Efficient Design of Selective Mapping and Partial Transmit Sequence using T-OFDM

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Abstract—Selective mapping (SLM) and partial transmit sequence (PTS) are attractive schemes for mitigating the high peak power inherent in orthogonal frequency division multiplexing (OFDM) signals. However, the high computational complexity and redundant side information (SI) bits have been identified as the main limitations for such techniques. The high computational complexity is mainly due to the need to perform several inverse fast Fourier transforms (IFFTs), and phase optimization process at the transmitter side. Therefore, this paper presents new SLM and PTS designs using a low complexity T-transform rather than IFFT. The use of the T-transform with SLM achieves a considerable computational complexity and peak-to-average power ratio (PAPR) reduction. Furthermore, we apply the T-transform to PTS and derive two different configurations that compromise the SI requirements and PAPR reduction. All the proposed schemes do not affect the original power spectrum of OFDM signals. The complexity analysis show that the proposed schemes have much lower complexity as compared to conventional schemes. Moreover, simulation results demonstrate that the proposed schemes are resilient to dispersion arising from multipath propagation, which is due to the frequency diversity introduced by the T-transform.

Index Terms—Orthogonal frequency division multiplexing (OFDM), peak-to-average poer ratio (PAPR), selective mapping (SLM), partial transmitted sequence (PTS), T-transform.

I. INTRODUCTION

DUE to its spectral efficiency and immunity to intersymbol interference (ISI) caused by multipath signal propagation, orthogonal frequency division multiplexing (OFDM) has been adopted by various high data rate standards for communication systems [1]. The spectral efficiency of OFDM is due to the spectra overlapping of adjacent subcarriers, and the immunity to ISI is gained by using the cyclic prefix (CP) as time-domain guard bands. Because the bandwidth of each subcarrier is much smaller than the coherence bandwidth of the channel, then each subcarrier experiences a flat fading even though the overall channel response is frequency-selective. Consequently, certain subcarriers can be severely attenuated by deep fades, and thus, OFDM signals do not offer any bit error

rate (BER) improvement over single carrier systems. However, as described in [2] and [3], OFDM can be exploited to provide full diversity without spectral losses by using the T-transform, which spreads each data symbol over all subcarriers.

Additionally, it is known that coherent superposition of a large number of subcarriers through the inverse fast Fourier transform (IFFT) may produce samples with very high peak values as compared to the OFDM average symbol power [4]–[5]. Consequently, the system design would be challenging if a high power amplifier (HPA) is used at the transmitter. Therefore, various schemes have been devised to eliminate the deleterious effect of the peak-to-average power ratio (PAPR), but at the expense of additional high complexity, data rate losses and BER degradation. Generally speaking, PAPR reduction schemes may cause distortion to the OFDM signal, or can be distortionless. Examples for techniques with distortion include peak cancellation [6], iterative clipping and filtering [7] and non linear companding [8]. Examples for distortionless techniques are the tone reservation (TR), active constellation extension (ACE) [9], coding technique [10], selective mapping (SLM) [11]–[22], and partial transmit sequence (PTS) [23]–[27].

Among distortionless techniques, SLM and PTS have received substantial attention because they can reduce the PAPR without BER degradation. However, such techniques have major limitations including high computational complexity and transmission of redundant side information (SI) bits. Complexity and PAPR reduction of SLM and PTS schemes are reported in [28] where it is shown that PTS outperforms SLM in terms of complexity, but SLM outperforms PTS in terms of PAPR reduction. Furthermore, many papers in the literature aim at alleviating the drawbacks associated with SLM and PTS while maintaining their PAPR reduction capabilities. For example, [29]–[37] attempted to solve the SI challenge inherent in the two schemes by utilizing a variety of techniques at the expense of substantial additional complexity.

PAPR reduction using linear precoding has also been considered in the literature. For example, [38] proposed a method for PAPR reduction in multicarrier systems by combining selective mapping (SLM) and dummy sequence iteration (DSI) with the Walsh Hadamard transform (WHT). Although this approach managed to reduce the PAPR efficiently, the PAPR reduction is achieved at the expense of high complexity caused by the repetitive computations of the IFFTs and WHTs. It is worth noting that employing the WHT with SLM or PTS actually doubles their complexity. The authors of [39] suggest a new PAPR reduction technique by including a WHT in the

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OFDM system. Although this technique improves the PAPR, its complexity is relatively high due to the cascaded WHT and IFFT.

Therefore, the complexity reduction of OFDM-based SLM and PTS has been the focus of many works [11]–[27]. Nevertheless, most of the work that considered the complexity reduction results in similar or inferior PAPR and BER performance, and the requirements for SI remain unchanged. Moreover, the complexity is reduced by either reducing the number of IFFTs, or by reducing the complexity of phase optimization process. Therefore, reducing the complexity and SI of the SLM and PTS systems is still of paramount importance.

Consequently, this paper presents an efficient approach to design low complexity SLM and PTS schemes. The proposed approach adopts the T-transform instead of the IFFT, which leads to PAPR and complexity reduction, as well as BER improvement. The T-transform was developed originally in [40] and used in OFDM to improve the BER over multipath channels, and jointly reduce the PAPR and complexity of the IFFT [2], [3]. Although the T-transform may achieve a noticeable PAPR reduction, the obtained reduction is very far from the PAPR of single carrier systems. Therefore, in the proposed T-SLM scheme, T-transform structure is combined with SLM and PTS to improve the PAPR reduction that can be achieved with the T-transform alone. Moreover, such combination is exploited to achieve substantial complexity reduction and BER improvement. It is worth noting that the PAPR reduction using SLM can be achieved with the same SI. On the other hand, the T-transform can also be utilized efficiently with PTS in two manners. The first scheme, T-PTS-I, achieves a considerable PAPR and complexity reduction with the same SI. We also show that the T-transform has a unique structure that can be exploited to implement the PTS efficiently. Thus, the second proposed scheme, T-PTS-II, may achieve a considerable complexity reduction when compared with the conventional PTS and reduces the SI by two bits. Therefore, instead of sending three bits as SI with four partitions and binary weight phase rotation, only one bit is required. Interestingly, these advantages are achieved for only a negligible PAPR reduction degradation. All the proposed systems have no deleterious effect on the power spectrum of the original OFDM signal. Furthermore, SLM-OFDM and PTS-OFDM systems can be exploited efficiently to provide full diversity by using the T-transform at the transmitter side.

Although the T-transform has several advantages, the transformation process spreads the deleterious effects of the deep fading in frequency-selective channels over all subcarriers, which may degrade the system performance severely. To avoid this problem, as illustrated in the forthcoming sections, the minimum mean square error (MMSE) equalizer will be used at the receiver instead of the zero-forcing (ZF) criterion. The MMSE equalizer has the ability to reduce the noise enhancement while equalizing the deep fading effects. However, it requires accurate estimate of the noise variance at the receiver. Moreover, similar to other WHT based OFDM systems, the proposed system has more stringent channel estimation and synchronization requirements [41], [42].

The rest of this paper is organized as follows. Section II

presents the analysis and complexity calculation of the inverse T-transform. The T-OFDM system description is introduced in Section III. Sections IV and V describe the proposed T-SLM, T-PTS-I and T-PTS-II schemes with complexity calculations. Section VI presents the simulation results, along with their discussions and Section VII concludes the paper.

II. INVERSE T-TRANSFORM STRUCTURE

As reported in [40], the size N inverse T-transform, $\hat{\mathbf{T}}^H$, can be expressed as,

$$\hat{\mathbf{T}}_N^H = \frac{1}{N} \mathbf{W} \hat{\mathbf{F}}^H \quad (1)$$

where $\mathbf{W}$ and $\hat{\mathbf{F}}^H$ are the normalized $N \times N$ WHT matrix and the IFFT matrix rearranged by column reversed order, respectively. As reported in [2], $\hat{\mathbf{T}}_N^H$ can be simplified as,

$$\hat{\mathbf{T}}_N^H = \frac{1}{N} \begin{bmatrix} 2\mathbf{W}_{\frac{N}{2}} \hat{\mathbf{A}}_{\frac{N}{2}} & 0 \\ 0 & 2\mathbf{W}_{\frac{N}{2}} \hat{\mathbf{B}}_{\frac{N}{2}} \end{bmatrix} \quad (2)$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the submatrices of $\hat{\mathbf{F}}^H$. Therefore, (2) can be factorized as,

$$\hat{\mathbf{T}}_N^H = \frac{1}{N} \begin{bmatrix} \frac{N}{2} \mathbf{W}_2 \hat{\mathbf{A}}_2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \frac{N}{2} \mathbf{W}_2 \hat{\mathbf{B}}_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \frac{N}{4} \mathbf{W}_4 \hat{\mathbf{B}}_4 & \cdots & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \cdots & 4\mathbf{W}_{\frac{N}{4}} \hat{\mathbf{B}}_{\frac{N}{4}} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 2\mathbf{W}_{\frac{N}{2}} \hat{\mathbf{B}}_{\frac{N}{2}} \end{bmatrix}. \quad (3)$$

For $N = 32$, (3) can be expressed as,

$$\hat{\mathbf{T}}_{32} = \frac{1}{32} \begin{bmatrix} 2\mathbf{W}_{16} \hat{\mathbf{A}}_{16} & 0 \\ 0 & 2\mathbf{W}_{16} \hat{\mathbf{B}}_{16} \end{bmatrix}. \quad (4)$$

In (3), $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ can be further simplified, which gives,

$$\hat{\mathbf{T}}_{32} = \frac{1}{32} \begin{bmatrix} 4\mathbf{W}_8 \hat{\mathbf{\Upsilon}}_8 & 0 & 0 & 0 \\ 0 & 4\mathbf{W}_8 \hat{\mathbf{\Theta}}_8 & 0 & 0 \\ 0 & 0 & \omega_8 \mathbf{W}_8 \hat{\mathbf{\beta}}_8 & \bar{\omega}_8 \mathbf{W}_8 \hat{\mathbf{\xi}}_8 \\ 0 & 0 & \bar{\omega}_8 \mathbf{W}_8 \hat{\mathbf{\beta}}_8 & \omega_8 \mathbf{W}_8 \hat{\mathbf{\xi}}_8 \end{bmatrix} \quad (5)$$

where $\omega_i \triangleq 2(1 + w^i)$, $\bar{\omega}_i \triangleq 2(1 - w^i)$, $\hat{\mathbf{\Upsilon}}$ and $\hat{\mathbf{\Theta}}$ are the submatrices of $\hat{\mathbf{A}}$, while $\hat{\mathbf{\beta}}$ and $\hat{\mathbf{\xi}}$ are the submatrices of $\hat{\mathbf{B}}$. Thus, $\hat{\mathbf{T}}_{32}$ can be factorized further as shown in (6), where the submatrices of the rearranged IFFT matrix $\hat{\mathbf{F}}$ are denoted by $\hat{\alpha}$, $\hat{\delta}$, $\hat{\zeta}$, $\hat{\eta}$, $\hat{\lambda}$, $\hat{\nu}$, $\hat{\varphi}$ and $\hat{\rho}$.

Ultimately, after calculating the elements of the submatrices in (6), the flowchart of the T-transform for $N = 32$ can be represented as shown in Fig. 1, where X_{th} denotes the frequency-domain samples, x_{th} denotes the time-domain samples, $a^n = (1 - w^n)/2$ and $w = e^{j2\pi/N}$. Consequently, an N -point T-transform requires only $\frac{1}{2} [N \log_2 N - (2N - 2)]$ butterflies.

The structure of each butterfly is shown in Fig. 2, where i_1 , i_2 , o_1 , o_2 are first input, second input, first output, and the second output, respectively, for each butterfly.

Based on the number of butterflies, the total number of complex operations in the T-transform of size N is com-

$$\hat{\mathbf{T}}_{32} = \frac{1}{32} \begin{bmatrix} 4\mathbf{W}_4\hat{\alpha}_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4\mathbf{W}_4\hat{\delta}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\omega_8\mathbf{W}_4\hat{\zeta}_4 & 2\bar{\omega}_8\mathbf{W}_4\hat{\eta}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\bar{\omega}_8\mathbf{W}_4\hat{\zeta}_4 & 2\omega_8\mathbf{W}_4\hat{\eta}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8\omega_4\mathbf{W}_4\hat{\lambda}_4 & \omega_8\bar{\omega}_4\mathbf{W}_4\hat{\nu}_4 & \bar{\omega}_8\omega_{12}\mathbf{W}_4\hat{\phi}_4 & \bar{\omega}_8\bar{\omega}_{12}\mathbf{W}_4\hat{\rho}_4 \\ 0 & 0 & 0 & 0 & \omega_8\bar{\omega}_4\mathbf{W}_4\hat{\lambda}_4 & \omega\omega_4\mathbf{W}_4\hat{\nu}_4 & \bar{\omega}_8\bar{\omega}_{12}\mathbf{W}_4\hat{\phi}_4 & \bar{\omega}_8\omega_{12}\mathbf{W}_4\hat{\rho}_4 \\ 0 & 0 & 0 & 0 & \bar{\omega}_8\omega_4\mathbf{W}_4\hat{\lambda}_4 & \bar{\omega}_8\bar{\omega}_4\mathbf{W}_4\hat{\nu}_4 & \omega_8\omega_{12}\mathbf{W}_4\hat{\phi}_4 & \omega_8\bar{\omega}_{12}\mathbf{W}_4\hat{\rho}_4 \\ 0 & 0 & 0 & 0 & \bar{\omega}_8\bar{\omega}_4\mathbf{W}_4\hat{\lambda}_4 & \bar{\omega}_8\omega_4\mathbf{W}_4\hat{\nu}_4 & \omega_8\bar{\omega}_{12}\mathbf{W}_4\hat{\phi}_4 & \omega_8\omega_{12}\mathbf{W}_4\hat{\rho}_4 \end{bmatrix}. \quad (6)$$

Fig. 1. T-transform flowchart, $N = 32$.

Fig. 2. The butterfly structure of the T-transform.

puted in [40], where each butterfly involves one complex multiplication and three complex additions. In general, each complex multiplication requires four real multiplications and two real additions, whereas a complex addition requires two real additions. Consequently, the calculation of the T-transform involves,

$$R_M = 2[N \log_2 N - (2N - 2)] \quad (7)$$

$$R_A = 4[N \log_2 N - (2N - 2)] \quad (8)$$

where R_M and R_A stand for real multiplications and additions, respectively. To make the complexity reduction more informative, it is more convenient to combine all operations into one equivalent metric [27], [43], [44]. Towards this end, we use the fact that a real multiplication is linearly proportion to the real addition, i.e., $R_M = f R_A$. Therefore, the total computational complexity in terms of real additions can be computed as,

$$R_{\text{Total}} = R_A + f R_M \quad (9)$$

where f represents the number of real additions equivalent to a real multiplication. Based on the results reported in [27], [43], [44], we consider $f = 4$ in all complexity analysis. Therefore, the total computational complexity of the T-transform can be evaluated as

$$R_{\text{Total}} = 12[N \log_2 N - (2N - 2)]. \quad (10)$$

Fig. 1 demonstrates the beneficial features of the N -point T-transform, where $\log_2(N) - 1$ stages are computed horizontally, compared with $\log_2(N)$ stages in the case of IFFT. Additionally, $\log_2(N) - 1$ independent sections with two direct paths are computed vertically. The lower summation process through the T-transform leads to the PAPR reduction of the transmitted signal, as will be shown in the proposed T-SLM and T-PTS-I schemes in the following sections. Furthermore, the unique butterfly structure of the $\log_2(N) - 1$ independent sections in the T-transform will be exploited in the proposed T-PTS-II scheme to significantly reduce the complexity of the PTS.

III. DESCRIPTION OF T-OFDM SYSTEM

To generate the time-domain samples of the T-OFDM, the modulating data vector $\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T$ is applied to an N -point inverse T-transform,

$$\mathbf{x} = \hat{\mathbf{T}}^H \mathbf{X} \quad (11)$$

where $\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T$, $\hat{\mathbf{T}}$ is the $N \times N$ T-transform unitary matrix, and $(\cdot)^H$ denotes the Hermitian operator. Thus, (11) can be rewritten as,

$$\begin{aligned} \mathbf{x} &= \hat{\mathbf{F}}^H \mathbf{W} \mathbf{X} \\ &= \hat{\mathbf{F}}^H \hat{\mathbf{X}} \end{aligned} \quad (12)$$

where $\hat{\mathbf{X}} = \mathbf{W} \mathbf{X}$ is the frequency-domain samples of Walsh-domain samples $\mathbf{X}$. A time-domain guard band is created by appending the last N_g samples of $\mathbf{x}$ as a preamble for the samples $x_0, \dots, x_{N-1}$. It should be taken into account that in order to prevent ISI, the length of the CP should be greater than the maximum delay spread of the channel.

At the receiver side, the received time-domain samples after removing the CP samples can be expressed as,

$$\mathbf{r} = \mathbf{h} \otimes \mathbf{x} + \mathbf{z} \quad (13)$$

where $\mathbf{z} = [z_0, z_1, \dots, z_{N-1}]^T$ denotes the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 = N_0/2$, $\mathbf{h}$ is a vector that consists of the discrete-time channel impulse response samples, and $\otimes$ denotes linear convolution operation. Therefore, $\mathbf{r}$ can be written as

$$\mathbf{r} = \mathbb{H} \mathbf{x} + \mathbf{z} \quad (14)$$

where $\mathbb{H}$ is a circulant $N \times N$ matrix that represents the channel matrix. To simplify the analysis, the T-transform is subdivided into its original transforms. Thus, the received signal in the frequency-domain is written as,

$$\mathbf{s} = \mathbf{F} \mathbf{r} = \mathbf{D} \hat{\mathbf{X}} + \mathbf{Z} \quad (15)$$

where $\mathbf{Z}$ is the FFT of the noise vector $\mathbf{z}$. Because $\mathbb{H}$ is circulant, it will be diagonalized by the IFFT and FFT operations, i.e., $\mathbf{H} = \mathbf{F} \mathbb{H} \mathbf{F}^H$, $\mathbf{H} = \text{diag} \{[H_0, H_1, \dots, H_{N-1}]\}$, $H_k = \sum_{l=0}^{L_D} h(l) e^{-j2\pi kl/N}$, where L_D represents the maximum normalized delay spread of the channel.

In order to eliminate the effects of the channel fading, the received signal should be equalized either in Walsh-domain or in frequency-domain. As it can be noted from (15), this can be achieved if the matrix $\mathbf{H}$, or an estimated version of it ($\hat{\mathbf{H}}$) is available. However, in frequency-domain, multiplying the FFT output by $\hat{\mathbf{H}}^{-1}$ also affects the noise; consequently we use the minimum mean square error (MMSE) equalizer to minimize the noise enhancement. Therefore,

$$\mathbf{X}'_k = \lambda_k s_k = \lambda_k H_{k,k} \hat{X}_k + \lambda_k Z_k \quad (16)$$

where λ_k is the MMSE coefficient at the k th subcarrier,

$$\lambda_k = \frac{\hat{H}_{k,k}^*}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} \quad (17)$$

where Γ_k is the signal-to-noise ratio (SNR) of k th subcarrier. As a result, the received equalized signal $\mathbf{X}''$ in Walsh-domain can be computed as,

$$\begin{aligned} \mathbf{X}'' &= \mathbf{W} \mathbf{X}' \\ &= \underbrace{\mathbf{W} \lambda \mathbf{H} \hat{\mathbf{X}}}_{\rho} + \mathbf{W} \lambda \mathbf{Z}. \end{aligned} \quad (18)$$

Fig. 3. Block diagram of OFDM-based T-SLM scheme.

IV. T-SLM-OFDM SYSTEM

As reported in the literature, the conventional SLM scheme is constructed by creating U branches of the frequency-domain symbols. Each branch is multiplied by an N distinct points phase rotation vector $\mathbf{s}^{(u)} = [s_0^{(u)}, s_1^{(u)}, \dots, s_{N-1}^{(u)}]$, $u \in \{0, 1, \dots, U-1\}$. The multiplication process can be expressed as

$$\mathbf{v}^{(u)} = \mathbf{s}^{(u)} \circ \mathbf{d} \quad (19)$$

where $\circ$ denotes the element-by-element multiplication process. The result of multiplication $\mathbf{v}^{(u)}$ at each branch is up-sampled by a factor $L = 4$, and applied to LN -point IFFT to generate the discrete-time samples and then the PAPR is computed. The branch with the lowest PAPR is selected for transmission. The side information to be transmitted given that the transmitter and receiver share a common knowledge of the vectors $\mathbf{s}^{(u)}$.

The SLM described above can be directly applied to the WHT-OFDM system by preceeding the IFFT with the WHT in order to reduce the PAPR at the expense of increasing the complexity owing to the WHTs, i.e.,

$$\mathbf{y}_{\text{WHT}}^{(u)} = \mathbf{W} \mathbf{v}^{(u)}. \quad (20)$$

The remaining operations are identical to those of the conventional SLM. Therefore, the T-transform can be applied to the SLM system by replacing the IFFT-WHT processes with the inverse T-transform, as shown in Fig. 3. Therefore,

$$\mathbf{y}_{\text{T-transform}}^{(u)} = \mathbf{T}^H \mathbf{v}^{(u)}. \quad (21)$$

In OFDM, oversampling can be achieved by padding the frequency-domain signal with $\mathbf{0}^{1 \times (L-1)N}$. Similarly, the same approach can be used with WHT-OFDM by padding the WHT output signal with zeros, however, an alternative method with similar effect can be used as well. This method can be achieved by duplicating the signal $\mathbf{v}^{(u)}$ in (19) L times as

$$\mathbf{r}^{(u)} = [\mathbf{v}_0^{(u)}, \mathbf{v}_1^{(u)}, \dots, \mathbf{v}_L^{(u)}] \quad (22)$$

where $\mathbf{v}_0^{(u)}, \mathbf{v}_1^{(u)}, \dots, \mathbf{v}_L^{(u)}$ have the same values, and L is the up-sampling factor. Thus, with oversampling, the samples of $\mathbf{r}^{(u)}$ are used instead of $\mathbf{v}^{(u)}$ as the input signals for each individual inverse T-transform shown in Fig. 3.

In SLM, the computational complexity consists of the following main three parts:

- LN -point IFFT operations.

b) phase factor multiplications in (19).

c) PAPR computation and comparison among candidate signals.

In OFDM, WHT-OFDM and T-OFDM systems, the computational complexity of the phase factor multiplications and PAPR computation are identical. Therefore, the computational complexity in a) compared to the other related transforms is mainly considered. Accordingly, the computational complexity of NL -point IFFT based on the full-butterfly IFFT design, and pruning IFFTs design, i.e. omitting multiplications or additions with zero [45], will be calculated in the forthcoming subsections.

A. Computational complexity based on full-butterfly IFFT design

Full-butterfly IFFT design requires $(UNL/2)\log_2(NL)$ and $(UNL)\log_2(NL)$ of complex multiplications and additions, respectively [16]. Thus, based on full-butterfly IFFT design, the total computational complexity of the considered systems in terms of real additions while substituting f in (9) by four [27], can be expressed as

$$R_{\text{Conv. SLM}} = 11UNL\log_2(NL) \quad (23)$$

$$R_{\text{WHT-IFFT-SLM}} = R_{\text{Conv. SLM}} + 2UN\log_2(N) \quad (24)$$

$$R_{\text{T-SLM}} = 12U[NL\log_2(NL) - (2NL - 2)]. \quad (25)$$

B. Computational complexity based on pruning IFFTs design

Based on the pruning IFFT design, the computational complexity of a) in the conventional SLM depends on the sparseness of data, i.e. non-zero data. Thus, the pruning IFFTs of SLM requires $\frac{1}{2}UNL\log_2 N + UN(L - 1)$ and $UNL\log_2 N$ complex multiplications and additions, respectively. In a sequel, based on (9), the total complexity of such scheme in terms of real additions (computed only for pruning-IFFTs) can be evaluated as

$$R_{\text{pruning-IFFT-SLM}} = 11UNL\log_2(N) + 18UN(L - 1). \quad (26)$$

Similarly, in the case of WHT-pruning-IFFT-SLM system with oversampling, the total computational complexity in terms of real additions can be expressed as

$$R_{\text{WHT-pruning-IFFT-SLM}} = R_{\text{pruning-IFFT-SLM}} + 2UN\log_2(N). \quad (27)$$

Interestingly, using the up-sampling described in (22) in the proposed T-SLM scheme gives the same data samples after the first stage (from left) of the T-transform. For example, assume that $N = 32$, therefore, the third duplicated data frame will be $X_{16}, X_{17}, \dots, X_{23}$ and the fourth duplicated data frame will be $X_{24}, X_{25}, \dots, X_{31}$. Based on (22), X_{16} should be equal to X_{24} , X_{17} should be equal to X_{25} , and so on for other samples. Thus, based on Fig. 2,

$$X'_{16} = X_{16} - (X_{16} - X_{24})W^8 \quad (28)$$

where X_{16} and X_{24} have the same values. Consequently,

$$X'_{16} = X_{16} \quad (29)$$

and

$$X'_{24} = X_{24} + (X_{16} - X_{24})W^8 \quad (30)$$

$$= X_{24}. \quad (31)$$

Therefore, $\frac{N}{4}$ butterflies, which are inside the rectangle shown in Fig. 1, will be pruned from the total computational complexity of oversampled T-SLM scheme. Thus, the total number of butterflies in the Pruning-T-SLM will be $\frac{1}{2}[NL\log_2 NL - (2NL - 2)] - \frac{NL}{4}$. In the sequel, the total computational complexity of T-SLM in terms of real additions can be expressed as

$$R_{\text{Pruning-T-SLM}} = 12U[NL\log_2(NL) - (2NL - 2)] - 6UNL. \quad (32)$$

The overall computational complexity reduction ratio (CCRR) can be computed as,

$$\eta = \left(1 - \frac{C_1}{C_2}\right) \times 100\% \quad (33)$$

where, in general, C_1 and C_2 denote the total real additions of the proposed systems and conventional systems, respectively.

V. CONVENTIONAL PTS, PROPOSED T-PTS-I AND PROPOSED T-PTS-II SCHEMES

A. Conventional PTS scheme

As reported in [23]-[27], the PTS scheme is based on partitioning the input data block $\mathbf{d}$, which consists of N symbols, into U disjoint subsets. Then each subset of $\mathbf{d}$ is padded with zeros at the left and right sides to obtain

$$\mathbf{d}'^{(u)} = [\mathbf{0}^{1 \times uN/U}, \{\mathbf{d}\}_{uN/U}^{(u+1)N/U-1}, \mathbf{0}^{1 \times [N-(u+1)N/U]}]. \quad (34)$$

Each u th block of $\mathbf{d}'$ is fed to an individual NL -points IFFT to generate the time-domain samples $\mathbf{x}^{(u)}$ as

$$\mathbf{x}^{(u)} = \mathbf{F}^H \mathbf{d}'^{(u)}. \quad (35)$$

The n th sample of $\mathbf{x}^{(u)}$ can be written as,

$$x_n^{(u)} = \frac{1}{\sqrt{NL}} \sum_{k=0}^{NL-1} d_k'^{(u)} e^{j\frac{2\pi kn}{NL}}, n = 0, 1, \dots, NL - 1. \quad (36)$$

Subsequently, each partially transmitted sequence $\mathbf{x}^{(u)}$ is multiplied by individual weighting phase factors, $b^{(u)} \in \{\pm 1, \pm j\}$, $u \in \{0, 1, \dots, U - 1\}$. Thus, by using P weighting phase factors of $b^{(u)}$, there are $B = P^{U-1}$ alternative representations for an OFDM symbol. Finally, the transmitted OFDM symbol is composed from adding U sequences as

$$\hat{d}_n = \sum_{u=0}^{U-1} b^{(u)} x_n^{(u)}. \quad (37)$$

At the receiver side, the received signal can be recovered as illustrated in [23]-[27]. The PTS described above can be directly applied to WHT-OFDM systems by preceeding the IFFT with the WHT. As will be shown in the next two subsections, the data partition can be achieved in two ways, before and after the WHT.

Similar to the SLM, the computational complexity of PTS results from the operation complexity of the three parts a), b), and c) mentioned formerly in Section IV. As reported in [43], the operations complexity for b) and c) in terms of real additions can be expressed as

$$D = NL(U(20B - 18) + 7B). \quad (38)$$

The complexity of D is identical in both full-butterfly and pruning IFFT design. Furthermore, the burden of IFFTs computational complexity in the PTS scheme will be computed based on the IFFT architecture design.

1) *PTS scheme complexity based on full-butterfly IFFT design*: Similar to Subsection IV-A, the burden of IFFTs in the conventional PTS and WHT-IFFT-PTS schemes require $11UNL \log_2 NL$ and $11UNL \log_2 NL + 2N \log_2 N$ real additions, respectively. Therefore, the total real operations in the conventional PTS and WHT-IFFT-PTS schemes can be computed as

$$R_{\text{Conv. PTS}} = 11UNL \log_2(NL) + D \quad (39)$$

$$R_{\text{WHT-IFFT-PTS}} = R_{\text{Conv. PTS}} + 2N \log_2(N) \quad (40)$$

where D is defined in (38).

2) *PTS scheme complexity based on pruning IFFT design*: The sparseness of data samples, i.e. non-zero-data in the oversampled PTS is lower than the oversampled SLM because the sparse data depends on the oversampling factor L and number of disjoint partitions U , whereas it depends on L in the case of SLM scheme. Consequently, the required complex multiplications M and additions A in the pruning IFFTs of the conventional PTS and WHT-IFFT-PTS can be computed as

$$M_{\text{pruning-PTS}} = \frac{1}{2}UNL \log_2 \left(\frac{N}{U} \right) + N(UL - 1) \quad (41)$$

$$A_{\text{pruning-PTS}} = UNL \log_2 \left(\frac{N}{U} \right) \quad (42)$$

$$M_{\text{pruning-WHT-IFFT-PTS}} = M_{\text{Prun. IFFT}}. \quad (43)$$

$$A_{\text{pruning-WHT-IFFT-PTS}} = A_{\text{Prun. IFFT}} + N \log_2(N), \quad (44)$$

Consequently, the total real additions in the conventional PTS and WHT-IFFT-PTS based on pruning-IFFT design can be expressed as

$$R_{\text{pruning-PTS}} = 11UNL \log_2 \left(\frac{N}{U} \right) + 18NUL - 18N + D \quad (45)$$

$$R_{\text{pruning-WHT-IFFT-PTS}} = R_{\text{pruning-PTS}} + 2N \log_2(N) \quad (46)$$

where D was previously defined in (38).

B. Proposed T-PTS-I scheme

In the first scheme, the WHT of the data symbols $\mathbf{d}$ is computed as

$$\mathbf{v} = \mathbf{W}\mathbf{d}. \quad (47)$$

Then, the U disjoint partitions of (47), $\mathbf{v}^{(u)}$, $u = 0, \dots, U - 1$, and the rest of the processes are evaluated in a similar way to those in the traditional PTS scheme, as illustrated in (34)-

Fig. 4. Block diagram of T-PTS-I scheme with $U = 4$.

(37). Alternatively, the same disjoint partitions $\mathbf{v}^{(u)}$ can be computed by copying U -times the data samples $\mathbf{d}$ as, $\mathbf{z}^{(u)} = \mathbf{d}$, $u = 0, 1, \dots, U - 1$, and each copy is multiplied with an individual normalized symmetrical new matrix $\mathbf{G}$. The U -versions of normalized $N \times N$ $\mathbf{G}$ are computed as

$$\mathbf{G}^{(u)} = \mathbf{P}^{(u)}\mathbf{W}, \quad (48)$$

where $\mathbf{P}^{(u)}$ are new matrices that have $\frac{N}{U}$ data samples, which are equivalent to u th quarter of WHT matrix, and $\frac{(U-1)}{U}N$ of zeros. Consequently, the elements of these matrices can be computed as

$$\mathbf{P}^{(u)} = [\mathbf{0}(:, 1:uN/U), \{\mathbf{W}\}(:, uN/U:(u+1)N/U-1), \mathbf{0}(:, 1:[N-(u+1)N/U])]. \quad (49)$$

Consequently, each individual normalized $\mathbf{G}^{(u)}$ matrix has U elements comprising of uniformly distributed ones and $N - U$ zeros in each column and row. Thus,

$$\mathbf{q}^{(u)} = \mathbf{G}^{(u)}\mathbf{z}^{(u)}. \quad (50)$$

In this sequel, the same disjoint sub-blocks partitions of (47) will be obtained by

$$\mathbf{v}^{(u)} = \mathbf{W}\mathbf{q}^{(u)}. \quad (51)$$

Then, the remaining steps are similar to the conventional PTS, which are illustrated in (34)-(37).

The up-sampling of (50) can be performed as in (22) by generating U copies of $\mathbf{q}$ divided by $\sqrt{L}$. This method can be achieved by duplicating the signals in (50) L times as

$$\mathbf{r}^{(u)} = [\mathbf{q}_0^{(u)}, \mathbf{q}_1^{(u)}, \dots, \mathbf{q}_L^{(u)}]. \quad (52)$$

Thus,

$$\hat{\mathbf{r}}^{(u)} = \mathbf{r}^{(u)} / \sqrt{L}. \quad (53)$$

The computational complexity burden of employing the WHT with PTS-OFDM is taken into account. Therefore, the WHT-IFFT can be simply replaced by the T-transform, as shown in Fig. 4. The U disjoint sub-blocks of a new scheme are obtained by passing each individual sequence of (53) to the inverse T-transform as

$$\mathbf{x}^{(u)} = \mathbf{T}^H \hat{\mathbf{r}}^{(u)}. \quad (54)$$

Then, the phase optimization and PAPR calculation steps are similar to that of conventional PTS scheme.

The computational complexity requirements of the proposed T-PTS-I scheme are evaluated based on the T-transform complexity, the complexity of phase optimization, PAPR calcula-

Fig. 5. Block diagram of T-PTS-II scheme with $U = 4$.

tions and the complexity of the $\mathbf{G}^{(u)}$ matrices. The complexity of phase optimization and PAPR calculations are identical in the T-PTS-I, conventional PTS and WHT-IFFT-PTS schemes. Therefore, the computational complexity of T-transform and $\mathbf{G}^{(u)}$ matrices compared to IFFTs and WHT-IFFTs are mainly considered. Utilizing U -matrices of $\mathbf{G}$ requires $2U(U-1)N$ real additions. In addition, the complexity of the T-transform is computed as in the T-SLM. Thus, the total real additions in the T-PTS-I (accounting T-transform and $\mathbf{G}^{(u)}$ matrices complexity) based on the full T-transform can be expressed as,

$$R_{\text{Full-T-PTS-I}} = 12U[NL \log_2(NL) - (2NL - 2)] + 2U(U-1)N. \quad (55)$$

On the other hand, the total real additions in the T-PTS-I based on pruning-T-transform, i.e. by omitting the butterflies of the same data symbols with the optimal oversampling factor $U = 4$ can be computed as,

$$R_{\text{Pruning-T-PTS-I}} = R_{\text{Full-T-PTS-I}} - 6UNL. \quad (56)$$

It is clear that, the proposed T-PTS scheme requires much lower computational complexity than conventional PTS and WHT-IFFT-PTS schemes when full-transforms design criterion is considered, whereas it requires slightly higher than the considered systems when pruning-transform design criterion is considered.

C. Proposed T-PTS-II scheme

In the second proposed scheme, T-PTS-II, the data symbols $\mathbf{d}$ are initially partitioned to U disjoint sub-blocks, and then fed to the T-transform. The unique flow chart structure of the T-transform, shown in Fig. 1, can be exploited to reduce the complexity of the PTS significantly, as shown in Fig. 5. The N -point T-transform has $\log_2(N) - 1$ uncorrelated butterfly sections and two direct paths. Consequently, up-sampling by zero padding the frequency-domain symbols is not suitable for T-PTS-II scheme. Alternatively, conventional up-sampling in the time-domain can be applied. All systems will be affected almost in the same way by omitting the up-sampling process, hence up-sampling is not considered in this scheme.

With the assumptions that $N = 16$ and $U = 4$, the data symbols $\mathbf{d}$ are partitioned into four disjoint subblocks, $\mathbf{d}^{(u)}$, $u = 0, \dots, 3$, therefore, each partition has N/U (four in our example) non-zero elements and $(U-1)N/U$ (twelve) zeros.

The first partition $\mathbf{d}^{(0)}$ contains the first four data symbols, which will be applied to the inverse T-transform,

$$\mathbf{v}^{(0)} = \mathbf{T}^H \mathbf{d}^{(0)} = [x_0, 0, 0, 0, x_8, 0, 0, 0, x_4, 0, 0, 0, x_{12}, 0, 0, 0]. \quad (57)$$

Therefore, $\mathbf{v}^{(0)}$ can be simply obtained by applying the non-zero elements of $\mathbf{d}^{(0)}$ to section S_0 of the butterfly; taking into consideration the right order of the samples. Arranging the samples in the proper order can be achieved by changing the position of the most significant bit from right to left. Similarly, $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$ can be obtained by applying $\mathbf{d}^{(1)}$, $\mathbf{d}^{(2)}$ and $\mathbf{d}^{(3)}$ to sections S_1 , S_2 and S_2 , respectively. Thus

$$\mathbf{v}^{(1)} = [0, 0, x_2, 0, 0, 0, x_6, 0, 0, 0, x_{10}, 0, 0, 0, x_{14}, 0] \quad (58)$$

$$\mathbf{v}^{(2)} = [0, a_1, 0, a_3, 0, a_5, 0, a_7, 0, a_9, 0, a_{11}, 0, a_{13}, 0, a_{15}] \quad (59)$$

$$\mathbf{v}^{(3)} = [0, b_1, 0, b_3, 0, b_5, 0, b_7, 0, b_9, 0, b_{11}, 0, b_{13}, 0, b_{15}]. \quad (60)$$

Due to the alignment of the nonzero elements in all the partitions, it is apparent that $\mathbf{v}^{(0)}$ and $\mathbf{v}^{(1)}$ branches cannot be used to reduce the PAPR because they do not add up with any other non-zero element in other branches. Moreover, to avoid superposition of samples having the same sign, which may lead to large peak power, multiplying either $\mathbf{v}^{(2)}$ or $\mathbf{v}^{(3)}$ by phase optimization vector, but not both, is sufficient to minimize the PAPR. Finally, all vectors are combined together to compose the transmitted data sequence,

$$\hat{\mathbf{d}} = \mathbf{v}^{(0)} + \mathbf{v}^{(1)} + \mathbf{v}^{(2)} + b\mathbf{v}^{(3)}. \quad (61)$$

Obviously, the effective disjoint sub-blocks number in the proposed T-PTS-II scheme is $U' = U - 3$ and the phase optimization factor, $B' = D^{U'-1}$. Consequently, the required SI redundant bits will be one when compared with three bits in the case of conventional PTS scheme with four sub-blocks, and binary weight phase rotation.

The computational complexity requirements of the proposed T-PTS-II scheme are evaluated based on the T-transform complexity, and the complexity of the partial sequence phase optimization process. For the case of $U = 4$, the implementation of sections, S_0 and S_1 of Fig. 1, require $\lceil \frac{N}{4} \log_2(\frac{N}{2}) - (\frac{N}{2} - 1) \rceil$ butterflies. Therefore, $\lceil \frac{N}{4} \log_2(\frac{N}{2}) - (\frac{N}{2} - 1) \rceil$ complex multiplications and $\lceil \frac{3}{4} N \log_2(\frac{N}{2}) - (\frac{N}{2} - 1) \rceil$ complex additions will be performed. Similarly, the computational complexity of section S_2 is $\lceil \frac{N}{4} \log_2(N) - \frac{N}{4} \rceil$ and $\lceil \frac{3}{4} N \log_2(N) - \frac{N}{4} \rceil$ complex multiplications and additions, respectively. Moreover, the same complexity is required to feed the fourth sub-block $\mathbf{d}^{(3)}$ into S_2 .

On the other hand, using $U = 4$, the phase optimization and PAPR calculations are required only for S_2 output signal with length of $\frac{N}{2}$. Thus, the phase optimization process requires $U'(B' - 1)\frac{N}{2}$ and $(U' - 1)(B' - 1)\frac{N}{2}$ complex multiplications and additions, respectively. Furthermore, PAPR calculation in such a scheme requires $B'N$ and $\frac{B'N}{2}$ real multiplications and additions, respectively. Thus, the total number of real additions of T-PTS-II scheme with four disjoint subblocks is

$$\Upsilon = 18N \log_2(N) - (30N - 28) + N\beta \quad (62)$$

where $\beta = \left\lceil \frac{9}{2}B' + (B' - 1) \left\lceil 9U' + (U' - 1) \right\rceil \right\rceil$.

The general formulas to compute the required butterflies number in T-PTS-II scheme are

$$\Omega = \frac{1}{2} \left[\sum_{i=0}^{\log_2 N-1} Q_i + \sum_{i=0}^{\log_2 U-1} \left(\frac{U}{2^i} - 1 \right) Q_i \right] \quad (63)$$

where $Q_i = \left\lceil \frac{N}{2^i} \log_2 \left(\frac{N}{2^i-1} \right) - \frac{N}{2^i} \right\rceil$. Thus, the required real additions in the T-PTS-II (counting in T-transform only) can be expressed as

$$R_T = 12 \left[\sum_{i=0}^{\log_2 N-1} Q_i + \sum_{i=0}^{\log_2 U-1} \left(\frac{U}{2^i} - 1 \right) Q_i \right]. \quad (64)$$

On the other hand, the computational complexity of phase rotation process can be expressed in general form as

$$C_M = \Gamma(B' - 1) \quad (65)$$

$$C_A = \Psi(B' - 1) \quad (66)$$

where C_M and C_A stand for the complex multiplications and additions, respectively, $\Gamma = \sum_{i=0}^{\log_2 U-1} \frac{UN}{2^i}$, and $\Psi = \sum_{i=0}^{\log_2 U-1} \left(\frac{UN}{2^i} - \frac{N}{2^i} \right)$. In addition, the PAPR calculation complexity can be computed as

$$R_M = 2\Lambda B' \quad (67)$$

$$R_A = \Lambda B' \quad (68)$$

where $\Lambda = \sum_{i=0}^{\log_2 U-1} \frac{N}{2^i}$. Thus, the total real additions of these two processes are

$$R_{\text{Opt and PAPR}} = 2(B' - 1)(8\Gamma + \Psi) + 9\Lambda B'. \quad (69)$$

Consequently, the total real addition of the T-PTS-II scheme can be expressed as

$$R_{\text{T-PTS-II}} = R_T + R_{\text{Opt and PAPR}}. \quad (70)$$

Eventually, the proposed T-PTS-II scheme achieves a significant complexity reduction compared with the conventional PTS and WHT-IFFT-PTS.

VI. PERFORMANCE ANALYSIS OF THE PROPOSED T-SLM-OFDM, T-PTS-OFDM-I AND T-PTS-OFDM-II SYSTEMS

Similar to the conventional SLM and PTS, the proposed T-SLM, T-PTS-I and T-PTS-II do not have any negative effect on the BER performance of T-OFDM system. Basically, performance analysis is achieved with the assumption that the input symbols are uncorrelated with the same variance E_s in real and imaginary parts. Also, the noise is assumed to be independent white Gaussian noise with variance N_0 .

The probability of error performance of M-PSK and M-QAM systems over a white complex Gaussian noise channels can be expressed as,

$$P_{e\text{M-PSK-OFDM}} = \frac{\varepsilon}{m} Q \left(\sqrt{2\Gamma} \sin(\pi/M) \right) \quad (71)$$

$$P_{e\text{M-QAM-OFDM}} = \frac{(4 - 2^{(2-m/2)})}{m} Q \left(\sqrt{3\Gamma/(M-1)} \right) \quad (72)$$

where ε stands for the average number of nearest neighbors signal points, $m = \log_2 M$ is the number of bits in each constellation sample, Γ is the signal-to-noise ratio (SNR) E_s/N_0 , E_s is the power per symbol, N_0 stands for the Gaussian noise average power, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$. In a sequel, the performance analysis of any system over any transmission media essentially depends on the calculation of the new values of SNR that appear in (71) and (72) which take the effect of transmission media into account.

Thus, the general form of the average SNR for the T-SLM, T-PTS-I or T-PTS-II received signal, which is shown in (18), can be expressed as

$$\Gamma = \frac{E[|\rho|^2]}{E[|\mathbf{X}'|^2] - E[|\rho|^2]} \quad (73)$$

where $E[\cdot]$ denotes the expectation process. Thus, parts of (73) can be recalculated using the MMSE criterion. Due to orthogonality of $\mathbf{W}$ and $\mathbf{F}$,

$$E[|\rho|^2] = \frac{E_s}{N} \left(\sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} \right)^2. \quad (74)$$

Also, the noise variance can be computed as in (75). Upon substituting (74) and (75) into (73), we obtain

$$\Gamma = \frac{\frac{1}{N} \sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}}}{1 - \frac{1}{N} \sum_{k=0}^{N-1} \frac{|\hat{D}_{k,k}|^2}{|\hat{D}_{k,k}|^2 + \frac{1}{\Gamma_k}}}. \quad (76)$$

By noting that

$$N - \sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} = \sum_{k=0}^{N-1} \frac{1}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} \quad (77)$$

then

$$\Gamma = \frac{\sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}}}{\sum_{k=0}^{N-1} \frac{1}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}}}. \quad (78)$$

Eventually, the BER performance of T-SLM, T-PTS-I and T-PTS-II systems over a multipath fading channel, using quadrature phase shift keying (QPSK) and 16 levels quadrature amplitude modulation (QAM), can be evaluated by substituting (78) into (71) and (72), respectively.

VII. SIMULATION RESULTS AND DISCUSSION

Without loss of generality, the results presented in this section are achieved based on assumptions of perfect knowledge of the channel response, perfect frequency and time synchronization and optimal SI transmission. Also, all the considered systems are uncoded.

A. Computational complexity

As shown in Table I, the proposed T-SLM requires lower computational complexity as compared to the conventional SLM and WHT-IFFT-SLM with full and pruning butterfly.

$$E|\mathbf{X}''|^2 - E|\boldsymbol{\rho}|^2 = E_s \left(\sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} \left(1 - \frac{1}{N} \sum_{k=0}^{N-1} \frac{|\hat{H}_{k,k}|^2}{|\hat{H}_{k,k}|^2 + \frac{1}{\Gamma_k}} \right) \right). \quad (75)$$

TABLE I

CCRR FOR THE T-SLM, CONVENTIONAL SLM AND WHT-IFFT-SLM SCHEMES BASED ON FULL-BUTTERFLY TRANSFORMS DESIGN CRITERION.

	CCRR			
	$L = 1$		$L = 4$	
N	η_1 (%)	η_2 (%)	η_1 (%)	η_2 (%)
64	26.76	37.98	18.08	20.78
128	21.84	33.86	15.10	18.00
256	18.08	30.68	12.71	15.77
1024	12.70	26.14	9.09	12.41

TABLE II

CCRR FOR THE T-SLM, CONVENTIONAL SLM AND WHT-IFFT-SLM SCHEMES BASED ON PRUNING-BUTTERFLY TRANSFORM DESIGN CRITERION.

	CCRR	
	η_1 (%)	η_2 (%)
N		
64	16.86	19.89
128	13.76	16.97
256	11.31	14.67
1024	9.69	11.28

Given that $N = 64$, $L = 1$ and $U = 4$, the T-SLM achieves a CCRR of about 26.76% and 38% over the conventional SLM and WHT-IFFT-SLM, when full-butterfly is used. Furthermore, using $L = 4$, the CCRR of the proposed T-SLM compared with the conventional SLM and WHT-IFFT-SLM is about 18.07% and 20.77%, respectively. The CCRR η_1 of the proposed T-SLM over conventional SLM is obtained by computing C_1 and C_2 in (33) using (25) and (23), respectively. Similarly, CCRR η_2 of the proposed T-SLM over WHT-IFFT-SLM is computed following the same approach used to compute η_1 , except that C_2 is computed using (24).

On the other hand, in the case of pruning-butterfly with oversampling factor $L = 4$, the obtained CCRRs of T-SLM over the pruning-IFFT-SLM and WHT-pruning-IFFT-SLM are 16.86% and 19.88% for $N = 64$, and 7.69% and 11.28% for $N = 1024$, respectively, as shown in Table II. The value of η_1 is obtained by computing C_1 and C_2 using (32) and (26), respectively. For η_2 , the same approach of η_1 is used, except that C_2 is computed using (27).

The values of phase optimization and PAPR calculation D are identical for the proposed T-PTS-I, conventional PTS and WHT-IFFT-PTS. Therefore, the computational complexity of D will be omitted in the complexity comparison among the considered schemes. The CCRR η_1 of the proposed T-PTS-I over the conventional PTS is obtained by computing C_1 and C_2 in (33) using (55) and (39), respectively. Similarly, the CCRR η_2 of the proposed T-PTS-I scheme over WHT-IFFT-PTS scheme is computed in the same way of computing

TABLE III

CCRR FOR THE T-PTS-I, CONVENTIONAL PTS AND WHT-IFFT-PTS SCHEMES BASED ON FULL-BUTTERFLY TRANSFORM DESIGN CRITERION.

	CCRR			
	$L = 1$		$L = 4$	
N	η_1 (%)	η_2 (%)	η_1 (%)	η_2 (%)
64	17.62	21.20	16.37	17.08
128	14.04	17.78	13.59	14.35
256	11.26	15.12	11.34	12.14
1024	7.25	11.28	7.95	8.82

TABLE IV

CCRR FOR THE T-PTS-II, CONVENTIONAL PTS AND WHT-IFFT-PTS SCHEMES.

	CCRR	
	η_1 (%)	η_2 (%)
N		
64	93.91	93.96
128	92.98	93.04
256	92.10	92.17
1024	90.45	90.56

η_1 except that C_2 is computed using (43). As shown in Table III, the T-PTS-I with full-butterfly transform design criterion provides CCRR that is about 17.62% and 21.20% with $N = 64$ and $L = 1$, and about 15.8% and 18.58% with $L = 4$ over the conventional PTS and WHT-IFFT-PTS, respectively. However, with pruning transforms design criterion, T-PTS-I requires slightly higher complexity about 8% compared to the conventional PTS.

Interestingly, as shown in Table IV, the proposed T-PTS-II achieves a significant CCRR that more than 90% over the considered systems. The CCRR η_1 of the proposed T-PTS-II over the conventional PTS is obtained by computing C_1 and C_2 in (33) using (70) and (39), respectively. Similarly, the CCRR η_2 of the proposed T-PTS-II scheme over WHT-IFFT-PTS is computed in the same way of computing η_1 except that C_2 is computed using (40). In addition, the data rate of T-PTS-II is higher than the conventional PTS scheme because the proposed scheme requires three bits less transmitted as SI than the conventional PTS.

B. PAPR reduction

A good estimation of the power density of the considered systems can be acquired from the histogram plots of the peak power of such systems, as shown in Figs. 6, 7, and 8. As can be noted from Fig. 6, the variance change of the T-OFDM peaks is smaller than conventional OFDM; therefore, this figure shows that the T-OFDM outperforms conventional

Fig. 6. Histogram for peak power of conventional OFDM and T-OFDM systems with $N=128$. (X-axis represents the peaks range in dB and Y-axis represents the number of peaks in each range in X-axis).

Fig. 7. Histogram for peak power of conventional SLM, and T-SLM schemes with $N=128$. (X-axis represents the peaks range in dB and Y-axis represents the number of peaks in each range in X-axis).

OFDM because it has fewer signals with high peaks, i.e. lower PAPR. Moreover, the low high peaks, i.e., lower PAPR of T-SLM, T-PTS-I systems compared to the conventional SLM and PTS, respectively, with the same value of SI, are clearly shown in Figs. 7, 8, respectively. The proposed T-SLM reduces the PAPR as a consequence of reducing the superposition of the subcarriers passed through the T-transform. As shown in Fig. 1, the number of stages in the T-transform is $\log_2 N - 1$ and the maximum number of subcarriers that will be processed together through T-transform is $\frac{N}{2}$; whereas in IFFT, it requires $\log_2 N$ stages and N subcarriers will be processed through the IFFT. Consequently, the T-SLM can achieve a low peak signal compared to conventional SLM when both are used with the OFDM. Moreover, the peak reduction is achieved with the preservation of the transmission average power because the T-transform is a unitary transform.

Fig. 8. Histogram for peak power of conventional PTS, T-PTS-I and T-PTS-II schemes with $N=128$. (X-axis represents the peaks range in dB and Y-axis represents the number of peaks in each range in X-axis).

Fig. 9. CCDF for conventional OFDM, SLM-OFDM, and T-SLM-OFDM systems with $N = 128$, $L = 4$ and $U = 4$ and 8.

Fig. 10. CCDF for conventional OFDM, PTS-OFDM and T-PTS-I-OFDM systems with $N = 128$, $L = 4$ and $U = 4, 8$.

Fig. 11. CCDF for conventional OFDM, PTS-OFDM, T-PTS-II-OFDM systems with $N = 128$, $L = 1$ and $U = 4, 8$.

Fig. 9 depicts the capability of the proposed T-SLM to reduce the PAPR. As can be noted from the figure, the T-SLM is about 0.8 dB less than conventional SLM and about 3.1 dB less than OFDM when $U = 4$ (Note: Dash line represents the CCDF of T-OFDM). For the case of $U = 8$, T-SLM is about 0.8 dB and 3.9 dB less than SLM and OFDM, respectively. These results are achieved for $N = 128$. Using the same parameters used with the T-SLM, Fig. 10 presents the PAPR of the proposed T-PTS-I, PTS and OFDM. As evident from Fig. 10, the proposed T-PTS-I achieves a considerable PAPR reduction with both values of U when using the same values of SI.

Conversely, Fig. 11 depicts the PAPR of the proposed T-PTS-II, PTS and OFDM using $N = 128$, $U = 4, 8$, and $L = 1$. Interestingly, the T-PTS-II achieves significant computational complexity reduction and reduces the data overhead by reducing the SI by three bits, however, with slight degradation in the PAPR performance, as depicted in Fig. 11.

Many PAPR reduction techniques cause in-band and out-of-band distortions for the spectrum of OFDM signal, and consequently causes slower spectrum roll-off, higher side-lobes, and adjacent channel interference. As shown in Fig. 12, the proposed T-SLM, T-PTS-I and T-PTS-II have no detrimental effect on the original power spectrum of OFDM signals.

C. BER performance

BER is a typical performance measure for quantifying the benefits of using the proposed T-SLM, T-PTS-I and T-PTS-II. To evaluate the BER performance of such systems over multipath fading channels, the following system parameters are used. The number of subcarriers $N = 1024$, the CP length is 256 samples, the sample time is 88 ns, and the system bandwidth is 10 MHz. The channel model follows the 6-tap ITU pedestrian channel B model [46] where the channel taps are considered fixed within the OFDM symbol period, but may change randomly for consecutive OFDM symbols, i.e., quasi-static channel.

Fig. 12. PSD for baseband signals of conventional OFDM, T-SLM-OFDM, T-PTS-I-OFDM and T-PTS-II-OFDM systems with $N = 128$, QPSK and $U = 4$.

Fig. 13. BER performance of (a) SLM-OFDM, T-SLM-OFDM systems and (b) PTS-OFDM, T-PTS-I-OFDM, T-PTS-II-OFDM systems, with QPSK and SSPA (IBO=7).

Fig. 14. BER performance of (a) SLM-OFDM, T-SLM-OFDM systems and (b) PTS-OFDM, T-PTS-I-OFDM, T-PTS-II-OFDM systems, with QPSK and SSPA (IBO=5).

Fig. 15. BER performance of (a) SLM-OFDM, T-SLM-OFDM systems and (b) PTS-OFDM, T-PTS-I-OFDM, T-PTS-II-OFDM systems, with 16-QAM and SSPA (IBO=7).

Fig. 16. BER performance of (a) SLM-OFDM, T-SLM-OFDM systems and (b) PTS-OFDM, T-PTS-I-OFDM, T-PTS-II-OFDM systems, with 16-QAM and SSPA (IBO=5).

The BER of the considered systems is presented in Figs. 13, 14, 15, 16, in which the Solid State Power Amplifier (SSPA) [47], which is a well-known class of HPAs, with a typical value of random positive integer parameter $P = 3$, and input-back off (IBO) with values 5 and 7 is considered. As can be noted from the figures, the T-SLM, T-PTS-I and T-PTS-II exhibit significant BER improvement with QPSK modulation over the multipath propagation, even in the presence of HPA, when compared with conventional SLM and PTS. Whereas, a noticeable improvement is achieved with 16-QAM. The performance improvement is a consequence of exploiting the WHT to achieve frequency diversity by spreading each data symbol over all subcarriers. Such configuration helps to mitigate the deep fade effect arising from the multipath channel on individual subcarriers, even in the presence of clipping distortion owing to using an SSPA. Consequently, the superiority of the OFDM-based T-SLM, T-PTS-I and T-PTS

is obtained. Moreover, the BER performance improvement of T-OFDM system increases with the reduction of the HPA clipping distortion by reducing the PAPR, which implies that a significant BER performance can be achieved by using T-OFDM even with high clipping factor, i.e. low IBO when an efficient PAPR reduction scheme is used. It should be noted that all BER results are achieved by utilizing the MMSE equalizer, which has the ability to reduce the noise enhancement and equalize the fading effect of the channel.

VIII. CONCLUSION

In this work, we proposed three efficient techniques to reduce the PAPR in OFDM systems, namely, T-SLM, T-PTS-I and T-PTS-II. The proposed schemes offer significant computational complexity reduction and PAPR improvement over conventional SLM and PTS schemes. Furthermore, the proposed T-PTS-II scheme reduced the number of side information bits required to be transmitted to the receiver. However, at the expense of small PAPR reduction degradation. Moreover, the use of the T-transform has the effect of spreading each subcarrier over other subcarriers which introduces frequency diversity. Thus, the proposed schemes achieved significant BER improvement over multipath fading channels, even in the presence of HPA. Simulation results demonstrated that the proposed systems require lower signal-to-noise ratio (SNR) compared to the traditional systems. Consequently, a multicarrier system utilizing the proposed schemes will benefit from the low computational complexity, low PAPR, and reduced SNR requirements over multipath transmission.

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