 10^4$ and $Pr = 10^3$ for both established (E.F.) and quiescent (Q.F.) flow initial conditions in the case of both CWT (left column) and CWHF (right column) boundary conditions.

Fig. 8: Contours of Ψ for different values of n and a) $AR = 0.5$, b) $AR = 2.0$ at $Ra = 5 \times 10^4$ and $Pr = 10^3$ for established flow (E.F.) initial condition in the case of both CWT (left column) and CWHF (right column) boundary conditions.

Fig. 9: Contours of Ψ for different values of n and a) $AR = 0.5$, b) $AR = 2.0$ at $Ra = 5 \times 10^4$ and $Pr = 10^3$ for quiescent flow (Q.F.) initial condition in the case of both CWT (left column) and CWHF (right column) boundary conditions.

Fig. 10: Contours of non-dimensional stream function Ψ for different values of AR for a) convergence criteria is 10^{-6} , b) convergence criteria is 10^{-7} at $r_i/L = 1$, $Ra = 10^5$, $n = 1$ and $Pr = 10^3$ for CWT boundary condition.

TABLE CAPTIONS

Table 1: Summary of the findings of existing analyses on natural convection in rectangular enclosures involving non-Newtonian fluids obeying power-law model of viscosity.

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Table 3: Summary of the non-uniform Cartesian meshes used in the current analysis for $0.125 \leq r_i/L \leq 16$, $0.25 \leq AR \leq 4$, $0.6 \leq n \leq 1.8$ and $10^3 \leq Ra \leq 10^6$ at $Pr = 10^3$ with non-dimensional minimum cell distance ($\Delta_{min,cell}/L$) and grid expansion ratio (r_e) values.

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