This is NOT final version

A method for the empirical formulation of current profile

D.K. Kim ^{a,b,c,*}, E.W.C. Wong ^{a,d}, S.Y. Yu ^{a,c} and Y.T. Kim ^e

^a *Ocean and Ship Technology Research Group, Universiti Teknologi PETRONAS,*

32610 Seri Iskandar, Perak, Malaysia;

^b *Graduate School of Engineering Mastership, POSTECH, 37673 Pohang, Republic of Korea;*

^c *Department of Civil and Environmental Engineering, Universiti Teknologi PETRONAS,*

32610 Seri Iskandar, Perak, Malaysia;

^d *Department of Mechanical Engineering, Universiti Teknologi PETRONAS,*

32610 Seri Iskandar, Perak, Malaysia;

^e *Technology strategy team, Daewoo Institute of Construction Technology, 16297 Suwon, Republic of Korea*

Abstract

In this study, an advanced method was proposed for the empirical formation of current profiles which can be directly used for the numerical analysis of offshore structures. A probabilistic approach was adopted to generate a reliable empirical model which can be expressed as a function of current velocity and water depth from the obtained best fit probability density function (PDF) with sub-parameters. It is recognised that the statistical scatter of current velocity at each normalised water depth is wide and requires a reliable method (or technique) with a refined manner to generate a simplified current profile model. From the probabilistic approach, the best-fit PDF of the current velocity distribution, including all ranges of normalised water depth is decided. In addition, sub-parameters of PDF (i.e., shape, scale, location parameters) can also be formulated as a function of normalised water depth through curve-fitting. For better understanding, three (3) main steps which are 1) individual, 2) overall, and 3) optimised outcomes have been highlighted in order to propose the empirical formulation of current profiles.

* Corresponding author. Tel.: +60 5 368 7312; Fax: +60 5 365 6716
E-mail address: do.kim@petronas.com.my or dokim@postech.ac.kr (D.K. Kim).

Applicability of the proposed method was verified by collecting 54 current profiles obtained from existing offshore fields, thus making it possible to generate a more accurate current profile model.

Keywords: current profile, current modelling, offshore, empirical formulation, probabilistic approach.

1. Introduction

In the design of marine structures, environmental loadings such as wind, wave, current, etc. need to be clearly defined. This is especially for marine risers which functions as the connecting structure between subsea and floating systems, where the effect of current loading on fatigue performance and design life should be carefully investigated in the FEED stage. On the other hand, current profile modelling is significantly considered in marine riser designs. The strength and vortex-induced vibration (VIV) fatigue strength analyses of marine structures are dependent on the current profile. Hence, a current profile has to be an accurate representation of current condition at a specific region. There are many methods being developed to define the designed current.

Traditionally, currents are designed based on its enveloping profile with the assumption that extreme speed happens concurrently at all meter depths. The current is measured at the proposed well site for each meter depth. The cumulative distribution of current speed at each meter depth is developed with probability distribution functions (PDFs) fitted to the current speed. From PDFs, the extreme speed at each meter depth is computed. The speeds of currents at each meter depth are then connected. The outcome of this method gives a conservative current profile ([Adams and Thorogood 1998](#)). The second tradition approach is by assuming that current load is a function of concurrent profiles at any instant time. The observed profiles are sorted into six relevant surface speed bands which are equally divided. The mean of the observed profiles in each speed band is computed ([Adams and Thorogood 1998](#)).

[Jeans et al. \(2002\)](#) stated that these two approaches disregard the vertical coherence of current profiles, thus do not provide appropriate results. To tackle this issue, several researchers had applied the

empirical orthogonal function (EOF) method (Forristall and Cooper 1997; Meling et al. 2002). EOF transforms data into a new coordinate system. The largest variance by data is projected on the first coordinate, the second largest variance by data is projected on the second coordinate and the residue is projected respectively as well. Minko et al. (2008) utilised the EOF approach to reduce current profiles and incorporate the EOF approach with probabilistic approaches for riser design. However, this approach becomes complex when the number of modes is excessive.

The EOF method was improved by identifying different time scales followed by the use of EOF method on each time scale. This provides a simpler model by using the superposition characteristic of currents of different physical processes. Guedes Soares and Neves (2005) used this approach together with the Fast Fourier Transform method to predict the time series power spectra for tidal current components. This approach is only suitable for complete data. In cases of insufficient data, the Empirical Modal Decomposition (EMD) method suggested by Huang et al. (1998) can be utilised. Another approach is by selecting a single average or several profiles that can represent each grouped profile that houses current profile categorised by current magnitude and direction (Minko et al. 2008).

In the present study, a method for the empirical formulation of current profiles was proposed by adopting the probabilistic approach. The obtained method can be applied to simplify the collected current profiles to be used in the design or numerical simulation of offshore structures. The detailed procedure for the empirical formulation was documented while the applicability of the proposed method was verified by applied example, i.e., current profile.

The obtained outcome may help offshore structural engineers who develop green fields without current profile measurement data. Instead, they might have current profiles as design input in the near field which can be used for the empirical formulation of the current profile model by adopting the proposed method in the present study.

2. Procedure for empirical formulation

A proposed procedure for the empirical formulation is described in the flowchart in Fig. 1. For the Health, Safety, and Environment (HSE) philosophy-based design of offshore structures, environmental conditions such as wind, current, wave, temperature, and many others were carefully identified and investigated. In addition, an advanced system (or tool) was required to simplify the design input, i.e., environmental loadings which can be measured from offshore fields. It also helped to design process simplification. Nowadays, data processing is one of the challenging areas which can also be applied to the engineering field as shown in Fig. A.1. In this regards, the probabilistic-based empirical formulation of current profile was targeted to be achieved in this study.

With regards to simplification and optimisation of data processing, recently new technology such as artificial intelligence (AI) including machine learning and deep learning is highlight as shown in Fig. A.1. Conventionally, probabilistic approaches were widely developed and the technique proposed in the present study can also be considered as one of the conventional method. Especially, empirical formulation shape is preferred to be used for the design of structures, estimation of the applied loading, etc.

Recently, Paik and Kim (2012) proposed an advanced method for the development of an empirical model by adopting the probabilistic approach to predict time-dependent corrosion wastage. The proposed method was verified by ballast tank structures. The proposed concept was also applied to oil well tubes and subsea gas pipeline structures by Mohd Hairil and Paik (2013) and Mohd Hairil et al. (2014), respectively.

In this study, a more refined method was proposed and applied to the empirical formulation of offshore current profiles. The obtained empirical formulation will be useful to estimate the current loading to be used in the structural design of offshore structures.

Figure 1. Procedure for the generation of a simplified empirical formulation of current profiles.

2.1. Collection of current profiles

For the 1st step of the generation or formulation of the empirical model, the collection of current profile data was required. It is well-recognised that many sampling applications (input data) may help draw reliable outcomes. It may be better if the current profiles in a specific sea area can be obtained. However, various current profiles were adopted in the present study for the generation of a current profile model due to lack of initial data samples.

2.2. Goodness of fit test

The obtained current profiles were required to be expressed with dimensionless water depth (normalised by maximum water depth = x/d) as shown in Fig. 2. From the normalised water depth, the number of scenarios can be decided. In this study, 0.05 (x/d) interval was adopted and a total of 21 scenarios were selected such as 0.0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, and 1.0. Then, a Goodness-of-fit test was performed for the selected 21 scenarios. There exist several types of Goodness-of-fit test techniques as follows:

- Anderson–Darling (A-D) test
- Akaike information criterion
- Chi-squared (C-S) test
- Cramér–von Mises criterion
- Hosmer–Lemeshow (H-L) test
- Kolmogorov–Smirnov (K-S) test
- Shapiro–Wilk (S-W) test

Figure 2. Sample of a normalised current profile.

Figure 3. Example of Anderson–Darling (A-D) test results.

The purpose of the abovementioned Goodness-of-fit test methods is to provide the best fit

probability density function (PDF) which is well-fitted with input data. In general, the PDFs are listed in ascending order of value of Goodness-of-fit test results obtained. Figure 3 shows one of the good examples of test results by adopting the Anderson–Darling (A-D) test. Four different A-D values, 0.808, 0.820, 0.989, and 0.1077 were obtained from four PDFs in Fig. 3. Once A-D values were obtained from the A-D Goodness-of-fit test, the best fit PDF (3-parameter Lognormal function) can be determined from the obtained minimum value (= 0.808) of the A-D test in the example case of Fig. 3.

2.3. Statistical analysis of current data

Once the goodness of fit test was done, determination of the best-interval (or best bin width) of histogram was conducted. In general, the methods below are highly recommended and adopted for the determination of best-interval of the histogram.

- Minimum coefficient of variation (COV) & Maximum mean value method
- R^2 method
- Sturges method
- Scott method
- Freedman method
- Shikamazi and Shinomoto method
- Etcetera

In the applied example section of the present study, the Scott's method ([Scott 1979](#)) was adopted to select the best-interval (or best-fit bin width) of histogram. Briefly, the Scott's method is optimal when input data is close to being normally distributed. However, it is also appropriate for most of the other probability density functions. The Scott's method can be expressed by Eq. (1).

$$W_b = \frac{3.49 SD}{\sqrt[3]{n}} \quad (1)$$

where W_b = bin width of histogram, SD = standard deviation, and n = sample size, and it emphasizes the dependence of the mesh and bin width on the sample size.

2.3.1. Individual outcome

Once the input (=current scenario mentioned in section 2.2) was decided, the distribution of probability density could be visualised in 3-dimension plots (3D) which show the relationship between current velocity, normalised depth (x/d), and probability density. In general, each normalised water depth scenario case produces a different type of best-fit PDF. In here, the obtained best-fit PDF by each normalised water depth scenario case is defined as “individual outcome”.

2.3.2. Overall outcome

Based on the obtained outcomes in the above stage (individual outcome), the overall outcome can be generated. The mean value of the obtained A-D test result in individual outcomes is calculated.

[Step 1] Calculation of mean value of the obtained A-D test from individual outcomes

[Step 2] Selection of best-fit PDF (Minimum A-D test result obtained from Step 1)

[Step 3] Application of obtained best-fit PDF to each normalised water depth

[Step 4] Comparison between individual and overall outcomes

2.3.4. Optimised outcome

Optimisation is conducted based on the overall outcome. For this, the empirical formulation technique should be adopted. The optimisation process can also proceed once the empirical formulation is achieved based on the overall outcome.

2.4. Empirical formulation of current equation

From the obtained overall outcome, sub-parameters of the selected best-fit PDF can be re-analysed.

In general, each PDF consists of two or three sub-parameters. For example, Eq. (2) shows the sample PDF which represents a 3-parameter Weibull function. For the sake of information sharing, other types of sub-parameters used in different PDFs are also presented in Table 1.

$$F(x) = \frac{A}{B} \left(\frac{x-C}{B} \right)^{A-1} \exp \left[- \left(\frac{x-C}{B} \right)^A \right] \quad \text{Eq. (2)}$$

where A =scale parameter, B = shape parameter, and C = location parameter.

Table 1. Example of probability density functions with sub-parameters.

Current profiles can be expressed as a function of water depth and current velocity. Therefore, once a best-fit PDF is selected, sub-parameters can also be plotted in the relationship between normalised water depth and sub-parameters as shown in Fig. 3. From this step, the empirical formulation of sub-parameters (=formulation of current equation) can be achieved and the accuracy of the obtained outcome can be evaluated using the calculated coefficient of determination ($R^2 > 0.9$).

Figure 4. Example of empirical formulation technique by curve-fitting to propose the empirical formula of sub-parameters.

3. Applied example

In this section, the applicability of the abovementioned procedure in Fig. 1 is verified by adopting an applied example.

3.1. Collection of current profiles

The generation of a current profile model started with the collection of current data from various

deep water regions with water depths greater than 500m. Figure 5(a) shows the collected current profiles which are the functions of current velocity and water depth. A total of 54 current data was collected from various regions as mentioned below. **Some of the information and references were summarised in Table A.1.** The return period of the current is ranged from 1 year to 1,000 years.

- Gulf of Mexico;
- North Sea;
- Santos Basin (southern Brazilian);
- Jubarte Field Brazil;
- Campos Basin (Brazil);
- Foz de Amazon (Brazil);
- Offshore Borneo;
- Girassol (West Africa);
- Atlantic Frontier; and
- West Seno.

The obtained current profiles are normalised by maximum water depth as shown in Fig. 1. In the first step of analysis of the current profile data, the current velocities were determined at each normalised water depth (x/d) starting at x/d of 0 with an increment of 0.05 until x/d of 1.0. As stated in section 2, it is assumed that the vertical axis (normalised water depth) can be divided into 21 sub-regions as shown in Fig. 5(b). This means that the variations of current velocities were analysed by probabilistic approaches based on 0.05 of normalised water depth. Therefore, a total of 21 current scenarios were selected as represented in Fig. 5(b).

Figure 5. Compilation of current profile.

3.2. Goodness-of-fit test

The best fit distribution for current data at each normalised depth was determined by adopting a goodness-of-fit test. The current velocities were statistically analysed at each normalised water depth to determine the probability distribution of the velocity at each normalised water depth using the goodness-of-fit test. In the present study, the best probability distribution function of the current velocities along the normalised water depth was determined using the Anderson-Darling test. The Anderson-Darling statistic can be expressed for normal distribution cases as follows (Anderson and Darling 1954):

$$A^2 = -n - S \quad \text{Eq. (3)}$$

where $S = \frac{1}{n} \sum_{i=1}^n (2i-1) [\log(\Phi(Y_i)) + \log(1-\Phi(Y_{n-i+1}))]$, Φ = representing cumulative distribution function (CDF) of the data, $Y_i = (X_i - \hat{\mu}) / SD$, $\hat{\mu}$ = mean value, and SD = standard deviation, $\Phi(Y_i)$.

Based on what are known and unknown about the normal distribution parameters such as mean (μ) and variance (σ^2), computation may differ and S can also be slightly revised. In the case of other probability density functions, it may be referred to as critical values to those PDFs. Details may be referred to Stephens (1986) for exponential, extreme value, Weibull, gamma, logistic, Cauchy, and von-Mises distributions, Pearson and Hartley (1972) for normal and exponential distributions, and Shorak and Wellner (1986) for Gumbel distribution. Details for the logistic distribution can also be found in Stephens (1979).

As stated earlier, there are several types of goodness-fit tests. The A-D test adopted in the present study is one of the options while other methods can also be adopted. From the results of the A-D test, the best fit distribution of the current data can be determined, and the following two (2) types of best fit distributions can be obtained.

- Individual best fit distribution: Best fit distribution of current velocity (with lowest AD statistics) at each normalised depth

- Overall best fit distribution: Overall best fit distribution that produces the lowest average AD statistics for overall normalised depth

The Anderson-Darling statistic shows how well-fitted the distribution is for data from the fitted line in a probability plot. Hence, by applying several probability distributions, the Anderson-Darling statistic was determined. The smaller the statistic, the better the distribution fits the data. Since current velocity data has different best-fitted distributions at each normalised water depth as shown in Table 2, the lowest average of the total Anderson-Darling statistics was determined to identify a single distribution that can represent the overall progress of current velocity along the normalised water depth. Current velocity at each normalised water depth is represented in the form of a probability density function and histogram.

Again, the A-D test results of the selected 21 current scenarios are illustrated in Table 2. As stated earlier, the smallest value of A-D test result represents the best-fit PDF as highlighted. In Table 2, for example, the Logistic PDF has been selected as the best-fit function for normalised water depth at 0.05, 0.10, and 0.15 based on their A-D test results of 0.063, 0.883, and 1.093, respectively.

Table 2. Results of goodness-of-fit test.

From the individual best fit distributions obtained by the A-D tests represented in Table 2, the mean value can be calculated. The smallest value among the calculated average values shown in Table 2 in the last column was then used for the determination of best-fit distribution. From this step, the overall best-fit distribution can be determined. In this applied example, the “Large Extreme Value” probability density function (PDF) was selected for the overall best fit distribution as shown in Table 3.

Table 3. Selected best fit PDFs from the results of goodness-of-fit test.

3.3. Statistical analysis of current data

Based on the Goodness-of-fit test results in the previous section, the best fit PDFs are decided for

the individual and overall best fit distributions shown in Figures 6(a) and 6(b), respectively. As stated in section 2.3.1, the statistical analysis result can be plotted in 3-dimension which shows the relationship between current velocity, normalised water depth, and probability density represented in Figures 6(a) and 6(b).

Once again, individual outcome is the best fit distribution at each normalised water depth, which means that different distributions are identified for current velocity at each normalised water depth (x/d). For instance, the best fit distribution at $x/d=0$ is a normal distribution, whereas the best fit distribution at $x/d=0.05$ is a logistic distribution. The overall outcome represents the overall best fit distribution after considering all normalised water depths determined from the lowest overall average Anderson-Darling statistic.

Figure 6. Obtained best fit probability density distributions of actual current velocity along normalised water depth.

In addition, the bin width of the histogram has a significant effect on the statistical properties of the data, hence the best bin width, W_b , for each normalised water depth was identified using Scott's normal reference rule calculated using Eq. (1). For the comparison of three (3) different outcomes, i.e., individual, overall, and optimised as introduced from sections 2.3.1 to 2.3.3, the goodness-of-fit test and best-interval results of individual histogram calculations are shown in PDFs together with the histogram in the next section.

3.4. Empirical formulation of current equation

Once the overall best-fit probability density function or distribution (PDF) was selected, optimisation can be performed by investigating the change of sub-parameter behaviour of PDF. The overall best fit probability density distribution that can represent the progress of current velocity along

the normalised water depth is the extreme value distribution. The extreme value distribution function used to plot the graph shown in Fig. 6(b) is represented in Eq. (4).

The selected best fit PDF “Largest Extreme Value” distribution has two sub-parameters which are location (A) and scale parameter (B) with current velocity as variable (v_c).

Selected best fit PDF (Largest Extreme Value PDF)

$$PDF = f(v_c) = \left(\frac{1}{A} \right) \cdot \exp \left[-\frac{(v_c - A)}{B} \right] \cdot \exp \left[-\exp \left\{ -\frac{(v_c - A)}{B} \right\} \right] \quad \text{Eq. (4)}$$

where A = location parameter, B = scale parameter, and v_c = current velocity.

Location (A) and scale parameter (B) are formulated as the function of normalised water depth as shown in Figures 7(a) and (b). These sub-parameters vary with the normalised water depth in a consistent trend that can be represented as an empirical formulation. A rational function was found to be the best fit to describe the relationship between location parameter (A) and normalised water depth, whereas a third-order polynomial function can well relate the normalised water depth to the scale parameter (B). This was proven by the high coefficient of determination, R-squared value (R^2). The closer the R^2 to 1.0, the better is the fit of the function. Hence, location (A) and scale parameter (B) can be estimated using the function of normalised water depth (x/d) as shown in Equations (5a) and (5b). The sub-parameter values obtained by the overall and optimised method are also represented in Table 4 and Fig. 7.

Empirical formulation of location parameter (A)

$$A = \frac{0.05222(x/d) + 0.08350}{(x/d) + 0.0670} \quad \text{Eq. (5a)}$$

Empirical formulation of scale parameter (B)

$$B = -1.163(x/d)^3 + 2.429(x/d)^2 - 1.743(x/d) + 0.6154 \quad \text{Eq. (5b)}$$

Figure 7. Empirical formulation of sub-parameters with normalised water depth.

Table 4. Comparison of parameters between overall and optimised outcomes.

By inserting the location and scale parameters into the extreme distribution function, the current profile model can be developed. The current profile model is approximated and developed using the equations as follows.

$$\text{Optimised outcome} = f(v_c) = \left(\frac{1}{A} \right) \cdot \exp \left[-\frac{(v_c - A)}{B} \right] \cdot \exp \left[-\exp \left\{ -\frac{(v_c - A)}{B} \right\} \right] \quad \text{Eq. (6)}$$

where v_c = current velocity, A = location parameter $(= \frac{0.05222(x/d) + 0.08350}{(x/d) + 0.0670})$, and B = scale parameter $(= -1.163(x/d)^3 + 2.429(x/d)^2 - 1.743(x/d) + 0.6154)$, x/d = normalised water depth.

The obtained PDF and sub-parameters illustrated in Eq. (6) are used to formulate the distribution function which is represented as the optimised distribution function (= optimised outcome) as shown in Fig. 8. From here, three different outcomes have been obtained, which are 1) individual, 2) overall, and 3) optimised PDFs as shown in Figures 6(a), 6(b), and Fig. 8, respectively. For comparison purposes, the three outcomes have been plotted together as shown in Figures 9(a) to 9(u).

Figure 8. Optimised best fit probability density distributions of actual current velocity along normalised water depth.

The best probability distribution and histogram to represent current velocity distribution at each normalised water depth are identified and presented in Figures 9(a) to 9(u). In each graph, the proposed

three types of outcome (individual, overall, and optimised) are presented. From the obtained results in Fig. 9, it was observed that the proposed method (optimised outcome) gave reasonable accuracy of the current model.

Figure 9. Best-fit probability density distributions using three methods for each normalised water depth.

The obtained optimised outcomes (optimised probability density distribution functions) are used to determine the probability of occurrence of a certain current velocity at the specified normalised water depth. The current profile model was then developed by combining all the optimised PDFs as illustrated in Fig. 10. For comparison purposes, three (3) methods, i.e., individual, overall, and optimised, were also presented. In addition, mean, standard deviation, 95 % and above band value were computed at each normalised water depth from the obtained optimised outcomes as shown in Fig. 8. The details of the computed values are presented in Table 5.

For user-friendliness purposes, several types of the current model can be generated such as mean model, mean + standard deviation (S.D.) model, mean + $2 \times$ S.D. model, and 95 % and above band model, as shown in Fig. 10. The obtained current profile model (optimised outcome) can be applied to determine the most representative current profile based on the engineering judgement of strength/fatigue assessment which was based on the mean and standard deviation of the statistical model.

Figure 10. Obtained current models.

3.5. Comparison

Through the mean and coefficients of variation (COV) values, the accuracy of the obtained outcomes (=optimised outcome or optimised empirical formulation) can be verified with two (2) other outcomes such as the individual and overall outcomes as shown in [Tables A.2\(a\) and A.2\(b\)](#). For comparison, the individual outcome which was used for the horizontal axis was considered and set as a

reference point as shown in Figures 11(a) and 11(c). In addition, a comparison between the overall and optimised outcomes was also investigated. Therefore, a total of three (3) cases were compared, such as 1) Individual versus Overall outcomes, 2) Overall versus Optimised outcomes, and 3) Individual versus Optimised outcomes in Figures 11(a), 11(b), and 11(c), respectively.

Once again, in this section, the mean values and coefficients of variation (COV) for the optimised empirical formulation and other methods were identified for each normalised water depth. In addition, the standard deviation between the methods is also summarised as reference.

Figure 11. Comparison of proposed three methods.

Table A.2(b) summarises the results of the comparison between the mean and COV obtained using the three methods proposed in this study. From the obtained outcomes, it is evident that the proposed method (=Optimised outcome) is in very good agreement with more refined methods (please refer obtained mean and COV values in Fig. 11) such as the Individual and Overall outcomes. For reader's information, cumulative density function (CDF) for each normalised water depth is also presented in Fig. A.2. As compared by 1) Mean and COV graphs as well as 2) CDF graphs, it is verified that proposed method shows good agreement with original input data. It is meaning that applicability of the proposed method is confirmed by applied example.

Lastly, the used input current data set generates empirical formulation based on extreme value distribution. However, this may not be considered as general type of PDF shape. It is meaning that obtained outcome will be depending on input data set. For example, in the case of hourly mean current speed and fir a distribution, probably Weibull distribution might be a good choice, while if we use 1-year current speed for the fitting, maybe an extreme value distribution might be the good choice. For the confirmation of the general type of PDF, additional researches are expected to be performed.

4. Conclusions

In the present study, the complete process of empirical formulation of a current profile model was proposed. The applicability of the proposed method was verified through an applied example based on real current profile data using a statistical approach. From the obtained outcome, it can be concluded that the proposed method can be considered as a reliable method which is simple and gives acceptable results. The obtained outcomes can be summarised as follows.

- Current velocity at each normalised water depth shows different statistical properties and can be best-fitted with different PDFs.
- Overall, the largest extreme value distribution is the average best fit PDF for the current velocity distribution based on Anderson-Darling statistics.
- Optimisation of parameters of the PDF for each water depth correlates the current velocity at each water depth through an empirical model of parameters as the function of water depth.
- Different current models such as mean model can be extracted from the optimised best fit probability density distributions to be used as the design of current for offshore structure design analysis based on engineering judgement.
- A comparison of the optimised outcome with individual outcome shows that the optimised outcome can represent the current profile with low variation well (Coefficient of Variation closed to 0.0).

In future, more studies will be done to improve the accuracy of the technique by incorporating the correlation of flow velocity along the water column and direction of the current. Additional current profiles will be collected from different regions to be used to validate the applicability of the proposed method at different sea regions.

Acknowledgements

This study was undertaken at Ocean and Ship Technology (OST) Research Group at Universiti Teknologi PETRONAS. This research was supported by the Technology Innovation Program (Grant No.: 10053121 and 10051279) funded by the Ministry of Trade, Industry & Energy (MI, Korea) and YUTP Grant (0153AA-E60, Malaysia). The authors would also like to thank for the great support of POSTECH, and Daewoo E&C, Republic of Korea. Some part of the present study was presented in The 1st International Conference on Architecture and Civil Engineering (ICACE 2017), 8-9 May, Kuala Lumpur, Malaysia.

References

- Adams A, Thorogood J. 1998. On the choice of current profiles for deep-water riser design. In: SPE Annual Technical Conference and Exhibition; New Orleans (USA).
- Adebayo A. 2011. Steel catenary risers supported by subsurface buoy [MSc thesis]. Stavanger (Norway): University of Stavanger.
- Anderson TW, Darling DA. 1954. A test of goodness of fit. Journal of the American Statistical Association 49(268):765-769.
- Bai Y, Bai Q. 2012. Subsea engineering handbook. Gulf Professional Publishing. Oxford (UK).
- Cao P, Cheng J. 2013. Design of steel lazy wave riser for disconnectable FPSO. Proceedings of the Offshore Technology Conference – OTC 24166; Houston (USA).
- Forristall GZ, Cooper CK. 1997. Design current profiles using empirical orthogonal function (EOF) and inverse FORM methods. Proceedings of the Offshore Technology Conference - OTC 8267; Houston (USA).
- Guedes Soares C, Neves SN. 2005. Modelling tidal current profiles by means of empirical orthogonal functions. J Offshore Mech Arct Eng. 128(3):184-190.
- HOE (2011). RPSEA 1402 Ultra Deepwater Dry Tree System for Drilling and Production (Document No.: H08130-G-DBN-GN-15001). Houston Offshore Engineering, Houston, USA.

- Huang NE, Shen Z, Long SR, Wu MC, Shih HH, Zheng Q, Yen NC, Tung CC, Liu HH. 1998. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. In: Proceedings of the royal society of London A: mathematical, physical and engineering sciences 454:903-995.
- Inoue M, Welsh SE, Rouse LJ, Weeks E. 2008. Deepwater currents in the eastern Gulf of Mexico: Observations at 25.5 N and 87 W. U.S. Department of the Interior, Minerals Management Service, Gulf of Mexico OCS Region.
- Jeans G, Grant C, Feld G. 2002. Improved current profile criteria for deepwater riser design. Proceedings of the 21st International Conference on Ocean, Offshore and Arctic Engineering - OMAE2002-28153; Oslo (Norway).
- Jeans G, Prevosto M, Harrington-Missin L, Maisondieu C, Herry C, Lima JAM. 2012. Deepwater current profile data sources for riser engineering offshore Brazil. Proceedings of the 31st International Conference on Ocean, Offshore and Arctic Engineering - OMAE2012-83400; Rio de Janeiro (Brazil).
- Lejlic E. 2013. Vortex induced fatigue damage of a steel catenary riser near the touchdown point [MSc thesis]. Trondheim (Norway): Norwegian University of Science and Technology.
- Li S. 2014. Response Analysis of the Deepwater Steel Lazy-wave Riser for the Turret Moored Floating Production Storage Off-loading System [Ph.D. thesis]. Florida (US): Florida Institute of Technology.
- Meling TS, Eik KJ, Nygaard E. 2002. An assessment of EOF current scatter diagrams with respect to riser VIV fatigue damage. Proceedings of the 21st International Conference on Ocean, Offshore and Arctic Engineering - OMAE2002-28062; Oslo (Norway).
- Minko IDM, Prevosto M, Damy G. 2008. Statistical Modelling of current profiles for a riser reliability design. Proceedings of the 27th International Conference on Ocean, Offshore and Arctic Engineering - OMAE2008-57933; Estoril (Portugal).
- Mohd MH, Paik JK. 2013. Investigation of the corrosion progress characteristics of offshore subsea oil well tubes. Corros Sci. 67:130-141.
- Mohd MH, Kim DK, Kim DW, Paik JK. 2014. A time-variant corrosion wastage model for subsea gas

- pipelines. *Ships Offshore Struc.* 9(2):161-176.
- Nurwanto T, Karunakaran D, Franciss R. 2013. COBRA Riser Concept for Ultra Deepwater Condition. *Proceedings of the 32nd International Conference on Ocean, Offshore and Arctic Engineering - OMAE2013-11161*; Nantes (France).
- Orimolade AP. 2014. Steel lazy wave risers from turret moored FPSO [MSc thesis]. Stavanger (Norway): University of Stavanger.
- Paik JK, Kim DK. 2012. Advanced method for the development of an empirical model to predict time-dependent corrosion wastage. *Corros Sci.* 63:51-58.
- Paik JK, Thayamballi AK. 2007. Ship-shaped offshore installations: design, building, and operation. Cambridge University Press. Cambridge (UK).
- Park KS. 2014. Fatigue Performance of Deepwater Steel Catenary Riser [MSc thesis]. South Korea: Pohang University of Science and Technology.
- Park KS, Kim YT, Kim DK, Yu SY, Choi HS. 2016. A new method for strake configuration design of Steel Catenary Risers. *Ships and Offshore Structures* 11(4): 385-404.
- Pearson ES, Hartley HO. 1972. *Biometrika tables for statisticians 2*. Cambridge University Press. Cambridge (UK).
- Ruswandi MI. 2009. Improvisation of deepwater weight distributed steel catenary riser [MSc thesis]. Stavanger (Norway): University of Stavanger.
- Scott DW. 1979. On optimal and data-based histograms. *Biometrika* 66(3):605-610.
- Sheikh R, Brown A (2010). Extreme vertical deepwater current profiles in the South China Sea, Offshore Borneo. *Proceedings of the 29th International Conference on Ocean, Offshore and Arctic Engineering - OMAE2010-21006*; Shanghai (China).
- Shorack GR, Wellner JA. 1986. *Empirical processes with applications to statistics*. Wiley. New York (USA).
- Stephens M. 1979. Tests of fit for the logistic distribution based on the empirical distribution function. *Biometrika* 66(3):591-595.
- Stephens M. 1986. Tests based on EDF statistics. In *Goodness-of-fit techniques* (Editors RB and

Stephens MA). New York (USA).

Utt ME, Emerson E, Yu J. 2004. West Seno Drilling and Production Riser Systems. Proceedings of the Offshore Technology Conference – OTC 16526; Houston (USA).

Welsh SE, Inoue M, Rouse Jr LJ, Weeks E. 2009. Observation of the deepwater manifestation of the loop current and loop current rings in the eastern Gulf of Mexico. Department of the Interior, Minerals Management Service, Gulf of Mexico OCS Region.

Appendix

Source: Samsung Securities Co. Ltd.

Figure A.1. Principle of 4th industrial revolution and importance of data processing.

(a) normalised water depth ($x/d = 0.00$)

(b) normalised water depth ($x/d = 0.05$)

(c) normalised water depth ($x/d = 0.10$)

(d) normalised water depth ($x/d = 0.15$)

(e) normalised water depth ($x/d = 0.20$)

(f) normalised water depth ($x/d = 0.25$)

(g) normalised water depth ($x/d = 0.30$)

(h) normalised water depth ($x/d = 0.35$)

(i) normalised water depth ($x/d = 0.40$)

(j) normalised water depth ($x/d = 0.45$)

(k) normalised water depth ($x/d = 0.50$)

(l) normalised water depth ($x/d = 0.55$)

(m) normalised water depth ($x/d = 0.60$)

(n) normalised water depth ($x/d = 0.65$)

(o) normalised water depth ($x/d = 0.70$)

(p) normalised water depth ($x/d = 0.75$)

(q) normalised water depth ($x/d = 0.80$)

(r) normalised water depth ($x/d = 0.85$)

Figure A.2. Best-fit cumulative density distributions using three methods for each normalised water depth.

Table A.1. Some of the information of collected current profiles with references.

References	Details	
	Region	Return period
Adebayo (2011)	North Sea (subsea7 project) /Norwegian Sea in accordance with Norsok N-003	[10-year, 100-year]
Lejlic (2013)	North Sea	N/A
Orimolade (2014)	West of Shetland (WoS) North sea	[10-year, 100-year]
Nurwanto et al. (2013)	Central Cluster of Santos Basin at Sao Paulo Plateau, southern Brazilian oceanic region	[10-year, 100-year]
Paik and Thayamballi (2007)	Norwegian Sea	[100-year extreme current]
Li (2014)	Gulf of Mexico (GOM)	GOM loop current [1-year, 10-year, 100-year]; GOM Hurricane [1-year, 10-year, 100-year, 1000-year]
Inoue et al. (2008)	Eastern Gulf of Mexico	N/A
Jeans et al. (2012)	Jubarte Field Brazil	N/A
Sheikh and Brown (2010)	Offshore Borneo	N/A
Bai and Bai (2012)	Brazil (Campos Basin, Foz de Amazon), GOM (Hurricane, Loop), West Africa (Girassol), Atlantic Frontier, Northern Norway	N/A
Cao and Cheng (2013)	GOM	N/A
Utt et al. (2004)	West Seno	[1-year, 10-year, 100-year]
Welsh et al. (2009)	GOM	N/A
Park (2014); Park et al. (2016)	GOM (Loop current, Hurricane)	N/A
Ruswandi (2009)	Northern Norway environment	N/A
HOE (2011)	RPSEA GOM	[1-year winter storm, 10-year winter storm, 100-year hurricane, Max loop, Hurricane plus loop current]

Table A.2(a). Data of the obtained current models.

No	Normalised water depth (x/d)	Individual outcome				Overall outcome				Optimised outcome			
		Mean	Mean + S.D.	Mean + 2S.D.	95% and above band	Mean	Mean + S.D.	Mean + 2S.D.	95% and above band	Mean	Mean + S.D.	Mean + 2S.D.	95% and above band
1	0.00	1.536	2.108	2.680	2.477	1.564	2.257	2.950	2.857	1.602	2.391	3.180	3.074
2	0.05	1.050	1.756	2.462	2.196	1.065	1.859	2.654	2.547	1.044	1.729	2.415	2.323
3	0.10	0.810	1.397	1.985	1.764	0.828	1.472	2.117	2.031	0.799	1.395	1.990	1.910
4	0.15	0.647	1.164	1.681	1.486	0.685	1.226	1.767	1.695	0.654	1.174	1.693	1.623
5	0.20	0.570	1.025	1.479	1.418	0.570	1.025	1.479	1.418	0.557	1.012	1.466	1.405
6	0.25	0.481	0.871	1.261	1.208	0.481	0.871	1.261	1.208	0.485	0.887	1.289	1.235
7	0.30	0.419	0.756	1.093	1.048	0.419	0.756	1.093	1.048	0.432	0.790	1.149	1.101
8	0.35	0.375	0.683	0.990	0.949	0.375	0.683	0.990	0.949	0.390	0.715	1.039	0.996
9	0.40	0.348	0.637	0.926	0.888	0.348	0.637	0.926	0.888	0.358	0.656	0.954	0.914
10	0.45	0.331	0.610	0.888	0.851	0.331	0.610	0.888	0.851	0.332	0.610	0.889	0.851
11	0.50	0.343	0.756	1.170	1.077	0.317	0.586	0.855	0.819	0.312	0.576	0.840	0.805
12	0.55	0.332	0.749	1.167	1.061	0.304	0.563	0.822	0.787	0.296	0.550	0.804	0.770
13	0.60	0.323	0.747	1.172	1.051	0.293	0.544	0.796	0.762	0.283	0.531	0.778	0.745
14	0.65	0.317	0.764	1.210	1.059	0.282	0.528	0.774	0.741	0.273	0.516	0.759	0.726
15	0.70	0.271	0.510	0.750	0.718	0.271	0.510	0.750	0.718	0.264	0.504	0.743	0.711
16	0.75	0.259	0.493	0.728	0.696	0.259	0.493	0.728	0.696	0.256	0.492	0.728	0.696
17	0.80	0.245	0.466	0.688	0.658	0.245	0.466	0.688	0.658	0.248	0.479	0.710	0.679
18	0.85	0.232	0.442	0.653	0.624	0.232	0.442	0.653	0.624	0.240	0.464	0.688	0.658
19	0.90	0.224	0.431	0.638	0.610	0.224	0.431	0.638	0.610	0.231	0.444	0.658	0.629
20	0.95	0.209	0.407	0.606	0.579	0.209	0.407	0.606	0.579	0.220	0.418	0.617	0.590
21	1.00	0.184	0.413	0.642	0.556	0.200	0.401	0.602	0.575	0.207	0.385	0.562	0.538

Table A.2(b). Statistical analysis result of proposed three methods for the comparison.

No	Normalised water depth (x/d)	Overall/Individual				Optimised/Overall				Optimised/Individual			
		Mean	Mean + S.D.	Mean + 2S.D.	95% and above band	Mean	Mean + S.D.	Mean + 2S.D.	95% and above band	Mean	Mean + S.D.	Mean + 2S.D.	95% and above band
1	0	1.0182	1.0707	1.1007	1.1534	1.0243	1.0594	1.0780	1.0760	1.0430	1.1343	1.1866	1.2410
2	0.05	1.0143	1.0587	1.0780	1.1598	0.9803	0.9301	0.9099	0.9121	0.9943	0.9846	0.9809	1.0578
3	0.1	1.0222	1.0537	1.0665	1.1514	0.9650	0.9477	0.9400	0.9404	0.9864	0.9986	1.0025	1.0828
4	0.15	1.0587	1.0533	1.0512	1.1406	0.9547	0.9576	0.9581	0.9575	1.0108	1.0086	1.0071	1.0922
5	0.2	1.0000	1.0000	1.0000	1.0000	0.9772	0.9873	0.9912	0.9908	0.9772	0.9873	0.9912	0.9908
6	0.25	1.0000	1.0000	1.0000	1.0000	1.0083	1.0184	1.0222	1.0224	1.0083	1.0184	1.0222	1.0224
7	0.3	1.0000	1.0000	1.0000	1.0000	1.0310	1.0450	1.0512	1.0506	1.0310	1.0450	1.0512	1.0506
8	0.35	1.0000	1.0000	1.0000	1.0000	1.0400	1.0469	1.0495	1.0495	1.0400	1.0469	1.0495	1.0495
9	0.4	1.0000	1.0000	1.0000	1.0000	1.0287	1.0298	1.0302	1.0293	1.0287	1.0298	1.0302	1.0293
10	0.45	1.0000	1.0000	1.0000	1.0000	1.0030	1.0000	1.0011	1.0000	1.0030	1.0000	1.0011	1.0000
11	0.5	0.9242	0.7751	0.7308	0.7604	0.9842	0.9829	0.9825	0.9829	0.9096	0.7619	0.7179	0.7474
12	0.55	0.9157	0.7517	0.7044	0.7418	0.9737	0.9769	0.9781	0.9784	0.8916	0.7343	0.6889	0.7257
13	0.6	0.9071	0.7282	0.6792	0.7250	0.9659	0.9761	0.9774	0.9777	0.8762	0.7108	0.6638	0.7088
14	0.65	0.8896	0.6911	0.6397	0.6997	0.9681	0.9773	0.9806	0.9798	0.8612	0.6754	0.6273	0.6856
15	0.7	1.0000	1.0000	1.0000	1.0000	0.9742	0.9882	0.9907	0.9903	0.9742	0.9882	0.9907	0.9903
16	0.75	1.0000	1.0000	1.0000	1.0000	0.9884	0.9980	1.0000	1.0000	0.9884	0.9980	1.0000	1.0000
17	0.8	1.0000	1.0000	1.0000	1.0000	1.0122	1.0279	1.0320	1.0319	1.0122	1.0279	1.0320	1.0319
18	0.85	1.0000	1.0000	1.0000	1.0000	1.0345	1.0498	1.0536	1.0545	1.0345	1.0498	1.0536	1.0545
19	0.9	1.0000	1.0000	1.0000	1.0000	1.0313	1.0302	1.0313	1.0311	1.0313	1.0302	1.0313	1.0311
20	0.95	1.0000	1.0000	1.0000	1.0000	1.0526	1.0270	1.0182	1.0190	1.0526	1.0270	1.0182	1.0190
21	1	1.0870	0.9709	0.9377	1.0342	1.0350	0.9601	0.9336	0.9357	1.1250	0.9322	0.8754	0.9676
Mean		0.992	0.960	0.952	0.979	1.002	1.001	1.000	1.000	0.994	0.961	0.953	0.980
S-D		0.047	0.115	0.136	0.137	0.030	0.037	0.043	0.043	0.064	0.126	0.149	0.142
COV		0.048	0.119	0.143	0.140	0.030	0.037	0.043	0.043	0.064	0.131	0.157	0.145

List of Figures

- Figure 1. Procedure for the generation of a simplified mathematical formulation of current profiles.
- Figure 2. Sample of a normalised current profile.
- Figure 3. Example of Anderson–Darling (A-D) test results.
- Figure 4. Example of mathematical formulation technique by curve-fitting to propose the empirical formula of sub-parameters.
- Figure 5. Compilation of current profile.
- Figure 6. Obtained best fit probability density distributions of actual current velocity along normalised water depth.
- Figure 7. Empirical formulation of sub-parameters with normalised water depth.
- Figure 8. Optimised best fit probability density distributions of actual current velocity along normalised water depth.
- Figure 9. Best-fit probability density distributions using three methods for each normalised water depth.
- Figure 10. Obtained current models.
- Figure 11. Comparison of proposed three methods.

List of Tables

- Table 1. Example of probability density functions with sub-parameters.
- Table 2. Results of goodness-of-fit test.
- Table 3. Selected best fit PDFs from the results of goodness-of-fit test.
- Table 4. Comparison of parameters between overall and optimised outcomes.

Figure 1. Procedure for the generation of a simplified empirical formulation of current profiles.

Figure 2. Sample of a normalised current profile.

Figure 3. Example of Anderson–Darling (A-D) test results.

Figure 4. Example of empirical formulation technique by curve-fitting to propose the empirical formula of sub-parameters.

(a) Collected current profiles – Normalised water depth version

(b) Normalised current profiles based on 0.05 (x/d)

Figure 5. Compilation of current profile.

(a) Individual best fit PDF

(b) Overall best fit PDF

Figure 6. Obtained best fit probability density distributions of actual current velocity along normalised water depth.

Figure 7. Empirical formulation of sub-parameters with normalised water depth.

Figure 8. Optimised best fit probability density distributions of actual current velocity along normalised water depth.

(a) normalised water depth (x/d) = 0.00

(b) normalised water depth (x/d) = 0.05

(c) normalised water depth (x/d) = 0.10

(d) normalised water depth (x/d) = 0.15

(e) normalised water depth (x/d) = 0.20

(f) normalised water depth (x/d) = 0.25

(g) normalised water depth (x/d) = 0.30

(h) normalised water depth (x/d) = 0.35

(i) normalised water depth (x/d) = 0.40

(j) normalised water depth (x/d) = 0.45

(k) normalised water depth (x/d) = 0.50

(l) normalised water depth (x/d) = 0.55

(m) normalised water depth (x/d) = 0.60

(n) normalised water depth (x/d) = 0.65

(o) normalised water depth (x/d) = 0.70

(p) normalised water depth (x/d) = 0.75

(q) normalised water depth (x/d) = 0.80

(r) normalised water depth (x/d) = 0.85

Figure 9. Best-fit probability density distributions using three methods for each normalised water depth.

(a) Mean and Mean + standard deviation model

(b) Mean and Several model

Figure 10. Obtained current models.

Figure 11. Comparison of proposed three methods.

Table 1. Example of probability density functions with sub-parameters.

Probability density functions		Parameter A	Parameter B	Parameter C
Birnbaum-Saunders	$\frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{\left(\sqrt{x/A} - \sqrt{A/x} \right)^2}{2B^2} \right\} \left(\frac{\sqrt{x/A} - \sqrt{A/x}}{2Bx} \right)$	scale parameter	shape parameter	-
Exponential	$\frac{1}{A} \exp \left(-\frac{x}{A} \right)$	mean	-	-
2-parameter Exponential	$\frac{1}{A} \exp \left(-\frac{x-B}{A} \right)$	scale parameter	threshold parameter	-
Gamma	$\frac{1}{B^A \Gamma(A)} x^{A-1} e^{-\frac{x}{B}}$	shape parameter	scale parameter	-
3-parameter Gamma	$\frac{1}{B^A \Gamma(A)} (x-C)^{A-1} e^{-\frac{(x-C)}{B}}$	shape parameter	scale parameter	threshold parameter
Largest Extreme Value	$B^{-1} \exp \left(\frac{x-A}{B} \right) \exp \left(-\exp \left(\frac{x-A}{B} \right) \right)$	shape parameter	scale parameter	-
Gen. Pareto	$\left(\frac{1}{B} \right) \left(1 + A \frac{(x-C)}{B} \right)^{-1-\frac{1}{A}}$	shape parameter	scale parameter	location parameter
Inv. Gaussian	$\sqrt{\frac{B}{2\pi x^3}} \exp \left\{ -\frac{B}{2A^2 x} (x-A)^2 \right\}$	scale parameter	shape parameter	-
Logistic	$\frac{\exp \left\{ \frac{x-A}{B} \right\}}{B \left(1 + \exp \left\{ \frac{x-A}{B} \right\} \right)^2}$	mean	scale parameter	-
Log-logistic	$\frac{1}{B} \frac{1}{x} \frac{e^{-\frac{\log(x)-A}{B}}}{\left(1 + e^{-\frac{\log(x)-A}{B}} \right)^2}$	log mean	log scale parameter	-
3-parameter Log-logistic	$\frac{1}{B} \frac{1}{(x-C)} \frac{e^{-\frac{\log(x-C)-A}{B}}}{\left(1 + e^{-\frac{\log(x-C)-A}{B}} \right)^2}$	log mean	log scale parameter	threshold parameter
Lognormal	$\frac{1}{xB\sqrt{2\pi}} \exp \left\{ -\frac{(\ln x - A)^2}{2B^2} \right\}$	log mean	log standard deviation	-
3-parameter Lognormal	$\frac{1}{(x-C)B\sqrt{2\pi}} \exp \left\{ -\frac{(\ln(x-C) - A)^2}{2B^2} \right\}$	log mean	log standard deviation	threshold parameter
Nakagami	$2 \left(\frac{A}{B} \right)^A \frac{1}{\Gamma(A)} x^{(2A-1)} e^{-\frac{A}{B} x^2}$	shape parameter	scale parameter	-
Normal	$\frac{1}{B\sqrt{2\pi}} e^{-\frac{(x-A)^2}{2B^2}}$	mean	standard deviation	-

Rayleigh	$\frac{1}{A^2} \exp\left(\frac{-x^2}{2A^2}\right)$	scale parameter	-	-
Rician	$I_0\left(\frac{xA}{B^2}\right) \frac{x}{B^2} e^{-\left(\frac{x^2+A^2}{2B^2}\right)}$	noncentrality parameter	scale parameter	-
t Location-Scale	$\frac{\Gamma\left(\frac{C+1}{2}\right)}{B\sqrt{C\pi}\Gamma\left(\frac{C}{2}\right)} \left[\frac{C + \left(\frac{x-A}{B}\right)^2}{C} \right]^{-\left(\frac{C+1}{2}\right)}$	location parameter	scale parameter	shape parameter
Weibull	$\frac{B}{A} \left(\frac{x}{A}\right)^{B-1} \exp\left[-\left(\frac{x}{A}\right)^B\right]$	scale parameter	shape parameter	-
3-parameter Weibull	$\frac{B}{A} \left(\frac{x-C}{A}\right)^{B-1} \exp\left[-\left(\frac{x-C}{A}\right)^B\right]$	scale parameter	shape parameter	location parameter

Table 2. Results of goodness-of-fit test.

No.	Probability density function	Goodness-of-fit test (Anderson-Darling)																					
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	Ave.
1	Normal	0.162	0.672	0.929	1.469	1.669	1.459	1.654	1.872	2.060	2.062	2.062	2.107	2.142	2.115	2.063	2.031	1.872	1.761	1.907	2.193	2.576	1.754
2	3-Parameter Lognormal	0.173	0.708	1.248	1.593	1.347	1.070	1.092	0.890	1.048	0.931	0.808	0.743	0.829	0.978	1.120	12.675	12.659	12.629	12.521	12.231	10.315	4.172
3	2-Parameter Exponential	5.667	3.986	3.931	3.402	2.638	1.984	1.991	1.471	1.445	1.242	1.077	0.971	1.009	1.092	1.165	2.348	2.367	2.502	2.767	3.537	9.423	2.667
4	3-Parameter Weibull	0.169	1.043	2.702	3.192	2.495	1.914	1.898	1.507	1.538	1.363	1.213	1.117	1.144	1.200	1.210	2.983	2.968	3.031	2.997	3.091	9.633	2.305
5	Smallest Extreme Value	0.598	0.798	1.724	3.534	3.565	3.186	3.979	4.237	4.432	4.327	4.238	4.200	4.187	4.106	4.051	4.012	3.483	3.277	3.142	3.384	3.501	3.427
6	Largest Extreme Value	0.667	1.575	1.779	1.509	1.153	0.860	0.857	0.728	0.893	0.856	0.820	0.865	0.978	1.079	1.077	1.069	1.083	1.157	1.430	1.823	2.551	1.181
7	3-Parameter Gamma	0.245	3.488	4.121	3.990	3.252	2.642	2.602	2.195	2.205	2.033	1.891	1.796	1.819	1.874	1.915	8.794	8.777	8.817	8.788	8.862	15.306	4.543
8	Logistic	0.227	0.663	0.883	1.093	1.179	1.049	1.010	1.116	1.265	1.290	1.316	1.396	1.469	1.497	1.462	1.437	1.441	1.449	1.714	2.031	2.469	1.307
9	3-Parameter Loglogistic	0.227	0.665	1.009	1.358	1.211	1.042	1.036	0.935	1.156	1.088	0.989	0.952	1.080	1.336	1.744	3.640	3.644	3.716	3.684	3.780	9.088	2.066

Table 3. Selected best fit PDFs from the results of goodness-of-fit test.

No	Normalised depth (x/d)	Individual Best Fit Distribution		Overall Best Fit Distribution	
		Type	AD statistic	Type	AD statistic
1	0.00	Normal	0.162	Large Extreme Value	0.667
2	0.05	Logistic	0.663	Large Extreme Value	1.575
3	0.10	Logistic	0.883	Large Extreme Value	1.779
4	0.15	Logistic	1.093	Large Extreme Value	1.509
5	0.20	Large Extreme Value	1.153	Large Extreme Value	1.153
6	0.25	Large Extreme Value	0.860	Large Extreme Value	0.860
7	0.30	Large Extreme Value	0.857	Large Extreme Value	0.857
8	0.35	Large Extreme Value	0.728	Large Extreme Value	0.728
9	0.40	Large Extreme Value	0.893	Large Extreme Value	0.893
10	0.45	Large Extreme Value	0.856	Large Extreme Value	0.856
11	0.50	3-Parameter lognormal	0.808	Large Extreme Value	0.820
12	0.55	3-Parameter lognormal	0.743	Large Extreme Value	0.865
13	0.60	3-Parameter lognormal	0.829	Large Extreme Value	0.978
14	0.65	3-Parameter lognormal	0.978	Large Extreme Value	1.079
15	0.70	Large Extreme Value	1.077	Large Extreme Value	1.077
16	0.75	Large Extreme Value	1.069	Large Extreme Value	1.069
17	0.80	Large Extreme Value	1.083	Large Extreme Value	1.083
18	0.85	Large Extreme Value	1.157	Large Extreme Value	1.157
19	0.90	Large Extreme Value	1.430	Large Extreme Value	1.430
20	0.95	Large Extreme Value	1.823	Large Extreme Value	1.823
21	1.00	Logistic	2.469	Large Extreme Value	2.551

Table 4. Comparison of parameters between overall and optimised outcomes.

No	Normalised water depth (x/d)	Overall		Optimised	
		A	B	A	B
1	0.00	1.2520	0.5405	1.2463	0.6154
2	0.05	0.7074	0.6194	0.7360	0.5342
3	0.10	0.5374	0.5028	0.5313	0.4642
4	0.15	0.4409	0.4221	0.4209	0.4047
5	0.20	0.3649	0.3547	0.3519	0.3547
6	0.25	0.3057	0.3039	0.3046	0.3133
7	0.30	0.2669	0.2630	0.2702	0.2797
8	0.35	0.2365	0.2399	0.2441	0.2530
9	0.40	0.2178	0.2255	0.2235	0.2324
10	0.45	0.2062	0.2170	0.2070	0.2169
11	0.50	0.1966	0.2094	0.1933	0.2058
12	0.55	0.1870	0.2021	0.1819	0.1980
13	0.60	0.1793	0.1963	0.1722	0.1928
14	0.65	0.1718	0.1915	0.1638	0.1893
15	0.70	0.1630	0.1867	0.1565	0.1866
16	0.75	0.1535	0.1827	0.1501	0.1838
17	0.80	0.1453	0.1726	0.1445	0.1801
18	0.85	0.1373	0.1640	0.1395	0.1746
19	0.90	0.1308	0.1614	0.1350	0.1664
20	0.95	0.1196	0.1548	0.1309	0.1546
21	1.00	0.1101	0.1564	0.1272	0.1384

Note: A = location parameter, B = scale parameter.

- A method for the mathematical formulation of current profile is proposed.
- Overall, the largest extreme value distribution is the average best fit PDF for the current velocity distribution based on Anderson-Darling statistics.
- Optimisation of parameters of the PDF for each water depth correlates the current velocity at each water depth through a mathematical model of parameters as the function of water depth.
- Different current models such as mean model can be extracted from the optimised best fit probability density distributions to be used as the design of current for offshore structure design analysis based on engineering judgement.
- A comparison of the optimised outcome with individual outcome shows that the optimised outcome can represent the current profile with low variation well (Coefficient of Variation closed to 0.0).

To: **Ships and Offshore Structures (SAOS)** Editor in Chief and reviewers

We are sending herewith a copy of the manuscript, which we would like to submit to **SAOS**.

The paper is entitled:

A method for the mathematical formulation of current profile

by Do Kyun Kim, Eileen Wee Chin Wong, Su Young Yu, and Young Tae Kim

Copy of the Abstract:

In this study, an advanced method was proposed for the mathematical formation of current profiles which can be directly used for the numerical analysis of offshore structures. A probabilistic approach was adopted to generate a reliable mathematical model which can be expressed as a function of current velocity and water depth from the obtained best fit probability density function (PDF) with sub-parameters. It is recognised that the statistical scatter of current velocity at each normalised water depth is wide and requires a reliable method (or technique) with a refined manner to generate a simplified current profile model. From the probabilistic approach, the best-fit PDF of the current velocity distribution, including all ranges of normalised water depth is decided. In addition, sub-parameters of PDF (i.e., shape, scale, location parameters) can also be formulated as a function of normalised water depth through curve-fitting. For better understanding, three (3) main steps which are 1) individual, 2) overall, and 3) optimised outcomes have been highlighted in order to propose the mathematical formulation of current profiles. Applicability of the proposed method was verified by collecting 54 current profiles obtained from existing offshore fields, thus making it possible to generate a more accurate current profile model.

Corresponding Author:

Dr. Do Kyun Kim

Assistant Professor

Civil Engineering Department, Universiti Teknologi PETRONAS, 31750 Tronoh, Perak, Malaysia

Offshore Engineering Division, Civil Engineering Department, Universiti Teknologi PETRONAS (UTP)

E-mail: do.kim@utp.edu.my or dokim@postech.ac.kr

We hereby certify that this paper consists of original, unpublished work which is not under consideration for publication elsewhere.

The original manuscript and figures will be transferred, following the instruction by the Editorial Committee when the paper is accepted. I hope your favorable consideration for publication to **SAOS**.

Sincerely yours,

Do Kyun Kim

On the behalf of co-authors

Attachment: PDF file of the manuscript.

List of Tables

Table 1. Example of probability density functions with sub-parameters.

Table 2. Results of goodness-of-fit test.

Table 3. Selected best fit PDFs from the results of goodness-of-fit test.

Table 4. Comparison of parameters between overall and optimised outcomes.

Table 1. Example of probability density functions with sub-parameters.

Probability density functions		Parameter A	Parameter B	Parameter C
Birnbaum-Saunders	$\frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{\left(\sqrt{x/A} - \sqrt{A/x} \right)^2}{2B^2} \right\} \left(\frac{\sqrt{x/A} - \sqrt{A/x}}{2Bx} \right)$	scale parameter	shape parameter	-
Exponential	$\frac{1}{A} \exp \left(-\frac{x}{A} \right)$	mean	-	-
2-parameter Exponential	$\frac{1}{A} \exp \left(-\frac{x-B}{A} \right)$	scale parameter	threshold parameter	-
Gamma	$\frac{1}{B^A \Gamma(A)} x^{A-1} e^{-\frac{x}{B}}$	shape parameter	scale parameter	-
3-parameter Gamma	$\frac{1}{B^A \Gamma(A)} (x-C)^{A-1} e^{-\frac{(x-C)}{B}}$	shape parameter	scale parameter	threshold parameter
Largest Extreme Value	$B^{-1} \exp \left(\frac{x-A}{B} \right) \exp \left(-\exp \left(\frac{x-A}{B} \right) \right)$	shape parameter	scale parameter	-
Gen. Pareto	$\left(\frac{1}{B} \right) \left(1 + A \frac{(x-C)}{B} \right)^{-1-\frac{1}{A}}$	shape parameter	scale parameter	location parameter
Inv. Gaussian	$\sqrt{\frac{B}{2\pi x^3}} \exp \left\{ -\frac{B}{2A^2 x} (x-A)^2 \right\}$	scale parameter	shape parameter	-
Logistic	$\frac{\exp \left\{ \frac{x-A}{B} \right\}}{B \left(1 + \exp \left\{ \frac{x-A}{B} \right\} \right)^2}$	mean	scale parameter	-
Log-logistic	$\frac{1}{B} \frac{1}{x} \frac{e^{-\frac{\log(x)-A}{B}}}{\left(1 + e^{-\frac{\log(x)-A}{B}} \right)^2}$	log mean	log scale parameter	-
3-parameter Log-logistic	$\frac{1}{B} \frac{1}{(x-C)} \frac{e^{-\frac{\log(x-C)-A}{B}}}{\left(1 + e^{-\frac{\log(x-C)-A}{B}} \right)^2}$	log mean	log scale parameter	threshold parameter
Lognormal	$\frac{1}{xB\sqrt{2\pi}} \exp \left\{ -\frac{(\ln x - A)^2}{2B^2} \right\}$	log mean	log standard deviation	-
3-parameter Lognormal	$\frac{1}{(x-C)B\sqrt{2\pi}} \exp \left\{ -\frac{(\ln(x-C) - A)^2}{2B^2} \right\}$	log mean	log standard deviation	threshold parameter
Nakagami	$2 \left(\frac{A}{B} \right)^A \frac{1}{\Gamma(A)} x^{(2A-1)} e^{-\frac{A}{B} x^2}$	shape parameter	scale parameter	-
Normal	$\frac{1}{B\sqrt{2\pi}} e^{-\frac{(x-A)^2}{2B^2}}$	mean	standard deviation	-

Rayleigh	$\frac{1}{A^2} \exp\left(\frac{-x^2}{2A^2}\right)$	scale parameter	-	-
Rician	$I_0\left(\frac{xA}{B^2}\right) \frac{x}{B^2} e^{-\left(\frac{x^2+A^2}{2B^2}\right)}$	noncentrality parameter	scale parameter	-
t Location-Scale	$\frac{\Gamma\left(\frac{C+1}{2}\right)}{B\sqrt{C\pi}\Gamma\left(\frac{C}{2}\right)} \left[\frac{C + \left(\frac{x-A}{B}\right)^2}{C} \right]^{-\left(\frac{C+1}{2}\right)}$	location parameter	scale parameter	shape parameter
Weibull	$\frac{B}{A} \left(\frac{x}{A}\right)^{B-1} \exp\left[-\left(\frac{x}{A}\right)^B\right]$	scale parameter	shape parameter	-
3-parameter Weibull	$\frac{B}{A} \left(\frac{x-C}{A}\right)^{B-1} \exp\left[-\left(\frac{x-C}{A}\right)^B\right]$	scale parameter	shape parameter	location parameter

Table 2. Results of goodness-of-fit test.

No.	Probability density function	Goodness-of-fit test (Anderson-Darling)																					
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	Ave.
1	Normal	0.162	0.672	0.929	1.469	1.669	1.459	1.654	1.872	2.060	2.062	2.062	2.107	2.142	2.115	2.063	2.031	1.872	1.761	1.907	2.193	2.576	1.754
2	3-Parameter Lognormal	0.173	0.708	1.248	1.593	1.347	1.070	1.092	0.890	1.048	0.931	0.808	0.743	0.829	0.978	1.120	12.675	12.659	12.629	12.521	12.231	10.315	4.172
3	2-Parameter Exponential	5.667	3.986	3.931	3.402	2.638	1.984	1.991	1.471	1.445	1.242	1.077	0.971	1.009	1.092	1.165	2.348	2.367	2.502	2.767	3.537	9.423	2.667
4	3-Parameter Weibull	0.169	1.043	2.702	3.192	2.495	1.914	1.898	1.507	1.538	1.363	1.213	1.117	1.144	1.200	1.210	2.983	2.968	3.031	2.997	3.091	9.633	2.305
5	Smallest Extreme Value	0.598	0.798	1.724	3.534	3.565	3.186	3.979	4.237	4.432	4.327	4.238	4.200	4.187	4.106	4.051	4.012	3.483	3.277	3.142	3.384	3.501	3.427
6	Largest Extreme Value	0.667	1.575	1.779	1.509	1.153	0.860	0.857	0.728	0.893	0.856	0.820	0.865	0.978	1.079	1.077	1.069	1.083	1.157	1.430	1.823	2.551	1.181
7	3-Parameter Gamma	0.245	3.488	4.121	3.990	3.252	2.642	2.602	2.195	2.205	2.033	1.891	1.796	1.819	1.874	1.915	8.794	8.777	8.817	8.788	8.862	15.306	4.543
8	Logistic	0.227	0.663	0.883	1.093	1.179	1.049	1.010	1.116	1.265	1.290	1.316	1.396	1.469	1.497	1.462	1.437	1.441	1.449	1.714	2.031	2.469	1.307
9	3-Parameter Loglogistic	0.227	0.665	1.009	1.358	1.211	1.042	1.036	0.935	1.156	1.088	0.989	0.952	1.080	1.336	1.744	3.640	3.644	3.716	3.684	3.780	9.088	2.066

Table 3. Selected best fit PDFs from the results of goodness-of-fit test.

No	Normalised depth (x/d)	Individual Best Fit Distribution		Overall Best Fit Distribution	
		Type	AD statistic	Type	AD statistic
1	0.00	Normal	0.162	Large Extreme Value	0.667
2	0.05	Logistic	0.663	Large Extreme Value	1.575
3	0.10	Logistic	0.883	Large Extreme Value	1.779
4	0.15	Logistic	1.093	Large Extreme Value	1.509
5	0.20	Large Extreme Value	1.153	Large Extreme Value	1.153
6	0.25	Large Extreme Value	0.860	Large Extreme Value	0.860
7	0.30	Large Extreme Value	0.857	Large Extreme Value	0.857
8	0.35	Large Extreme Value	0.728	Large Extreme Value	0.728
9	0.40	Large Extreme Value	0.893	Large Extreme Value	0.893
10	0.45	Large Extreme Value	0.856	Large Extreme Value	0.856
11	0.50	3-Parameter lognormal	0.808	Large Extreme Value	0.820
12	0.55	3-Parameter lognormal	0.743	Large Extreme Value	0.865
13	0.60	3-Parameter lognormal	0.829	Large Extreme Value	0.978
14	0.65	3-Parameter lognormal	0.978	Large Extreme Value	1.079
15	0.70	Large Extreme Value	1.077	Large Extreme Value	1.077
16	0.75	Large Extreme Value	1.069	Large Extreme Value	1.069
17	0.80	Large Extreme Value	1.083	Large Extreme Value	1.083
18	0.85	Large Extreme Value	1.157	Large Extreme Value	1.157
19	0.90	Large Extreme Value	1.430	Large Extreme Value	1.430
20	0.95	Large Extreme Value	1.823	Large Extreme Value	1.823
21	1.00	Logistic	2.469	Large Extreme Value	2.551

Table 4. Comparison of parameters between overall and optimised outcomes.

No	Normalised water depth (x/d)	Overall		Optimised	
		A	B	A	B
1	0.00	1.2520	0.5405	1.2463	0.6154
2	0.05	0.7074	0.6194	0.7360	0.5342
3	0.10	0.5374	0.5028	0.5313	0.4642
4	0.15	0.4409	0.4221	0.4209	0.4047
5	0.20	0.3649	0.3547	0.3519	0.3547
6	0.25	0.3057	0.3039	0.3046	0.3133
7	0.30	0.2669	0.2630	0.2702	0.2797
8	0.35	0.2365	0.2399	0.2441	0.2530
9	0.40	0.2178	0.2255	0.2235	0.2324
10	0.45	0.2062	0.2170	0.2070	0.2169
11	0.50	0.1966	0.2094	0.1933	0.2058
12	0.55	0.1870	0.2021	0.1819	0.1980
13	0.60	0.1793	0.1963	0.1722	0.1928
14	0.65	0.1718	0.1915	0.1638	0.1893
15	0.70	0.1630	0.1867	0.1565	0.1866
16	0.75	0.1535	0.1827	0.1501	0.1838
17	0.80	0.1453	0.1726	0.1445	0.1801
18	0.85	0.1373	0.1640	0.1395	0.1746
19	0.90	0.1308	0.1614	0.1350	0.1664
20	0.95	0.1196	0.1548	0.1309	0.1546
21	1.00	0.1101	0.1564	0.1272	0.1384

Note: A = location parameter, B = scale parameter.

List of Figures

- Figure 1. Procedure for the generation of a simplified mathematical formulation of current profiles.
- Figure 2. Sample of a normalised current profile.
- Figure 3. Example of Anderson–Darling (A-D) test results.
- Figure 4. Example of mathematical formulation technique to propose the empirical formula of sub-parameters.
- Figure 5. Compilation of current profile.
- Figure 6. Obtained best fit probability density distributions of actual current velocity along normalised water depth.
- Figure 7. Empirical formulation of sub-parameters with normalised water depth.
- Figure 8. Optimised best fit probability density distributions of actual current velocity along normalised water depth.
- Figure 9. Best-fit probability density distributions using three methods for each normalised water depth.
- Figure 10. Obtained current models.
- Figure 11. Comparison of proposed three methods.

Figure 1. Procedure for the generation of a simplified mathematical formulation of current profiles.

Figure 2. Sample of a normalised current profile.

Figure 3. Example of Anderson–Darling (A-D) test results.

Figure 4. Example of mathematical formulation technique to propose the empirical formula of sub-parameters.

(a) Collected current profiles – Normalised water depth version

(b) Normalised current profiles based on 0.05 (x/d)

Figure 5. Compilation of current profile.

(a) Individual best fit PDF

(b) Overall best fit PDF

Figure 6. Obtained best fit probability density distributions of actual current velocity along normalised water depth.

Figure 7. Empirical formulation of sub-parameters with normalised water depth.

Figure 8. Optimised best fit probability density distributions of actual current velocity along normalised water depth.

(a) normalised water depth (x/d) = 0.00

(b) normalised water depth (x/d) = 0.05

(c) normalised water depth (x/d) = 0.10

(d) normalised water depth (x/d) = 0.15

(e) normalised water depth (x/d) = 0.20

(f) normalised water depth (x/d) = 0.25

(g) normalised water depth (x/d) = 0.30

(h) normalised water depth (x/d) = 0.35

(i) normalised water depth (x/d) = 0.40

(j) normalised water depth (x/d) = 0.45

(k) normalised water depth (x/d) = 0.50

(l) normalised water depth (x/d) = 0.55

(m) normalised water depth (x/d) = 0.60

(n) normalised water depth (x/d) = 0.65

(o) normalised water depth (x/d) = 0.70

(p) normalised water depth (x/d) = 0.75

(q) normalised water depth (x/d) = 0.80

(r) normalised water depth (x/d) = 0.85

Figure 9. Best-fit probability density distributions using three methods for each normalised water depth.

(a) Mean and Mean + standard deviation model

(b) Mean and Several model

Figure 10. Obtained current models.

Figure 11. Comparison of proposed three methods.