Design of an Individual Pitch Controller for Offshore Wind Turbines Based on Neuro-adaptive Control

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Abstract: For large offshore wind turbines (WTs), pitch control is usually used for regulating generated power to the rated value and for mitigating the dynamic loads that at the wind speeds above the rated speeds. However, tracking the pitch angle accurately and quickly can hardly be realized due to complex operating environments, uncertain system parameters, various disturbances, and coupled effect between wind, wave, and turbine structure. In this paper, an individual pitch control system based on a neural adaptive strategy is proposed to address the problems related to uncertain system parameters and various disturbances. The proposed control method can achieve zero error tracking for the pitch angle in a predefined finite time. The design and stability analysis for the proposed method is elaborated. A simulation model is established in Matlab/Simulink, and by comparing it with the traditional PID control method, the merit of the proposed control scheme is verified.

1. Introduction

Wind Energy has been developed rapidly in the last decades and is already playing a role as a mainstream source of electricity[1-2]. In recent years, the research on large offshore floating WT's has become one of the important issues in the wind industry as floating WT's can harvest more reliable wind energy in deep water areas[3]. However, there exist grand challenges in the research and development of floating WT's.

For a large-scale offshore floating wind turbine(WT), it is typically required that the generated power should be limited to its rated value at above the rated wind speed to guarantee its safe operation[4-6]. Besides, the dynamic loads should be mitigated to prolong the service life of the WT. Such goals can be achieved by adjusting the pitch angle. However, the pitch control strategy for the floating WT may be quite complicated as the pitch system is a complex and fast-changing non-linear dynamic system, which has uncertain and changing system parameters and various disturbances [7]. So far, numerous control strategies have been deemed promising solutions for developing a pitch control system. Most of these strategies are traditional control algorithms, such as proportion integration differentiation control [8], disturbance accommodating control [9], and full-state feedback control [10]. These traditional control algorithms are usually designed for one fixed operating point, and the controller parameters cannot be self-regulated; Besides, these control methods cannot deliver excellent performance in a non-linear time-varying WT system [11]. Advanced control strategies have also been deemed promising solutions [12]- [22].

Currently, individual pitch control (IPC) is usually adopted for offshore floating WT's, as it can help mitigate the dynamic loads caused by wind shear, tower shadow, wave effects, etc. So far, lots of advanced IPC strategies have been

proposed. Navalkar et al. combines repetitive control (RC) with system identification for the IPC of WT's, which enables the IPC system to start from zero knowledge of the system dynamics and operating conditions, and be able to converge to a steady-state with high reductions in blade loads [23]. However, the algorithm is too complicated to be implemented in a real application. Zafer et al. propose a fuzzy logic controller based IPC method. Due to the use of FLC, the pitch control strategy does not rely on a detailed mathematical model of the WT's pitch system. However, the effectiveness of this strategy highly depends on the use of fuzzy rules and membership functions, which may be very hard to obtain [24]. In [25], a pitch regulation system is proposed, based on an advanced proportional and integral (PI) controller, whose proportional and integral coefficients are adjusted by a radial basis function (RBF) neural network (NN). Although effective, the proposed RBF based PI method is not able to deal with the problems related to unknown and time-varying system parameters. Mahmood et al. proposed a model-predictive-control (MPC) based IPC system for WT's in [26]. It combines LIDAR measurement with MPC to effectively reduce unbalanced loads on the turbine blades. The problem of this method is that the LIDAR may work improperly due to errors in the measurement of wind speed. Besides, IPC methods based on the generic algorithm are also widely seen in literature [27]. However, it should be noted that time-varying uncertain system parameters and various disturbances are not adequately considered in the design of these IPC methods. Moreover, the tracking of the pitch angle in the predefined finite-time cannot be achieved. In all, these methods can hardly guarantee the performance of pitch angle regulation.

The main contribution of this paper is to propose an advanced IPC method to deal with the pitch angle tracking problem of offshore floating WT's in a finite and predefined time, on the condition that both time-varying uncertain

system parameters and various disturbances exist. The rest of this paper is organized as follows: In Section 2, a non-linear dynamic model of the pitch system with uncertain parameters and unknown load disturbances is established. In Section 3, an individual pitch neural adaptive tracking controller with finite time tracking ability is proposed. Then, the proof of the system stability is elaborated. In Section 4, a simulation model is developed in Matlab/Simulink. Through comparison with the PID control method, the simulation results verify the effectiveness of the control strategy. Finally, conclusions are made in Section 5.

2. Modeling of the pitch system

The mechanical schematic diagram of a single blade pitch mechanism of a WT is shown in Figure 1 [28]. In the figure, β_i is the pitch angle for the i th blade; T_{dri} is the driving torque of the i th blade pitch adjustment; T_{twi} is the sum of the torsional torque of the i th blade and the disturbance torque generated by the aerodynamic force, including the thrust moment of the blade, the lifting moment generated by the lifting force of the blade, the torsional reset moment of the blade, the tilting moment caused by the rotor tilting of the blade, the bending moment caused by the bending of the blade and the unbalanced aerodynamic load.

Fig. 1. Pitch mechanism schematic diagram for the i th blade

Derived from the differential form of the momentum moment theorem, for the blade i , the dynamic equation of the blade can be obtained as,

$$\frac{d(J_i \frac{d\beta_i}{dt})}{dt} = T_{dri} - \frac{d((k_{Di} + k_{Fi})\beta_i)}{dt} - T_{twi} \quad (1)$$

Equation (1) can be further rewritten as:

$$J_i \ddot{\beta}_i + \left(\frac{dJ_i}{dt} + k_{Di} + k_{Fi} \right) \dot{\beta}_i + \left(\frac{dk_{Di}}{dt} + \frac{dk_{Fi}}{dt} \right) \beta_i + T_{twi} = T_{dri} \quad (2)$$

Where J_i is the moment of inertia of the i th blade along its axis; k_{Di} is the damping coefficient of the i th blade; k_{Fi} is the friction coefficient of the i th blade. These parameters may change over time during the operation of the blade system.

The relationship between the motor current and the blade driving torque can be written as $T_{dri} = f(I_{dri})$. For simplicity, a linear relationship is used, $f(I_{dri}) = C_{Ti} I_{dri}$, where C_{Ti} is an electromechanical transformation constant between the motor driving current of the i th blade and the blade driving torque.

To facilitate the design of the controller, (2) is further rewritten as

$$J(\beta) \ddot{\beta} + M(\beta, \dot{\beta}) \dot{\beta} + Q(\beta, \dot{\beta}) \beta + T_{tw}(\beta, \dot{\beta}) = C_T u \quad (3)$$

Where $\beta = [\beta_1, \beta_2, \beta_3]^T \in \mathbf{R}^3$, $u = [u_1, u_2, u_3]^T =$

$[I_{dri}, I_{dri2}, I_{dri3}]^T \in \mathbf{R}^3$, $T_{tw}(\beta, \dot{\beta}) = [T_{tw1}, T_{tw2}, T_{tw3}]^T \in \mathbf{R}^3$, $J(\beta) = \text{diag}[J_1, J_2, J_3] \in \mathbf{R}^{3 \times 3}$, $M(\beta, \dot{\beta}) = \text{diag}[M_1, M_2, M_3] \in \mathbf{R}^{3 \times 3}$, $Q(\beta, \dot{\beta}) = \text{diag}[Q_1, Q_2, Q_3] \in \mathbf{R}^{3 \times 3}$, and $M_i = \frac{dJ_i}{dt} + k_{Di} + k_{Fi}$, $i = 1, 2, 3$; $Q_i = \frac{dk_{Di}}{dt} + \frac{dk_{Fi}}{dt}$, $i = 1, 2, 3$; $C_T = \text{diag}[C_{T1}, C_{T2}, C_{T3}] \in \mathbf{R}^{3 \times 3}$ is a positive constant diagonal matrix. Define $x_1 = \beta$, $x_2 = \dot{\beta}$, (3) can be reorganized in the following form:

$$\begin{cases} \dot{x}_1 = F_1(\bar{x}_1) + G_1(\bar{x}_1)x_2 + D_1(\bar{x}_1, t) \\ \dot{x}_2 = F_2(\bar{x}_2) + G_2(\bar{x}_2)B(\xi)u + D_2(\bar{x}_2, t) \\ y = x_1. \end{cases} \quad (4)$$

Where $x_i \in \mathbf{R}^3$, $\bar{x}_i = [x_1^T, \dots, x_i^T]^T \in \mathbf{R}^{i \times 3}$, $i = 1, 2$ are the states vectors, it is noted that those states are available for the controller design. $F_i(\bar{x}_i)$ represents the uncertainties. $G_2(\bar{x}_2) = J^{-1}(x_1)C_T$, $G_1(x_1) = I_{3 \times 3}$ represents the unknown state-independent control gain matrix. Moreover, $D_1(x_1, t)$ and $D_2(\bar{x}_2, t)$ represents the unknown external disturbances. u represents the actual control input. $B(\xi) = \text{diag}\{B_i(\xi_i)\}$ is a bounded dialog matrix related to the transformed error ξ , which will be discussed later. Besides, there exist some positive constants $\underline{B}_i$ and $\bar{B}_i$ such that $0 < \underline{B}_i \leq B_i(\xi_i) \leq \bar{B}_i$.

In order for the system to realize a feasible performance, the following assumptions are established.

Assumption 1: The desired pitch angle signal. is a smooth function of t , and $\dot{y}_d, \ddot{y}_d$ are known and bounded.

Assumption 2: For any given positive constants $E_1 > 0$, it holds that $V(0) \leq E_1$, where $V(0)$ is the initial of the Lyapunov function, which is detailed in Section 3.

Assumption 3: For the unknown and uncertain disturbance D_i , there exists an unknown constant $\bar{d}_i$ and a known continuous scalar function $F_{di}(\bar{x}_i)$ such that $\|D_i\| \leq \bar{d}_i F_{di}(\bar{x}_i)$ for all $t \geq 0$.

Assumption 4: The leading minors $\Delta_1, \Delta_2, \dots, \Delta_m$ for the 3×3 control gain matrix $G(\bar{x}_2)$ are nonzero. Besides, the sign of $G(\bar{x}_2)$ is known and available for the controller design.

Assumption 5: The desired signal y_d satisfy that $\|y_d\| \leq Y_0 \leq k_{c1}$, where k_{c1} and Y_0 are some known positive constants[31].

Remark 1: Assumptions 1 and 2 are widely used in dynamic surface control methods[32]. From Assumption 1, there exists a positive constant E_0 such that $\Pi_0 = \{\|y_d, \dot{y}_d, \ddot{y}_d\|: \|y_d\|^2 + \|\dot{y}_d\|^2 + \|\ddot{y}_d\|^2 \leq E_0\}$. Assumption 3 denotes that the external disturbance is bounded by an unknown constant $\bar{d}_i$ and a known continuous scalar function $F_{di}(\bar{x}_i)$, where $F_{di}(\bar{x}_i)$ is bounded only when the system states $\bar{x}_i(x_1, x_2, \dots, x_i)$ are bounded. Assumption 4 is common in the controller design for a multi-input multi-output (MIMO) system. Assumption 3 on the trajectory is needed to ensure that the output constraints are not violated.

Lemma 1 [29]: for the 3×3 control gain matrix G with nonzero leading minors $\Delta_1, \Delta_2, \Delta_3$ can be factored as $G = SCU$, where S is symmetric and positive definite, U is unity upper triangular, and $C = QR$ is diagonal, where $R = \text{diag}\{\text{sign}(\Delta_1), \text{sign}(\Delta_2 \Delta_1^{-1}), \text{sign}(\Delta_3 \Delta_2^{-1})\}$ and Q being an arbitrary positive diagonal matrix.

3. Controller Design

For the individual pitch controller of WT, the goal is to ensure the pitch angles $\beta = [\beta_1, \beta_2, \beta_3]^T$ of the three blades independently track the desired pitch angles $\beta^* = [\beta_1^*, \beta_2^*, \beta_3^*]^T$. To proceed, a tracking error model introduced [30],

$$\begin{cases} \mathbf{z}_1 = \mathbf{x}_1 - \mathbf{y}_d \\ \mathbf{z}_2 = \mathbf{x}_2 - \alpha_{2f} \\ \mathbf{y}_2 = \alpha_{2f} - \alpha_1 \end{cases} \quad (5)$$

Where α_{2f} is the output of the first-order filter with α_1 as the input, and $\mathbf{y}_d = \beta^*$.

Note that from (5), we have:

$$\mathbf{x}_2 = \mathbf{z}_2 + \mathbf{y}_2 + \alpha_1 \quad (6)$$

To make sure that the error vector $\mathbf{z}_1$ converges within a finite time, we define the following rate function[20]:

$$\beta(t) = \begin{cases} \frac{T^3 \kappa(t)}{(1-b_f)(T-t)^3 + b_f T^3 \kappa(t)}, 0 \leq t < T \\ \frac{1}{b_f}, t \geq T \end{cases} \quad (7)$$

where $0 < b_f \ll 1$ is a design parameter; $T > 0$ is a finite settling time chosen by the designer; $\kappa(t) \in C^\infty$ is a strictly increasing function, which is relevant to the convergence rate of the tracking error.

Remark 2: Note that $\beta(t) \in [1, 1/b_f]$ for all $t \geq 0$. β^{-1} , $\dot{\beta}$, and $\ddot{\beta}$ are bounded for $t \geq 0$. Moreover, there exists a positive constant E_β such that $\Pi_\beta = \{(\beta, \dot{\beta}, \ddot{\beta}) : \beta^2 + \dot{\beta}^2 + \ddot{\beta}^2 \leq E_\beta\}$.

Integrate (5) with (7) yields:

$$\begin{cases} \zeta_1 = \beta \mathbf{z}_1 \\ \zeta_2 = \beta \mathbf{z}_2 \end{cases} \quad (8)$$

where ζ_1 and ζ_2 are the transformed error used in the controller design to obtain the desired tracking performance.

From (4), the unknown non-linear vector function $(\mathbf{G}_1 - \mathbf{I})\mathbf{x}_2$ is approximated by an RBFNN as follows:

$$\mathbf{F}_1 + (\mathbf{G}_1 - \mathbf{I})\mathbf{x}_2 = \omega_1^T \boldsymbol{\varphi}_1(\mathbf{Z}_1) + \boldsymbol{\delta}_1(\mathbf{Z}_1) \quad (9)$$

Where $\mathbf{I} \in \mathbf{R}^{3 \times 3}$ is an identity matrix, $\mathbf{Z}_1 = \bar{\mathbf{x}}_2 \in \mathbf{R}^6$ is an obtainable stimulating signal vector, $\omega_1 \in \mathbf{R}^{89 \times 6}$ is the ideal weight matrix; $\boldsymbol{\varphi}_1(\mathbf{Z}_1)$ is the bounded basis function vector; and $\boldsymbol{\delta}_1(\mathbf{Z}_1)$ denotes the intrinsic approximation error satisfying $\|\boldsymbol{\delta}_1(\mathbf{Z}_1)\| \leq \bar{\delta}_1 < \infty$, with $\bar{\delta}_1$ being a small unknown constant.

To further simplify the tedious online computation process, the following virtual parameter is introduced,

$$\theta = \max\{\|\omega_1\|^2\} \quad (10)$$

Now, we proceed to design the virtual and actual control laws.

Step 1: The virtual control law will be derived in this step.

Note that $\mathbf{z}_1 = \mathbf{x}_1 - \mathbf{y}_d$, then we have

$$\begin{aligned} \dot{\mathbf{z}}_1 &= \mathbf{F}_1 + \mathbf{G}_1 \mathbf{x}_2 + \mathbf{D}_1 - \dot{\mathbf{y}}_d \\ &= \mathbf{F}_1 + (\mathbf{G}_1 - \mathbf{I})\mathbf{x}_2 + \mathbf{x}_2 + \mathbf{D}_1 - \dot{\mathbf{y}}_d \end{aligned} \quad (11)$$

From (9), we have

$$\dot{\mathbf{z}}_1 = \omega_1^T \boldsymbol{\varphi}_1 + \boldsymbol{\delta}_1 + \mathbf{z}_2 + \mathbf{y}_2 + \alpha_1 + \mathbf{D}_1 - \dot{\mathbf{y}}_d \quad (12)$$

For $i = 1$, from (8), we can further get the following transformed dynamics:

$$\begin{aligned} \dot{\zeta}_1 &= \dot{\beta} \mathbf{z}_1 + \beta \dot{\mathbf{z}}_1 \\ &= \beta(\beta^{-1} \dot{\beta} \mathbf{z}_1 + \omega_1^T \boldsymbol{\varphi}_1 + \boldsymbol{\delta}_1 + \mathbf{z}_2 + \mathbf{y}_2 + \alpha_1 + \mathbf{D}_1 - \dot{\mathbf{y}}_d) \\ &= \beta(\beta^{-1} \dot{\beta} \mathbf{z}_1 + \mathbf{z}_2 + \alpha_1 - \dot{\mathbf{y}}_d) + \beta(\omega_1^T \boldsymbol{\varphi}_1 + \boldsymbol{\delta}_1 + \mathbf{y}_2 + \mathbf{D}_1) \end{aligned} \quad (13)$$

By introducing a new state variable α_1 and letting α_{2f} pass through a first-order filter with a positive constant τ_2 yields [32],

$$\tau_2 \dot{\alpha}_{2f} + \alpha_{2f} = \alpha_1, \alpha_{2f}(0) = \alpha_1(0) \quad (14)$$

Where $\alpha_{2f}(0)$ is the initial value of α_{2f} .

To ensure that the constraint $\mathbf{x}_1$ is not violated, the following barrier Lyapunov function (BLF) is introduced:

$$V_{10} = \frac{1}{2} \log \frac{k_a^2}{k_a^2 - \zeta_1^T \zeta_1} \quad (15)$$

To meet the restriction of $\|\mathbf{x}_1\| < k_{c1}$, with k_{c1} being given, k_a must be chosen such that

$$k_a = k_{c1} - Y_0 \quad (16)$$

where Y_0 is defined as in assumption 3. Consider the following Lyapunov function candidate:

$$V_1 = V_{10} + \frac{1}{2} \mathbf{y}_2^T \mathbf{y}_2 \quad (17)$$

Then, the derivative of V_1 along (17) is

$$\begin{aligned} \dot{V}_1 &= M_1 \zeta_1^T \dot{\zeta}_1 + \mathbf{y}_2^T \dot{\mathbf{y}}_2 \\ &= M_1 \zeta_1^T \beta(\beta^{-1} \dot{\beta} \mathbf{z}_1 + \mathbf{z}_2 + \alpha_1 - \dot{\mathbf{y}}_d) \\ &\quad + M_1 \zeta_1^T \beta(\omega_1^T \boldsymbol{\varphi}_1 + \boldsymbol{\delta}_1 + \mathbf{y}_2 + \mathbf{D}_1) + \mathbf{y}_2^T \dot{\mathbf{y}}_2 \end{aligned} \quad (18)$$

where $M_1 = (1/(k_a^2 - \zeta_1^T \zeta_1))$, and using the following inequalities:

$$\begin{aligned} M_1 \zeta_1^T \beta \omega_1^T \boldsymbol{\varphi}_1 &\leq \frac{M_1^2 \beta^2 \|\zeta_1\|^2 \|\omega_1\|^2 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} + \frac{r_1^2}{2} \\ &\leq \frac{\theta M_1^2 \beta^2 \|\zeta_1\|^2 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} + \frac{r_1^2}{2} \end{aligned} \quad (19)$$

$$M_1 \zeta_1^T \beta \boldsymbol{\delta}_1 \leq \frac{M_1^2 \beta^2 \|\zeta_1\|^2}{2} + \frac{\bar{\delta}_1^2}{2} \quad (20)$$

$$M_1 \zeta_1^T \beta \mathbf{y}_2 \leq \frac{M_1^2 \beta^2 \|\zeta_1\|^2}{2} + \frac{1}{2} \|\mathbf{y}_2\|^2 \quad (21)$$

$$\begin{aligned} M_1 \zeta_1^T \beta \mathbf{D}_1 &\leq M_1 \beta \|\zeta_1\| \|\mathbf{D}_1\| \leq M_1 \beta \|\zeta_1\| d_1 F_{d1} \\ &\leq \frac{M_1^2 \beta^2 \|\zeta_1\|^2 F_{d1}^2}{2} + \frac{d_1^2}{2} \end{aligned} \quad (22)$$

where $r_1 > 0$ is the design parameter, $\bar{d}_1$ and F_{d1} are defined in assumption 2

From (18), we have,

$$\begin{aligned} \dot{V}_1 \leq & M_1 \zeta_1^T \beta (\beta^{-1} \dot{\beta} \mathbf{z}_1 + \mathbf{z}_2 + \boldsymbol{\alpha}_1 - \dot{\mathbf{y}}_d + \frac{\theta M_1 \beta \mathbf{z}_1 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} \\ & + M_1 \beta \zeta_1 (1 + \frac{1}{2} F_{d1}^2)) + \mathbf{y}_2^T \dot{\mathbf{y}}_2 + d \frac{1}{2} \|\mathbf{y}_2\|^2 \\ & + \frac{r_1^2}{2} + \frac{\bar{\delta}_1^2}{2} + \frac{\bar{d}_1^2}{2} \end{aligned} \quad (23)$$

Then, a virtual control law $\boldsymbol{\alpha}_1$ can be constructed as,

$$\begin{aligned} \boldsymbol{\alpha}_1 = & -k_1 \mathbf{z}_1 - \beta^{-1} \dot{\beta} \mathbf{z}_1 + \dot{\mathbf{y}}_d - \frac{\bar{\theta} M_1 \beta \zeta_1 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} \\ & - M_1 \beta \zeta_1 (1 + \frac{1}{2} F_{d1}^2) \end{aligned} \quad (24)$$

where $k_1 > 0$ is the design parameters and $\hat{\theta}$ is the estimation of θ to be specified later.

By substituting (24) into (23), we obtain

$$\begin{aligned} \dot{V}_1 \leq & -k_1 M_1 \zeta_1^T \zeta_1 + M_1 \zeta_1^T \zeta_2 + \frac{\bar{\theta} M_1^2 \beta^2 \|\zeta_1\|^2 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} \\ & + \mathbf{y}_2^T \dot{\mathbf{y}}_2 + \frac{1}{2} \|\mathbf{y}_2\|^2 + D_1 \end{aligned} \quad (25)$$

where $\bar{\theta} = \theta - \hat{\theta}$ is the estimation error, and $\bar{D}_1 = (r_1^2/2) + (\bar{\delta}_1^2/2) + (\bar{d}_1^2/2)$.

By using (5) and (14), it is seen that

$$\dot{\mathbf{y}}_2 = \dot{\boldsymbol{\alpha}}_{2f} - \dot{\boldsymbol{\alpha}}_1 = -\frac{\mathbf{y}_2}{\tau_2} - \dot{\boldsymbol{\alpha}}_1 \quad (26)$$

Note that $\dot{\boldsymbol{\alpha}}_1 \boldsymbol{\varepsilon}(\mathbf{u}) \|\dot{\boldsymbol{\alpha}}_1\| \leq a_1(\mathbf{x}_1, \mathbf{x}_2, \beta, \dot{\beta}, \hat{\theta}, \mathbf{y}_d, \dot{\mathbf{y}}_d)$ is the function of $\mathbf{x}_1, \mathbf{x}_2, \beta, \dot{\beta}, \hat{\theta}, \mathbf{y}_d, \dot{\mathbf{y}}_d$, and there exists a continuous function $a_1(\mathbf{x}_1, \mathbf{x}_2, \beta, \dot{\beta}, \hat{\theta}, \mathbf{y}_d, \dot{\mathbf{y}}_d)$, such that

$$\|\dot{\boldsymbol{\alpha}}_1\| \leq a_1(\mathbf{x}_1, \mathbf{x}_2, \beta, \dot{\beta}, \hat{\theta}, \mathbf{y}_d, \dot{\mathbf{y}}_d) \quad (27)$$

Hence, from (26), and by using Young's inequality, we have

$$\mathbf{y}_2^T \dot{\mathbf{y}}_2 = \mathbf{y}_2^T \left(-\frac{\mathbf{y}_2}{\tau_2} - \dot{\boldsymbol{\alpha}}_1 \right) \leq -\frac{1}{\tau_2} \|\mathbf{y}_2\|^2 + \frac{a_1^2}{2} \quad (28)$$

Thus, from (25), we get

$$\begin{aligned} \dot{V}_1 \leq & -k_1 M_1 \zeta_1^T \zeta_1 + M_1 \zeta_1^T \zeta_2 + \frac{\bar{\theta} M_1^2 \beta^2 \|\zeta_1\|^2 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} \\ & - \frac{1}{\tau_2} \|\mathbf{y}_2\|^2 + \|\mathbf{y}_2\|^2 + \frac{a_1^2}{2} + D_1 \\ \leq & -k_1 M_1 \zeta_1^T \zeta_1 + M_1 \zeta_1^T \zeta_2 + \frac{\bar{\theta} M_1^2 \beta^2 \|\zeta_1\|^2 \|\boldsymbol{\varphi}_1\|^2}{2r_1^2} \\ & - \tau_2 \|\mathbf{y}_2\|^2 + \frac{a_1^2}{2} + D_1 \end{aligned} \quad (29)$$

where the design parameter τ_2 satisfies that $\frac{1}{\tau_2} \geq 1 + \tau_2^*$

, with τ_2^* a positive constant.

Step 2: The final control law $\mathbf{u}$ will be derived in this step.

Note that $\mathbf{z}_2 = \mathbf{x}_2 - \boldsymbol{\alpha}_{2f}$ we have

$$\begin{aligned} \dot{\mathbf{z}}_2 = & \dot{\mathbf{x}}_2 - \dot{\boldsymbol{\alpha}}_{2f} \\ = & \mathbf{F}_2 + \mathbf{G}_2 \mathbf{B} \mathbf{u} + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2 - \dot{\boldsymbol{\alpha}}_{2f}. \end{aligned} \quad (30)$$

From (8), we have,

$$\begin{aligned} \dot{\zeta}_2 = & \dot{\beta} \mathbf{z}_2 + \beta \dot{\mathbf{z}}_2 \\ = & \beta \mathbf{G}_2 \mathbf{B} \mathbf{u} + \beta (\beta^{-1} \dot{\beta} \mathbf{z}_2 + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2 - \dot{\boldsymbol{\alpha}}_{2f}) \end{aligned} \quad (31)$$

Where $\mathbf{G}_2(\bar{x}_2)$ can be decomposed as[33],

$$\mathbf{G}_2(\bar{x}_2) = \mathbf{S}(\bar{x}_2) \mathbf{C} \mathbf{U}(\bar{x}_2) \quad (32)$$

where the matrix $\mathbf{S}(\bar{x}_2)$ is symmetric and positive definite, $\mathbf{U}(\bar{x}_2)$ is unity upper triangular, and $\mathbf{C} = \mathbf{Q} \mathbf{R}$ is diagonal, where $\mathbf{R} = \text{diag}\{\text{sign}(\Delta_1), \text{sign}(\Delta_2 \Delta_1^{-1}), \text{sign}(\Delta_3 \Delta_2^{-1})\}$ and $\mathbf{Q}$ being an arbitrary positive diagonal matrix.

Then, we choose

$$\mathbf{Q} = \text{diag} \left\{ \frac{1}{\underline{B}_1}, \dots, \frac{1}{\underline{B}_3} \right\} \quad (33)$$

The elements of $\mathbf{C}$ denoted by C_i satisfying [34]

$$|C_i| B_i \geq 1, (i = 1, 2, 3) \quad (34)$$

Since the sign of Δ_i is known, $\text{sign}(C_i)$ is known. Then, from (31), we have

$$\begin{aligned} \mathbf{S}^{-1} \dot{\zeta}_2 = & \beta \mathbf{C} \mathbf{U} \mathbf{B} \mathbf{u} + \mathbf{S}^{-1} \beta (\beta^{-1} \dot{\beta} \mathbf{z}_2 + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2 - \dot{\boldsymbol{\alpha}}_{2f}) \\ = & \beta \mathbf{C} \mathbf{B} \mathbf{u} + \beta \mathbf{C} (\mathbf{U} - \mathbf{I}) \mathbf{B} \mathbf{u} + \mathbf{S}^{-1} \beta \\ & \times (\beta^{-1} \dot{\beta} \mathbf{z}_2 + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2 - \dot{\boldsymbol{\alpha}}_{2f}) \end{aligned} \quad (35)$$

Note that $\mathbf{U} - \mathbf{I}$ is strictly upper triangular and $\mathbf{B}$ is a positive diagonal matrix, and we get

$$\beta \mathbf{C} (\mathbf{U} - \mathbf{I}) \mathbf{B} \mathbf{u} = [\Lambda_1^T \boldsymbol{\gamma}_1, \Lambda_2^T \boldsymbol{\gamma}_2, 0]^T \quad (36)$$

where $\Lambda_1^T = \beta C_1 [U_{1,2} B_2, U_{1,3} B_3]$, $\Lambda_2^T = \beta C_2 U_{2,3} B_3$, with $U_{i,j}$ being the element of the matrix $\mathbf{U}$, and $\boldsymbol{\gamma}_1 = [u_2, u_3]^T \in \mathbf{R}^2$, $\boldsymbol{\gamma}_2 = [u_3]^T \in \mathbf{R}$. For convenience, let $\Lambda_3 = 0$ and $\boldsymbol{\gamma}_3 = 0$. Consider the Lyapunov function candidate

$$V_2 = \frac{1}{2} \zeta_2^T \mathbf{S}^{-1} \zeta_2 \quad (37)$$

Integrated (37) with (35) and (36), the time derivative of V_2 is given by

$$\begin{aligned} & (\beta^{-1} \dot{\beta} \mathbf{z}_2 + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2 - \dot{\boldsymbol{\alpha}}_{2f}) + \frac{1}{2} \zeta_2^T \dot{\mathbf{S}}^{-1} \zeta_2 \\ = & -k_2 \zeta_2^T \mathbf{S}^{-1} \zeta_2 + \beta \zeta_2^T \mathbf{C} \mathbf{B} \mathbf{u} + \sum_{i=1}^3 \zeta_{3i} \Lambda_i^T \boldsymbol{\gamma}_i + \frac{1}{2} \zeta_2^T \dot{\mathbf{S}}^{-1} \zeta_2 \\ & + \zeta_2^T \mathbf{S}^{-1} \beta (k_2 \mathbf{z}_2 + \beta^{-1} \dot{\beta} \mathbf{z}_2 - \dot{\boldsymbol{\alpha}}_{2f} + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2) \end{aligned}$$

where $k_2 > 0$ is a user-defined parameter. We denote the i th rows of $\mathbf{S}^{-1}$ and $\dot{\mathbf{S}}^{-1}$ as $\boldsymbol{\eta}_i$ and $\dot{\boldsymbol{\eta}}_i$, respectively. Noting that $\mathbf{B}_i$, and disturbance $\mathbf{D}_2$ are bounded, and it is readily obtained that there exist continuous but unknown functions $L_i(\bar{x}_2) \in \mathbf{R}, i = 1, 2, 3$ such that $\max\{\|\Lambda_i\|, \|\boldsymbol{\eta}_i\|, \|\dot{\boldsymbol{\eta}}_i\| \|\mathbf{F}_2\|, \|\boldsymbol{\eta}_i\| \|\mathbf{G}_2\| \|\boldsymbol{\varepsilon}\|, d_2 \|\boldsymbol{\eta}_i\|, \|\dot{\boldsymbol{\eta}}_i\|\} \leq L_i(\bar{x}_2)$. Then, from (38), we further have

$$\begin{aligned} \dot{V}_2 = & -k_2 \zeta_2^T \mathbf{S}^{-1} \zeta_2 + \beta \zeta_2^T \mathbf{C} \mathbf{B} \mathbf{u} + \sum_{i=1}^3 \zeta_{2i} \\ & \times (\Lambda_i^T \boldsymbol{\gamma}_i + \boldsymbol{\eta}_i \beta \times (k_2 \mathbf{z}_2 + \beta^{-1} \dot{\beta} \mathbf{z}_2 \\ & - \dot{\boldsymbol{\alpha}}_{2f} + \mathbf{F}_2 + \mathbf{G}_2 \boldsymbol{\varepsilon} + \mathbf{D}_2) + \frac{1}{2} \boldsymbol{\eta}_i \zeta_2) \\ \leq & -k_2 \zeta_2^T \mathbf{S}^{-1} \zeta_2 + \beta \zeta_2^T \mathbf{C} \mathbf{B} \mathbf{u} + \sum_{i=1}^3 |\zeta_{2i}| \\ & \times (\|\Lambda_i\| \|\boldsymbol{\gamma}_i\| + \|\boldsymbol{\eta}_i\| \beta \times (\|k_2 \mathbf{z}_2 + \beta^{-1} \dot{\beta} \mathbf{z}_2 - \dot{\boldsymbol{\alpha}}_{2f}\| \\ & + \|\mathbf{F}_2\| + \|\mathbf{G}_2\| \|\boldsymbol{\varepsilon}\| + \bar{d}_2 F_{d2}) + \frac{1}{2} \|\boldsymbol{\eta}_i\| \|\zeta_2\|) \\ \leq & -k_2 \zeta_2^T \mathbf{S}^{-1} \zeta_2 + \beta \zeta_2^T \mathbf{C} \mathbf{B} \mathbf{u} + \sum_{i=1}^3 L_i \psi_i |\zeta_{2i}| \end{aligned} \quad (39)$$

where

$$\psi_i = \|\gamma_i\| + \beta \|k_2 z_2 + \beta^{-1} \dot{\beta} z_2 - \dot{\alpha}_{2f}\| + 2\beta + \beta F_{d2} + \frac{1}{2} \|\zeta_2\| \quad (40)$$

$L_i(\bar{x}_2)$ ($i = 1, 2, 3$) is an unknown non-linear continuous function, which can be approximated by an RBFNN with

$$L_i(\bar{x}_2) = \mathbf{N}_i^T \mathbf{P}_i(\bar{x}_2) + \varepsilon_i(\bar{x}_2) \quad (41)$$

where $\mathbf{N}_i \in \mathbf{R}^{89}$ is an ideal weight vector, $\mathbf{P}_i(\bar{x}_2)$ is the basis function vector of NN, and $\varepsilon_i(\bar{x}_2)$ is the intrinsic approximation error and satisfies $|\varepsilon_i(\bar{x}_2)| \leq \bar{\varepsilon}_i < \infty$ ($i = 1, 2, 3$), with $\bar{\varepsilon}_i$ being a small unknown constant.

Substituting (41) into (39), we have

$$\dot{V}_2 \leq -k_2 \zeta_2^T S^{-1} \zeta_2 + \beta \zeta_2^T \mathbf{C} \mathbf{B} + \sum_{i=1}^3 (\mu_i |\zeta_{2i}| - \tilde{\mathbf{N}}_i^T \mathbf{P}_i \psi_i |\zeta_{2i}| + \varepsilon_i \psi_i |\zeta_{2i}|) \quad (42)$$

where $\mu_i = \hat{\mathbf{N}}_i^T \mathbf{P}_i \psi_i$, and $\tilde{\mathbf{N}}_i = \hat{\mathbf{N}}_i - \mathbf{N}_i$ is the estimation error.

Thus, the actual control law is designed as:

$$u_i = -\left(\frac{M_1}{2} + \frac{l_i}{2} \mu_i^2 + \frac{c_i}{2} \psi_i^2\right) \text{sign}(C_i) \zeta_{2i} \quad (43)$$

where $i = 1, 2, 3, l_i > 0$ and $c_i > 0$ are the design parameters, and $\hat{\mathbf{N}}_i, \hat{\theta}$ are updated by the following formula:

$$\dot{\hat{\mathbf{N}}}_i = \mathbf{F}_i \mathbf{P}_i \psi_i |\zeta_{2i}| - \sigma_i \mathbf{F}_i \hat{\mathbf{N}}_i \quad (44)$$

$$\dot{\hat{\theta}} = \rho \frac{M_1^2 \beta^2 \|\zeta_1\|^2 \|\varphi_1\|^2}{2r_1^2} - \sigma_\theta \hat{\theta} \quad (45)$$

where $\mathbf{F}_i = \mathbf{F}_i^T$, $\sigma_i > 0$, $\rho > 0$ and $\sigma_\theta > 0$ are user-defined design parameters.

The structure of the proposed controller is illustrated in Fig.2. Wherein the function 1 comprise (9), (10), (16) and (24), function 2 comprise (40),(43)-(45). The principle of RBFNN is to approximate $L_i(\bar{x}_2)$ ($i = 1, 2, 3$) and $\mathbf{F}_1 + (\mathbf{G}_1 - \mathbf{I})\mathbf{x}_2$. Under the control input $\mathbf{u}$, the pitch system can realize a favorable tracking performance.

Fig. 2. The diagram of the proposed control method.

The following control objectives are achieved:

- 1) All the internal signals in the closed-loop system are bounded; the output constraints are not violated, i.e. $\|\mathbf{x}_1\| < k_{c1}, \forall t \geq 0$.
- 2) The given tracking performance is achieved, such that the tracking error converges to a prescribed compact set around zero within preassigned finite time T at a preset converging rate when $t \in [0, T)$.

Proof: Construct the Lyapunov function candidate as

$$V = V_1 + V_2 + \frac{1}{2\rho} \tilde{\theta}^2 + \frac{1}{2} \sum_{i=1}^3 \tilde{\mathbf{N}}_i^T \mathbf{F}_i^{-1} \tilde{\mathbf{N}}_i \quad (46)$$

Integrating (29) with (42), the time derivative of V is given by

$$\begin{aligned} \dot{V} &\leq -k_1 M_1 \zeta_1^T \zeta_1 + M_1 \zeta_1^T \zeta_2 + \frac{\hat{\theta} M_1^2 \beta^2 \|\zeta_1\|^2 \|\varphi_1\|^2}{2r_1^2} \\ &\quad + \tau_2^* \|\mathbf{y}_2\|^2 + \frac{a_2^2}{2} + \bar{D}_1 - k_2 \zeta_2^T S^{-1} \zeta_2 \\ &\quad + \sum_{i=1}^3 (\mu_i |\zeta_{2i}| - \tilde{\mathbf{N}}_i^T \mathbf{P}_i \psi_i |\zeta_{2i}| + \beta \zeta_2^T \mathbf{C} \mathbf{B} \mathbf{u} \\ &\quad + \varepsilon_i \psi_i |\zeta_{2i}|) - \frac{1}{\rho} \tilde{\theta} \dot{\hat{\theta}} + \sum_{i=1}^3 \tilde{\mathbf{N}}_i^T \mathbf{F}_i^{-1} \dot{\hat{\mathbf{N}}}_i \\ &\leq -k_1 M_1 \zeta_1^T \zeta_1 + M_1 \zeta_1^T \zeta_2 - k_2 \zeta_2^T S^{-1} \zeta_2 \\ &\quad + \frac{1}{\rho} \left(\rho \tilde{\theta} \frac{\hat{\theta} M_1^2 \beta^2 \|\zeta_1\|^2 \|\varphi_1\|^2}{2r_1^2} - \dot{\hat{\theta}} \right) \\ &\quad + \sum_{i=1}^3 \beta \zeta_{2i} C_i B_i u_i + \sum_{i=1}^3 (\mu_i |\zeta_{2i}| \\ &\quad + \tilde{\mathbf{N}}_i^T \mathbf{F}_i^{-1} (\dot{\hat{\mathbf{N}}}_i - \mathbf{F}_i \mathbf{P}_i \psi_i |\zeta_{2i}| + \varepsilon_i \psi_i |\zeta_{2i}|) \\ &\quad - \tau_2^* \|\mathbf{y}_2\|^2 + \frac{a_2^2}{2} + \bar{D}_1 \end{aligned} \quad (47)$$

Using the following inequalities:

$$M_1 \zeta_1^T \zeta_2 \leq \frac{M_1}{2} \zeta_1^T \zeta_1 + \frac{M_1}{2} \zeta_2^T \zeta_2 \quad (48)$$

$$\mu_i |\zeta_{2i}| \leq \frac{l_i}{2} \mu_i^2 \zeta_{2i}^2 + \frac{1}{2l_i} \quad (49)$$

$$\varepsilon_i \psi_i |\zeta_{2i}| \leq \frac{c_i}{2} \psi_i^2 \zeta_{2i}^2 + \frac{\varepsilon_i^2}{2c_i} \leq \frac{c_i}{2} \psi_i^2 \zeta_{2i}^2 + \frac{\bar{\varepsilon}_i^2}{2c_i} \quad (50)$$

Substituting (44) and (45) into (47), we then have

$$\begin{aligned} \dot{V} &\leq -k_1 M_1 \zeta_1^T \zeta_1 + \frac{M_1}{2} \zeta_1^T \zeta_1 - k_2 \zeta_2^T S^{-1} \zeta_2 \\ &\quad + \frac{\sigma_\theta}{\rho} \tilde{\theta} \hat{\theta} + \sum_{i=1}^3 \left(\frac{1}{2l_i} - \sigma_i \tilde{\mathbf{N}}_i^T \hat{\mathbf{N}}_i + \frac{\bar{\varepsilon}_i^2}{2c_i} \right) \\ &\quad - \tau_2^* \|\mathbf{y}_2\|^2 + \frac{a_2^2}{2} + \bar{D}_1. \end{aligned} \quad (51)$$

From (44), it is deduced that,

$$\beta \zeta_{2i} C_i B_i u_i = -\beta |C_i| |B_i| \left(\frac{M_1}{2} \zeta_{2i}^2 + \frac{l_i}{2} \mu_i^2 \zeta_{2i}^2 + \frac{c_i}{2} \psi_i^2 \zeta_{2i}^2 \right) \quad (52)$$

From (35) and $\beta \geq 1$, we have $\beta |C_i| |B_i| \geq 1$, then we obtain:

$$\beta \zeta_{2i} C_i B_i u_i \leq -\left(\frac{M_1}{2} \zeta_{2i}^2 + \frac{l_i}{2} \mu_i^2 \zeta_{2i}^2 + \frac{c_i}{2} \psi_i^2 \zeta_{2i}^2 \right) \quad (53)$$

Substituting it into (52), we have

$$\begin{aligned} \dot{V} &\leq -k_1 M_1 \zeta_1^T \zeta_1 + \frac{M_1}{2} \zeta_1^T \zeta_1 - k_2 \zeta_2^T S^{-1} \zeta_2 \\ &\quad + \frac{\sigma_\theta}{\rho} \tilde{\theta} \hat{\theta} + \sum_{i=1}^3 \left(\frac{1}{2l_i} - \sigma_i \tilde{\mathbf{N}}_i^T \hat{\mathbf{N}}_i + \frac{\bar{\varepsilon}_i^2}{2c_i} \right) \\ &\quad - \tau_2^* \|\mathbf{y}_2\|^2 + \frac{a_2^2}{2} + \bar{D}_j \end{aligned} \quad (54)$$

Using Young's inequality, we have:

$$\tilde{\theta} \hat{\theta} \leq -\frac{1}{2} \tilde{\theta}^2 + \frac{1}{2} \theta^2 \quad (55)$$

$$-\tilde{\mathbf{N}}_i^T \tilde{\mathbf{N}}_i \leq -\frac{1}{2} \tilde{\mathbf{N}}_i^T \tilde{\mathbf{N}}_i + \frac{1}{2} \|\mathbf{N}_i\|^2 \quad (56)$$

Note that $\mathbf{x}_1 = \mathbf{z}_1 + \mathbf{y}_d$ and $\mathbf{x}_2 = \mathbf{z}_2 + \mathbf{y}_2 + \boldsymbol{\alpha}_1$, we define a compact set as

$$\Pi_1 = \{\zeta_1, \zeta_2, \mathbf{y}_2, \hat{\theta}, \hat{\mathbf{N}}_1, \hat{\mathbf{N}}_2, \hat{\mathbf{N}}_3 : V \leq E_1\} \quad (57)$$

As $\Pi_0 \times \Pi_1 \times \Pi_\beta$ is a compact set in $\mathbf{R}^{2 \times (3+1) \times 3 + 3 \times 89 + 4}$, it is readily concluded that the continuous function a_i has a maximum $\bar{a}_i$ on $\Pi_0 \times \Pi_1 \times \Pi_\beta$, with $\bar{a}_i$ being some unknown positive constant.

If $V = C_1$ then $a_i \leq \bar{a}_i$, and substituting (56) and (57) into (55), we can further have

$$\begin{aligned} \dot{V}_2 &\leq -k_1 M_1 \zeta_1^T \zeta_1 + \frac{M_1}{2} \zeta_1^T \zeta_1 - k_2 \zeta_2^T S^{-1} \zeta_2 - \frac{\sigma_\theta}{2\rho} \tilde{\theta}^2 \\ &\quad + \sum_{i=1}^3 \left(\frac{1}{2\iota_i} - \frac{\sigma_i}{2} \tilde{\mathbf{N}}_i^T \tilde{\mathbf{N}}_i + \frac{\sigma_i}{2} \|\mathbf{N}_i\|^2 + \frac{\bar{\varepsilon}_i^2}{2c_i} \right) \\ &\quad + \frac{\sigma_\theta}{2\rho} \theta^2 - \tau_2^* \|\mathbf{y}_2\|^2 + \frac{\bar{a}_1^2}{2} + \bar{D}_1 \\ &\leq -\left(k_1 - \frac{1}{2}\right) M_1 \zeta_1^T \zeta_1 - \tau_2^* \|\mathbf{y}_2\|^2 - k_2 \zeta_2^T S^{-1} \zeta_2 \\ &\quad - \frac{\sigma_\theta}{2\rho} \tilde{\theta}^2 - \frac{1}{2} \sum_{i=1}^3 \sigma_i \tilde{\mathbf{N}}_i^T \tilde{\mathbf{N}}_i + \frac{\sigma_\theta}{2\rho} \theta^2 \\ &\quad + \sum_{i=1}^3 \left(\frac{1}{2\iota_i} + \frac{\sigma_i}{2} \|\mathbf{N}_i\|^2 + \frac{\bar{\varepsilon}_i^2}{2c_i} \right) + \frac{\bar{a}_1^2}{2} + \bar{D}_1 \end{aligned} \quad (58)$$

Note that

$$-k_1 M_1 \zeta_1^T \zeta_1 \leq -k_1 \log \frac{k_a^2}{k_a^2 - \zeta_1^T \zeta_1} \quad (59)$$

Thus, from (58), we get

$$\dot{V} \leq -\lambda V + \Theta \quad (60)$$

Where $\lambda = \min\{2k_q, 2(k_1 - (1/2)), 2\tau_j^*, \sigma_\theta, \sigma_i T_{imax}\}$ ($q = 1, j = 2, i = 1, 2, 3$), with T_{imax} being the largest eigenvalue of the matrix $\mathbf{F}_i$ and

$$\begin{aligned} \Theta &= \left(\frac{\sigma_\theta}{2\rho} \right) \theta^2 + \sum_{i=1}^3 \left(\frac{1}{2\iota_i} \right) + \left(\frac{\sigma_i}{2} \right) \|\mathbf{N}_i\|^2 \\ &\quad + \left(\bar{\varepsilon}_i^2 / 2c_i \right) + \bar{a}_1^2 / 2 + \bar{D}_1 < \infty \end{aligned} \quad (61)$$

Besides, we can choose $k_1 > (1/2)$ and $\lambda = \Theta / C_1$, and based on that, if $V = C_1$, then $V(0) = C_1, V(0) = C_1, \dot{V} \leq 0, \forall t \geq 0$. Furthermore, we get

$$V \leq \frac{\Theta}{\lambda} + \left(V(0) - \frac{\Theta}{\lambda} \right) e^{-\lambda t} \quad (62)$$

Therefore, we can conclude that $V(t) \in \ell_\infty$ then it follows $\|\zeta_1(0)\| < k_a$ that ζ_1 always remain in the set as long as the initial value $\|\zeta_1(t)\| < k_a$; $\zeta_2 \in \ell_\infty, \mathbf{y}_2 \in \ell_\infty, \hat{\mathbf{N}}_i \in \ell_\infty, i = 1, 2, 3$, and $\hat{\theta} \in \ell_\infty$. As β is bounded, then $\zeta_i \in \ell_\infty (i = 1, 2)$ imply that $F_{d1} \in \ell_\infty$. Since $\mathbf{z}_1 = \mathbf{x}_1 - \mathbf{y}_d$ and $\mathbf{y}_d$ are bounded, so is $\mathbf{x}_1$, which further implies that $F_{d1} \in \ell_\infty$, since β^{-1} and $\hat{\beta}$ are bounded as mentioned before, we get $\boldsymbol{\alpha}_1 \in \ell_\infty$. Since $\mathbf{x}_2 = \mathbf{z}_2 + \mathbf{y}_2 + \boldsymbol{\alpha}_1$, $\mathbf{x}_2$ is bounded, F_{d2} is also bounded, which further implies that $\boldsymbol{\alpha}_2$ is bounded. Then, in the same way, we can prove that $\hat{\theta}, \hat{\mathbf{N}}_i (i = 1, 2, 3)$, and the actual control inputs $\mathbf{u}_i (i = 1, 2, 3)$ are bounded. Therefore, all the signals in the closed-loop system are bounded.

4. Simulation results

Simulations are conducted in Matlab/Simulink in this section to verify the effectiveness of the proposed method. The references of the pitch angles, which are determined by the pitch regulation system to ensure the WT can achieve constant power operation and mitigated dynamic loads, are as shown in Fig.3. (Refer to [15] for more information on the generation of the pitch angle references). Suppose that the reference pitch angle would vary in the simulation, which is consistent with the actual application. The proposed IPC controller must make sure that the real pitch angles can track the references within a predefined finite time under time-varying unknown system parameters and unknown disturbances.

Fig. 3. Pitch angle of three blades

In the simulation, the uncertainties of the parameters in the pitch system are simulated by the equations provided in (63). Besides, a sinusoidal signal is used to simulate the aerodynamic load caused by wind shear effects and tower shadow effects as these are periodical. Meanwhile, white noise is used to simulate the random disturbance, as shown in (64). In the simulation, the initial values of the desired trajectory are $\mathbf{x}_1(0) = \mathbf{x}_2(0) = [0, 0, 0]^T$, and $\hat{\theta}(0) = 0$; the RBFNN used in the controller contains 89 nodes with centers evenly distributed in the span of input space $[-11, 11]$. The parameters of the RBFNN controller are listed in Table 1.

Table 1 Simulation parameters

Parameters	Value
b_f	0.18
T	6
$k_i, (i = 1, 2)$	6.5, 1.1
$\iota_i, (i = 1, 2, 3)$	0.1, 0.1, 0.1
$\varsigma_i, (i = 1, 2, 3)$	1.5, 1.5, 1.5
ς_θ	1.5
$\mathbf{F}_1$	$\mathbf{I}_{89 \times 89}$
$\mathbf{F}_2$	$2\mathbf{I}_{89 \times 89}$
$\sigma_i, (i = 1, 2, 3)$	0.03, 0.08, 0.08
ρ_1	1
r_1	0.001
$c_i, (i = 1, 2, 3)$	0.1
k_a	6.5
τ_2	0.03

$$\begin{aligned}
J(\beta) &= \text{diag}(\sin(0.5\beta_1 t) + 2, \sin(0.5\beta_2 t) + 2, \\
&\quad \sin(0.5\beta_3 t + 2)) \\
M(\beta, \dot{\beta}) &= \text{diag}(\cos(0.5\beta_1 + \dot{\beta}_1) + 2, \cos(0.5\beta_2 \\
&\quad + \dot{\beta}_2) + 2, \cos(0.5\beta_3 + \dot{\beta}_3) + 2) \\
Q(\beta, \dot{\beta}) &= \text{diag}(\cos(0.5\beta_1 + \dot{\beta}_1)/3 + 1, \cos(0.5\beta_2 \\
&\quad + \dot{\beta}_2)/3 + 1, \cos(0.5\beta_3 + \dot{\beta}_3)/3 + 1)
\end{aligned} \quad (63)$$

$$\begin{aligned}
D_1 &= [0.5\cos(0.5\beta_1 t) + \sin(0.05t); \\
&\quad 0.5\cos(0.5\beta_2 t) + \sin\left(0.05\left(t + \frac{2\pi}{3}\right)\right); \\
&\quad 0.5\cos(0.5\beta_3 t) + \sin(0.05(t + 4\pi/3));] \\
D_2 &= [0.5\cos(0.5\beta_1 t) + 3\sin(0.5\dot{\beta}_1) + \sin(0.05t); \\
&\quad 0.5\cos(0.5\beta_2 t) + 3\sin(0.5\dot{\beta}_2) + \sin\left(0.05\left(t + \frac{2\pi}{3}\right)\right); \\
&\quad 0.5\cos(0.5\beta_3 t) + 3\sin(0.5\dot{\beta}_3) + \sin(0.05(t + 4\pi/3));]
\end{aligned} \quad (64)$$

In this simulation, three scenarios are set to verify the merits of the proposed method:

Scenario 1: Step change of the reference pitch angle happens at $t=10s$. In this transient process, the user-defined time T is selected as 1s, 2s, and 3s, respectively.

Scenario 2: Influence of parameter change. Specifically, the value of the parameter matrix $J(\beta)$ became half of the original value at $t=40s$, which is shown in Fig. 5.

Scenario 3: Influences of disturbances. The disturbance matrices D_1 and D_2 , which are shown in Fig. 6, are applied to the system at time $t=70s$;

In scenario 1, to verify that the proposed controller can tracking signal within a finite time, we chosen different settling time for the controller, which are 1s, 2s, and 3s, respectively. Besides, the convergence rate function $\kappa(t) = e^t$ is chosen in the design. We assume the reference pitch angle as 15° with a step change to 20° at $t=10s$. Noting that y_d is bounded in this study as beforementioned in Assumption 1, smooth processing on the stepping change signal has been conducted. Fig. 4 shows the tracking performance of the pitch angle β_1 in scenario 1. As can be observed in this figure, before step change in reference pitch angle, both three controllers can ensure that the pitch angle β_1 can track the reference β_1^* closely, with a tracking error of nearly 0. At $t=10s$, the reference pitch angle suddenly steps from 15° to 20° , and it is observed that the pitch angle β_1 can track the reference signal within the predefined times. Moreover, the transient performance of the controller improved effectively by choosing a shorter settling time T properly. This keeps pace with the theoretical analysis.

Fig. 4. A stepping change of the reference pitch angle

In scenario 2, the tracking performance of the proposed controller for blade 1 (the other two blades shares the same pitch angle adjustment mechanism and not shown here) under the influence of system parameter change is depicted in Fig. 5. Meanwhile, a traditional PID control method was introduced for comparison. As shown in Fig. 5, before the value of the parameter matrix $J(\beta)$ is changed, the pitch angle β_1 can track the reference angle β_1^* closely, and the tracking error is nearly 0. Meanwhile, the tracking performance under the traditional PID control is relatively poor. From Fig. 5, it is readily known that the RBFNN based controller enjoys a faster error convergence and better rejection capability on parameter change effects compared to the traditional PID controller. After the parameter matrix $J(\beta)$ suddenly changed, it is seen that the pitch angle β_1 can still track β_1^* closely. The tracking error still remains at around 0. This verifies the declaration that the proposed controller can address the control problems related to parameter changes.

Fig. 5. The pitch angle tracking error of blade1 the parameters change on the system.

In scenario 3, the pitch angle tracking performance for blade 1 is depicted in Fig. 6. A traditional PID controller is also introduced for comparison. As can be observed, before

disturbances D_1 and D_2 are introduced into the system, the proposed controller ensure that the pitch angle β_1 can track the reference β_1^* closely, with a tracking error of nearly 0. The tracking error under the traditional PID controller is relatively greater, especially in the first 25s of simulation. After adding the disturbances at $t=70$ s, it is evidenced that the tracking performance of pitch angle 1 is not heavily affected: the tracking error remains at around 0. Compared with the traditional PID controller, the fluctuation range of the RBFNN controller error is smaller. This indicates that the proposed IPC method can reject the disturbances and avoid their influences on tracking performance.

Hence, the validity of the proposed IPC method is verified.

Fig. 6. The pitch angle tracking error of blade1 under system disturbances D_1 and D_2

5. Conclusion

In this paper, a neuro-adaptive based individual pitch tracking controller for offshore floating WTs is proposed to regulate the power output and mitigate dynamic loads. The neural-adaptive tracking controller designed in this study can track the desired pitch angle precisely and quickly, and can ensure that all the internal signals is bounded. The controller does not depend on the parameters of the system model, so there is no need to install additional sensors to measure the uncertain time-varying parameters of the system, which is economical. Compared with the traditional PID controller, under system parameter uncertainty and disturbances, this control method can achieve zero error tracking within a predefined finite time. Thus the performance of pitch angle regulation is guaranteed. On this basis, one can conclude that the superior pitch tracking ability realized in this work can ensure that the floating WTs can produce rated power output and mitigate loads in the presence of and disturbances.

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